UNIFIED FIELD MECHANICS II

Formulations and Empirical Tests

Editors

Richard L. Amoroso

Louis H. Kauffman

Peter Rowlands

Gianni Albertini

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Proceedings of the Xth Symposium Honoring Noted French Mathematical Physicist Jean-Pierre Vigier

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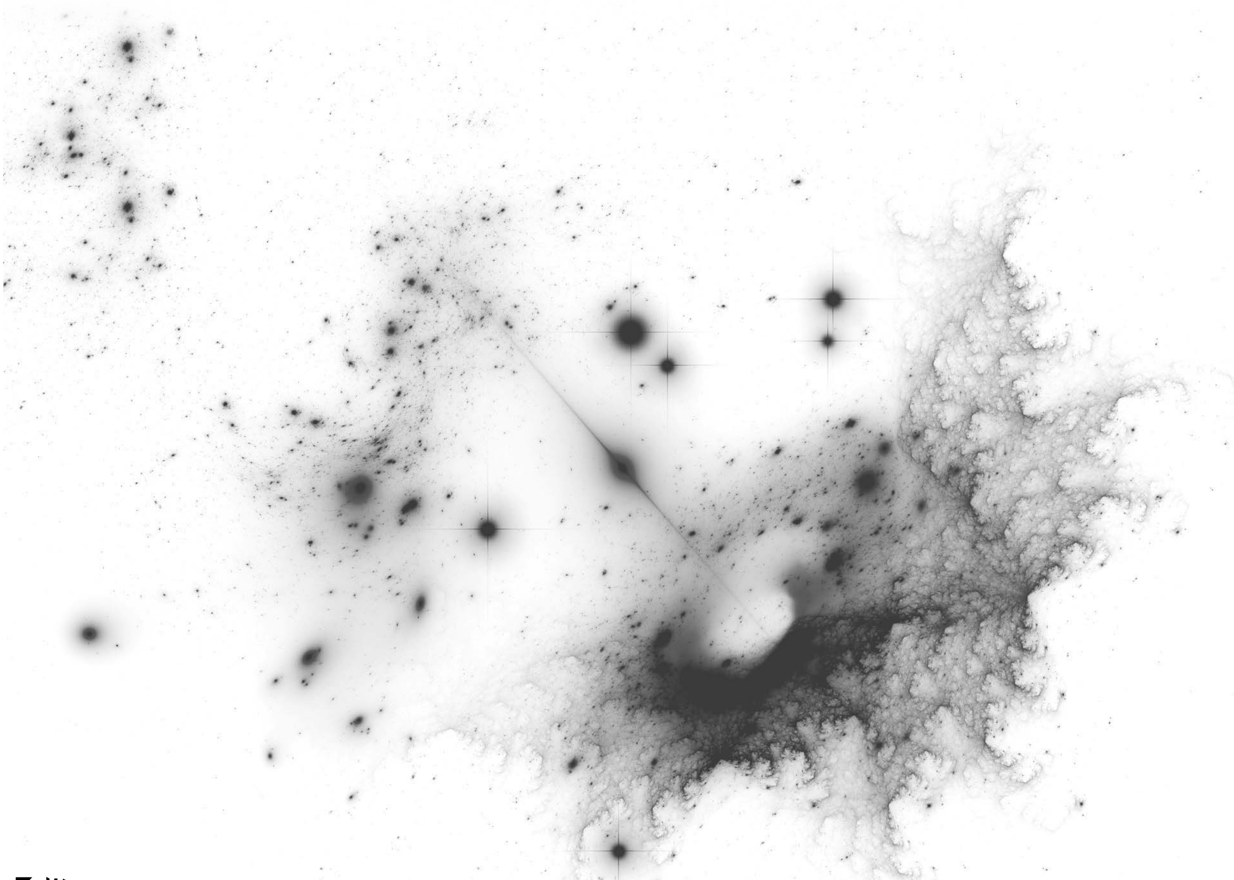

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Dedication

"*I want to do Physics*!"

Paris, Jean-Pierre Vigier, 2003

Nobel Laureate Brian Josephson receiving The Noetic Medal of Consciousness and Brain Research for lifetime achievement. From L to R: Gianni Albertini, Louis H. Kauffman, Laureate Brian Josephson, Richard L. Amoroso & Peter Rowlands.

Group photo, Many delegates of the Tenth Vigier Symposium, Porto Novo, Italy, 25 July to 28 July 2016

Photos courtesy Bruno Bassani: (www.fotograficabb.com) photographer

Foreword

The 10th Vigier Symposium was the 1st at a beautiful international seaside resort – PortoNovo, Italy on the Adriatic; and also the 1st to host a Nobel Laureate – Brian Josephson, famous for the 'Josephson-Junction', still a very active item of research and new technology.

Just as infinities in the Raleigh-Jeans Law called the ultraviolet catastrophe, led to the discovery of quantum mechanics during the turn of the 20th Century; likewise, infinities in quantum field theory point the way to the imminent discovery of Unified Field Mechanics (UFM) – The 3rd regime of natural science. QED is being routinely violated at the σ^5 level (minimum for threshold of statistical significance) [1,2], and CERN is busy searching for additional dimensions and superpartners beyond the Standard Model. In lieu of the inertia inherent in our team's ability to perform our proposed experimental tests [3]; we predict a likely arena breaking through the *crack in the cosmic egg (coined by Lemaitre)* to UFM, will be associated with Majorana topological phase transitions in anyon quasiparticle graphene bi-layers. Indeed, Majorana modes are the most active research area of physics; and the Princeton IAS where Einstein was, is devoting 2018 as a *year of topological phase*.

In the prior VIIIth & VIX Vigier Symposia 'The Physics of Reality: Space, Time, Matter, Cosmos', Unified Field Mechanics: Natural Science Beyond the Veil of Uncertainty; a crack was made in the so-called Cosmic Egg' so to speak in the ongoing program to fulfill Einstein's final quest to understand the nature of reality in terms of developing a Unified Field Theory. This crack creates a small but albeit sufficient space within which to place a crowbar to begin 'opening the door' to the next step in the historical progression:

1) Classical Newtonian Mechanics,
2) Quantum Mechanics, and now to
3) Unified Field Mechanics a 3rd regime of reality - Natural Science Beyond the Veil of Spacetime.

Come, join us on the adventure of the new Millennium!

This Symposium shed light on the ongoing challenges of 'Local' Lorentz Invariance (LLI) as a fundamental first principle, verifying for the first time a violation of LLI phenomena in support of the imminent paradigm shift leading to the pragmatic demonstrtation of Unified Field Mechanics (UFM).

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Richard L. Amoroso
25 May, 2017
Tel Aviv-Yafo, Israel

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Iterants, Braiding and the Dirac Equation

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1. Introduction

The simplest discrete system corresponds directly to the square root of minus one, when the square root of minus one is seen as an oscillation between plus and minus one. This way thinking about the square root of minus one as an *iterant* is explained below. More generally, by starting with a discrete time series of positions, one has immediately a non-commutativity of observations and this non-commutativity can be encapsulated in an iterant algebra as defined in Section 2 of the present paper. Iterant algebra generalizes matrix algebra and we shall see how it can be used to formulate the algebra of the framed Artin Braid Group, the Lie algebra $su(3)$ for the Standard Model for particle physics, the framed braid representations for Fermions of Sundance Bilson-Thompson and the Clifford algebra for Majorana Fermions. This paper is a sequel to[9] and[6–9] and it uses material from these papers. This paper incorporates new results of the author that first appear in the joint paper of the author and Rukhsan Ul-Haq.[10]

Distinction and processes arising from distinction are at the base of the described world. Distinctions are elemental bits of awareness. The world is composed not of things but processes and observations. We will discuss how basic Clifford algebra comes from very elementary processes such as an alternation of $+-+-+-\cdots$ and the fact that one can think of $\sqrt{-1}$ itself as a temporal iterant, a product of an ϵ and an η where the ϵ is the $+-+-+-\cdots$ and the η is a time shift operator. Clifford algebra is at the base of this mathematical world, and the fermions are composed of these things.

Sections 2 and 3 are an introduction to the process algebra of iterants and how the square root of minus one arises from an alternating process. Section 4 shows how iterants give an alternative way to do 2×2 matrix algebra. The section ends with the construction of the split quaternions. Section 5 considers iterants of arbitrary period (not just two) and shows, with the example of the cyclic group, how the ring of all $n\times n$ matrices can be seen as a faithful representation of an iterant algebra based on the cyclic group of order n. We then generalize this construction to arbitrary non-commutative finite groups G. Such a group has a multiplication table ($n\times n$ where n is the order of the group G.). We show that by rearranging the multiplication table so the identity element appears on the diagonal, we get a set of permutation matrices that represent the group faithfully as $n\times n$ matrices. This gives a faithful representation of the iterant algebra associated with the group G onto the ring of $n\times n$ matrices. As a result we see that iterant algebra is fundamental to all matrix algebra. Section 5 ends with a number of classical examples including iterant represtations for quaternion algebra.

Section 6 discusses the iterant structure of the framed Artin braid group. In Section 7 we apply this to a formulation of the particle model of Sundance Bilson-Thompson,[31] using framed braids. In Section 7 we give an iterant interpretation of the $su(3)$ Lie algebra for the Standard Model using.[30] In Section 8 we apply this point of view on the Standard Model to obtain an embedding of the framed braid algebra for the Sundance Bilson-Thompson model into the iterant version of $su(3)$. These three sections are an account of research of the author and Rukhsan Ul-Haq in.[10]

Section 9 discusses how Clifford algebras are fundamental to the structure of Fermions. We show how the simple algebra of the split quaternions, the very first iterant algebra that appears in relation to the square root of minus one, is in back of the structure of the operator algebra of the electron. The underlying Clifford structure describes a pair of Majorana Fermions, particles that are their own antiparticles. These Majorana Fermions can be symbolized by Clifford algebra generators a and b such that $a^2 = b^2 = 1$ and $ab = -ba$. One can take a as the iterant corresponding to a period two oscillation, and b as the time shifting operator. Then their product ab is a square root of minus one in a non-commutative context. These are the Majorana Fermions that underlie an electron. The electron can be symbolized by $\phi = a+ib$ and the anti-electron by $\phi^\dagger = a-ib$. These form the operator algebra for an electron. Note that

$$\phi^2 = (a+ib)(a+ib) = a^2-b^2+i(ab+ba) = 0+i0 = 0.$$

This nilpotent structure of the electron arises from its underlying Clifford structure in the form of a pair of Majorana Fermions. Section 9 then shows how braiding is related to the Majorana Femions. Section 10 discusses the structure of the Dirac equation and how the nilpotent and the Majorana operators arise naturally in this context. This section provides a link between our work and the work on nilpotent structures and the Dirac equation of Peter Rowlands.[33] We end this section with an expression in split quaternions for the Majorana Dirac equation in one dimension of time and three dimensions of space. The Majorana Dirac equation can be written as follows:

$$(\partial/\partial t + \hat{\eta}\eta\partial/\partial x + \epsilon\partial/\partial y + \hat{\epsilon}\eta\partial/\partial z - \hat{\epsilon}\hat{\eta}\eta m)\psi = 0$$

where η and ϵ are the simplest generators of iterant algebra with $\eta^2 = \epsilon^2 = 1$ and $\eta\epsilon + \epsilon\eta = 0$, and $\hat{\epsilon}, \hat{\eta}$ form a copy of this algebra that commutes with it. This combination of the simplest Clifford algebra with itself is the underlying structure of Majorana Fermions, forming indeed the underlying structure of all Fermions.

This paper is a stopping-place along the way in a larger story of mathematics and physics that we are in the process of telling and exploring. To begin the story, we conclude this introduction with a formulation of the Schroedinger equation that can motivate the iterants.

1.1. *Iterants and the Schroedinger Equation*

We begin with the Diffusion Equation

$$\partial\psi/\partial t = \kappa\partial^2\psi/\partial x^2.$$

Consider the possibility of putting a "plus or minus" ambiguity into this equation, like so:

$$\pm\partial\psi/\partial t = \kappa\partial^2\psi/\partial x^2.$$

The $\pm$ coefficient should be lawful not random, for then we can follow an algebraic formulation of the process behind the equation. We shall take $\pm$ to mean the alternating sequence

$$\pm = \cdots + - + - + - + - \cdots$$

and time will be discrete. Then the equation will become a difference equation in space and time

$$\psi_{t+1} - \psi_t = (-1)^t\kappa(\psi_t(x-dx) - 2\psi_t(x) + \psi_t(x+dx))$$

where

$$\partial_x^2\psi_t = \psi_t(x-dx) - 2\psi_t(x) + \psi_t(x+dx).$$

But we wish to consider the continuum limit. However, there is no meaning to

$$(-1)^t$$

in the realm of continuous time. What to do? In the discrete world the wave function ψ divides into ψ_e and ψ_o where the (discrete) time is either even or odd. So we can write

$$\partial_t\psi_e = \kappa\partial_x^2\psi_o$$

$$\partial_t\psi_o = -\kappa\partial_x^2\psi_e.$$

We take the continuum limit of ψ_e and ψ_o *separately.*

In fact we can interpret the $\{\pm\}$ as the complex number i. Recall that the complex number i has the property that $i^2 = -1$ so that

$$i(A + iB) = iA - B$$

when A and B are real numbers,

$$i = -1/i,$$

and so if $i = 1$ then $i = -1$, and if $i = -1$ then $i = 1$. So i can be interpreted as oscillating between $+1$ and -1, but it does it lawfully and so we shall regard i as a definition of ±1.

$$i = \pm 1.$$

In fact, when we multiply $ii = (\pm1)(\pm1)$, we get -1 because the i takes a little time to oscillate and so by the time this second term multiplies the first term, they are just out of phase and so we get either

$(+1)(-1) = -1$ or $(-1)(+1) = -1$. We will formalize this point of view later in the paper.

Now ± 1 behaves quite lawfully and we can write

$$\psi = \psi_e + i\psi_o$$

so that

$$i\partial_t\psi = i\partial_t(\psi_e + i\psi_o) = i\partial_t\psi_e - \partial_t\psi_o$$

$$= i\kappa\partial_x^2\psi_o + \kappa\partial_x^2\psi_e = \kappa\partial_x^2(\psi_e + i\psi_o)$$

$$= \kappa\partial_x^2\psi.$$

Thus

$$i\partial\psi/\partial t = \kappa\partial^2\psi/\partial x^2.$$

This is the Schroedinger equation. Instead of the simple diffusion equation, we have a mutual dependency where the temporal variation of ψ_e is mediated by the spatial variation of ψ_o and vice-versa.

$$\psi = \psi_e + i\psi_o$$

$$\partial_t\psi_e = \kappa\partial_x^2\psi_o$$

$$\partial_t\psi_o = -\kappa\partial_x^2\psi_e.$$

$$i\partial\psi/\partial t = \kappa\partial^2\psi/\partial x^2.$$

Remark. The discrete recursion at the beginning of this section, can actually be implemented to approximate solutions to the Schroedinger equation. This will be studied in a separate paper. The reader may wish to point out that the playing of dice in quantum mechanics has nothing to do with the deterministic evolution of the Schroedinger equation, and everything to do with the measurement postulate that interprets $\psi\psi^\dagger$ as a probability density.

Probability and generalizations of classical probability are necessary for doing science. One should keep in mind that the quantum mechanics is based on a model that takes the solution of the Schroedinger equation to be a superposition of all possible observations of a given observer. The solution has norm equal to one in an appropriate vector space. That norm is the integral of the absolute square of the wave function over all of space. The absolute square of the wavefunction is seen as the associated probability density. This extraordinary and concise recipe for the probability of observed events is at the core of this subject. It is natural to ask, in relation to our fable, what is the relationship of probability for the diffusion process and the probability in quantum theory. This will have to be the subject of another paper.

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2. Iterants and Idempotents

An iterant is a sum of elements of the form

$$[a_1, a_2, ..., a_n]\sigma$$

where $[a_1, a_2, ..., a_n]$ is a vector of elements that are scalars (usually real or complex numbers) and σ is a permutation on n letters. Such elements are themselves sums of elements of the form

$$[0, 0, ...0, 1, 0, ..., 0]\sigma = e_i\sigma$$

where the 1 is in the i-th place. The elements e_i are the basic idempotents that generate the iterants with the help of the permutations.

Note that if $a = [a_1, a_2, ..., a_n]$, then we let a^σ denote the vector with its elements permuted by the action of σ. If a and b are vectors then ab denotes the vector where $(ab)_i = a_ib_i$, and $a + b$ denotes the vector where $(a + b)_i = a_i + b_i$. Then

$$(a\sigma)(b\tau) = (ab^\sigma)\sigma\tau,$$

$$(ka)\sigma = k(a\sigma)$$

for a scalar k, and

$$(a + b)\sigma = a\sigma + b\sigma$$

where vectors are multiplied as above and we take the usual product of the permutations. All of matrix algebra and more is naturally represented in the iterant framework, as we shall see in the next sections.

For example, if η is the order two permutations of two elements, then $[a, b]^\eta = [b, a]$. We can define

$$i = [1, -1]\eta$$

and then

$$i^2 = [1,-1]\eta[1,-1]\eta = [1,-1][1,-1]^{\eta}\eta^2$$

$$= [1,-1][-1,1] = [-1,-1] = -1.$$

In this way the complex numbers arise naturally from iterants. One can interpret $[1,-1]$ as an oscillation between $+1$ and -1 and η as denoting a temporal shift operator. The $i = [1,-1]\eta$ is a time sensitive element and its self-interaction has square minus one. In this way iterants can be interpreted as a formalization of elementary discrete processes.

Note that we can write $a = [1,0], b = [0,1]$ and $A = a\eta, B = b\eta$ where η denotes the transposition so that $[x,y]\eta = \eta[y,x]$ and $\eta^2 = 1$. Then we have

$$aa = a, bb = b, ab = 0, a + b = 1, AA = 0 = BB,$$

$$AB = a, BA = b.$$

This is the mixed idempotent and permutation algebra for $n = 2$. Then we have

$$i = A - B$$

as we can see by

$$ii = (A-B)(A-B) = AA - AB - BA + BB$$

$$= -a - b = -1.$$

This is the beginning of the relationships between idempotents, iterants and Clifford algebras.

Note that we construct an elementary Clifford algebra via

$$\alpha = [1,-1] = a - b$$

and

$$\beta = \eta.$$

Then we have

$$\alpha^2 = \beta^2 = 1$$

and

$$\alpha\beta + \beta\alpha = 0.$$

Note also that the non-commuting of α and β is directly related to the interaction of the idempotents and the permutations.

$$\alpha\beta = [1,-1]\eta = \eta[-1,1] = -\eta[1,-1] = -\beta\alpha.$$

Iterant algebra is generated by the elements

$$e_i\sigma$$

where e_i is a vector with a 1 in the i-th place and zeros elsewhere, and σ is an abritrary element of the symmetric group S_n. We have that

$$e_i\sigma = \sigma e_{\sigma^{-1}(i)}$$

so that the multiplication of iterants is defined in terms of the action of the symmetric group. We have

$$e_i\sigma e_j\tau = e_i e_{\sigma(j)}\sigma\tau = \delta(i,\sigma(j))e_i\sigma\tau.$$

By themselves, the elements e_i are idempotent and we have

$$1 = e_1 + \cdots e_n.$$

The iterant algebra is generated by these combinations of idempotents and permutations.

3. Iterants, Discrete Processes and Matrix Algebra

The primitive idea behind an iterant is a periodic time series or "waveform"

$$\cdots abababababab \cdots .$$

The elements of the waveform can be any mathematically or empirically well-defined objects. We can regard the ordered pairs $[a,b]$ and $[b,a]$ as abbreviations for the waveform or as two points of view about the waveform (a first or b first). Call $[a,b]$ an *iterant*. One has the collection of transformations of the form $T[a,b] = [ka, k^{-1}b]$ leaving the product ab invariant. This tiny model contains the seeds of special relativity, and the iterants contain the seeds of general matrix algebra! For related discussion see.[1–7,10,11,13,14]

Define products and sums of iterants as follows

$$[a,b][c,d] = [ac, bd]$$

and

$$[a,b] + [c,d] = [a+c, b+d].$$

The operation of juxtaposition of waveforms is multiplication while $+$ denotes ordinary addition of ordered pairs. These operations are natural with respect to the structural juxtaposition of iterants:

$$...abababababab...$$

$$...cdcdcdcdcdcd...$$

Structures combine at the points where they correspond. Waveforms combine at the times where they correspond. Iterants combine in juxtaposition.

If $\bullet$ denotes any form of binary composition for the ingredients (a,b,...) of iterants, then we can extend $\bullet$ to the iterants themselves by the definition $[a, b] \bullet [c, d] = [a \bullet c, b \bullet d]$.

The appearance of a square root of minus one unfolds naturally from iterant considerations. Define the "shift" operator η on iterants by the equation

$$\eta[a, b] = [b, a]\eta$$

with $\eta^2 = 1$. Sometimes it is convenient to think of η as a delay operator, since it shifts the waveform $...ababab...$ by one internal time step. Now define

$$i = [-1, 1]\eta.$$

We see at once that

$$ii = [-1, 1]\eta[-1, 1]\eta = [-1, 1][1, -1]\eta^2$$

$$= [-1, 1][1, -1] = [-1, -1] = -1.$$

Thus

$$ii = -1.$$

Here we have described i in a *new* way as the superposition of the waveform $\epsilon = [-1, 1]$ and the temporal shift operator η. By writing $i = \epsilon\eta$ we recognize an active version of the waveform that shifts temporally when it is observed. This theme of including the result of time in observations of a discrete system occurs at the foundation of our construction.

In the next section we show how all of matrix algebra can be formulated in terms of iterants.

4. Matrix Algebra via Iterants

Matrix algebra has some strange wisdom built into its very bones. Consider a two dimensional periodic pattern or "waveform."

....

$$...abababababababab...$$

$$...cdcdcdcdcdcdcdcd...$$

$$...abababababababab...$$

$$...cdcdcdcdcdcdcdcd...$$

$$...abababababababab...$$

....

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \begin{pmatrix} b & a \\ d & c \end{pmatrix}, \begin{pmatrix} c & d \\ a & b \end{pmatrix}, \begin{pmatrix} d & c \\ b & a \end{pmatrix}.$$

Above are some of the matrices apparent in this array. Compare the matrix with the "two dimensional waveform" shown above. A given matrix freezes out a way to view the infinite waveform. In order to keep track of this patterning, lets write

$$[a, b] + [c, d]\eta = \begin{pmatrix} a & c \\ d & b \end{pmatrix}$$

where

$$[x, y] = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$$

and

$$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Recall the definition of matrix multiplication.

$$\begin{pmatrix} a & c \\ d & b \end{pmatrix} \begin{pmatrix} e & g \\ h & f \end{pmatrix} = \begin{pmatrix} ae + ch & ag + cf \\ de + bh & dg + bf \end{pmatrix}.$$

Compare this with the iterant multiplication.

$$([a, b] + [c, d]\eta)([e, f] + [g, h]\eta) =$$

$$[a, b][e, f] + [c, d]\eta[g, h]\eta + [a, b][g, h]\eta + [c, d]\eta[e, f] =$$

$$[ae, bf] + [c, d][h, g] + ([ag, bh] + [c, d][f, e])\eta =$$

$$[ae, bf] + [ch, dg] + ([ag, bh] + [cf, de])\eta =$$

$$[ae + ch, dg + bf] + [ag + cf, de + bh]\eta.$$

Thus matrix multiplication is identical with iterant multiplication. The concept of the iterant can be used to motivate matrix multiplication.

The four matrices that can be framed in the two-dimensional wave form are all obtained from the two iterants $[a,d]$ and $[b,c]$ via the shift operation $\eta[x,y]=[y,x]\eta$ which we shall denote by an overbar as shown below

$$\overline{[x,y]}=[y,x].$$

Letting $A=[a,d]$ and $B=[b,c]$, we see that the four matrices seen in the grid are

$$A+B\eta, B+A\eta, \overline{B}+\overline{A}\eta, \overline{A}+\overline{B}\eta.$$

The operator η has the effect of rotating an iterant by ninety degrees in the formal plane. Ordinary matrix multiplication can be written in a concise form using the following rules:

$$\eta\eta=1$$

$$\eta Q=\overline{Q}\eta$$

where Q is any two element iterant. Note the correspondence

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}+\begin{pmatrix} b & 0 \\ 0 & c \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$=[a,d]1+[b,c]\eta.$$

This means that $[a,d]$ corresponds to a diagonal matrix.

$$[a,d]=\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix},$$

η corresponds to the anti-diagonal permutation matrix.

$$\eta=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

and $[b,c]\eta$ corresponds to the product of a diagonal matrix and the permutation matrix.

$$[b,c]\eta=\begin{pmatrix} b & 0 \\ 0 & c \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}=\begin{pmatrix} 0 & b \\ c & 0 \end{pmatrix}.$$

Note also that

$$\eta[c,b]=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} c & 0 \\ 0 & b \end{pmatrix}=\begin{pmatrix} 0 & b \\ c & 0 \end{pmatrix}.$$

This is the matrix interpretation of the equation

$$[b,c]\eta=\eta[c,b].$$

The fact that the iterant expression $[a,d]1+[b,c]\eta$ captures the whole of 2×2 matrix algebra corresponds to the fact that a two by two matrix is combinatorially the union of the identity pattern (the diagonal) and the interchange pattern (the antidiagonal) that correspond to the operators 1 and η.

$$\begin{pmatrix} * & @ \\ @ & * \end{pmatrix}$$

In the formal diagram for a matrix shown above, we indicate the diagonal by $*$ and the anti-diagonal by @.

In the case of complex numbers we represent

$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}=[a,a]+[-b,b]\eta=a1+b[-1,1]\eta=a+bi.$$

In this way, we see that all of 2×2 matrix algebra is a hypercomplex number system based on the symmetric group S_2. In the next section we generalize this point of view to arbitrary finite groups.

We have reconstructed the square root of minus one in the form of the matrix

$$i=\epsilon\eta=[-1,1]\eta=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

In this way, we arrive at this well-known representation of the complex numbers in terms of matrices. Note that if we identify the ordered pair (a,b) with $a+ib$, then this means taking the identification

$$(a,b)=\begin{pmatrix} a & -b \\ b & a \end{pmatrix}.$$

Thus the geometric interpretation of multiplication by i as a ninety degree rotation in the Cartesian plane,

$$i(a,b)=(-b,a),$$

takes the place of the matrix equation

$$i(a,b)=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a & -b \\ b & a \end{pmatrix}=\begin{pmatrix} -b & -a \\ a & -b \end{pmatrix}$$

$$=b+ia=(-b,a).$$

In iterant terms we have

$$i[a,b]=\epsilon\eta[a,b]=[-1,1][b,a]\eta=[-b,a]\eta,$$

and this corresponds to the matrix equation

$$i[a,b]=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}=\begin{pmatrix} 0 & -b \\ a & 0 \end{pmatrix}=[-b,a]\eta.$$

All of this points out how the complex numbers, as we have previously examined them, live naturally in the context of the non-commutative algebras of iterants and matrices. The factorization of i into a product $\epsilon\eta$ of non-commuting iterant operators is closer both to the temporal nature of i and to its algebraic roots.

More generally, we see that

$$(A+B\eta)(C+D\eta) = (AC+B\overline{D}) + (AD+B\overline{C})\eta$$

writing the 2×2 matrix algebra as a system of hypercomplex numbers. Note that

$$(A+B\eta)(\overline{A}-B\eta) = A\overline{A} - B\overline{B}.$$

The formula on the right equals the determinant of the matrix. Thus we define the *conjugate* of $Z = A + B\eta$ by the formula

$$\overline{Z} = \overline{A+B\eta} = \overline{A} - B\eta,$$

and we have the formula

$$D(Z) = Z\overline{Z}$$

for the determinant $D(Z)$ where

$$Z = A + B\eta = \begin{pmatrix} a & c \\ d & b \end{pmatrix}$$

where $A = [a,b]$ and $B = [c,d]$. Note that

$$A\overline{A} = [ab, ba] = ab1 = ab,$$

so that

$$D(Z) = ab - cd.$$

Note also that we assume that a, b, c, d are in a commutative base ring.

Note also that for Z as above,

$$\overline{Z} = \overline{A} - B\eta = \begin{pmatrix} b & -c \\ -d & a \end{pmatrix}.$$

This is the classical adjoint of the matrix Z.

We leave it to the reader to check that for matrix iterants Z and W,

$$Z\overline{Z} = \overline{Z}Z$$

and that

$$\overline{ZW} = \overline{W}\,\overline{Z}$$

and

$$\overline{Z+W} = \overline{Z} + \overline{W}.$$

Note also that

$$\overline{\eta} = -\eta,$$

whence

$$\overline{B\eta} = -B\eta = -\eta\overline{B} = \overline{\eta}\overline{B}.$$

We can prove that

$$D(ZW) = D(Z)D(W)$$

as follows

$$D(ZW) = ZW\overline{ZW} = ZW\overline{W}\,\overline{Z}$$

$$= Z\overline{Z}W\overline{W} = D(Z)D(W).$$

Here the fact that $W\overline{W}$ is in the base ring which is commutative allows us to remove it from in between the appearance of Z and $\overline{Z}$. Thus we see that iterants as 2×2 matrices form a direct non-commutative generalization of the complex numbers.

It is worth pointing out the first precursor to the quaternions (the so-called *split quaternions*): This precursor is the system

$$\{\pm 1, \pm\epsilon, \pm\eta, \pm i\}.$$

Here $\epsilon\epsilon = 1 = \eta\eta$ while $i = \epsilon\eta$ so that $ii = -1$. The basic operations in this algebra are those of epsilon and eta. Eta is the delay shift operator that reverses the components of the iterant. Epsilon negates one of the components, and leaves the order unchanged. The quaternions arise directly from these two operations once we construct an extra square root of minus one that commutes with them. Call this extra root of minus one $\sqrt{-1}$. Then the quaternions are generated by

$$I = \sqrt{-1}\epsilon, J = \epsilon\eta, K = \sqrt{-1}\eta$$

with

$$I^2 = J^2 = K^2 = IJK = -1.$$

The "right" way to generate the quaternions is to start at the bottom iterant level with boolean values of 0 and 1 and the operation EXOR (exclusive or). Build iterants on this, and matrix algebra from these iterants. This gives the square root of negation. Now take pairs of values from this new algebra and build 2×2 matrices again. The coefficients include square roots of negation that commute with constructions at the next level and so quaternions appear in the third level of this hierarchy. We will return to the quaternions after discussing other examples that involve matrices of all sizes.

5. Iterants of Arbitrarily High Period

As a next example, consider a waveform of period three.

$$\cdots abcabcabcabcabcabc \cdots$$

Here we see three natural iterant views (depending upon whether one starts at a, b or c).

$$[a, b, c], \ \ [b, c, a], \ \ [c, a, b].$$

The appropriate shift operator is given by the formula

$$[x, y, z]S = S[z, x, y].$$

Thus, with $T = S^2$,

$$[x, y, z]T = T[y, z, x]$$

and $S^3 = 1$. With this we obtain a closed algebra of iterants whose general element is of the form

$$[a, b, c] + [d, e, f]S + [g, h, k]S^2$$

where $a, b, c, d, e, f, g, h, k$ are real or complex numbers. Call this algebra $\mathbb{V}ect_3(\mathbb{R})$ when the scalars are in a commutative ring with unit $\mathbb{F}$. Let $M_3(\mathbb{F})$ denote the 3×3 matrix algebra over $\mathbb{F}$. We have the

Lemma. The iterant algebra $\mathbb{V}ect_3(\mathbb{F})$ is isomorphic to the full 3×3 matrix algebra $M_3((\mathbb{F})$.

Proof. Map 1 to the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Map S to the matrix

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

and map S^2 to the matrix

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

Map $[x, y, z]$ to the diagonal matrix

$$\begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}.$$

Then it follows that

$$[a, b, c] + [d, e, f]S + [g, h, k]S^2$$

maps to the matrix

$$\begin{pmatrix} a & d & g \\ h & b & e \\ f & k & c \end{pmatrix},$$

preserving the algebra structure. Since any 3×3 matrix can be written uniquely in this form, it follows that $\mathbb{V}ect_3(\mathbb{F})$ is isomorphic to the full 3×3 matrix algebra $M_3(\mathbb{F})$. //

We can summarize the pattern behind this expression of 3×3 matrices by the following symbolic matrix.

$$\begin{pmatrix} 1 & S & T \\ T & 1 & S \\ S & T & 1 \end{pmatrix}.$$

Here the letter T occupies the positions in the matrix that correspond to the permutation matrix that represents it, and the letter $T = S^2$ occupies the positions corresponding to its permutation matrix. The 1's occupy the diagonal for the corresponding identity matrix. The iterant representation corresponds to writing the 3×3 matrix as a disjoint sum of these permutation matrices such that the matrices themselves are closed under multiplication. In this case the matrices form a permutation representation of the cyclic group of order 3, $C_3 = \{1, S, S^2\}$.

Remark. Note that a permutation matrix is a matrix of zeroes and ones such that some permutation of the rows of the matrix transforms it to the identity matrix. Given an $n \times n$ permutation matrix P, we associate to it a permutation

$$\sigma(P) : \{1, 2, \cdots, n\} \longrightarrow \{1, 2, \cdots, n\}$$

via the following formula

$$i\sigma(P) = j$$

where j denotes the column in P where the i-th row has a 1. Note that an element of the domain of a permutation is indicated to the left of the symbol for the permutation. It is then easy to check that for permutation matrices P and Q,

$$\sigma(P)\sigma(Q) = \sigma(PQ)$$

given that we compose the permutations from left to right according to this convention.

It should be clear to the reader that this construction generalizes directly for iterants of any period and hence for a set of operators forming a cyclic group of any order. In fact we shall generalize further to any finite group G. We now define $\mathbb{V}ect_{n(}G, \mathbb{F})$ for any finite group G.

Definition. Let G be a finite group, written multiplicatively. Let $\mathbb{F}$ denote a given commutative ring with unit. Assume that G acts as a group of permutations on the set $\{1, 2, 3, \cdots, n\}$ so that given an element $g \in G$ we have (by abuse of notation)

$$g : \{1, 2, 3, \cdots, n\} \longrightarrow \{1, 2, 3, \cdots, n\}.$$

We shall write

$$ig$$

for the image of $i \in \{1, 2, 3, \cdots, n\}$ under the permutation represented by g. Note that this denotes functionality from the left and so we ask that $(ig)h = i(gh)$ for all elements $g, h \in G$ and $i1 = i$ for all i, in order to have a representation of G as permutations. We shall call an n-tuple of elements of $\mathbb{F}$ a *vector* and denote it by $a = (a_1, a_2, \cdots, a_n)$. We then define an action of G on vectors over $\mathbb{F}$ by the formula

$$a^g = (a_{1g}, a_{2g}, \cdots, a_{ng}),$$

and note that $(a^g)^h = a^{gh}$ for all $g, h \in G$. We now define an algebra $\mathbb{V}ect_n(G, \mathbb{F})$, the *iterant algebra for G,* to be the set of finite sums of formal products of vectors and group elements in the form ag with multiplication rule

$$(ag)(bh) = ab^g(gh),$$

and the understanding that $(a + b)g = ag + bg$ and for all vectors a, b and group elements g. It is understood that vectors are added coordinatewise and multiplied coordinatewise. Thus $(a + b)_i = a_i + b_i$ and $(ab)_i = a_i b_i$.

Theorem. Let G be a finite group of order n. Let $\rho : G \longrightarrow S_n$ denote the right regular representation of G as permutations of n things where we list the elements of G as $G = \{g_1, \cdots, g_n\}$ and let G act on its own underlying set via the definition $g_i\rho(g) = g_i g$. Here we describe $\rho(g)$ acting on the set of elements g_k of G. If we wish to regard $\rho(g)$ as a mapping of the set $\{1, 2, \cdots n\}$ then we replace g_k by k and $i\rho(g) = k$ where $g_i g = g_k$.

Then $\mathbb{V}ect_n(G, \mathbb{F})$ is isomorphic to the matrix algebra $M_n((\mathbb{F})$. In particular, we have that $\mathbb{V}ect_{n!}(S_n, \mathbb{F})$ is isomorphic with the matrices of size $n! \times n!$, $M_{n!}((\mathbb{F})$.

Proof. Consider the $n \times n$ matrix consisting in the multiplication table for G with the columns and rows listed in the order $[g_1, \cdots, g_n]$. Permute the rows of this table so that the diagonal consists in all 1's. Let the resulting table be called the *G-Table*. The *G-Table* is labeled by elements of the group. For a vector a, let $D(a)$ denote the $n \times n$ diagonal matrix whose entries in order down the diagonal are the entries of a in the order specified by a. For each group element g, let P_g denote the permutation matrix with 1 in every spot on the *G-Table* that is labeled by g and 0 in all other spots. It is now a direct verification that the mapping

$$F(\Sigma_{i=1}^n a_i g_i) = \Sigma_{i=1}^n D(a_i) P_{g_i}$$

defines an isomorphism from $\mathbb{V}ect_n(G, \mathbb{F})$ to the matrix algebra $M_n((\mathbb{F})$. The main point to check is that $\sigma(P_g) = \rho(g)$. We now prove this fact.

In the *G-Table* the rows correspond to

$$\{g_1^{-1}, g_2^{-1}, \cdots g_n^{-1}\}$$

and the columns correspond to

$$\{g_1, g_2, \cdots g_n\}$$

so that the i-i entry of the table is $g_i^{-1} g_i = 1$. With this we have that in the table, a group element g occurs in the i-th row at column j where

$$g_i^{-1} g_j = g.$$

This is equivalent to the equation

$$g_i g = g_j$$

which, in turn is equivalent to the statement

$$i\rho(g) = j.$$

This is exactly our functional interpretation of the action of the permutation corresponding to the matrix P_g. Thus

$$\rho(g) = \sigma(P_g).$$

The remaining details of the proof are straightforward and left to the reader. //

Examples.

(1) We have already implicitly given examples of this process of translation. Consider the cyclic group of order three.

$$C_3 = \{1, S, S^2\}$$

with $S^3 = 1$. The multiplication table is

$$\begin{pmatrix} 1 & S & S^2 \\ S & S^2 & 1 \\ S^2 & 1 & S \end{pmatrix}.$$

Interchanging the second and third rows, we obtain

$$\begin{pmatrix} 1 & S & S^2 \\ S^2 & 1 & S \\ S & S^2 & 1 \end{pmatrix},$$

and this is the *G-Table* that we used for $\mathbb{V}ect_3(C_3, \mathbb{F})$ prior to proving the Main Theorem.

The same pattern works for abitrary cyclic groups. for example, consider the cyclic group of order 6. $C_6 = \{1, S, S^2, S^3, S^4, S^5\}$ with $S^6 = 1$. The multiplication table is

$$\begin{pmatrix} 1 & S & S^2 & S^3 & S^4 & S^5 \\ S & S^2 & S^3 & S^4 & S^5 & 1 \\ S^2 & S^3 & S^4 & S^5 & 1 & S \\ S^3 & S^4 & S^5 & 1 & S & S^2 \\ S^4 & S^5 & 1 & S & S^2 & S^3 \\ S^5 & 1 & S & S^2 & S^3 & S^4 \end{pmatrix}.$$

Rearranging to form the *G-Table*, we have

$$\begin{pmatrix} 1 & S & S^2 & S^3 & S^4 & S^5 \\ S^5 & 1 & S & S^2 & S^3 & S^4 \\ S^4 & S^5 & 1 & S & S^2 & S^3 \\ S^3 & S^4 & S^5 & 1 & S & S^2 \\ S^2 & S^3 & S^4 & S^5 & 1 & S \\ S & S^2 & S^3 & S^4 & S^5 & 1 \end{pmatrix}.$$

The permutation matrices corresponding to the positions of S^k in the *G-Table* give the matrix representation that gives the isomorphism of $\mathbb{V}ect_6(C_6, \mathbb{F})$ with the full algebra of six by six matrices.

(2) Now consider the symmetric group on six letters,

$$S_6 = \{1, R, R^2, F, RF, R^2F\}$$

where $R^3 = 1, F^2 = 1, FR = RF^2$. Then the multiplication table is

$$\begin{pmatrix} 1 & R & R^2 & F & RF & R^2F \\ R & R^2 & 1 & RF & R^2F & F \\ R^2 & 1 & R & R^2F & F & RF \\ F & R^2F & RF & 1 & R^2 & R \\ RF & F & R^2F & R & 1 & R^2 \\ R^2F & RF & F & R^2 & R & 1 \end{pmatrix}.$$

The corresponding *G-Table* is

$$\begin{pmatrix} 1 & R & R^2 & F & RF & R^2F \\ R^2 & 1 & R & R^2F & F & RF \\ R & R^2 & 1 & RF & R^2F & F \\ F & R^2F & RF & 1 & R^2 & R \\ RF & F & R^2F & R & 1 & R^2 \\ R^2F & RF & F & R^2 & R & 1 \end{pmatrix}.$$

Here is a rewritten version of the *G-Table* with

$$R = \Delta, R^2 = \Theta, F = \Psi, RF = \Omega, R^2F = \Sigma.$$

$$\begin{pmatrix} 1 & \Delta & \Theta & \Psi & \Omega & \Sigma \\ \Theta & 1 & \Delta & \Sigma & \Psi & \Omega \\ \Delta & \Theta & 1 & \Omega & \Sigma & \Psi \\ \Psi & \Sigma & \Omega & 1 & \Theta & \Delta \\ \Omega & \Psi & \Sigma & \Delta & 1 & \Theta \\ \Sigma & \Omega & \Psi & \Theta & \Delta & 1 \end{pmatrix}.$$

This *G-Table* is the keystone for the isomorphism of $\mathbb{V}ect_6(S_3, \mathbb{F})$ with the full algebra of six by six matrices. At this point it may occur to the reader to wonder about $\mathbb{V}ect_3(S_3, \mathbb{F})$ since S_3 does act on vectors of length three. We will discuss $\mathbb{V}ect_n(S_n, \mathbb{F})$ in the next section. We see from this example how it will come about that $\mathbb{V}ect_{n!}(S_n, \mathbb{F})$ is isomorphic with the full algebra of $n! \times n!$ matrices. In particular, here are the permutation matrices that form the non-identity elements of this representation of the symmetric group on three letters.

$$R = \Delta = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$R^2 = \Theta = \begin{pmatrix} 0&0&1&0&0&0 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 0&0&0&1&0&0 \end{pmatrix}$$

$$F = \Psi = \begin{pmatrix} 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \end{pmatrix}$$

$$FR = \Omega = \begin{pmatrix} 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \end{pmatrix}$$

$$FR^2 = \Sigma = \begin{pmatrix} 0&0&0&0&0&1 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 1&0&0&0&0&0 \end{pmatrix}$$

(3) In this example we consider the group $G = C_2 \times C_2$, often called the "Klein 4-Group." We take $G = \{1, A, B, C\}$ where $A^2 = B^2 = C^2 = 1, AB = BA = C$. Thus G has the multiplication table, which is also its G-*Table* for $\mathbb{V}ect_4(G, \mathbb{F})$.

$$\begin{pmatrix} 1 & A & B & C \\ A & 1 & C & B \\ B & C & 1 & A \\ C & B & A & 1 \end{pmatrix}.$$

Thus we have the following permutation matrices that I shall call E, A, B, C :

$$E = \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix},$$

$$A = \begin{pmatrix} 0&1&0&0 \\ 1&0&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{pmatrix},$$

$$B = \begin{pmatrix} 0&0&1&0 \\ 0&0&0&1 \\ 1&0&0&0 \\ 0&1&0&0 \end{pmatrix},$$

$$C = \begin{pmatrix} 0&0&0&1 \\ 0&0&1&0 \\ 0&1&0&0 \\ 1&0&0&0 \end{pmatrix}.$$

The reader will have no difficulty verifying that $A^2 = B^2 = C^2 = 1, AB = BA = C$. Recall that $[x, y, z, w]$ is iterant notation for the diagonal matrix

$$[x, y, z, w] = \begin{pmatrix} x & 0 & 0 & 1 \\ 0 & y & 1 & 0 \\ 0 & 1 & z & 0 \\ 1 & 0 & 0 & w \end{pmatrix}.$$

Let

$$\alpha = [1, -1, -1, 1], \beta = [1, 1, -1, -1],$$

$$\gamma = [1, -1, 1, -1].$$

And let

$$I = \alpha A, J = \beta B, K = \gamma C.$$

Then the reader will have no trouble verifying that

$$I^2 = J^2 = K^2 = IJK = -1, IJ = K, JI = -K.$$

Thus we have constructed the quaternions as iterants in relation to the Klein Four Group. In Figure 1 we illustrate these quaternion generators with string diagrams for the permutations. The reader can check that the permutations correspond to the permutation matrices constructed for the Klein Four Group. For example, the permutation for I is $(12)(34)$ in cycle notation, the permutation for J is $(13)(24)$ and the permutation for K is $(14)(23)$. In the Figure we attach signs to each string of the permutation. These "signed permutations" act exactly as the products of vectors and permutations that we use for the iterants. One can see that the quaternions arise naturally from the Klein Four Group by attaching signs to the generating permutations as we have done in this Figure.

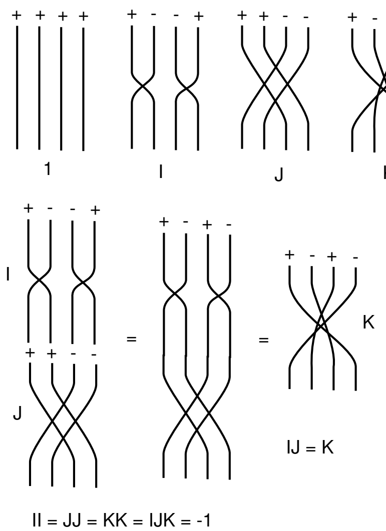

Fig. 1. **Quaternions From Klein Four Group**

(4) One can use the quaternions as a linear basis for 4×4 matrices just as our theorem would use the permutation matrices $1, A, B, C$. If we restrict to real scalars a, b, c, d such that $a^2+b^2+c^2+c^2 = 1$, then the set of matrices of the form $a1 + bI + cJ + dK$ is isomorphic to the group $SU(2)$. To see this, note that $SU(2)$ is the set of matrices with complex entries z and w with determinant 1 so that $z\bar{z} + w\bar{w} = 1$.

$$M = \begin{pmatrix} z & w \\ -\bar{w} & \bar{z} \end{pmatrix}.$$

Letting $z = a + bi$ and w $= c + di$, we have

$$M = \begin{pmatrix} a+bi & c+di \\ -c+di & a-bi \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b \begin{pmatrix} i & 0 \\ o & -i \end{pmatrix}$$

$$+c \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + d \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

If we regard $i = \sqrt{-1}$ as a commuting scalar, then we can write the generating matrices in terms of size two iterants and obtain

$$I = \sqrt{-1}\epsilon, J = \epsilon\eta, K = \sqrt{-1}\eta$$

as described in the previous section. IF we regard these matrices with complex entries as shorthand for 4×4 matrices with i interpreted as a 2×2 matrix as we have done above, then these 4×4 matrices representing the quaternions are exactly the ones we have constructed in relation to the Klein Four Group.

Since complex numbers commute with one another, we could consider iterants whose values are in the complex numbers. This is just like considering matrices whose entries are complex numbers. For this purpose we shall allow given a version of i that commutes with the iterant shift operator η. Let this commuting i be denoted by ι. Then we are assuming that

$$\iota^2 = -1$$

$$\eta\iota = \iota\eta$$

$$\eta^2 = +1.$$

We then consider iterant views of the form $[a+b\iota, c+d\iota]$ and $[a+b\iota, c+d\iota]\eta = \eta[c+d\iota, a+b\iota]$. In particular, we have $\epsilon = [1, -1]$, and $i = \epsilon\eta$ is quite distinct from ι. Note, as before, that $\epsilon\eta = -\eta\epsilon$ and that $\epsilon^2 = 1$. Now let

$$I = \iota\epsilon$$

$$J = \epsilon\eta$$

$$K = \iota\eta.$$

We have used the commuting version of the square root of minus one in these definitions, and indeed we find the quaternions once more.

$$I^2 = \iota\epsilon\iota\epsilon = \iota\iota\epsilon\epsilon = (-1)(+1) = -1,$$

$$J^2 = \epsilon\eta\epsilon\eta = \epsilon(-\epsilon)\eta\eta = -1,$$

$$K^2 = \iota\eta\iota\eta = \iota\iota\eta\eta = -1,$$

$$IJK = \iota\epsilon\epsilon\eta\iota\eta = \iota 1\iota\eta\eta = \iota\iota = -1.$$

Thus

$$I^2 = J^2 = K^2 = IJK = -1.$$

This construction shows how the structure of the quaternions comes directly from the non-commutative structure of period two iterants. In other, words, quaternions can be represented by 2×2 matrices. This is the way it has been presented in standard language. The group $SU(2)$ of 2×2 unitary matrices of determinant one is isomorphic to the quaternions of length one.

(5) Similarly,

$$H = [a, b] + [c + d\iota, c - d\iota]\eta = \begin{pmatrix} a & c + d\iota \\ c - d\iota & b \end{pmatrix}.$$

represents a Hermitian 2×2 matrix and hence an observable for quantum processes mediated by $SU(2)$. Hermitian matrices have real eigenvalues.

If in the above Hermitian matrix form we take $a = T + X, b = T - X, c = Y, d = Z$, then we obtain an iterant and/or matrix representation for a point in Minkowski spacetime.

$$H = [T + X, T - X] + [Y + Z\iota, Y - Z\iota]\eta$$

$$= \begin{pmatrix} T + X & Y + Z\iota \\ Y - Z\iota & T - X \end{pmatrix}.$$

Note that we have the formula

$$Det(H) = T^2 - X^2 - Y^2 - Z^2.$$

It is not hard to see that the eigenvalues of H are $T \pm \sqrt{X^2 + Y^2 + Z^2}$. Thus, viewed as an observable, H can observe the time and the invariant spatial distance from the origin of the event (T, X, Y, Z). At least at this very elementary juncture, quantum mechanics and special relativity are reconciled.

(6) Hamilton's Quaternions are generated by iterants, as discussed above, and we can express them purely algebraicially by writing the corresponding permutations as shown below.

$$I = [+1, -1, -1, +1]s$$

$$J = [+1, +1, -1, -1]l$$

$$K = [+1, -1, +1, -1]t$$

where

$$s = (12)(34)$$

$$l = (13)(24)$$

$$t = (14)(23).$$

Here we represent the permutations as products of transpositions (ij). The transposition (ij) interchanges i and j, leaving all other elements of $\{1, 2, ..., n\}$ fixed.

One can verify that

$$I^2 = J^2 = K^2 = IJK = -1.$$

For example,

$$I^2 = [+1, -1, -1, +1]s[+1, -1, -1, +1]s$$

$$= [+1, -1, -1, +1][-1, +1, +1, -1]ss$$

$$= [-1, -1, -1, -1]$$

$$= -1.$$

and

$$IJ = [+1, -1, -1, +1]s[+1, +1, -1, -1]l$$

$$= [+1, -1, -1, +1][+1, +1, -1, -1]sl$$

$$= [+1, -1, +1, -1](12)(34)(13)(24)$$

$$= [+1, -1, +1, -1](14)(23)$$

$$= [+1, -1, +1, -1]t.$$

Nevertheless, we must note that making an iterant interpretation of an entity like $I = [+1, -1, -1, +1]s$ is a conceptual departure from our original period two iterant (or cyclic period n) notion. Now we are considering iterants such as $[+1, -1, -1, +1]$ where the permutation group acts to produce other orderings of a given sequence. The iterant itself is not necessarily an oscillation. It can represent an implicate form that can be seen in any of its possible orders. These orders are subject to permutations that produce the possible views of the iterant. Algebraic structures such as the quaternions appear in the explication of such implicate forms.

The reader will also note that we have moved into a different conceptual domain from an original emphasis in this paper on eigenform in relation to recursion. That is, we take an *eigenform* to mean a fixed point for a transformation. Thus i is an eigenform for $R(x) = -1/x$. Indeed, each generating quaternion is an eigenform for the transformation $R(x) = -1/x$. The richness of the quaternions arises from the closed algebra that arises with its infinity of eigenforms that satisfy the equation $U^2 = -1$:

$$U = aI + bJ + cK$$

where $a^2 + b^2 + c^2 = 1$. This kind of significant extra structure in the eigenforms comes from paying attention to specific aspects of implicate and explicate structure, relationships with geometry and ideas and inputs from the perceptual, conceptual and physical worlds. Just as with our other examples of phenomena arising in the course of the recursion, we see the same phenomena here in the evolution of mathematical and theoretical physical structures in the course of the recursion that constitutes scientific conversation.

(7) In all these examples, we have the opportunity to interpret the iterants as short hand for matrix algebra based on permutation matrices, or as indicators of discrete processes. The discrete processes become more complex in proportion to the complexity of the groups used in the construction. We began with processes of order two, then considered cyclic groups of arbitrary order, then the symmetric group S_3 in relation to 6×6 matrices, and the Klein Four Group in relation to the quaternions. In the case of the quaternions, we know that this structure is intimately related to rotations of three and four dimensional space and many other geometric themes. It is worth reflecting on the possible significance of the underlying discrete dynamics for this geometry, topology and related physics.

6. The Framed Braid Group

The reader should recall that the symmetric group S_n has presentation

$$S_n = (T_1, \cdots T_{n-1} | T_i^2 = 1, T_iT_{i+1}T_i = T_{i+1}T_iT_{i+1}, T_iT_j = T_jT_i; |i-j| > 1).$$

The Artin Braid Group B_n is a relative of the symmetric group that is obtained by removing the condition that each generator has square equal to the identity.

$$B_n = (\sigma_1, \cdots \sigma_{n-1} | \sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}, \sigma_i\sigma_j = \sigma_j\sigma_i; |i-j| > 1).$$

In Figure 2 we illustrate the the generators $\sigma_1, \sigma_2, \sigma_3$ of the 4-strand braid group and we show the topological nature of the relation $\sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2$ and the commuting relation $\sigma_1\sigma_3 = \sigma_3\sigma_1$. Topological braids are represented as collections of always descending strings, starting from a row of points and ending at another row of points. The strings are embedded in three dimensional space and can wind around one another. The elementary braid generators σ_i correspond to the i-th strand interchanging with the $i+1$-th strand. Two braids are multiplied by attaching the bottom endpoints of one braid to the top endpoints of the other braid to form a new braid.

There is a fundamental homomorphism

$$\pi : B_n \longrightarrow S_n$$

defined on generators by

$$\pi(\sigma_i) = T_i$$

in the language of the presentations above. In term of the diagrams in Figure 2, a braid diagram is a permutation diagram if one forgets about its weaving structure of over and under strands at a crossing.

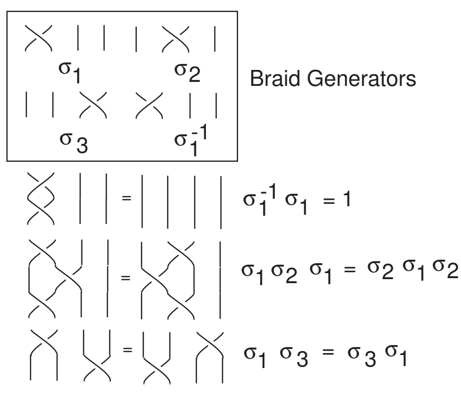

Fig. 2. **Braid Generators**

We now turn to a generalization of the braid group, the *framed braid group*. In this generalization, we associate elements of the form t^a to the top of each braid strand. For these purposes it is useful to take t as an algebraic variable and a as an integer. To interpret this framing geometrically replace each braid strand by a ribbon and interpret t^a as a $2\pi a$ twist in the ribbon. In Figure 3 we illustrate how to multiply two framed braids. In our formalism the braids A and B in this figure are given by the formulas

$$A = [t^a, t^b, t^c]\sigma_1\sigma_2\sigma_3,$$

$$B = [t^d, t^e, t^f]\sigma_2\sigma_3$$

in the framed braid group on three strands, denoted FB_3. As the Figure 3 illustrates, we have the basic formula

$$v\sigma = \sigma v^{\pi(\sigma)}$$

where v is a vector of the form $v = [t^a, t^b, t^c]$ (for $n = 3$) and $v^{\pi(\sigma)}$ denotes the action of the permutation associated with the braid σ on the vector v. In the figure the permutation is accomplished by sliding the algebra along the strings of the braid.

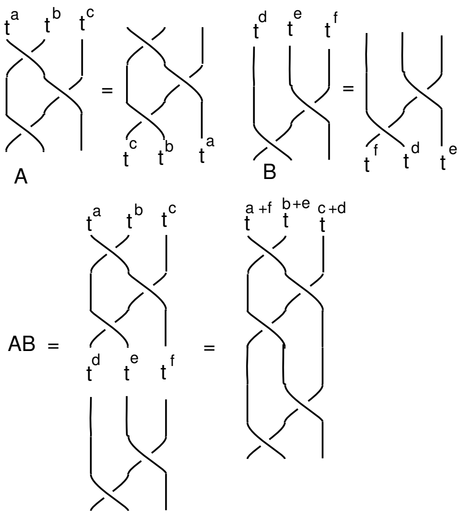

Fig. 3. **Framed Braids**

We can form an algebra $Alg[FB_n]$ by taking formal sums of framed braids of the form $\sum c_k v_k G_k$ where c_k is a scalar, v_k is a framing vector and G_k is an element of the Artin Braid group B_n. Since braids act on framing vectors by permutations, this algebra is a generalization of the iterant algebras we have defined so far. The algebra of framed braids uses an action of the braid group based on its representation to the symmetric group. Furthermore, the representation $\pi : B_n \longrightarrow S_n$ induces a map of algebras

$$\hat{\pi} : Alg[FB_n] \longrightarrow Alg[FS_n]$$

where we recognize $Alg[FS_n]$ as exactly an iterant algebra based in S_n.

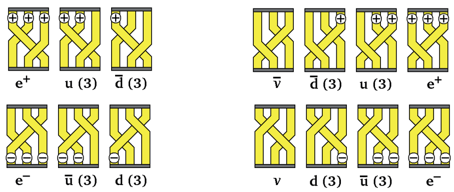

Fig. 4. **Sundance Bilson Thompson Framed Braid Fermions**

In[31] Sundance Bilson-Thompson represents Fermions as framed braids. See Figure 4 for his diagrammatic representations. In this theory each fermion is associated with a framed braid. Thus from the figure we see that the positron and the electron are given by the framed braids

$$e^+ = [t, t, t]\sigma_1\sigma_2^{-1},$$

and

$$e^- = \sigma_2\sigma_1^{-1}[t^{-1}, t^{-1}, t^{-1}].$$

Here we use $[t^a, t^b, t^c]$ for the framing numbers (a, b, c). Products of framed braids correspond to particle interactions. Note that $e^+e^- = [1, 1, 1] = \gamma$ so that the electron and the positron are inverses in this algebra. In Figure 5 are illustrated the representations of bosons, including γ, a photon and the identity element in this algebra. Other relations in the algebra correspond to particle interactions. For example in Figure 6 is illustrated the muon decay

$$\mu \rightarrow \nu_\mu + W_- \rightarrow \nu_\mu + \bar{\nu_e} + e^-.$$

The reader can see the definitions of the different parts of this decay sequence from the three figures we have just mentioned. Note the strictly speaking the muon decay is a multiplicative identity in the braid algebra:

$$\mu = \nu_\mu W_- = \nu_\mu \bar{\nu_e} e^-.$$

Particle interactions in this model are mediated by factorizations in the non-commutative algebra of the framed braids.

By using the representation $\hat{\pi} : Alg[FB_3] \longrightarrow Alg[FS_3]$ we can image the structure of Bilson-Thompson's framed braids in the iterant algebra corresponding to the symmetric group. However, we propose to change this map so that we have a non-trivial representation of the Artin braid group. This can be accomplished by defining

$$\rho : Alg[FB_3] \longrightarrow Alg[FS_3]$$

where

$$\rho(\sigma_k) = [t, t]T_k$$

and

$$\rho(\sigma_k^{-1}) = [t^{-1}, t^{-1}]T_k$$

for $k = 1, 2$. The reader will find that we have now embedded the braid group in the iterant algebra $Alg[FS_3]$ and extended the embedding to the framed braid group algebra. Thus *the Sundance Bilson-Thompson representation of elementary particles as framed braids is embedded inside the iterant algebra for the symmetric group on three letters.* In Section 10 we carry this further and place the representation inside the Lie Algebra $su(3)$.

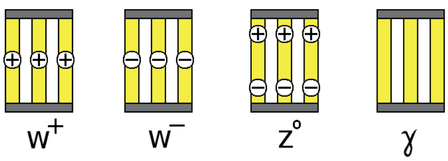

Fig. 5. **Bosons**

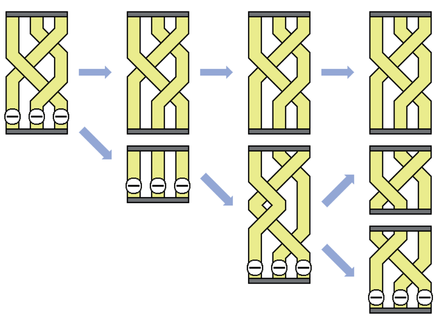

Fig. 6. **Representation of** $\mu \to \nu_\mu + W_- \to \nu_\mu + \bar{\nu_e} + e^-$

7. Iterants and the Standard Model

In this section we shall give an iterant interpretation for the Lie algebra of the special unitary group $SU(3)$. The Lie algebra in question is denoted as $su(3)$ and is often described by a matrix basis. The Lie algebra $su(3)$ is generated by the following eight Gell-Mann Matrices.[29]

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

The group $SU(3)$ consists in the matrices $U(\epsilon_1, \cdots, \epsilon_8) = e^{i \sum_a \epsilon_a \lambda_a}$ where $\epsilon_1, \cdots, \epsilon_8$ are real numbers and a ranges from 1 to 8. The Gell-Mann matrices satisfy the following relations.

$$tr(\lambda_a \lambda_b) = 2\delta_{ab},$$

$$[\lambda_a/2, \lambda_b/2] = i f_{abc} \lambda_c/2.$$

Here we use the summation convention – summing over repeated indices, and tr denotes standard matrix trace, $[A, B] = AB - BA$ is the matrix commutator and δ_{ab} is the Kronecker delta, equal to 1 when $a = b$ and equal to 0 otherwise. The structure coefficients f_{abc} take the following non-zero values.

$$f_{123} = 1, f_{147} = 1/2, f_{156} = -1/2, f_{246} = 1/2,$$

$$f_{257} = 1/2,$$

$$f_{345} = 1/2, f_{367} = -1/2, f_{458} = \sqrt{3/2}, f_{678} = \sqrt{3/2}.$$

We now give an iterant representation for these matrices that is based on the pattern

$$\begin{pmatrix} 1 & A & B \\ B & 1 & A \\ A & B & 1 \end{pmatrix}$$

as described in the previous section. That is, we use the cyclic group of order three to represent all 3×3

matrices at iterants based on the permutation matrices

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

Recalling that $[a, b, c]$ as an iterant, denotes a diagonal matrix

$$[a, b, c] = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix},$$

the reader will have no difficulty verifying the following formulas for the Gell-Mann Matrices in the iterant format:

$$\lambda_1 = [1, 0, 0]A + [0, 1, 0]B$$

$$\lambda_2 = [-i, 0, 0]A + [0, i, 0]B$$

$$\lambda_3 = [1, -1, 0]$$

$$\lambda_4 = [1, 0, 0]B + [0, 0, 1]A$$

$$\lambda_5 = [i, 0, 0]B + [0, 0, -i]A$$

$$\lambda_6 = [0, 1, 0]A + [0, 0, 1]B$$

$$\lambda_7 = [0, -i, 0]A + [0, 0, i]B$$

$$\lambda_8 = \frac{1}{\sqrt{3}}[1, 1, -2].$$

Letting $F_a = \lambda_a/2$, we can now rewrite the Lie algebra into simple iterants of the form $[a, b, c]G$ where G is a cyclic group element. Compare with.[30] Let

$$T_\pm = F_1 \pm iF_2,$$

$$U_\pm = F_6 \pm iF_7,$$

$$V_\pm = F_4 \pm iF_5,$$

$$T_3 = F_3,$$

$$Y = \frac{2}{\sqrt{3}}F_8.$$

Then we have the specific iterant formulas

$$T_+ = [1, 0, 0]A,$$

$$T_- = [0, 1, 0]B,$$

$$U_+ = [0, 1, 0]A,$$

$$U_- = [0, 0, 1]B,$$

$$V_+ = [0, 0, 1]A,$$

$$V_- = [1, 0, 0]B,$$

$$T_3 = [1/2, -1/2, 0],$$

$$Y = \frac{1}{\sqrt{3}}[1, 1, -2].$$

We have that $A[x, y, z] = [y, z, x]A$ and $B = A^2 = A^{-1}$ so that $B[x, y, z] = [z, y, x]B$. Thus we have reduced the basic $su(3)$ Lie algebra to a very elementary patterning of order three cyclic operations. In a subsequent paper, we will use this point to view to examine the irreducible representations of this algebra and to illuminate the Standard Model's Eightfold Way.

8. Iterants, Braiding and the Sundance-Bilson Thompson Model for Fermions

In the last section we based our iterant representations on the following patterns and matrices. The pattern,

$$\begin{pmatrix} 1 & A & B \\ B & 1 & A \\ A & B & 1 \end{pmatrix},$$

using the cyclic group of order three to represent all 3×3 matrices at iterants based on the permutation matrices

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

Recalling that $[a, b, c]$ as an iterant, denotes a diagonal matrix

$$[a, b, c] = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}.$$

In fact there are six 3×3 permutation matrices: $\{I, A, B, P, Q, R\}$ where

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, Q = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

We then have $A = QP, B = PQ, R = PQP = QPQ$. The two transpositions P and Q generate the entire group of permutations S_3. It is usual to think of the order-three transformations A and B as expressed in terms of these transpositions, but we can also use the iterant structure of the 3×3 matrices to express P, Q and R in terms of A and B. The result is as follows:

$$P = [0,0,1] + [1,0,0]A + [0,1,0]B,$$

$$Q = [1,0,0] + [0,1,0]A + [0,0,1]B,$$

$$R = [0,1,0] + [0,0,1]A + [1,0,0]B.$$

Recall from the previous section that we have the iterant generators for the $su(3)$ Lie algebra:

$$T_+ = [1,0,0]A,$$

$$T_- = [0,1,0]B,$$

$$U_+ = [0,1,0]A,$$

$$U_- = [0,0,1]B,$$

$$V_+ = [0,0,1]A,$$

$$V_- = [1,0,0]B.$$

Thus we can express these transpositions P and Q in the iterant form of the Lie algebra as

$$P = [0,0,1] + T_+ + T_-,$$

$$Q = [1,0,0] + U_+ + U_-,$$

$$R = [0,1,0] + V_+ + V_-.$$

The basic permutations receive elegant expressions in the iterant Lie algebra.

Now that we have basic permutations in the Lie algebra we can take the map from section 7.1

$$\rho : Alg[FB_3] \longrightarrow Alg[FS_3]$$

with

$$\rho(\sigma_k) = [t,t]T_k$$

and

$$\rho(\sigma_k^{-1}) = [t^{-1}, t^{-1}]T_k$$

for $k = 1,2$ and send T_1 to P and T_2 to Q. Then we have

$$\rho(\sigma_1) = [t,t]P$$

and

$$\rho(\sigma_1^{-1}) = [t^{-1}, t^{-1}]P$$

and

$$\rho(\sigma_2) = [t,t]Q$$

and

$$\rho(\sigma_1^{-1}) = [t^{-1}, t^{-1}]Q.$$

By choosing $t \neq 1$ on the unit circle in the complex plane, we obtain representations of the Sundance Bilson-Thompson constructions of Fermions via framed braids *inside* the $su(3)$ Lie algebra. This brings the Bilson-Thompson formalism in direct contact with the Standard Model via our iterant representations. We shall return to these relationships in a sequel to the present paper.

9. Clifford Algebra, Majorana Fermions and Braiding

Recall fermion algebra. One has fermion annihiliation operators ψ and their conjugate creation operators $\psi^\dagger$. One has $\psi^2 = 0 = (\psi^\dagger)^2$. There is a fundamental commutation relation

$$\psi\psi^\dagger + \psi^\dagger\psi = 1.$$

If you have more than one of them say ψ and ϕ, then they anti-commute:

$$\psi\phi = -\phi\psi.$$

The Majorana fermions c that satisfy $c^\dagger = c$ so that they are their own anti-particles. There is a lot of interest in these as quasi-particles and they are related to braiding and to topological quantum computing. A group of researchers[28] claims, at this writing, to have found quasiparticle Majorana fermions in edge effects in nano-wires. (A line of fermions could have a Majorana fermion happen non-locally from one end of the line to the other.) The Fibonacci model that we discuss is also based on Majorana particles, possibly related to collective electronic excitations. If P is a Majorana fermion particle, then P can interact with itself to either produce itself or to annihilate itself. This is the simple "fusion algebra" for this particle. One can write $P^2 = P + 1$ to denote the two possible self-interactions the particle P. The patterns of interaction and braiding of such a particle P give rise to the Fibonacci model.

Majoranas are related to standard fermions as follows: The algebra for Majoranas is $c = c^\dagger$ and $cc' = -c'c$ if c and c' are distinct Majorana fermions with $c^2 = 1$ and $c'^2 = 1$. One can make a standard fermion from two Majoranas via

$$\psi = (c + ic')/2,$$

$$\psi^\dagger = (c - ic')/2.$$

Similarly one can mathematically make two Majoranas from any single fermion. Now if you take a set of Majoranas

$$\{c_1, c_2, c_3, \cdots, c_n\}$$

then there are natural braiding operators that act on the vector space with these c_k as the basis. The operators are mediated by algebra elements

$$\tau_k = (1 + c_{k+1}c_k)/\sqrt{2},$$

$$\tau_k^{-1} = (1 - c_{k+1}c_k)/\sqrt{2}.$$

Then the braiding operators are

$$T_k : Span\{c_1, c_2, \cdots, , c_n\} \longrightarrow Span\{c_1, c_2, \cdots, , c_n\}$$

via

$$T_k(x) = \tau_k x \tau_k^{-1}.$$

The braiding is simply:

$$T_k(c_k) = c_{k+1},$$

$$T_k(c_{k+1}) = -c_k,$$

and T_k is the identity otherwise. This gives a very nice unitary representaton of the Artin braid group and it deserves better understanding. See Figure 7 for an illustration of this braiding of Fermions in relation to the topology of a belt that connects them. The relationship with the belt is tied up with the fact that in quantum mechanics we must represent rotations of three dimensional space as unitary transformations. See[12] for more about this topological view of the physics of Fermions. In the Figure, we see that the belt does not know which of the two Fermions to annoint with the phase change, but the clever algebra above makes this decision. There is more to be done in this domain.

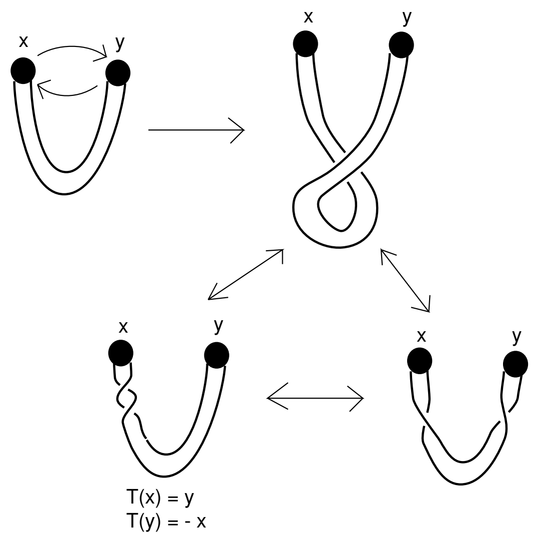

Fig. 7. **Braiding Action on a Pair of Fermions**

It is worth noting that a triple of Majorana fermions say a, b, c gives rise to a representation of the quaternion group. This is a generalization of the well-known association of Pauli matrices and quaternions. We have $a^2 = b^2 = c^2 = 1$ and they anticommute. Let $I = ba, J = cb, K = ac$. Then

$$I^2 = J^2 = K^2 = IJK = -1,$$

giving the quaternions. The operators

$$A = (1/\sqrt{2})(1 + I)$$

$$B = (1/\sqrt{2})(1 + J)$$

$$C = (1/\sqrt{2})(1 + K)$$

braid one another:

$$ABA = BAB, BCB = CBC, ACA = CAC.$$

This is a special case of the braid group representation described above for an arbitrary list of Majorana fermions. These braiding operators are entangling and so can be used for universal quantum computation, but they give only partial topological quantum computation due to the interaction with single qubit operators not generated by them.

Recall that in discussing the beginning of iterants, we introduce a *temporal shift operator* η such that

$$[a,b]\eta = \eta[b,a]$$

and

$$\eta\eta = 1$$

for any iterant $[a,b]$, so that concatenated observations can include a time step of one-half period of the process

$$\cdots abababab \cdots .$$

We combine iterant views term-by-term as in

$$[a,b][c,d] = [ac,bd].$$

We now define i by the equation

$$i = [1,-1]\eta.$$

This makes i both a value and an operator that takes into account a step in time.

We calculate

$$ii = [1,-1]\eta[1,-1]\eta = [1,-1][-1,1]\eta\eta$$

$$= [-1,-1] = -1.$$

Thus we have constructed a square root of minus one by using an iterant viewpoint. In this view i represents a discrete oscillating temporal process and it is an eigenform for $T(x) = -1/x$, participating in the algebraic structure of the complex numbers. In fact the corresponding algebra structure of linear combinations $[a,b]+[c,d]\eta$ is isomorphic with 2×2 matrix algebra and iterants can be used to construct $n\times n$ matrix algebra, as we have already discussed.

Now we can make contact with the algebra of the Majorana fermions. Let $e = [1,-1]$. Then we have $e^2 = [1,1] = 1$ and $e\eta = [1,-1]\eta = [-1,1]\eta = -e\eta$. Thus we have

$$e^2 = 1,$$

$$\eta^2 = 1,$$

and

$$e\eta = -\eta e.$$

We can regard e and η as a fundamental pair of Majorana fermions.

Note how the development of the algebra works at this point. We have that

$$(e\eta)^2 = -1$$

and so regard this as a natural construction of the square root of minus one in terms of the phase synchronization of the clock that is the iteration of the reentering mark. Once we have the square root of minus one it is natural to introduce another one and call this one i, letting it commute with the other operators. Then we have the $(ie\eta)^2 = +1$ and so we have a triple of Majorana fermions:

$$a = e, b = \eta, c = ie\eta$$

and we can construct the quaternions

$$I = ba = \eta e, J = cb = ie, K = ac = i\eta.$$

With the quaternions in place, we have the braiding operators

$$A = \frac{1}{\sqrt{2}}(1+I), B = \frac{1}{\sqrt{2}}(1+J), C = \frac{1}{\sqrt{2}}(1+K),$$

and can continue as we did above.

10. The Dirac Equation and Majorana Fermions

We now construct the Dirac equation. This may sound circular, in that the fermions arise from solving the Dirac equation, but in fact the algebra underlying this equation has the same properties as the creation and annihilation algebra for fermions, so it is by way of this algebra that we will come to the Dirac equation. If the speed of light is equal to 1 (by convention), then energy E, momentum p and mass m are related by the (Einstein) equation

$$E^2 = p^2 + m^2.$$

Dirac constructed his equation by looking for an algebraic square root of p^2+m^2 so that he could have a linear operator for E that would take the same role as the Hamiltonian in the Schroedinger equation. We will get to this operator by first taking the case where p is a scalar (we use one dimension of space and one dimension of time). Let $E = \alpha p + \beta m$ where α and β are elements of a a possibly non-commutative, associative algebra. Then

$$E^2 = \alpha^2 p^2 + \beta^2 m^2 + pm(\alpha\beta + \beta\alpha).$$

Hence we will satisfy $E^2 = p^2 + m^2$ if $\alpha^2 = \beta^2 = 1$ and $\alpha\beta + \beta\alpha = 0$. This is our familiar Clifford algebra pattern and we can use the iterant algebra generated by e and η if we wish. Then, because the quantum operator for momentum is $-i\partial/\partial x$ and the operator for energy is $i\partial/\partial t$, we have the Dirac equation

$$i\partial\psi/\partial t = -i\alpha\partial\psi/\partial x + \beta m\psi.$$

Let

$$\mathcal{O} = i\partial/\partial t + i\alpha\partial/\partial x - \beta m$$

so that the Dirac equation takes the form

$$\mathcal{O}\psi(x,t) = 0.$$

Now note that

$$\mathcal{O}e^{i(px-Et)} = (E - \alpha p - \beta m)e^{i(px-Et)}.$$

We let

$$\Delta = (E - \alpha p - \beta m)$$

and let

$$U = \Delta\beta\alpha = (E - \alpha p - \beta m)\beta\alpha = \beta\alpha E + \beta p - \alpha m,$$

then

$$U^2 = -E^2 + p^2 + m^2 = 0.$$

This nilpotent element leads to a (plane wave) solution to the Dirac equation as follows: We have shown that

$$\mathcal{O}\psi = \Delta\psi$$

for $\psi = e^{i(px-Et)}$. It then follows that

$$\mathcal{O}(\beta\alpha\Delta\beta\alpha\psi) = \Delta\beta\alpha\Delta\beta\alpha\psi = U^2\psi = 0,$$

from which it follows that

$$\psi = \beta\alpha U e^{i(px-Et)}$$

is a (plane wave) solution to the Dirac equation.

In fact, this calculation suggests that we should multiply the operator $\mathcal{O}$ by $\beta\alpha$ on the right, obtaining the operator

$$\mathcal{D} = \mathcal{O}\beta\alpha = i\beta\alpha\partial/\partial t + i\beta\partial/\partial x - \alpha m,$$

and the equivalent Dirac equation

$$\mathcal{D}\psi = 0.$$

In fact for the specific ψ above we will now have $\mathcal{D}(Ue^{i(px-Et)}) = U^2 e^{i(px-Et)} = 0$. This idea of reconfiguring the Dirac equation in relation to nilpotent algebra elements U is due to Peter Rowlands.[33] Rowlands does this in the context of quaternion algebra. Note that the solution to the Dirac equation that we have found is expressed in Clifford algebra or iterant algebra form. It can be articulated into specific vector solutions by using an iterant or matrix representation of the algebra.

We see that $U = \beta\alpha E + \beta p - \alpha m$ with $U^2 = 0$ is really the essence of this plane wave solution to the Dirac equation. This means that a natural non-commutative algebra arises directly and can be regarded as the essential information in a Fermion. It is natural to compare this algebra structure with algebra of creation and annihilation operators that occur in quantum field theory. To this end, let

$$U^\dagger = \alpha\beta E + \alpha p - \beta m.$$

Here we regard $U^\dagger$ as a formal counterpart to complex conjugation, since in the split quaternion algebra we have not yet constructed commuting square roots of negative one. We then find that with

$$A = U + U^\dagger = (\alpha + \beta)(p - m)$$

and

$$B = U - U^\dagger = 2\beta\alpha E + (\beta - \alpha)(p - m)$$

that

$$[\frac{A}{\sqrt{2}(p-m)}]^2 = 1$$

and

$$[\frac{iB}{\sqrt{2}(p+m)}]^2 = 1,$$

with i a commuting square root of negative one, giving the underlying Majorana Fermion operators for our Dirac Fermion. The operators U and $U^\dagger$ satisfy the usual commutation relations for the annihilation and creation operators for a Fermion.

It is worth noting how the Pythgorean relationship $E^2 = p^2 + m^2$ interacts here with the Clifford algebra of α and β. We have

$$U^\dagger = p\alpha + m\beta + \alpha\beta E$$

$$U = p\beta + m\alpha + \beta\alpha E$$

with

$$(U^\dagger)^2 = U^2 = 0,$$

$$U + U^\dagger = (p+m)(\alpha+\beta),$$

$$U - U^\dagger = (p-m)(\alpha-\beta) + 2E\alpha\beta.$$

This implies that

$$(U+U^\dagger)^2 = 2(p+m)^2$$

$$(U-U^\dagger)^2 = 2(p-m)^2 - 4E^2$$

$$= 2[p^2 + m^2 - 2pm - 2p^2 - 2m^2] = -2(p+m)^2.$$

From this we easily deduce that

$$UU^\dagger + U^\dagger U = 2(p+m)^2,$$

and this can be normalized to equal 1.

10.1. *Another version of U and $U^\dagger$*

We start with $\psi = e^{i(px-Et)}$ and the operators

$$\hat{E} = i\partial/\partial t$$

and

$$\hat{p} = -i\partial/\partial x$$

so that

$$\hat{E}\psi = E\psi$$

and

$$\hat{p}\psi = p\psi.$$

The Dirac operator is

$$\mathcal{O} = \hat{E} - \alpha\hat{p} - \beta m$$

and the modified Dirac operator is

$$\mathcal{D} = \mathcal{O}\beta\alpha = \beta\alpha\hat{E} + \beta\hat{p} - \alpha m,$$

so that

$$\mathcal{D}\psi = (\beta\alpha E + \beta p - \alpha m)\psi = U\psi.$$

If we let

$$\tilde{\psi} = e^{i(px+Et)}$$

(reversing time), then we have

$$\mathcal{D}\tilde{\psi} = (-\beta\alpha E + \beta p - \alpha m)\psi = U^\dagger\tilde{\psi},$$

giving a definition of $U^\dagger$ corresponding to the anti-particle for $U\psi$.

We have

$$U = \beta\alpha E + \beta p - \alpha m$$

and

$$U^\dagger = -\beta\alpha E + \beta p - \alpha m.$$

Note that here we have

$$(U+U^\dagger)^2 = (2\beta p + \alpha m)^2 = 4(p^2+m^2) = 4E^2,$$

and

$$(U-U^\dagger)^2 = -(2\beta\alpha E)^2 = -4E^2.$$

We have that

$$U^2 = (U^\dagger)^2 = 0$$

and

$$UU^\dagger + U^\dagger U = 4E^2.$$

Thus we have a direct appearance of the Fermion algebra corresponding to the Fermion plane wave solutions to the Dirac equation. Furthermore, the decomposition of $U$and $U^\dagger$ into the corresponding Majorana Fermion operators corresponds to $E^2 = p^2+m^2$. Normalizing by dividing by $2E$ we have

$$A = (\beta p + \alpha m)/E$$

and

$$B = i\beta\alpha$$

so that

$$A^2 = B^2 = 1$$

and

$$AB + BA = 0$$

then

$$U = (A+Bi)E$$

and

$$U^\dagger = (A-Bi)E,$$

showing how the Fermion operators are expressed in terms of the simpler Clifford algebra of Majorana operators (split quaternions once again).

10.2. *Writing in the Full Dirac Algebra*

We have written the Dirac equation so far in one dimension of space and one dimension of time. We give here a way to boost the formalism directly to three dimensions of space. We take an independent Clifford algebra generated by $\sigma_1, \sigma_2, \sigma_3$ with $\sigma_i^2 = 1$ for $i = 1, 2, 3$ and $\sigma_i \sigma_j = -\sigma_j \sigma_i$ for $i \neq j$. Now assume that α and β as we have used them above generate an independent Clifford algebra that commutes with the algebra of the σ_i. Replace the scalar momentum p by a 3-vector momentum $p = (p_1, p_, p_)$ and let $p \bullet \sigma = p_1\sigma_1 + p_2\sigma_2 + p_3\sigma_3$. We replace $\partial/\partial x$ with $\nabla = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_2)$ and $\partial p/\partial x$ with $\nabla \bullet p$.

We then have the following form of the Dirac equation.

$$i\partial\psi/\partial t = -i\alpha\nabla \bullet \sigma\psi + \beta m\psi.$$

Let

$$\mathcal{O} = i\partial/\partial t + i\alpha\nabla \bullet \sigma - \beta m$$

so that the Dirac equation takes the form

$$\mathcal{O}\psi(x, t) = 0.$$

In analogy to our previous discussion we let

$$\psi(x, t) = e^{i(p\bullet x - Et)}$$

and construct solutions by first applying the Dirac operator to this ψ. The two Clifford algebras interact to generalize directly the nilpotent solutions and Fermion algebra that we have detailed for one spatial dimension to this three dimensional case. To this purpose the modified Dirac operator is

$$\mathcal{D} = i\beta\alpha\partial/\partial t + \beta\nabla \bullet \sigma - \alpha m.$$

And we have that

$$\mathcal{D}\psi = U\psi$$

where

$$U = \beta\alpha E + \beta p \bullet \sigma - \alpha m.$$

We have that $U^2 = 0$ and $U\psi$ is a solution to the modified Dirac Equation, just as before. And just as before, we can articulate the structure of the Fermion operators and locate the corresponding Majorana Fermion operators. We leave these details to the reader.

10.3. *Majorana Fermions at Last*

There is more to do. We will end with a brief discussion making Dirac algebra distinct from the one generated by $\alpha, \beta, \sigma_1, \sigma_2, \sigma_3$ to obtain an equation that can have real solutions. This was the strategy that Majorana[26] followed to construct his Majorana Fermions. A real equation can have solutions that are invariant under complex conjugation and so can correspond to particles that are their own anti-particles. We will describe this Majorana algebra in terms of the split quaternions ϵ and η. For convenience we use the matrix representation given below. The reader of this paper can substitute the corresponding iterants.

$$\epsilon = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Let $\hat{\epsilon}$ and $\hat{\eta}$ generate another, independent algebra of split quaternions, commuting with the first algebra generated by ϵ and η. Then a totally real Majorana Dirac equation can be written as follows:

$$(\partial/\partial t + \hat{\eta}\eta\partial/\partial x + \epsilon\partial/\partial y + \hat{\epsilon}\eta\partial/\partial z - \hat{\epsilon}\hat{\eta}\eta m)\psi = 0.$$

To see that this is a correct Dirac equation, note that

$$\hat{E} = \alpha_x \hat{p_x} + \alpha_y \hat{p_y} + \alpha_z \hat{p_z} + \beta m$$

(Here the "hats" denote the quantum differential operators corresponding to the energy and momentum.) will satisfy

$$\hat{E}^2 = \hat{p_x}^2 + \hat{p_y}^2 + \hat{p_z}^2 + m^2$$

if the algebra generated by $\alpha_x, \alpha_y, \alpha_z, \beta$ has each generator of square one and each distinct pair of generators anti-commuting. From there we obtain the general Dirac equation by replacing $\hat{E}$ by $i\partial/\partial t$, and $\hat{p_x}$ with $-i\partial/\partial x$ (and same for y, z).

$$(i\partial/\partial t + i\alpha_x\partial/\partial x + i\alpha_y\partial/\partial y + i\alpha_z\partial/\partial y - \beta m)\psi = 0.$$

This is equivalent to

$$(\partial/\partial t + \alpha_x\partial/\partial x + \alpha_y\partial/\partial y + \alpha_z\partial/\partial y + i\beta m)\psi = 0.$$

Thus, here we take

$$\alpha_x = \hat{\eta}\eta, \alpha_y = \epsilon, \alpha_z = \hat{\epsilon}\eta, \beta = i\hat{\epsilon}\hat{\eta}\eta,$$

and observe that these elements satisfy the requirements for the Dirac algebra. Note how we have a significant interaction between the commuting square root of minus one (i) and the element $\hat{\epsilon}\hat{\eta}$ of square minus one in the split quaternions. This brings us back to our original considerations about the source of the square root of minus one. Both viewpoints combine in the element $\beta = i\hat{\epsilon}\hat{\eta}\eta$ that makes this Majorana algebra work. Since the algebra appearing in the Majorana Dirac operator is constructed entirely from two commuting copies of the split quaternions, there is no appearance of the complex numbers, and when written out in 2×2 matrices we obtain coupled real differential equations to be solved. Clearly this ending is actually a beginning of a new study of Majorana Fermions. That will begin in a sequel to the present paper.

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Representations of the Nilpotent Dirac Matrices

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A variety of mathematical representations of the nilpotent Dirac algebra is presented. Each of these can be seen as a group of order 64, with 4 terms equivalent to the units of complex algebra, 1, –1, i, –i, and the remaining 60 divisible into 12 collections of 5 basic units, each of which becomes a generator of the entire algebra. Any set of 5 such units can be used as the coefficients of the energy, momentum and mass operators or eigenvalues in the nilpotent Dirac equation or wavefunction. The squares of the sums of the eigenvalues multiplied by the coefficients are always zero, whatever version of the coefficients is used. This is relatively easy to show using purely algebraic symbols, but has not been previously demonstrated explicitly for the matrices. An example is given to illustrate how the matrix sets show the same nilpotent properties as the algebraic symbols

Keywords: Nilpotent, Dirac equation, Dirac algebra, Gamma matrices, Multivariate vector quaternions.

1. Introduction

The nilpotent method of relativistic quantum mechanics and quantum field theory introduced and developed by Rowlands in many publications is both a particularly powerful method of quantum calculation and one that is especially significant in terms of physical interpretation.[1-14] It requires only an operator to define and to characterize an entire quantum system, and makes meaningful many aspects of fundamental physics, such as fermions, bosons, baryons, vacuum, Pauli exclusion, *CPT* symmetry, and the broken symmetry between the three gauge interactions. It relies, however, on mathematical structures which, however simple in principle, are relatively little known. Different representations of these structures give different insights into the subject, and, for future progress in this area, it is convenient to find as many representations as possible for comparison.

The conventional Dirac equation

$$(\gamma^0\partial / \partial t + \gamma^1\partial / \partial x + \gamma^2\partial / \partial y + \gamma^3\partial / \partial z + im)\psi = 0 \quad (1)$$

(with $c = 1$, $\hbar = 1$) linearises the quadratic Klein-Gordon equation

$$(-\partial^2 / \partial t^2 + \partial^2 / \partial x^2 + \partial^2 / \partial y^2 + \partial^2 / \partial z^2 - m^2)\psi = 0 \quad (2)$$

by using a set of five 4 × 4 gamma matrices, each of which are anticommutative to each other. Three of these (γ^1, γ^2, γ^3, the components of a vector $\boldsymbol{\gamma}$) are square roots of the unit 4 × 4 matrix, I, and the other two (γ^0, γ^5) are square roots of –I.

The matrix γ^5 is not used in the conventional Dirac equation but is required for closure in the algebra as the product $-i\gamma^0\gamma^1\gamma^2\gamma^3$. An alternative way of describing the algebra in a closed form is to specify its five units as i, γ^0, γ^1, γ^2, γ^3 or complexified γ^0, γ^1, γ^2, γ^3. Yet another way is to use i and the original $\boldsymbol{\alpha}$ and β matrices of Dirac, where $\beta = \gamma^0$ and $\boldsymbol{\alpha}\beta = \boldsymbol{\gamma}$, or $\alpha^1\beta = \gamma^1$, $\alpha^2\beta = \gamma^2$, $\alpha^3\beta = \gamma^3$. The γ algebra can also be seen as a product of two commutative sets of 2 × 2 sigma or Pauli matrices, say $\boldsymbol{\sigma}_1$, $\boldsymbol{\sigma}_2$, $\boldsymbol{\sigma}_1$ and $\boldsymbol{\Sigma}_1$, $\boldsymbol{\Sigma}_2$, $\boldsymbol{\Sigma}_3$, with I now standing for the 2 × 2 unit matrix.

$$\mathrm{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix};\ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3a)$$

$$\Sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\ \Sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix};\ \Sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3b)$$

The full algebras of these representations and others will be worked out in sections 2 and 3.

2. The Algebraic Formulations

The key fact about the algebra is that it is composed of 64 units, which can be represented as a group, with 5 generators. The 64 units are organized into 1, –1, *i*, –*i*, or equivalent, and 12 sets of 5 units, each of which sets can be taken as the generators of the group.

1	*i*			
i**i**	***i*****j**	***i*****k**	*i****k***	***j***
j**i**	***j*****j**	***j*****k**	*i****i***	***k***
k**i**	***k*****j**	***k*****k**	*i****j***	***i***
*i***i*****i***	*i***i*****j***	*i***i*****k***	*i***k**	**j**
*i***j*****i***	*i***j*****j***	*i***j*****k***	*i***i**	**k**
*i***k*****i***	*i***k*****j***	*i***k*****k***	*i***j**	**i**
–1	–*i*			
–***i*****i**	–***i*****j**	–***i*****k**	–*i****k***	–***j***
–***j*****i**	–***j*****j**	–***j*****k**	–*i****i***	–***k***
–***k*****i**	–***k*****j**	–***k*****k**	–*i****j***	–***i***
–*i***i*****i***	–*i***i*****j***	–*i***i*****k***	–*i***k**	–**j**
–*i***j*****i***	–*i***j*****j***	–*i***j*****k***	–*i***i**	–**k**
–*i***k*****i***	–*i***k*****j***	–*i***k*****k***	–*i***j**	–**i**

Since (if we disregard sign changes) vectors become quaternions by multiplying by *i* and vice versa, we can use complexified quaternions instead of vectors:

1	*i*			
iI	***iJ***	***iK***	*i****k***	***j***
jI	***jJ***	***jK***	*i****i***	***k***
kI	***kJ***	***kK***	*i****j***	***i***
*i****Ii***	*i****Ij***	*i****Ik***	*i****K***	***J***
*i****Ji***	*i****Jj***	*i****Jk***	*i****I***	***K***
*i****Ki***	*i****Kj***	*i****kk***	*i****J***	***I***
–*1*	–*i*			
– ***iI***	–***iJ***	–***iK***	–*i****k***	–***j***
– ***jI***	–***jJ***	–***jK***	–*i****i***	–***k***
– ***kI***	–***kJ***	–***kK***	–*i****j***	–***i***
–*i****Ii***	–*i****Ij***	–*i****Ik***	–*i****K***	–***J***
–*i****Ji***	–*i****Jj***	–*i****Jk***	–*i****I***	–***K***
–*i****Ki***	–*i****Kj***	–*i****kk***	–*i****J***	–***I***

Reversing the process, we can also represent the group using double vectors:

1	*i*			
iI	**iJ**	**iK**	*i***k**	**j**
jI	**jJ**	**jK**	*i***i**	**k**
kI	**kJ**	**kK**	*i***j**	**i**
*i***Ii**	*i***Ij**	*i***Ik**	*i***K**	**J**
*i***Ji**	*i***Jj**	*i***Jk**	*i***I**	**K**
*i***Ki**	*i***Kj**	*i***kk**	*i***J**	**I**
–1	–*i*			
–**iI**	–**iJ**	–**iK**	–*i***k**	–**j**
–**jI**	–**jJ**	–**jK**	–*i***i**	–**k**
–**kI**	–**kJ**	–**kK**	–*i***j**	–**i**
–*i***Ii**	–*i***Ij**	–*i***Ik**	–*i***K**	–**J**
–*i***Ji**	–*i***Jj**	–*i***Jk**	–*i***I**	–**K**
–*i***Ki**	–*i***Kj**	–*i***kk**	–*i***J**	–**I**

3. The Matrix Formulations

The most familiar representation is in terms of 4 × 4 γ matrices. In algebraic symbolism, this becomes:

I	i			
γ^1	γ^2	γ^3	γ^0	$\gamma^0\gamma^1\gamma^2\gamma^3$
$-i\gamma^0\gamma^1$	$-i\gamma^0\gamma^2$	$-i\gamma^0\gamma^3$	$-\gamma^1\gamma^2\gamma^3$	$-i\gamma^0$
$\gamma^0\gamma^2\gamma^3$	$-\gamma^0\gamma^1\gamma^3$	$\gamma^0\gamma^1\gamma^2$	$-i\gamma^0\gamma^1\gamma^2\gamma^3$	$-i\gamma^1\gamma^2\gamma^3$
$-i\gamma^1$	$\gamma^0\gamma^1$	$i\gamma^0\gamma^2\gamma^3$	$-\gamma^1\gamma^2$	$-i\gamma^1\gamma^3$
$i\gamma^2$	$\gamma^0\gamma^2$	$-i\gamma^0\gamma^1\gamma^3$	$\gamma^2\gamma^3$	$i\gamma^1\gamma^2$
$-i\gamma^3$	$\gamma^0\gamma^3$	$i\gamma^0\gamma^1\gamma^2$	$-\gamma^1\gamma^3$	$i\gamma^2\gamma^3$
–I	$-i$			
$-\gamma^1$	$-\gamma^2$	$-\gamma^3$	$-\gamma^0$	$-\gamma^0\gamma^1\gamma^2\gamma^3$
$i\gamma^0\gamma^1$	$i\gamma^0\gamma^2$	$i\gamma^0\gamma^3$	$\gamma^1\gamma^2\gamma^3$	$i\gamma^0$
$-\gamma^0\gamma^2\gamma^3$	$\gamma^0\gamma^1\gamma^3$	$-\gamma^0\gamma^1\gamma^2$	$i\gamma^0\gamma^1\gamma^2\gamma^3$	$i\gamma^1\gamma^2\gamma^3$
$i\gamma^1$	$-\gamma^0\gamma^1$	$-i\gamma^0\gamma^2\gamma^3$	$\gamma^1\gamma^2$	$i\gamma^1\gamma^3$
$-i\gamma^2$	$-\gamma^0\gamma^2$	$i\gamma^0\gamma^1\gamma^3$	$-\gamma^2\gamma^3$	$-i\gamma^1\gamma^2$

$i\gamma^3$	$-\gamma^0\gamma^3$	$-i\gamma^0\gamma^1\gamma^2$	$\gamma^1\gamma^3$	$-i\gamma^2\gamma^3$

However, as we have previously stated, we can also represent the algebra, using $\boldsymbol{\gamma}^5$ as the fifth unit, rather than i. In this case, we have:

I	$-\gamma^0\gamma^1\gamma^2\gamma^3\gamma^5$			
γ^1	γ^2	γ^3	γ^0	$\gamma^0\gamma^1\gamma^2\gamma^3$
$\gamma^2\gamma^3\gamma^5$	$-\gamma^1\gamma^3\gamma^5$	$\gamma^1\gamma^2\gamma^5$	$-\gamma^1\gamma^2\gamma^3$	$\gamma^1\gamma^2\gamma^3\gamma^5$
$\gamma^0\gamma^2\gamma^3$	$-\gamma^0\gamma^1\gamma^3$	$\gamma^0\gamma^1\gamma^2$	γ^5	$\gamma^0\gamma^5$
$-\gamma^0\gamma^2\gamma^3\gamma^5$	$\gamma^0\gamma^1$	$\gamma^1\gamma^5$	$-\gamma^1\gamma^2$	$\gamma^0\gamma^2\gamma^5$
$\gamma^0\gamma^1\gamma^3\gamma^5$	$\gamma^0\gamma^2$	$\gamma^2\gamma^5$	$\gamma^2\gamma^3$	$\gamma^0\gamma^3\gamma^5$
$-\gamma^0\gamma^1\gamma^2\gamma^5$	$\gamma^0\gamma^3$	$\gamma^3\gamma^5$	$-\gamma^1\gamma^3$	$\gamma^0\gamma^1\gamma^5$

–I	$\gamma^0\gamma^1\gamma^2\gamma^3\gamma^5$			
$-\gamma^1$	$-\gamma^2$	$-\gamma^3$	$-\gamma^0$	$-\gamma^0\gamma^1\gamma^2\gamma^3$
$-\gamma^2\gamma^3\gamma^5$	$\gamma^1\gamma^3\gamma^5$	$-\gamma^1\gamma^2\gamma^5$	$\gamma^1\gamma^2\gamma^3$	$-\gamma^1\gamma^2\gamma^3\gamma^5$
$-\gamma^0\gamma^2\gamma^3$	$\gamma^0\gamma^1\gamma^3$	$-\gamma^0\gamma^1\gamma^2$	$-\gamma^5$	$-\gamma^0\gamma^5$
$\gamma^0\gamma^2\gamma^3\gamma^5$	$-\gamma^0\gamma^1$	$-\gamma^1\gamma^5$	$\gamma^1\gamma^2$	$-\gamma^0\gamma^2\gamma^5$
$-\gamma^0\gamma^1\gamma^3\gamma^5$	$-\gamma^0\gamma^2$	$-\gamma^2\gamma^5$	$-\gamma^2\gamma^3$	$-\gamma^0\gamma^3\gamma^5$
$\gamma^0\gamma^1\gamma^2\gamma^5$	$-\gamma^0\gamma^3$	$-\gamma^3\gamma^5$	$\gamma^1\gamma^3$	$-\gamma^0\gamma^1\gamma^5$

If we replace the Dirac 4 × 4 γ matrices by two commutative sets of 2 × 2 Pauli matrices, we obtain a representation of the form:

I	iI			
$\sigma_1\Sigma_1$	$\sigma_1\Sigma_2$	$\sigma_1\Sigma_3$	$i\mathrm{I}\sigma_3$	$\mathrm{I}\sigma_2$
$\sigma_2\Sigma_1$	$\sigma_2\Sigma_2$	$\sigma_2\Sigma_3$	$i\mathrm{I}\sigma_1$	$\mathrm{I}\sigma_3$
$\sigma_3\Sigma_1$	$\sigma_3\Sigma_2$	$\sigma_3\Sigma_3$	$i\mathrm{I}\sigma_2$	$\mathrm{I}\sigma_1$
$i\sigma_1\Sigma_1$	$i\sigma_1\Sigma_2$	$i\sigma_1\Sigma_3$	$i\mathrm{I}\Sigma_3$	$\mathrm{I}\Sigma_2$
$i\sigma_2\Sigma_1$	$i\sigma_2\Sigma_2$	$i\sigma_2\Sigma_3$	$i\mathrm{I}\Sigma_1$	$\mathrm{I}\Sigma_3$
$i\sigma_3\Sigma_1$	$i\sigma_3\Sigma_2$	$i\sigma_3\Sigma_3$	$i\mathrm{I}\Sigma_2$	$\mathrm{I}\Sigma_1$

–I	$-i$I			
$-\sigma_1\Sigma_1$	$-\sigma_1\Sigma_2$	$-\sigma_1\Sigma_3$	$-i\mathrm{I}\sigma_3$	$-\mathrm{I}\sigma_2$
$-\sigma_2\Sigma_1$	$-\sigma_2\Sigma_2$	$-\sigma_2\Sigma_3$	$-i\mathrm{I}\sigma_1$	$-\mathrm{I}\sigma_3$
$-\sigma_3\Sigma_1$	$-\sigma_3\Sigma_2$	$-\sigma_3\Sigma_3$	$-i\mathrm{I}\sigma_2$	$-\mathrm{I}\sigma_1$
$-i\sigma_1\Sigma_1$	$-i\sigma_1\Sigma_2$	$-i\sigma_1\Sigma_3$	$-i\mathrm{I}\Sigma_3$	$-\mathrm{I}\Sigma_2$
$-i\sigma_2\Sigma_1$	$-i\sigma_2\Sigma_2$	$-i\sigma_2\Sigma_3$	$-i\mathrm{I}\Sigma_1$	$-\mathrm{I}\Sigma_3$
$-i\sigma_3\Sigma_1$	$-i\sigma_3\Sigma_2$	$-i\sigma_3\Sigma_3$	$-i\mathrm{I}\Sigma_2$	$-\mathrm{I}\Sigma_1$

There is also a version using Dirac's original α and β matrices and the complex unit i, which leads to a group representation of the form:

I	i			
$\alpha^1\beta$	$\alpha^2\beta$	$\alpha^3\beta$	β	$\alpha^1\alpha^2\alpha^3$
$-i\beta\alpha^1$	$-i\beta\alpha^2$	$-i\beta\alpha^3$	$-\alpha^1\alpha^2\alpha^3$	$-i\beta$
$-\alpha^2\alpha^3\beta$	$-\alpha^1\alpha^3\beta$	$-\alpha^1\alpha^2\beta$	$-i\alpha^1\alpha^2\alpha^3$	$i\alpha^1\alpha^2\alpha^3\beta$
$-i\alpha^1$	$\beta\alpha^1$	$-i\alpha^2\alpha^3\beta$	$-\alpha^1\alpha^2$	$i\alpha^1\alpha^3$
$i\alpha^2$	$\beta\alpha^2$	$-i\alpha^1\alpha^3\beta$	$\alpha^2\alpha^3$	$i\alpha^1\alpha^2$
$-i\alpha^3$	$\beta\alpha^3$	$-i\alpha^1\alpha^2\beta$	$\alpha^1\alpha^3$	$i\alpha^2\alpha^3$

–I	$-i$			
$-\alpha^1\beta$	$-\alpha^2\beta$	$-\alpha^3\beta$	$-\beta$	$-\alpha^1\alpha^2\alpha^3$
$i\beta\alpha^1$	$i\beta\alpha^2$	$i\beta\alpha^3$	$\alpha^1\alpha^2\alpha^3$	$i\beta$
$\alpha^2\alpha^3\beta$	$\alpha^1\alpha^3\beta$	$\alpha^1\alpha^2\beta$	$i\alpha^1\alpha^2\alpha^3$	$-i\alpha^1\alpha^2\alpha^3\beta$
$i\alpha^1$	$-\beta\alpha^1$	$i\alpha^2\alpha^3\beta$	$\alpha^1\alpha^2$	$-i\alpha^1\alpha^3$
$-i\alpha^2$	$-\beta\alpha^2$	$i\alpha^1\alpha^3\beta$	$-\alpha^2\alpha^3$	$-i\alpha^1\alpha^2$
$i\alpha^3$	$-\beta\alpha^3$	$i\alpha^1\alpha^2\beta$	$-\alpha^1\alpha^3$	$-i\alpha^2\alpha^3$

Typical representations of the α and β matrices in the algebraic formulation might be $\alpha^1 = -i\mathbf{j}\boldsymbol{i}$, $\alpha^2 = -i\mathbf{j}\boldsymbol{j}$, $\alpha^3 = -i\mathbf{j}\boldsymbol{k}$, $\beta = -i\boldsymbol{k}$.

4. The Dirac Equation

The Dirac equation can be written using any of these representations. For example, the conventional form (1) can be transformed into a nilpotent form by multiplying from the left by γ^5.

$$(\gamma^0\partial / \partial t + \gamma^1\partial / \partial x + \gamma^2\partial / \partial y + \gamma^3\partial / \partial z + i\gamma^5 m)\psi = 0 \quad (4)$$

A nilpotent (in this sense) is a square root of zero, and we can show that an expression such as $(i\boldsymbol{k}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m)$ is a nilpotent because

$$(i\boldsymbol{k}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m)^2 = 0 \quad (5)$$

where we can identify equation (2) as Einstein's relativistic energy equation

$$E^2 - p^2 - m^2 = 0 \quad (6)$$

or, in its more usual form,

$$E^2 - p^2c^2 - m^2c^4 = 0. \quad (7)$$

The Dirac equation simply quantizes the nilpotent equation (2), using differentials in time and space applied to a phase factor for E and p. So (2) or (3) becomes

$$(-\boldsymbol{k}\partial / \partial t - \boldsymbol{i}\mathbf{i}\nabla + \boldsymbol{j}m)\,(i\boldsymbol{k}E + \mathbf{ip} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p.r})} = 0. \quad (8)$$

Assuming the usual four solutions, we obtain

$$(\mp\, \boldsymbol{k}\partial / \partial t \mp \boldsymbol{i}\mathbf{i}\nabla + \boldsymbol{j}m)\,(\pm\, i\boldsymbol{k}E \pm \mathbf{ip} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p.r})} = 0$$

which is equivalent to a nilpotent Dirac equation of the form

$$(\mp \boldsymbol{k}\partial / \partial t \mp \boldsymbol{i}\mathbf{i}\nabla + \boldsymbol{j}m)\psi = 0. \quad (9)$$

We can also express it in operator form as

$$(\pm\, i\boldsymbol{k}E \pm \mathbf{ip} + \boldsymbol{j}m)\,(\pm\, i\boldsymbol{k}E \pm \mathbf{ip} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p.r})} = 0. \quad (10)$$

where the operators E and $\mathbf{p}$ become $i\partial / \partial t$ and $-i\nabla$ as in the usual canonical quantization for a free particle, while potentials can be added to $i\partial / \partial t$ and $-i\nabla$, with appropriate changes in the phase factor from $e^{-i(Et - \mathbf{p.r})}$, when the particle is interacting with fields.

5. The Complete Set of Matrices

The final formulation is in terms of the 4 × 4 matrices, given explicitly. Discussions of the Dirac gamma matrices usually give only partial representations of the 4 × 4 matrices. Occasionally, they state that alternative formulations are available, but without specifying the complete set or their grouping into 12 sets of 5 which each become sources of the nilpotent formulation.

$$\mathrm{I} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \iota\mathrm{I} = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{pmatrix}$$

$$-\mathrm{I} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad -\iota\mathrm{I} =$$

$$\begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

$\Sigma_1\boldsymbol{\sigma}_1$; $\Sigma_1\boldsymbol{\sigma}_2$; $\Sigma_1\boldsymbol{\sigma}_3$; $\iota\mathrm{I}\Sigma_3$; $\mathrm{I}\Sigma_2$

$$\Sigma_1\boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \Sigma_1\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\Sigma_1\boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\iota\mathrm{I}\Sigma_3 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} \quad \mathrm{I}\Sigma_2 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$\Sigma_2\boldsymbol{\sigma}_1$; $\Sigma_2\boldsymbol{\sigma}_2$; $\Sigma_2\boldsymbol{\sigma}_3$; $\iota\mathrm{I}\Sigma_1$; $\mathrm{I}\Sigma_3$

$$\Sigma_2\boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \Sigma_2\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\Sigma_2\boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}$$

$$\iota\mathrm{I}\Sigma_1 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \mathrm{I}\Sigma_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$\Sigma_3\boldsymbol{\sigma}_1$; $\Sigma_3\boldsymbol{\sigma}_2$;$\Sigma_3\boldsymbol{\sigma}_3$;$\iota\mathrm{I}\Sigma_2$;$\mathrm{I}\Sigma_1$

$$\Sigma_3\boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \Sigma_3\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}$$

$$\Sigma_3\boldsymbol{\sigma}_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\iota \mathrm{I}\Sigma_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \mathrm{I}\Sigma_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$\iota\boldsymbol{\sigma}_1\Sigma_1$; $\iota\boldsymbol{\sigma}_1\Sigma_2$; $\iota\boldsymbol{\sigma}_1\Sigma_3$; $\iota\mathrm{I}\boldsymbol{\sigma}_3$; $\mathrm{I}\boldsymbol{\sigma}_2$

$$\iota\boldsymbol{\sigma}_1\Sigma_1 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \iota\boldsymbol{\sigma}_1\Sigma_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\iota\boldsymbol{\sigma}_1\Sigma_3 = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{pmatrix}$$

$$\iota\mathrm{I}\boldsymbol{\sigma}_3 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} \mathrm{I}\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$\iota\boldsymbol{\sigma}_2\Sigma_1$; $\iota\boldsymbol{\sigma}_2\Sigma_2$; $\iota\boldsymbol{\sigma}_2\Sigma_3$; $\iota\mathrm{I}\boldsymbol{\sigma}_1$; $\mathrm{I}\boldsymbol{\sigma}_3$

$$\iota\boldsymbol{\sigma}_2\Sigma_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \iota\boldsymbol{\sigma}_2\Sigma_2 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$\iota\boldsymbol{\sigma}_2\Sigma_3 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\iota\mathrm{I}\boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \mathrm{I}\boldsymbol{\sigma}_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$\iota\boldsymbol{\sigma}_3\Sigma_1$; $\iota\boldsymbol{\sigma}_3\Sigma_2$; $\iota\boldsymbol{\sigma}_3\Sigma_3$; $\iota\mathrm{I}\boldsymbol{\sigma}_2$; $\mathrm{I}\boldsymbol{\sigma}_1$

$$\iota\boldsymbol{\sigma}_3\Sigma_1 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix} \iota\boldsymbol{\sigma}_3\Sigma_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\iota\boldsymbol{\sigma}_3\Sigma_3 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \end{pmatrix}$$

$$\iota\mathrm{I}\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \mathrm{I}\boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$-\mathrm{I} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} -\iota\mathrm{I} = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

$-\Sigma_1\boldsymbol{\sigma}_1$; $-\Sigma_1\boldsymbol{\sigma}_2$; $-\Sigma_1\boldsymbol{\sigma}_3$; $-\iota\mathrm{I}\Sigma_3$; $-\mathrm{I}\Sigma_2$

$$-\Sigma_1\boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} -\Sigma_1\boldsymbol{\sigma}_2$$

$$\begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$-\Sigma_1\boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$-\iota \mathbf{I}\Sigma_3 = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \quad -\mathbf{I}\Sigma_2 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}$$

$-\Sigma_2\boldsymbol{\sigma}_1$; $-\Sigma_2\boldsymbol{\sigma}_2$; $-\Sigma_2\boldsymbol{\sigma}_3$; $-\iota\mathbf{I}\Sigma_1$; $-\mathbf{I}\Sigma_3$

$$-\Sigma_2\boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad -\Sigma_2\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$-\Sigma_2\boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}$$

$$-\iota\mathbf{I}\Sigma_1 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix} \quad -\mathbf{I}\Sigma_3 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$-\Sigma_3\boldsymbol{\sigma}_1$; $-\Sigma_3\boldsymbol{\sigma}_2$; $-\Sigma_3\boldsymbol{\sigma}_3$; $-\iota\mathbf{I}\Sigma_2$; $-\mathbf{I}\Sigma_1$

$$-\Sigma_3\boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad -\Sigma_3\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$$-\Sigma_3\boldsymbol{\sigma}_3 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$-\iota\mathbf{I}\Sigma_2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad -\mathbf{I}\Sigma_1 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$-\iota\boldsymbol{\sigma}_1\Sigma_1$; $-\iota\boldsymbol{\sigma}_1\Sigma_2$; $-\iota\boldsymbol{\sigma}_1\Sigma_3$; $-\iota\mathbf{I}\boldsymbol{\sigma}_3$; $-\mathbf{I}\boldsymbol{\sigma}_2$

$$-\iota\boldsymbol{\sigma}_1\Sigma_1 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$-\iota\boldsymbol{\sigma}_1\Sigma_2 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad -\iota\boldsymbol{\sigma}_1\Sigma_3 = \begin{pmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \end{pmatrix}$$

$$-\iota\mathbf{I}\boldsymbol{\sigma}_3 = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \quad -\mathbf{I}\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}$$

$-\iota\boldsymbol{\sigma}_2\Sigma_1$; $-\iota\boldsymbol{\sigma}_2\Sigma_2$; $-\iota\boldsymbol{\sigma}_2\Sigma_3$; $-\iota\mathbf{I}\boldsymbol{\sigma}_1$; $-\mathbf{I}\boldsymbol{\sigma}_3$

$$-\iota\boldsymbol{\sigma}_2\Sigma_1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$-\iota\boldsymbol{\sigma}_2\Sigma_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad -\iota\boldsymbol{\sigma}_2\Sigma_3 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$-\iota\mathbf{I}\boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix} \quad -\mathbf{I}\boldsymbol{\sigma}_3 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$-\iota\sigma_3\Sigma_1$; $-\iota\sigma_3\Sigma_2$; $-\iota\sigma_3\Sigma_3$; $-\iota\mathrm{I}\sigma_2$; $-\mathrm{I}\sigma_1$

$$-\iota\sigma_3\Sigma_1 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} -\iota\sigma_3\Sigma_2 =$$

$$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$-\iota\sigma_3\Sigma_3 = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

$$-\iota\mathrm{I}\sigma_2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} -\mathrm{I}\sigma_1 =$$

$$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

6. The Matrices as Sources of Nilpotency

The matrices have never been discussed previously as sources of nilpotency. Discussions of the Dirac gamma matrices usually give only partial representations. The nilpotency comes from two sources. The squared values, E^2, p^2, m^2, disappear in every case because the matrices have + and – terms arranged in such a way that the square of the total matrix combining energy, momentum and mass values only ever has squared terms in the form $E^2 - p^2 - m^2$, or equivalent, which automatically zero. However, the 'cross' terms, such as multiples of Em and pm, disappear only when the 4 terms based on $\pm E$ and $\pm p$, are added together, exactly as in the algebraic representation. This corresponds to the algebraic versions where the cross terms require the addition of the squares of all four solutions to be eliminated.

Let us take a simplified example to represent the methodology. We take the first set of 5 matrices, but, for convenience, take only one component of momentum, p_y, that is, assume the total momentum p is in the direction y.

$$\iota\mathrm{I}\Sigma_3 E = \begin{pmatrix} iE & 0 & 0 & 0 \\ 0 & iE & 0 & 0 \\ 0 & 0 & -iE & 0 \\ 0 & 0 & 0 & -iE \end{pmatrix}$$

$$\Sigma_1\sigma_2 p = \begin{pmatrix} 0 & 0 & 0 & -ip \\ 0 & 0 & -ip & 0 \\ 0 & ip & 0 & 0 \\ ip & 0 & 0 & 0 \end{pmatrix}$$

$$\mathrm{I}\Sigma_2 m = \begin{pmatrix} 0 & 0 & -im & 0 \\ 0 & 0 & 0 & -im \\ im & 0 & 0 & 0 \\ 0 & im & 0 & 0 \end{pmatrix}$$

The total matrix, represented by the algebraic equivalent ($\pm\iota\mathrm{I}\Sigma_3 E \pm \Sigma_1\sigma_2 p + \mathrm{I}\Sigma_2 m$) becomes the four sign variations of

$$\begin{pmatrix} iE & 0 & -im & -ip \\ 0 & iE & -ip & -im \\ im & ip & -iE & 0 \\ ip & im & 0 & -iE \end{pmatrix} = i \begin{pmatrix} E & 0 & -m & -p \\ 0 & E & -p & -m \\ m & p & -E & 0 \\ p & im & 0 & -E \end{pmatrix}$$

For nilpotency, we require

$$\begin{pmatrix} E & 0 & -m & -p \\ 0 & E & -p & -m \\ m & p & -E & 0 \\ p & im & 0 & -E \end{pmatrix} \begin{pmatrix} E & 0 & -m & -p \\ 0 & E & -p & -m \\ m & p & -E & 0 \\ p & im & 0 & -E \end{pmatrix} +$$

$$\begin{pmatrix} E & 0 & -m & p \\ 0 & E & p & -m \\ m & -p & -E & 0 \\ -p & im & 0 & -E \end{pmatrix} \begin{pmatrix} E & 0 & -m & p \\ 0 & E & p & -m \\ m & -p & -E & 0 \\ -p & im & 0 & -E \end{pmatrix} +$$

$$\begin{pmatrix} -E & 0 & -m & -p \\ 0 & -E & -p & -m \\ m & p & E & 0 \\ p & im & 0 & E \end{pmatrix} \begin{pmatrix} -E & 0 & -m & -p \\ 0 & -E & -p & -m \\ m & p & E & 0 \\ p & im & 0 & E \end{pmatrix} +$$

$$\begin{pmatrix} -E & 0 & -m & p \\ 0 & -E & -p & -m \\ m & -p & E & 0 \\ -p & im & 0 & E \end{pmatrix} \begin{pmatrix} -E & 0 & -m & p \\ 0 & -E & -p & -m \\ m & -p & E & 0 \\ -p & im & 0 & E \end{pmatrix} =$$

$$0.$$

In this case, we have

$$\begin{pmatrix} 0 & -2mp & -2mE & 0 \\ -2mp & 0 & 0 & 0 \\ 0 & 0 & 0 & -2mp \\ 0 & 0 & -2mp & 0 \end{pmatrix} +$$

$$\begin{pmatrix} 0 & 2mp & -2mE & 0 \\ 2mp & 0 & 0 & 0 \\ 0 & 0 & 0 & 2mp \\ 0 & 0 & 2mp & 0 \end{pmatrix} +$$

$$\begin{pmatrix} 0 & -2mp & 2mE & 0 \\ -2mp & 0 & 0 & 0 \\ 0 & 0 & 0 & -2mp \\ 0 & 0 & -2mp & 0 \end{pmatrix} +$$

$$\begin{pmatrix} 0 & 2mp & 2mE & 0 \\ 2mp & 0 & 0 & 0 \\ 0 & 0 & 0 & 2mp \\ 0 & 0 & 2mp & 0 \end{pmatrix} = 0.$$

The same result will be found for any choice of the momentum matrices, and for any of the 12 sets of 5.

7. Conclusion

In principle, the 64 units of the Dirac algebra can be represented in at least eight different ways: using Dirac's original α and β symbols, using a complexified version of the four γ matrices from the Dirac equation; using the γ matrices with γ^5; using a double sigma algebra; using complexified double quaternions; using vector quaternions; using double vectors; and using 64 4 × 4 matrices derived from the double sigma algebra. Each of these adds to the physical picture created by the Dirac equation. Matrices tell us that the Dirac equation needs four solutions and a 4-vector space-time. Using the algebraic methods tells us that point particles can only be created within a double space. The 4 × 4 matrices incorporated within a nilpotent equation provide details of how the nilpotency is worked out, in particular how it requires all 4 solutions at once. All of the representations show the primacy of 12 sets of 5 generators, each indicating a broken symmetry for the Dirac point particle.

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Spacetime Emergence, Inertia, and Rowlands Duality

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Quantum mechanics and general relativity have been separately and accurately confirmed experimentally. No experiments, however, have been conducted in regions where both theories simultaneously predict significant results. In order to achieve a compatibility between the two theories at small length scales a quantum geometric theory of space was introduced into field theory. The assumption that space is smooth and continuous has been abandoned in favor of a Planck-scale structure. New ideas for experimental agreement at all distance scales include emergent spacetime, induced gravity, and quantized inertia. Along with *f*(R) metric theories of gravity the newer approaches also appeal to the equivalence principle. Duality, mirror symmetry, and equivalence are sometimes used interchangeably. Duality has many unrelated philosophical and physical meanings and the term finds its origins in Euclidean geometry but also refers to morphisms in category theory. This paper will show how Rowlands notion of duality (a) eliminates the terminological confusion associated with the term, (b) provides an intuitive way for understanding the relationship between continuous gravitational fields and discrete inertial forces, and (c) offers a more foundational approach at bridging the quantum-relativistic divide.

Keywords: Newtonian Potentials, Newton-Hooke Law, Hubble Law, *f*(R) theories, de Sitter universe, Schwarzschild universe, Kottler metric, Dirac-Milne universe, Nilpotent Quantum Mechanics, Transverse Waves, Emergence: spacetime, gravity, inertia, WMAP, Gravity Probe B, Quantized Inertia, Rowlands Dualities

1. Overview

Forty years have passed since a quantized geometric theory of space was introduced into quantum field theory (QFT). The old assumption, that space is smooth and continuous, has been replaced with a discrete Planck sized scale structure. The assumption has resulted in limited theoretical success at bridging the gap between quantum mechanics (QM) and general relativity (GR). Experimental justification must always accompany the theory developments. The issue is that quantum gravitational effects only become apparent near the Planck scale which is a scale smaller in distance and larger in energy than what is currently producible at facilities hosting high-energy particle accelerators. Attempts to link gravitational forces to the actions of the quantum vacua and electromagnetic zero-point fields and examining how spacetime emerges from quantum entanglement may be a more fruitful approach at unification. A question that arises in this context is, Does spacetime curve matter? We know that in GR matter curves space. The new theories appear to be suggesting it is space that curves matter? This study will appeal to Rowlands definition of duality [1] to argue that quantized inertial spacetime emerges with induced curvature while satisfying a gauge/gravity zero-totality condition. Curvature in GR minimizes notions of mass or gravity and introduce singularities, nonlinearities, and unrenormalizable infinities. GR is understood to mean not just the mathematical formalism but also the physical assumptions which are used in deriving them. GR is a mathematical theory of space-time curvature described by equations which do not correspond to any physical principle and assumes only that gravity is an expression of the local curvature of Lorentzian space-time. There is nothing physical that is regarded as the cause of this curvature. In the works of de Sitter of Godel, however, mathematical solutions have been produced allowing spacetime to be devoid of matter. Their studies will be examined in a later section. The creation of mass appears to be a vacuum process, similar to the creation process of the Standard Model. Inertial mass is understood to be generated by an interaction of matter sources of strong, weak and electric charge within a vacuum Higgs field.

2. Spacetime Emergence

The universe is accelerating. Particle, quantum, and astro-physicists, agree on this and have joined forces with cosmologists in trying to find reasons for the acceleration. There is agreement that the energy for the expansion resides in spacetime. There are several proposals on how this happens. Sakharov in an early treatment of the spacetime action integral in GR considered the "*metrical elasticity of space* [2]." Spacetime was thought of as possessing properties of an elastic continuum and the deformations were being produced by torsion and spin. Vigier, on the other hand, suggested that inertia could be produced from the weak/strong interactions of a Dirac vacuum [3]. Dirac's real covariant ether model was the physical origin of inertial forces being produced by local interactions. Neither dark matter nor dark energy are relevant in the early models nor other contemporary vacuum field models like zero-point fields (ZPF) [4], Rowlands nilpotent quantum fields [1], or the Dirac-Milne matter-antimatter model [5]. Inertial effects are linked to properties of the quantum field and different scenarios have been proposed which produce them including (a) a deceleration in the vacuum generating a reaction force, (b) an interaction between the localized fermionic state and the unlocalized vacuum, and (c) emergent inertial mass as a property of field/matter interactions. In quantum electrodynamics (QED) vacuum polarization is treated as an electromagnetic field that produces electron–positron pairs, considered as the self-energy of the gauge boson (photon), that change the distribution of charges. The effects of vacuum polarization were observed experimentally before they were theoretically calculated and resulted in higher accuracy for measuring the energy levels of the hydrogen atom, called the Lamb shift, and the anomalous magnetic dipole moment of the electron predicted by the Dirac equation. The Dirac nilpotent structure proposed by Rowlands [1] goes far beyond a mere discussion of a quantum vacuum and related questions. The quantum vacuum becomes a background in which group-theoretic structures, a duality principle, and a zero-totality condition are proposed as foundational principles for establishing physical law. We will reconsider these ideas in the following sections. Rowlands' sharp distinction between discrete-continuous variables also uncover the important differences between equivalences and dualities.

Galileo was the first to affirm that any two observers who move at a constant speed and direction *relative* to one another will obtain *equivalent* results when conducting the same mechanical experiment. The law is codified in Newton's first law of motion. It is not called the "law of relativity" but rather the "law of inertia." The special and general theories of relativity derive their names from a generalization of the law of relative motion. It seems appropriate to call Galileo's observation about inertial motion a principle of *equivalence*, since equivalent results are obtained when taking measurements. Newton demonstrated a form of geometrical *equivalence* when relating terrestrial and celestial motions to conic sections. Maxwell proved that a mathematical *equivalence* exists between the electric field and the magnetic field and the speed of the electric/magnetic wave equals the speed of light $1/\sqrt{\mu_0\varepsilon_0}$. Einstein's assumption that there is a "*complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system*" and this has been experimentally demonstrated to an accuracy of 10^{-12}. The calculations allow us to assert that e*quivalent* results are obtained when comparing inertial and gravitational masses. Equivalences provide extensions of physical reality to geometrical shapes in higher dimensions and for correspondences between comparable mathematical formulations. Equivalence has also served as a driving force behind finding grand unified theories.

3. Equivalences and Dualities

Equivalences, mathematical or physical, are sometimes confused with dualities such as wave-particle. Waves describe a continuous phenomenon; particles are discrete physical objects. Particles are obviously not *equivalent* to waves and two theories are needed to explain each type of phenomenon. Bohr regarded the duality paradox as a fundamental law of nature. Einstein considered them as two contradictory pictures of reality; separately neither fully explains the phenomena of light, but together they do. Wave theory could not provide an explanation why electrons received energy from the electric field in discrete amounts, quanta which are called photons. Einstein was awarded the Noble Prize in 1921 for describing the particle property of light phenomenon called the photoelectric effect. There are still, however, fundamental interactions that take place between particles and waves that are not completely understood. The interaction between gravitational fields and inertial forces also needs further investigation. What is the exactly meant by duality?

Duality, in the physics community, refers to an equivalence that exists between two theories which are different descriptions of the same physical phenomenon. Unifying relativity theory with quantum mechanics has led to a correspondence anti-de Sitter/conformal field theory (AdS/CFT) correspondence. The attempt to correlate gravitational fields and gauge forces has also been referred to as a *gauge/gravity duality*. It should more accurately be called an *equivalence* since it is a strongly-coupled four-dimensional CFT gauge theory *equivalent* to a gravitational theory in five-dimensional AdS spacetime. Boundary conditions are significant in the correspondence. The quantum fields are strongly interacting, the ones in the gravitational theory are weakly interacting making them more mathematically tractable in what is called a strong-weak duality. Duality, in the Rowlands sense [4, p. 148], appears to depict a similar type of interaction but provides a clearer relationship between relativistic and quantum phenomenon. Rowlands duality has a meaning of "negation" requiring that a "zero-totality condition" be satisfied and there is no attempt to unify the properties of large/small scale phenomena but, rather, describes how particles and fields interact with each other. The idea of correspondence or equivalence is replaced with a stronger condition, viz. a requirement that interaction results in a nilpotent condition. As a nilpotent, the Dirac state vector becomes a precise expression of a fundamentally dualistic process of resulting in a zero-totality through self-interaction.

Recall that de Broglie used the equations of special relativity to show that particles can exhibit wave-like characteristics and that waves can exhibit particle-like characteristics. Two applications of de Broglie's thesis resulted in a particle description of the hydrogen atom by Heisenberg called "matrix mechanics" and a wave description by Schrödinger called "wave mechanics." Schrödinger later showed that these two approaches were *equivalent* mathematically. Dirac combined the two approaches into a single mathematical framework where operators in a Hilbert space are associated with the measurable quantities used to describe a physical system. Dirac introduced the delta-function in this unification. The Heisenberg-Schrödinger model of the atom gives a view of the electron as a boundary wave around the nucleus of an atom which is closed. The boundary conditions restrict the energy to discrete values where only whole numbers are allowed, explaining why energy levels are quantized. The *equivalence* of the two mathematical approaches has possibly led to the contemporary use of the term *duality* being treated as an equivalence and, later, to the AdS/CFT correspondence being called a *gauge-gravity duality*. A quantum theory has two classical limits and the original wave-particle duality embodies this idea. In QFT one limit yields classical fields, the other limit produces classical particles.

4. Inertial Forces and Gravitational Fields

Newton proved that planets were controlled by gravitational forces, moving in an absolute space and time. Einstein argued that matter shaped space-time. But Newton never revealed his private thoughts about the origin of gravitational forces. Einstein thought that the "victory over the concept of absolute space or over that of the inertial system became possible only because the concept of the material object was gradually replaced as the fundamental concept of physics by that of the field." The equivalence principle and $E = mc^2$ gives us deep epistemological insights into the meaning of mass, inertial forces, and gravitational fields but Einstein continued to question the role that inertia played in the universe and modified his equations more than once to accommodate experimental discoveries. Over the past twenty-five years, quantum field theorists, astrophysicists, and particle physicists have been assuming that inertia emerges in a quantum vacuum, produced by spin. Gravity, as an emergent entropic force, keeps it in equilibrium.

Newtonian potentials are solutions of general relativistic field equations and bind the *curvature tensor* with the effects of the *mass* parameter. The connection between curvature and gravity becomes apparent when the classical Newtonian potential for the radial field around a point source produces the Schwarzschild solution of the Einstein-Hilbert (EH) field equations. For over two and a quarter centuries Newtonian formulations have given scientists a way to describe and manipulate forces of gravity that are caused by the interaction of material objects, even though the origin of those forces remain unknown. Since 1916, however, the concept of force has been replaced with notion of field. Massive objects shape the fabric of spacetime. Special and general relativity have done away with absolute space and absolute time. Inertial systems and inertial forces have even been relegated to a somewhat illusory status. They are called fictitious, Coriolis, d'Alembert, or pseudo mainly because they do not arise from physical interactions of material objects. The imperative of asking about or looking for the origin of forces has essentially been abandoned. Gravitational forces have become geometric fields and the correspondence is reflected in the Maxwell-like form of the gravitational field tensor and the energy-momentum tensor of the dynamical equations.

Idealized geometrical objects, rather than abstract symbols, is where most of classical physics theorems are proved. Launched in a natural Euclidean background some twenty-three centuries ago, Newton utilized triangles and parallelograms to prove many of his theorems, privately reserving the fluxions to validate his conclusions. Physicists today are comfortable with curved non-Euclidean geometries used in general relativistic settings and manipulating abstract mathematical symbols, devoid of reality, to draw conclusions. Forces have been replaced with curvature of spacetime. Logical rules behind symbol manipulation is a requirement for drawing inferences about the physical world. The physics places restrictions on the mathematical symbols. It has been called an unreasonably effectiveness that operates in both directions, an entanglement between mathematics and physics. The search for the origin of gravitational forces has been abandoned but it has emerged in a new form. Physicists are now searching for the origin of inertia and ask: Where do inertial reaction forces which oppose the action of acceleration come from? Why does matter resist acceleration? The question exists even after the detection of the Higgs particle. Is inertia truly an intrinsic property of matter? Physicists no longer search for the origin of gravitational *forces* but the search for the origin of inertia is in full acceleration mode; even though it is labelled fictitious.

Space scientists have applied Newton's second law with such precision that men have landed on the moon and satellites have been placed in orbit around Jupiter. Earth bound satellites calculate GPS time using Einstein's field equations. Classical mechanics and general relativity have been separately confirmed in quite dramatic ways but the question of origins remains. We know that inertia acts in a direction which opposes gravity, but it is still a mysterious notion. Inertia is absent in an inertial frame of reference but acts on all masses whose motions can only be described in non-inertial (spinning or rotating) reference frames. Inertial properties of planets, the de Sitter geodetic effect, and the Lense-Thirring dragging effect of inertial frames around rotating bodies have been studied in satellite-based experiments. They will be discussed in the next section. In order to solve the mystery of inertia and the accelerating expansion of the universe discovered in 1998 astrophysicists have suggested a new kind of matter which initiates the acceleration. The dark forms of matter and energy have introduced other problems that are as yet unresolved, including the cosmological constant problem, the coincidence problem, and value of the equation of state.

Newton regarded reaction forces as being an internal property of all inertial matter. A similar inertial force, vacuum reaction in the elementary components of matter, occurs in the fermions which are made up of quarks and electrons. The effects of the vacuum energy have been experimentally observed in spontaneous photon emission, the Lamb shift, and Casimir effect. The cosmological constant yields an upper limit of the vacuum energy of free space to be 10^{-9} joules per cubic meter. In quantum electrodynamics (QED) and stochastic electrodynamics (SED) the Lorentz covariance calculations shows that the constant has an exceedingly higher value of 10^{+113} joules per cubic meter. There is a large unexplained discrepancy between the two theories.

Cosmological models of the origin of the universe accommodate features of gravitation, inertia, as well as dark matter and energy. The theory must also satisfy an assumption called the "Cosmological Principle (CP)." The current non-rotational version of this theory is called "Lambda cold dark matter (ΛCDM)." This model of the universe requires a cosmological constant, Λ, in the equations of general relativity to account for an accelerating expansion of the cosmos. Dark energy and dark matter were introduced into the theory to account for observed characteristics of the universe, such as (a) the distribution of the galaxies, (b) the cosmic microwave background (CMB), and (c) the abundance of hydrogen, helium, deuterium, and lithium. The ΛCDM model has been extended by adding cosmological inflation, quintessence, and other speculative notions.

There are alternative models of the which universe challenge the assumptions of the standard ΛCDM model including (a) theories of large-scale variations in matter density, (b) extended and modified *f*(R) gravity [6], and (c) non-relativistic models which revise classical Newtonian laws. All metric theories of gravity describe gravity as a manifestation of curved spacetime. This study will look at some relevant features of metric theories that resemble Newtonian potentials. The three *f*(R) metrics examined in this paper are Schwarzschild, de Sitter, Kottler. The *f*(R) approach to investigating questions relating to the accelerating universe requires a modification to the action of gravity. An extension to Einstein's field equations in general relativity relaxes the hypothesis that the EH action, $\int d^4x\sqrt{-g}\ R$, for the gravitational field is strictly linear in the Ricci curvature scalar. The modified and extended gravity theories come under the generic name "*f*(R) theories" [7] and are generalizations of the Lagrangian in the EH action $\int d^4x\sqrt{-g}\, f(R)$, where the Ricci scalar, R, is replaced by a function of *f*(R).

A fundamental assumption of the ΛCDM model is that CP holds true, viz. over large enough distance scales the universe is homogeneous and isotropic. The principle treats large-scale observational positions in the universe as being the same in all directions (isotropy) and also from every location (homogeneity). Observations of distant galaxies suggest that a universe must be non-static if it adheres to the cosmological principle. In 1923 Alexander Friedmann described the dynamics of such a homogeneous isotropic universe. A few years later Georges Lemaître derived the equations of an expanding universe from general relativistic equations calling it *L'Atom primitive*. It is now commonly called the Big Bang (BB) theory. When applying the cosmological principle to general relativity, however, a non-static universe emerges independent of observations of distant galaxies.

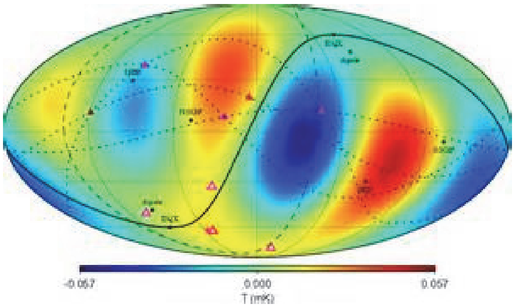

Figure 1. Quadrupole / octopole alignment from Tegmark map, Image credit: www.phys.cwru.edu/projects/mpvectors/.

The CP assumption has been challenged in recent years with a breakdown of symmetry on cosmic levels. The publication of the Wilkinson Microwave Anisotropy Probe (WMAP), a NASA Explorer mission launched in 2001, with measurements of the Cosmic Background Radiation (CBR) has revealed an unexpected property. When the map of the remnant radiation intensity is projected back onto the spherical plane of the sky and analyzed for its lowest order moments, an unexpected large quadrupole and co-aligned octupole moments were found [8], produced by a dipole resulting in the distribution of spiral galaxy handedness for redshifts <0.085. The probe shows an alignment axis pointing towards a large low-temperature structure in the background radiation [9].

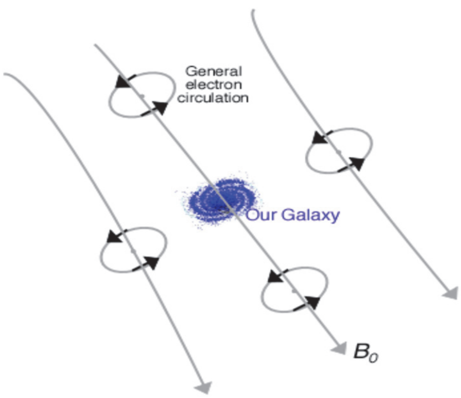

Figure 2. Image credit: www.physics.ohio-state.edu/~astro/ccapp/workshops/GLCW8/glcw8/talks/mLongo.pdf.

Another study revealed that the spins of spiral galaxies catalogued in the Sloan Digital Sky Survey display a statistically significant alignment. The observed spiral structure in galaxies prefers a handedness along the same alignment axis [10]. The asymmetry in spiral galaxy handedness is in approximate agreement with the spin alignment axis of the WMAP qudrupole/octopole axes and suggests that the special axis spans the universe. The spin asymmetry appears to be independent of redshift and suggests that it is not connected to simply local structure. The explanation given for the observed alignment of spiral galaxy spins [11] is that a large scale magnetic field exists. Ionized matter in the proto-galaxy condenses electrons and protons move in cyclotron orbits about the field lines, but in opposite senses. The associated energy loss with synchrotron radiation damps the angular momentum. Spiral handedness in the observed galaxies suggests that a rotational property emerged at the time of the big bang. The cosmic microwave background radiation discovered in 1965 also provides crucial evidence in favor of the model and it was the big bang theory that predicted the existence of such background radiation before it was even discovered. The model does not, however, take into account that galaxies and their emergent properties somehow acquire spin. Spin in the quantum vacuum may be the origin of inertia and it appears to give spacetime its curvature.

5. The *f*(R) Incentive in General Relativity

Answers to questions relating to the big bang singularity, flatness, and horizon problems have been proposed in general relativity, quantum field theory, particle and astrophysics. But GR and the Standard Model (SM) of particle physics are inadequate to describe the universe at extreme distances. GR as a classical theory does not work as a foundational theory if one wants to achieve a full quantum description of spacetime and gravity. Early efforts at making GR cosmologically friendly, called *f*(R) metric theories of gravity, can be traced to the Equivalence Principle (EP) [12]. Godel provided a rotating universe solution to Einstein's field equations interpreting "dust particles" as galaxies but his model exhibited no Hubble expansion. In Godel's cosmological model matter is rotating and world lines twist about each other suggesting a preferred direction, with no distinguished axis of rotation being identified. Remarkably, WMAP data has revealed such a preferred direction of galaxies and astrophysicists as well as cosmologists are suggesting that *f*(R) theories and other theories assuming the CP may need to be reexamined, if not reformulated.

In emergent *f*(R) spacetimes inertia and gravity play an important role. Arguments can be found in the literature for emergent spacetimes [13], quantized inertia [14], and emergent gravity [15] as well as extensions to de Sitter space which provide explanations on how entropy and temperature are associated with the cosmological horizon. An idea that follows from this is that emergent gravity contains an additional dark gravitational force describing the 'elastic' (Hooke-like) reaction due to the entropy displacement. We note that an idea of *force* seems to have relevance again. We will return to these ideas in a later section. Emergent properties have also entered into discussions among particle physicists. In modern QFT the symmetry between absolute and relational views of spacetime is violated as a virtual, empty, vacuum space can be inhabited by gravitational waves and also matter in the form of virtual particles. The equivalence principle and general covariance are two foundational principles of general relativity. Einstein, in 1918, tried to integrate a third metaphysical principle into GR called Mach's principle. He wanted the new theory of gravitation to "*secure the relativization of inertia*" by requiring space and time to be so tightly entangled to matter that it would be impossible for one to exist without the other. Einstein believed that matter and space were essentially the same thing. Interestingly, a solution to the equations of GR can satisfy spacetimes that contain no matter at all. Minkowski (flat) spacetime is the obvious example. De Sitter in 1916 proved that empty spacetime can also be curved. Godel, in 1949, showed that spacetimes exist whose distant nebulae rotate endlessly around the universe relative to an observer's local inertial frame. The existence of such spacetime solutions in GR means that it cannot be Machian. Data from the Gravity Probe B was examined in its relation to the geodetic effect, and it was observed that GR is "*nearly as relational as Mach might have wished* " and that "*matter and spacetime remain logically independent.*" The absolute spacetime of Newtonian mechanics is retained but "*endowed with a more flexible mathematical skeleton (the metric tensor)" meaning that "spacetime behaves relationally but exists absolutely* [16]."

The geodetic precession effect was introduced in 1916 by de Sitter. Within the background of the gravito-electromagnetic analogy it arises partly as a spin-orbit interaction between the spin of the test body, the gyroscope in the case of GP-B, and the mass-current of the earth. The term geodetic effect has two meanings: for non-spinning bodies the movement follows geodesics whereas for spinning bodies movement follow different orbits [17]. This is the analog of Thomas precession in electromagnetism. The electron experiences an induced magnetic field in its own rest frame due to the apparent motion of the nucleus. In the gravito-magnetic case, the orbiting gyroscope senses the massive earth racing around it and experiences an induced torque, causing its spin vector to precess. This spin-orbit interaction accounts for only one third of the total geodetic precession. The other two thirds arise due to ***space curvature*** alone and can be understood geometrically if flat space is modeled as a two-dimensional sheet. Removing a section of the flat spacetime sheet reveals curvature associated with the mass of the earth and has been referred to as the "missing inch."

Other approaches at extending relativistic equations of gravity to the cosmic scale are also relevant. Correct relativistic theories are mostly derived by modifying the EH action. The derivations have come from standard tests of GR in the vicinity of our Sun in which the curved geodesics have been experimentally tested and are in agreement with the Schwarzschild line element metric. The geodesics are fixed by the fact that the energy-momentum tensor is covariantly conserved. New research has followed alternative curvature paths. Rather than using second-order EH equations of motion analysis is based on higher-order curvature invariance principles [18] allowing explanations of observed galactic rotation curves without appealing to dark matter or dark energy. Similar gravity theories [19] introduce torsion into the EH-action where GR is modified to include the intrinsic angular momentum (spin) of matter. This leads to the Einstein-Cartan-Kibble-Sciama (ECKS) theory of gravity and, without invoking inflation, a universe emerges that is spatially flat, homogeneous and isotropic. We recall that it was Wilhelm de Sitter who, as early as 1911, applied a "principle of relativity" to planetary motion. De Sitter noted that "*in general relativity there is no essential difference between inertia and gravitation*" and declares but for his study it "*will be convenient to make this difference.*" De Sitter later identifies the line-element that "*produces 'no gravitation' but only 'inertia.'*" His

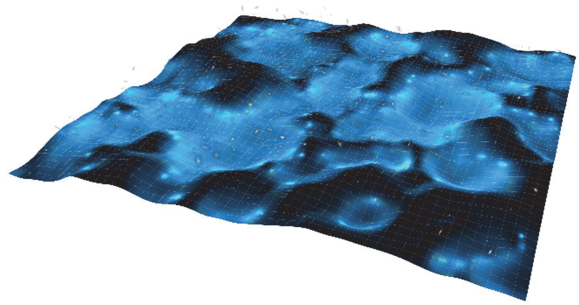

Figure 3. Simulation of galaxies on curved spacetime
Image credit: www.kenyon.edu/files/resources/highresuniv-1.png

analysis disregards gravitation and considers only the inertial field which induces curvature [20].

6. Schwarzschild, de Sitter, and Kottler Metrics

Newton investigated central force power laws of the form $F = ar^n$ and determined that the only two values for which spherically symmetrical masses can be treated as if all the mass were located at the central point are the values n = +1 and n = −2. The positive value of n yields a Hooke-like force law $F = ar$ and the negative value yields the familiar inverse square law $F = a/r^2$ which is Newton's second law of motion found in Propositions LXXVII and LXXVIII in Book I of the Principia, 1687. The two central force laws are also the only ones for which the shape of orbits in a two-body system are conic sections and are described in Proposition X. It is important to note that in the case of a force which is in a "simple ratio of the distances" the center of force is at the center of the conic section, rather than at the focal point. It was in the text of the Scholium following the discussion of spherically symmetrical bodies that Newton elaborates on this "*remarkable*" conclusion.

> *I have now explained the two principal cases of attractions; to wit, when the centripetal forces decrease as the square of the ratio of the distances, or increase in a simple ratio of the distances, causing the bodies in both cases to revolve in conic sections, and composing spherical bodies whose centripetal forces observe the same law of increase or decrease in the recess from the center as the forces from the particles themselves do; which is very remarkable.*

Newton's "increase or decrease in the recess from the center" is understood to imply that both centripetal and centrifugal forces are operating in the solar system and they satisfy geometric laws established by Kepler. In 1822 Jean Pierre Laplace, re-examined the Newton-Hooke law of attraction in "A Treatise of Celestial Mechanics" [Part I, Book II, p. 69]. Laplace writes that

> *…all the laws of attraction, in which a sphere acts on an exterior point, placed at the distance r from its centre, as if the entire mass was collected in this centre, are comprised in the general formula*

$$A\,r + B/r^2 \tag{1}$$

Rejecting the first term as not being a natural law, Laplace adds, "*If we suppose A = 0 we shall have the law of nature*" in agreement with Newton's law of gravitational attraction. Some seventy years later, in 1873, Joseph Bertrand [21] revisited the "laws of attraction" question that was initially proposed by Newton and re-formulated by Laplace. Bertrand proved that among central force potentials with bound orbits there are only two types having the property that the orbits are also closed orbits. The first type is the inverse-square central force associated with the gravitational and electrostatic potentials $V(r) = -k/r$ and the second type is the radial harmonic oscillator potential $V(r) = k\,r^2/2$, a complete full force Newton 2nd law of motion. Laplace's general formula is also called the Newton-Hooke attractive-repulsive law [22].

The first term in (1) was rejected by both Newton and Laplace as not representing a valid physical law of motion. Bertrand used Lagrange's equation to describe the motion of a particle of mass m with radius r moving in a central potential $V(r)$. He proved that the force law must have the form $F(r) = -k/r^{3-\beta^2}$.

Bertrand concluded that the only potentials that produce stable, closed, and non-circular orbits are β = 1 the inverse-square force central force and β = 2 the radial harmonic oscillator potential.

In 1915 K. Schwarzschild [23], reflecting on Einstein's work on the perihelion of Mercury, produced a solution to the Einstein field equations in the form of a spherically symmetric region of vacuum spacetime. His solution describes a gravitational field outside a spherical mass and assumes that the electric charge, angular momentum, and the cosmological constant are all zero. The geometry outside of a spherical, nonrotating, gravitating body is given by the Schwarzschild metric. Both the timelike and spacelike Schwarzschild metrics are of the form

$$f_S(R) = a_1 + 2\,b\,M / r \tag{2}$$

(a_1, b constants). The $f_S(R)$ metric provides the solution for non-rotating curved empty spacetime on a plane through the center of a spherically symmetric center of gravitational attraction. When de Sitter looked at the curvature requirements of general relativity he derived the metric

$$f_{dS}(R) = a_2 + \Lambda\,d\,r^2 / 3 \tag{3}$$

(a_2, d constants) and found a static solution to the modified field equations with zero matter saying, "*we will neglect gravitation and consider only the inertial field* [24]." It has been noted that de Sitter's *inertial field* does not prevent the occurrence of inertia relative to space but does undermine the principle of the relativity of inertia [16]. A positive cosmological constant in an expanding universe will approach a de Sitter model; the universe will become homogeneous and isotropic whatever its initial conditions may be. The question, "From whence cometh the *inertial field*?" may have the answer, "Perchance, emergent spacetime!"

Adding the de Sitter and Schwarzschild metrics yields the Kottler metric. It is the relativistic analog of Newton-Laplace full force law derived in 1918. The Kottler metric $f_K(R) = a + 2\ b\ M / r + \Lambda\ d\ r^2 / 3$ (a, d constants) is seen to have the same form as a Newton-Laplace potential (α, β, δ consts.)

$$\Phi = \alpha + \beta / r + \delta r^2. \tag{4}$$

Bertrand's theorem states that among central force potentials with bound orbits, there are only two types of central force potentials with the property that all bound orbits are also closed: an inverse-square central force such as the gravitational or electrostatic potential and the radial harmonic oscillator potential. Newton's conical orbits of spherically symmetric masses were derived from geometrical considerations and values for '*n*' were found such that bound orbits would also be closed conic sections. Birkhoff's theorem [25-26] proves a similar condition in relativistic settings showing that any spherically symmetric solution of the vacuum field equations must be static and asymptotically flat. Birkhoff showed that the Schwarzschild solution for the metric from a point particle was also valid in the *a priori* non-static case as long as spherical symmetry was maintained. This theorem was actually discovered and published two years earlier by an unknown Norwegian physicist [27]. Names and dates of valid formulas, are summarized in the following table.

potentials/ metrics	**newtonian potentials**	**general relativity *f*(R) metrics**
attractive	Newton 1687	Schwarzschild 1916
repulsive	Hooke 1660	de Sitter 1917
combined	Laplace 1822	Kottler 1918
proof	Bertrand 1873	Birkhoff [28] 1923

7. Newton-Hooke Potentials

A definitive account of the attractive-repulsive nature of gravitational forces of the Newtonian-Hooke type can be found in Chapter XV, pg. 293, of Milne's 1935 book [29] where he puts forward his kinematic structure "*erected with Newtonian and relativistic mechanics and their associated cosmologies*." Milne's model of "*a given nebular nucleus in the vicinity of a given group of nebulae … is unaccelerated and ... the equation of motion of the nebula must be*

$$0 = -\gamma \frac{M(r)}{r^2} + \frac{1}{3} c^2 \lambda r \tag{5}$$

where $\frac{1}{3}\gamma c^2$ is the repulsive acceleration which is the resultant of all the repulsive actions at a distance arising from the matter of totality present." The "matter of totality present" sounds Machian and the Hooke-like term is identified as being the "cosmical repulsion" with the associated "cosmical constant" λ. Milne calculates the value of $\lambda = \frac{3}{c^2 t^2}$ to be "*about* 10^{-54} cm^{-2}" which is "*approximately the value assigned to the 'cosmical constant' λ in Einstein's original static universe*." The present value of the cosmological constant is 1.19×10^{-54} cm^{-2}. Milne, like Einstein, initially believed that the acceleration of the cosmos must be zero. He introduces a repulsive inertial term to balance the attractive gravitational term. It is instructive to follow Milne's kinematical analysis of a static universe.

> *In our kinematic treatment of gravitational problems no constants of nature are introduced at all. Our comparison affords great insight, however, into the manner in which an empirical procedure such as Einstein's can lead to a belief in the objective existence of a quantity which a more deep-going analysis shows to be merely subjective. Thus do we dignify with the name '**laws of nature**' regularities or relations inserted into the situation by the observer – a view held and forcibly expressed in his Glifford lectures by Eddington, but a view from which his own practice, in relation to the 'cosmical constant', has somewhat deviated. If the observer chooses to believe in action at a distance, he will inevitably be led to introduce 'constants' of nature, but these are purely products of his own imagination; at least in this context.*† *Our analysis not only introduces no cosmical constant, but makes it highly improbable that any such constant has any part in the ultimate description of phenomena.*

The passage has relevance not only to cosmology but how physics is done in general. The reference to "laws of nature" reminds us of Laplace's 1822 rejection of the *Ar* term in Newton's second law as not being a '***law of nature***.' But Milne, in a footnote, identifies a rather significant role that the constant plays; it allows a transition from particle to mass measurement. It will be seen in a later section that this idea is foundational in Rowlands' treatment of mathematical variables and physical parameters that are combined (entangled) in a discrete-continuous, conserved-nonconserved, and/or conjugated/nonconjugated manner.

> † *The constant γ, of course, serves a useful purpose – it enables us to pass from particle counts to mass*

measures, just as c enables us to pass from time-measures to length measures, the value of γ fixes the gramme, the value of c the centimeter.

Milne makes an important distinction between variables that are continuous, like *time* and *mass*, and those that are discrete, like *space* and *charge*. On p. 319, he shows that the solution to the cosmological problem using "*Newtonian principles*" is derived by adding a "*term proportional to r to the Newtonian accelerations*." The Newtonian equation of motion is "*identical in form to relativistic cosmology.*"

$$\frac{Dv}{Dt} = -\gamma \frac{M(r)}{r^2} + \frac{1}{3}\lambda c^2 r$$

Milne continues saying that "*there is a complete correspondence between the relativistic universes and the Newtonian universes as modified by the λ-term…*" But Milne, along with Newton and Laplace, believed that there is "*no theoretical or observational justification for the adoption of a cosmic term λr as expressing an objective* ***law of nature***." It is now rejected for the third time as not being a law of nature.

The $f_K(R)$ metric (4) derived by Kottler for the relativistic setting can also be called the Newton-Laplace-Milne (NLM) potential.

$$\Phi = \frac{GM}{r} + \frac{\Lambda r^2}{6} \quad (6)$$

The classical Newtonian force per unit mass due to the cosmological constant is shown [30] to be $F = \frac{-GM}{r^2} + \frac{c^2 \Lambda r}{3}$ which can be derived from the NLM potential (6). This form of the full force Newtonian second law is also derived by Rowlands where, ignoring the sign, $c^2\Lambda/3 = H_0^2$ where H_0 is the Hubble constant. Euclidean geometries are approximations to curved spacetimes. Newtonian systems are approximations to relativistic spacetimes. It is these extensions that allow for global constants like the Hubble one. The second term in (6) represents a repulsive acceleration and the full force law "obeys Hooke's law, exactly as would be expected for a term equivalent to the cosmological constant [4, p. 162]." The inertial term is related to the cosmological constant and links a continuous gravitational term with a discrete inertial one. This type of linkage, or we could say entanglement, of discrete and continuous physical parameters and their associated variables leads to mathematical formulations of physical reality that seem to be more certain. Quantum entanglement surely links discrete particle behavior with continuous wave behavior. The wave-particle property of light and gravity-inertia property of mater are two striking examples of this kind of entanglement.

The Newtonian potential outside of a spherical shell can be obtained by regarding the complete mass as being concentrated at the center of the shell, the force inside the shell being zero. A general solution [31] of all potentials ϕ(r), for a point mass M(a), was shown to be a linear combination of

$$1/r,\ 1,\ r,\ e^{kr}/r,\ e^{-kr}/r,\ \sin kr/r,\ \cos kr/r \quad (7)$$

where $k > 0$. Note that the potential r^2 was either overlooked or not considered. Of the possible solutions only 1/r and e^{-kr}/r were considered to be of "*any possible physical significance.*" The former obviously represents the Newtonian potential and the latter is the Yukawa potential used to explain meson behavior. In a later mathematical treatment it is noted that "*potentials differing by a constant lead to the same force law*" and "*there are only three classes of potentials with the spherical property* [32]." The three potentials are

(i) $\phi(r) = A\, r^{-1} e^{kr} + B\, r^{-1} e^{-kr}/r + C$
(ii) $\phi(r) = A\, r^{-1} \sin kr + B\, r^{-1} \cos kr + C$
(iii) $\phi(r) = A\, r^{-1} + B\, r^2 + C$

…where A, B, and C are arbitrary constants. The constant C may be set to zero since it produces no contribution to the force of attraction." The inertial Hooke term, potential (iii), remarkably makes its reappearance for the fourth time. In the previous years, 1687, 1822, and 1935, the Hooke-like potential was rejected as not representing to a real physical. In 1983 it was finally given a full and proper mathematical, if not physical, treatment. The full force Newton second law may not have had significance for classical mechanics but it has been relevant in GR as seen above when discussing the Schwarzschild-de Sitter metric introduced by Kottler. An examination of Newtonian potentials and their application in GR has led one independent researcher to conclude that the "*gravitational field in spacetime comes about as a consequence of the electromagnetic constitutive properties of spacetime* [33]." This seems to be a euphemism for saying that spacetime emerges with curvature. The nilpotent quantum vacuum described by Rowlands also links the continuous gravitational field to the discrete weak, strong, and electric vacua.

One interpretation of vacuum is as 'the rest of the universe', the 'reaction' half of Newton's third law. This is how we can define it by reference to the 'image' charge or 'reflection' of a discrete source.

*For the discrete weak, strong and electric vacua, it means that part of the rest of the universe recognized by the appropriate charge, and it is an effective negation of that component. The total vacuum, however, is the continuous vacuum produced by the real (gravitational) component, and, for any given fermion, produces a state vector equivalent to –1(**ik**E + **ip** + **j**m), with negative energy. The combination of fermion plus total vacuum then produces a zero totality and zero state vector. 'Continuity', in this context, can only mean the absence of discrete energy levels, and it is this property which gives rise to the infinite virtual energy density and virtual energy of ½hw for every possible mode of vibration, the so-called zero-point energy. The continuous vacuum is thus constituted out of the mirror image states of all possible fermion states, and it is this continuous vacuum which makes possible the nonlocal connection required by Pauli* exclusion [34, p. 312]."

8. Rowlands' Version of Duality

Milne, as well as Newton, regarded the ability of going from discrete particles to continuous mass, and the other way around, as necessary for doing physics. Rowlands [4] has characterized four parameters (space, time, mass, charge) as being the most fundamental ones, providing a structure of mathematical duality required for physical application, all combined them into one equation, the nilpotent Dirac equation. From the generalized Dirac equation and its associated algebra Rowlands, Cullerne, and White have derived the Standard Model of particle physics and have shown how the helical structure of DNA satisfies the mathematical structure of nilpotency. In applying a zero-totality and duality condition a foundational approach to viewing all physical law [4] has been codified. Please note that all references in this section will display only page numbers being quotes taken from Rowlands' 2007 book [34].

Rowlands [pp. 436-438] identifies three distinct dualities used to define physical systems found in both classical and quantum mechanics. The dualities conserved/ nonconserved, conjugated/nonconjugated, and +/– manifest themselves as pairs of *conjugate variables.* In each case, a conserved quantity is paired with a nonconserved one. In classical mechanics conserved quantities are paired with nonconserved ones, e.g. momentum is paired with space, and energy with time. In QM, limits set by the Heisenberg uncertainty relations are the conjugated variables. Conjugation implies a Noether conservation and requires the simultaneous application the +/– duality at once. This is the case when potential, rather than kinetic energy equations are used or both action and reaction sides of Newton's third law. The real / complex duality is the relativistic one allowing transformations to be made between space and time. It also apparent in the well-known duality between electric and magnetic fields in Maxwell's equations. A third type of duality identified by Rowlands is the discrete/continuous type and it will be the focus of the next section.

The most significant feature of the Dirac nilpotent equation is that it combines the four physical variables, two of which are continuous and two discrete, into a single equation with all information represented within the four parameter group [pp. 111-136]. Rowlands derives this form of the Dirac equation, including its inherent symmetries, from "*first principles*." Space/time links an infinitely divisible parameter with an absolutely continuous one, as does the mass/charge linkage. The significance of the linkage is for taking *measurements.* The Lorentz relation requires the fixing of space and time as well as mass and charge and their interaction to each other. The symmetry between the four parameters is exact and is represented in terms of a group structure. Rowlands writes that

> *... the Lorentz invariance between **space** and **time**, and the parallel connection between **mass** and **charge** – the ultimate source of the process and units of 'measurement' as independent of the laws of physics – which forces us to link one quantity which is continuous (that is, time or mass) with one which is discrete (that is, space or charge)* [p. 223].

Mass-charge can be described by quaternions and by symmetry space-time can be treated as a 4-vector. The…

> *…symmetry between **space-time** and **mass-charge** is so exact that any reversal of role between space and time is likely also to produce a corresponding reversal of role between mass and charge. An example of this effect is seen in the prediction of negative energy or mass states by the Dirac equation, which subsequent theory has to interpret as referring to opposite charge states (or antiparticles), with the assumption that the negative energy states are all filled, a possibility which is not present in the equation itself. In fact, the Dirac theory fails to accommodate antiparticles, though Dirac himself actually predicted them, after overcoming some initial conceptual difficulties* [p. 89].

Masses are responsible for gravitational effects and are "*elements in continuous fields, while charges are* *singularities. The difficulties can only be resolved when we have a true understanding of the concept of inertia* [p. 455]." Although linking quantities that are continuous and those that are discrete allows us to do physics, mixing of these variables inevitably leads to paradoxes. In Chapter 9 "*The Resolution of Paradoxes"* reasons underlying the dilemmas and the rationale for resolving them is provided. There paradoxes extend particularly to the two *versions* of general relativity for which there exists "*a duality between fundamentally continuous and fundamentally discontinuous approaches.*" This following excerpt is included because it includes a list dualities which highlight the difficulties encountered for the past century. These dual versions of doing physics are…

> *…relevant to the relationship and relative status of the Einstein-Minkowski version of special relativity with respect to the aether-based theories associated with Poincaré and Lorentz. This problem has been one of the most contentious in recent scientific history, but we can now see that the relationship between the two relativity theories is very clearly related to the problem of duality in the physical nature of radiation. And, in fact, there are many sets of alternatives in fundamental physics which are merely different ways of making the same basic choice. The choice, as we have seen, also extends to areas of pure mathematics; for example, to that between Leibnizian differentials (based on space) and Newtonian fluxions (based on time), or to that between real numbers, representing space, which are ultimately based on a countable number of algorithmic processes, and the uncountable set of real numbers defined by Cantor, representing time. So, we have:*

particles	*waves*
relativity	*aether*
quantum mechanics	*wave mechanics*
QED	*SED*
h	*h / 2*
potential energy	*kinetic energy*
charge-like	*mass-like*
space-like	*time-like*
momentum-related	*energy-related*
spin 1 exchange	*spin ½ exchange*
boson exchange	*fermion exchange*

> *There is undoubtedly a truly continuous real distribution of energy or mass in the vacuum, but matter, on the other hand (representing 'charged' particles), is discrete. Einstein, Minkowski, Heisenberg and QED, taken to their logical conclusions, would deny the existence of real continuous mass; Lorentz, Poincaré, Schrödinger and SED, taken to their logical conclusions, would deny the existence of real discontinuous charged matter; each, of course, has to accommodate the alternative possibilities in a virtual form. Essentially, to maintain Lorentz-invariance for the purpose of measurement, we have to assume, either that continuous mass is discrete, or discontinuous charge is continuous; either way, the choice represents a deviation from fundamental 'reality'* [p. 232]."

Other dualities, such as inertia-gravity, and entropy-evolution seem to fit into Rowlands' duality schema. Most of Rowlands dualities can be treated as conjugated variables satisfying a Noether symmetry [35] and, if they satisfy a zero-totality condition, fit precisely into a generalized version of Newton's third law [36]. Discrete processes are entangled, in the sense of quantum entanglement, to continuous processes. *Entropy*, [p. 207] is a bifurcating process that is complexified and disordered has an associated dual, establishing a zero-totality condition. *Evolution*, a continuous organizing process, is a most likely candidate. Evolution is a continuous process that leads to self-aggregation, complexity and emergence in higher-order systems. On the other hand, "*entropy always increases can be taken as evidence that the rewrite system is always bifurcating* [4]." It seems that the discrete bifurcating increase of entropy is inversely entangled with evolution. Rowlands' description of dualistic phenomena, split "*the universe into two halves that are mathematically and physically, if not observationally, equivalent* [p. 144]." Rowlands does not mean equivalent in the sense of a correspondence but rather a dualistic process. He writes, "*Further dualling is possible on the same basis, but it is clear that only three fundamental principles are required to continue the dualling to infinity – opposite signs (or equivalent), the distinction between real and imaginary components, and the introduction of cyclic dimensionality – and to establish every conceivable combination of these, that is to establish every type of dualling, requires a group of 64 elements* [p. 16, 34]."

This is a truly revolutionary way of resolving the inherent problems in doing physics and addresses the foundational role that dualistic variables play in taking measurements. It appears that successful theories in physics require the inclusion of discrete and continuous phenomena and/or variables to have a complete description of the phenomena under investigation. The wave-particle duality theory of light exhibits the same dualistic requirement, a discrete particle description requires a linkage to a continuous wave phenomenon.

The conservation of energy, likewise, combines discrete potential energy with continuous kinetic energy. Rowlands explains that…

> *… the kinetic energy relation is used when we consider a particle as an object in itself, described by a rest mass m_0, undergoing a continuous change. The potential energy relation is used when we consider a particle within its 'environment', with 'relativistic mass', in an equilibrium state requiring a discrete transition for any change. We can consider the kinetic energy relation to be concerned with the action side of Newton's third law, while the potential energy relation concerns both action and reaction. Because of the necessary relation between them, each of these approaches is a proper and complete expression of the conservation of energy* [p. 174].

The field equations of GR directly incorporate inertial effects while preserving the classical structure of gravity itself, i.e. it is a linkage of classical gravity and relativistic inertia. Newton's second law includes the attractive continuous inverse-square gravitational term but the four-time rejected Hooke-like term, *Ar*, representing the discrete repulsive inertial variable, if added, will produce a more complete dual-type theory of the type that Rowlands has created in the nilpotent equation.

A Newtonian result is not easily achieved in inertial cases, spinning bodies are non-inertial frames, and are problematic outside the boundaries of Newtonian laws. Rowlands resolves this dilemma saying that…

> *…although Newtonian theory is defined only for inertial frames, all experimental observations use frames which are noninertial. The way round this, of course, is to assume that the noninertial frames are actually inertial, and then add on purely fictitious centrifugal and Coriolis forces to accommodate the inertial effects. In the present case, then, there is no reason why, if we assume Newtonian gravitational theory to be correct, we cannot simply add on the inertial force terms needed to incorporate the aberration of space* [p. 471].

Field equations in GR describe the effect of gravity on spacetime. It appears that the gravitational aberration of space is an inertial force correction and giving spacetime its curvature. The repulsive inertial force needs to be was initially derived by Newton. If the discrete inertial Hooke term is added to Newton's second law a full proper attractive-repulsive continuous-discontinuous complete law is seen to emerge. The Kottler treatment of this type of law was introduced early in relativity studies. Rowlands explains why this has to be done.

> *Such 'aberration' effects would be entirely analogous to those produced by 'inertial forces' in classical physics, and with this interpretation we can choose either to regard the GR field equations as the full gravity plus inertia package, and work, by successive approximations, in the simplest physical system, of a spherically symmetric point source, towards the Schwarzschild solution, or we can begin with the classical gravitational potential and then add inertial effects (de Sitter solution), as required, by minimally relativistic corrections. The latter approach yields easy derivations of gravitational redshift, the gravitational deflection and time-delay of electromagnetic radiation perihelion and periastron precession, gravo-magnetic effects and gravitational waves* [p. 444].
>
> (Note: *de Sitter solution* added by the author.)

Besides GR there exists in non-relativistic quantum mechanics two versions, the Heisenberg and Schrödinger**.** The first theory is discrete and directly based on observables, the Schrödinger formalism is continuous and therefore unmeasurable. Wave-particle…

> *…duality arises from the fact that, when we mathematically combine space and time in Minkowski's 4-vector formalism, as symmetry apparently requires us to do, we have two options: we can either make time space-like (or discrete) or space time-like (or continuous). Using the discrete options, we obtain particles, special relativity and Heisenberg's quantum mechanics. Using the continuous options, we obtain waves, Lorentzian relativity and Schrödinger's wave mechanics. Heisenberg makes everything discrete, so mass becomes charge-like quanta in quantum mechanics; the parallel combination of mass and charge in the quaternion structure then produces the discrete concept of rest mass, as opposed to the continuous source of gravity. Schrödinger, by contrast, makes everything continuous, so charge becomes mass-like wavefunctions in wave mechanics. In measurement, the true situations are restored, for Heisenberg's uncertainty principle and virtual vacuum effectively reintroduce continuous mass, while the collapse of the wavefunction in Schrödinger's formulation (see 7.2, 9.3) is an effective restoration of discreteness to particle states involving 'charge'* [p. 50].

The quantum vacuum discussed in the introduction of this paper is readily understood to function within this framework. Rowlands adds that…

> *…the key driving mechanism in all Casimir calculations is that they are the result of separating out discrete objects from a continuous background, and that they only have meaning in the context of object pairs. Creating a discrete object pair at some finite separation generates a force because it creates a discrete space which is shielded from some of the modes of vacuum vibration outside this space. In principle, therefore, all interactions between discrete charged objects, and even the values of the charged coupling constants, can be seen as resulting from the existence of the rest of the universe as a vacuum state, exactly in line with renormalization and Mach's principle for the parallel case of inertial mass.*
>
> *In this interpretation, the Casimir and related effects become the way in which the discrete charged vacua manifest themselves in relation to the continuous total vacuum background; they represent the partitioning of the vacuum through the three types of charge state* [p. 320].

Rowlands' analysis of inertia as being the result of the "*interaction between discrete matter and the continuous gravitational vacuum suggests that it is gravitational inertia rather than gravity which is subject to quantization* [4, p. 163]" has profound implications for unification efforts. Support for this idea comes in the form of a new quantum mechanical model for inertial mass [37] called a "modification of inertia resulting from a Hubble-scale Casimir effect (MiHsC)" or Quantised Inertia, for short. MiHsC assumes that the Unruh radiation is subject to a Casimir effect on a Hubble-scale. The inertial mass of an object is caused by a drag from Unruh radiation. Inertia apparently seems to be reduced for low accelerations. The model predicts that the inertial mass (m_I) varies as

$$m_I = m_g(1 - \beta\pi^2c^2/|a|\Theta) \qquad (8)$$

where m_g is the gravitational mass, $\beta = 0.2$ being derived from Wien's law, c = speed of light, $\Theta = 2.7 \times 10^{26}$m the Hubble diameter. McCulloch starts with Newton's gravity laws for a star with gravitational mass mg and inertial mass m_I which orbit a galaxy with gravitational mass M and gets $F = m_I a = GMm_g\, r^2$. The inertial mass is then replaced with (8) where |a| is considered to be the average mutual acceleration of all mass in the universe. Simplifying and rearranging McCullouh gets

$$a = GM / r^2 + (2c^2 / \Theta)\, \hat{a} \qquad (9)$$

where $\hat{a} = a/|a|$, a unit acceleration vector. The acceleration term, *a*, represents the acceleration of the Pioneer spacecraft's relative to their main attractor, the Sun. Invisible baryonic (dark matter) was suggested as a possible source of the extra gravitational pull. Galaxies, as early as 1933, were observed to be have more rotational energy [38] than the calculated strength of gravitational force of visible matter had to hold them together. Quantized inertia is an attempt to explain the missing mass of the anomalous galaxy phenomena and, assuming that the zero point field in quantum mechanics and relativistic horizons interact, precludes any need for dark energy/matter. It should be noted that the second term in (9) is equivalent to the second term in (5). Milne describes the term as being*…the repulsive acceleration which is the resultant of all the repulsive actions at a distance arising from the matter of totality present.*

This is a clear reference to Mach's principle with no appeal to an arbitrary cosmological constant. The repulsive force is entirely explicable in terms of similar fictitious forces found throughout physics.

9. Discreteness/Continuity Duality

Rowlands' third duality is especially important for implications about how successful theories emerge not only in physics but in other disciplines as well. Two types of Rowlands' dualities were discussed in the last section. The third duality…

> *…can be represented in terms of the discrete /continuous, or the dimensional/nondimensional options. A classic case of the first representation is the well-known wave-particle duality, where waves represent continuous options and particles discrete ones. The reason why it is an option is that the theories apply continuity or discreteness to the entire system, instead of only to those components which are fundamentally continuous or discrete; and the balance has to be restored by allowing the possibility of the alternative option. The same applies to the Heisenberg and Schrödinger theories of quantum mechanics, which are, respectively, discrete and continuous, but which each have to incorporate some aspect of the excluded property when applied to a real physical system. The factor 2 is also an expression of the discreteness of both material particles (or charges) and the spaces between them, as opposed to the continuity of the vacuum in terms of energy. The same discreteness further implies, though more*

subtly, the concept of dimensionality, which is responsible for the noncommutativity of the momentum operator, as well as the discreteness of the division of rectangles into triangles [34, p. 438].

The mixing of real and imaginary numbers reveals whether a variable is discrete or continuous. Constant terms produce effects which are twice the changing terms; the real produce ones which are twice the imaginary, and the discrete produces ones which are twice the continuous. Examples of the first include…

…action + reaction, absorption + emission, radiation + reaction, potential v. kinetic energy, relativistic v. rest mass, uniform v. uniformly accelerated motion, and rectangles v. triangles. Examples of the second include bosons v. fermions, and space-like v. time-like systems. Examples of the third include fermion + 'environment' (Aharonov-Bohm, Berry phase, Jahn Teller, etc.), space-like v. time-like systems, particles v. waves, Heisenberg v. Schrödinger / the harmonic oscillator, quantum mechanics v stochastic electrodynamics / zero point energy; 4p v. 2p rotation, and all cases in which physical dimensionality or noncommutativity is involved [34, p. 439].

A common feature of space and time in special and general relativity, as well as particle physics, is the notion of *continuity*. Rowlands has argued that space is discrete and time is continuous. Einstein's kinematical theory, based on two physical assumptions, explained a whole series of facts in a 1905 in the paper called 'Electrodynamics of Moving Bodies.' The special theory of relativity was made precise by embodying Minkowski's spacetime theory which incorporated Einstein's assumptions about the speed of light and the laws of physics being the same in all inertial frames. Trying to include quantized gravity in unified theory gives a solid reason to require maybe space, but not time, to be discrete. The requirement in some new spacetime structure at small length scales was made tenable after the discovery of ultraviolet divergences in quantum field theory. In 1947 non-commutative operators for spacetime coordinates, which were Lorentz invariant but not translation invariant, were introduced along with a minimum length for quantizing spacetime [39]. An excellent bibliographical review with exhaustive references can be found in [40]. Minkowski united discrete space with continuous time making special relativity a viable theory which can be characterized as a Rowlands' duality condition satisfying a generalized version of Newton's third law. It is important to recall that GR was motivated by three principles: the principle of equivalence, the principle of general covariance, and Mach's principle. None of these, however, are preserved in the final theory. Rowlands equates the inertial reaction with gravitational attraction and obtaining, as a result, gravomagnetic effects, redshift, acceleration of the redshift, and the cosmic microwave background radiation (CMB).

Spacetime, as an emergent phenomenon, might also be considered as a matterless de Sitter universe having inertia, no gravity, but an inherent rotation. In 1935 P.A.M. Dirac considered the "*de-Sitter space (with no gravitational fields)*" as most suitable for his "*study of the equations of atomic physics.*" He regarded schemas which remained invariant under all transformations to be of minimal interest because they simply carried "*the space-time over into itself* [41]." Dirac's preference for a de Sitter space established "*a connexion between physics and the mathematical theory of groups.*" Rowlands has taken this idea to a formidable conclusion finding that the symmetric groups of the Dirac algebra incorporate all the discrete and continuous groups of interest in the Standard Model of physics from C_2 to E_8.

If it is true that spacetime emerges with a rotational spin in a matterless de Sitter space then it may be that rotational inertia, of the Dirac zitterbewegung type, gives *space* its curvature and, and for that (dark) matter, its inertial properties. We know that the inertia of a body changes when it emits or absorbs energy. This implies that inertia is discrete, unlike gravity which is continuous. GR can be regarded as a "*combination of classical gravity and relativistic inertia, its field equations directly incorporating inertial effects while preserving the classical structure of gravity itself* [34, p. 481]." If we add the Hooke-like inertial component to Newton's second law it becomes as valid as GR in explaining gravitational/inertial phenomena. Accepting the argument that inertia, like charge, is quantizable [4, 163] and that gravity acts instantaneously as a continuous wave we can infer the following: laws of physics seem to have more certainty when a discrete entity (parameter, variable, phenomenon, property, etc.) is paired with one that is continuous. Minkowski space, wave-particle duality, and conservation of energy are prototypes of these types of relationships. Rowlands insights and analysis of the dual versions of QM and GR leads to more coherent theories in physics, or maybe all sciences for that (dark) matter. A foundational approach in bridging the quantum-relativistic divide is needed because…

…the two major representations of quantum mechanics, like the two major physical interpretations of relativity theory offer choices between discontinuous and continuous formalisms, and the same choice is offered between quantum

mechanics itself and a stochastic theory based on a classical vacuum field. Heisenberg's representation of quantum mechanics is a discontinuous theory, with integral spin h boson exchange (equivalent to fixed potential energy) as the mechanism for interaction, & unit values of h in the commutators; while Schrödinger's formulation is continuous, with gradualistic energy exchange, just as the classical kinetic energy $mv^2/2$ *or* $p^2/2m$ *represents integration over a continuous range of energy values. In the same way, Einstein's special relativity is a* ***discontinuous theory****, with events caused by a localized exchange of particles with unit h; while the Lorentz-Poincaré aether provides an alternative model in which the emphasis is on continuity provided by the delocalised energy provided by a* ***continuous vacuum field*** [34, p. 231]." (Note: emphasis added by author.)

Combining Einstein's discontinuous special theory with Lorentz-Poincare continuous results in a more complete description of local/non-local energy phenomena.

Rowlands gauge/gravity is a true duality balancing local discrete inertial gauge forces with non-local continuous instantaneous gravitational ones satisfying Newton's third law. The relationship also defines an equivalence since the "*inertial reaction*" is numerically equated with "*undetectable gravitational attraction*" and justifies Mach's principle.

The zero-point energy becomes a kind of 'antigravity'. Energy is not transferred in gravity because the vacuum is already full of 'negative energy' states. The energy is positive, the energy states are negative, and the vacuum is full because there is no negative energy. There is, nevertheless, a discrete vacuum representation related to mass. This is the inertial component, related to the discrete rest mass, which itself originates in the fermionic or bosonic charge structure. In the Higgs mechanism, it is signalled by a nonlocal finite energy level for the weak vacuum. The inertial component may be seen as a discrete local reaction specified by $1(\pm ikE \pm ip + jm)$ *to the continuous nonlocal gravitational energy specified by* $-1(\pm ikE \pm ip + jm)$*, and this can be quantized (cf 18.8). The total zero energy of the 'universe' (its zero wavefunction) could then be said to come from the combination of a positive nilpotent (inertia, sum of charges) with a negative one (gravity).* [34, p. 318-319].

Topological fields and duality concepts play an important role in emergent spacetime. Amoroso [42] correlates Verlinde's theory of gravity [15] with Mach's principle and proposes a "*dynamical M-Theoretic anthropic multiverse explanation for the influence of gravity as an inherent 'force of coherence' in >4D brane interactions as an emergent topological field.*"

This approach is an attempt to integrate an observed duality between "Newtonian instantaneity and Einstein's relativity." Applied areas of physics, nanotechnology and spintronics, have been successful at manipulating atoms at the nanometer and quantum levels. One nanotechnology study shows curvature is induced by corrugations or periodic ripples in single-layer graphenes and two types of effective spin-orbit coupling have been generated [43]. In spintronics scientists manipulate spins of electrons using both magnetic and electrical fields and has application in the semiconductor industry. The electron field exerts a transverse force on electron spin ½ particles. The spin force is found to be perpendicular to the electric field. The spin-orbit coupling force has been used to explain zitterbewegung phenomenon in the electron wave packet and is also relevant in the Hall effect [44].

10. Conclusions

The main conclusions of this study are listed below.

- Rowlands theme on foundational principles compellingly demonstrates that formulations of theories in physics that combine continuous and discrete variables produce a more complete mathematical representation of physical reality.
- A full force Newton-Hooke attractive-repulsive law, H_0 = Hubble constant, can be written

$$F = \frac{-GMm}{r^2} + {H_0}^2 rm$$

- One way to resolve the paradox between inertia and gravity, which arises in the context of mixing discrete and continuous variables, is to say that it is discrete inertial space that curves matter rather than weak gravitational forces curving spacetime.
- CFT/AdS (gravity/gauge) is an equivalence and not a duality in the Rowlands sense.
- Inertial forces are produced when objects are rotating. Inertial waves are transverse and their vibrations are perpendicular to the direction of wave travel and their phase velocity, the movement of the *crests* and *troughs* of the wave, are *perpendicular* to direction of the propagation of energy.
- Spin/rotation in the quantum vacuum gives spacetime its inertial properties and subsequent curvature.
- Gravity is very weak and it has a negligible effect at the atomic level. Emergent spacetime spin and inertial

effects generate the transverse properties observed in electromagnetic, seismic, and oceanic waves as well as in galaxy formations.

- Spacetime curves matter. Transverse waves are moving waves that consist of oscillations occurring orthogonal to the direction of energy transfer. Ripples in a pond, vibrations in strings membranes, and solid objects, and animal locomotion are examples of transverse waves.
- Discrete inertial forces are to particles as continuous gravitational fields are to waves.
- A unified approach to the paradoxical world of discrete forces and continuous fields, can be understood in a generalized form of Newton's law action (+) reaction = Rowlands' zero-totality condition.
- There is no conclusive nor compelling experiment which demonstrates that the origin of inertia is caused by the gravitational action of matter in the universe. Gravitational waves are transverse waves, not dipole like electromagnetic waves, but rather quadrupole waves. They squeeze and stretch matter, like the Newton-Hooke potential, in mutually perpendicular directions. Gravitational waves propagate in a fixed direction and the effect on matter is perpendicular to the direction of motion.
- Rowlands' argument that gravity *"acts a dual to the combined gauge theories of the electric, strong, and weak interactions* [4, p. 148]" is an application of Newton's third law:
gauge forces (+) gravity = Z
- Inertia is an interaction between discrete matter and the continuous gravitational vacuum and satisfies a zero-totality condition.
- Quaternion charge operators ***i,j,k*** satisfy a zero-totality condition, i j + j i = 0, giving spacetime an entangled dynamic 3-D structure that is non-commutative.

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Non-Conventional Effects Induced by Energy Density in Materials An Introduction to Deformed Space-Time Reactions

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Since about three decades, Italian researchers are realising nuclear reactions in solid and liquid materials, in conditions that are considered non-suitable according to the commonly accepted theories. These results were obtained by following the suggestion of the so-called Deformed Space-Time theory. An overview of some of these experimental achievements is reported together with a short introduction to fundamental aspects of the theory. Recent results implying a breakdown of the Lorentz Local Invariance are also reported and suggestions toward a systematic use of DST-reactions are given.

Keywords: Deformed spacetime, Lorentz invariance, Neutron emission, Ultrasound irradiation

1. Introduction

Since about three decades, emissions of neutrons and nuclear particles are obtained in Italy in conditions that are considered non-suitable according to the commonly accepted theories. These results are not fortuitous but they were searched for, following the suggestion of the so-called Deformed Space-Time (DST) theory. An overview of some of these experimental achievements is reported in this paper. Finally, recent results obtained in this field are reported, together with a short introduction to fundamental aspects of the DST-theory.

2. Ultrasound Irradiation

Four cylindrical samples, with 20 cm height and 2 cm diameter, were separately irradiated with 19W-20kHz ultrasounds [1]. Two of them were steel, surface hardened by Carbon Steel, and two were Ferrite.

After about five minutes, neutron emissions were detected in all cases. They got peculiar unusual characteristics. In fact:

- They were not continuous in time, as impulsive bursts of particles were detected
- They were not isotropic in space, as the particle bursts were emitted in some particular directions
- They were not accompanied by gamma ray radiation above the background level, as it is usually observed during neutron emission (at least in the great majority of cases)
- Circular damage spots were found on the lateral surface of the samples at the end of the experiment. The spots were whitish in center, brownish in the periphery and with 2-3 mm diameter in the two samples of Steel; dark with about 1 mm diameter in the two Ferrite samples.

In the Steel samples, the elemental composition of the spots was compared with that of regions without spots by using techniques of electron microscopy with x-ray energy-dispersive analysis. Elements not detected in the matrix were found in the spots: Oxygen, Sodium, Magnesium, Aluminum, Sulphur, Chlorine, Potassium and Calcium.

Besides, the Carbon content was higher in the spots. The increased concentration of elements (very high in the case of Oxygen and Carbon) is balanced by Iron, main component of the matrix, with concentration decreased of a factor not less than 2.

A more detailed investigation was performed in the treated Ferrite samples with the aim of also investigating the internal parts [2]: two smaller samples were obtained from each cylinder by using a wire-EDM (Electric Discharge Machining) technique (fmrom Startec Ltd.); they are parallelepipeds with height corresponding to a cylinder diameter (2 cm) and bases, of 2 mm × 2 mm, corresponding to small regions of the lateral surface. One of the four bases obtained from each cylinder contained a spot, while the other three corresponded to apparently non-damaged regions.

Thanks to this cut, the region under the damage spot could be investigated and a morphology different than in the other regions was observed by optical and electron microscopy. In fact, a large number of cavities (about 1300/mm^2) were found at about 300 microns below the spot. Their size was few microns, while some larger ones had diameters up to 10 microns. They were of irregular shape and contained amorphous material, maybe an assemblage of sub-micron amorphous particles.

A massive explosion in this region appeared to be responsible of the surface spot: The Ferrite fused, the surface fractured and was expelled out, as suggested by its small thickness. From the x-ray energy-dispersion spectra, the internal cavities were found to contain material with lower concentration of Iron but richer of Carbon, Chromium and Manganese with respect to the matrix, while the surface spot was poorer of Manganese and Iron but richer, among the others, of Oxygen, Chlorine, Potassium and Copper, which are not constitutive elements of Ferrite.

The presence of Copper was more deeply investigated by Neutron Activation Analysis (NAA) techniques [3], which confirmed its occurrence in the spot region. Thanks to the higher sensitivity of this technique, small Copper traces were also found in the treated matrix. However, its isotopic ratio ^{63}Cu/^{65}Cu, which is 2.2 in nature, ranged between 0.3 and 0.6 (i.e.: an inverted ratio) in the greatest parts of the two samples, apart from a region of one sample where it was 1.7±0.2. In any case, it is a value lower than the natural ratio. As a consequence, the presence of Copper was considered related to nuclear reactions, which occurred in the region under the damage spots, rather than to diffusion of spurious elements.

The occurrence of nuclear reactions is further supported by the distribution of ^{64}Zn isotope, which is more concentrated in the damage spot. It could be produced after the chain of reactions:

$$^{63}\text{Cu} + \text{neutron} \rightarrow {}^{64}\text{Cu} \rightarrow (\text{Beta-decay})\ {}^{64}\text{Zn}$$

Neutrons used in the NAA treatment could induce this chain of nuclear transformations. However, the concentration of ^{64}Zn is higher in the damaged region. This fact supports that neutrons are produced in those nuclear reactions that also produced the damage spots. The production of these neutrons is in agreement with the neutron bursts detected during the ultrasound irradiation.

3. Mechanical Presses

The reactions produced by ultrasounds are slowed down if pressure is created at lower rate by using mechanical presses, either with a monotonic increase or with cycles made of increase and subsequent relaxation of stress and strain. This slowing down can be considered as similar to that produced in (95% enriched Uranium) fast neutron reactors with respect to an atomic bomb.

3.1. *Monotonic Increase of Pressure*

Two samples of Carrara Marble (Calcite) and two samples of green Luserna granite (Gneiss) were compressed at controlled displacement rate of the piston: 10 micron/s [4]. The formers, with a lower Iron content, underwent ductile failure and no neutron intensity higher than the background was detected all along the processes. The latter underwent a brittle failure with abrupt energy release: a neutron intensity one order of magnitude higher than the background level was detected in correspondence of their failure.

Two types of detectors were used for neutrons: Helium-3 detectors, enclosed in a polystyrene case, and bubble detectors. The latter are elastic polymer gels containing 3 small drops of superheated liquid. If neutrons hit them, characteristic gas bubbles are visible, trapped in the gel. In this case, bubbles were observed only in a limited zone of the detectors, thus indicating that anisotropic neutron beams were emitted from the Granite samples.

3.2. *Cyclic Stress-Strain*

Three cylindrical samples, 20 cm high with 2 cm diameter, were submitted to cyclic stress: two were AISI304 Steel, the other was a reference sample made of Teflon (i.e. Polytetrafluoroethylene: PTFE) [5]. Each of them was separately submitted to five cycles of increasing stress from 300N to 130kN followed by a sudden stress release to 300N. The strain rate was 1 micron/s in the first cycle, 2 micron/s in the second, 3 micron/s in the 3rd, 4 micron/s in the 4th, and 5 micron/s in the last one.

The amplified output from 3 alpha counters (ZnS-Ag scintillators), 1 neutron counter (^{3}He Proportional counter) and 1 Geiger counter with related detector for Alpha, Beta and Gamma radiation were automatically register together with the corresponding values of stress and strain in the same computer.

For each type of detector, the highest value of counts per second obtained during the treatment of Teflon was assumed as background level. This assumption is severe in general but in particular for alpha counters: in fact, in this case the highest value obtained from the three instruments was considered for all of them, without taking into account the different sensitivity.

During the treatment of the AISI samples, all the detectors but the neutron counter registered peaks of counts higher than the background level. Absence of neutron emission and occurrence of alpha emission is the suggested interpretation of these experimental results.

This suggestion is confirmed by alpha tracks observed in PADC Polycarbonate (Poly-allyl-diglycol-carbonate, also known as CR39) plate detectors. In fact, alpha tracks were only recorded in four plates that were put close to AISI samples during treatment. No alpha track was found neither in the two plates close to the Teflon reference nor in a plate registering the background signals integrated over all the measurements, far from the samples [6].

3.3. *Alpha and Neutron Emissions*

Neutron emissions were both observed from metallic samples after ultrasound irradiation [1] and from natural rock containing iron at abrupt failure under stress induced by a mechanical press [4]. On the other hand, alpha emissions were detected from metallic samples cyclically stressed without rupture in a mechanical press [5, 6].

In order to check if different emissions are produced at different concentration of energy, 60 cylindrical steel rods were studied during tension rupture tests. Their height was 1 m and the diameter was 8, 10 or 12 mm [7]. A ZnS-Ag alpha counter, a neutron counter (^{3}He Proportional counter) and a Geiger counter with related detector for Alpha, Beta and Gamma radiation were used to detect the emissions during the tests.

For every detector, the highest value of counts per second obtained from each of the sample was considered. The intensity registered from Geiger and that from alpha counter were in agreement with those obtained in the previous experiment with cyclic stress [5, 6]; the neutron detector, on his side, besides some samples with a maximum of zero or one count per second, in agreement with the previous experiment, in many cases also registered either some count per second or some tenths of counts per second.

Higher energy concentration in space and time, obtained in these fractured samples, was considered responsible for the neutron emission, thus also justifying the neutron emissions reported in correspondence of the ultrasound irradiation and of the dramatic failure observed in Granite.

In these cases, it is interesting to note that a single count of a detector can also correspond to a single burst of many particles, as they were observed to occur in these nuclear transformations.

4. Relevance of Iron

All samples where nuclear particle emissions were observed contained Iron. This fact is not fortuitous, as the importance of Iron in these anomalous nuclear reactions were previously put in evidence in sonicated water solutions. In fact, a reference sample of pure water and four water solutions 1 ppm in deionized and bi-distilled water were considered [8]:

- Lithium Chloride,
- Aluminium Chloride,
- Iron Chloride,
- Iron Nitrate.

The α, β, γ and neutron intensities were measured during cavitation induced by 20 kHz ultrasounds. At the same time, detectors of the same type measured the background intensities in a suitable separated room.

The experimental conditions were the same for the three types of ions, but neutron emissions clearly above the background level were only recorded in the last two solutions, containing Iron. In both cases, the first detections were recorded after 40–50 minutes from the beginning of the ultrasound irradiation and in no case the gamma radiation was higher than the background level. After these results, a systematic study of Iron Nitrate in 250 ml deionized and bi-distilled water was performed. 20 kHz ultrasounds of 100W or 130 W powers were used in 90-minute-lasting sessions with the following samples:

(a) Reference water – 100 W – Total Energy: 0.54 MJ
(b) Reference water – 130 W – Total Energy: 0.70 MJ
(c) Water solution of 1 ppm $Fe(NO_3)_3$ – 100W – Total Energy: 0.54 MJ
(d) Water solution of 10 ppm $Fe(NO_3)_3$ – 100W – Total Energy: 0.54 MJ
(e) Water solution of 1 ppm $Fe(NO_3)_3$ – 130W – Total Energy: 0.70 MJ
(f) Water solution of 10 ppm $Fe(NO_3)_3$ – 130W – Total Energy: 0.70 MJ

During ultrasound irradiation, alpha, beta and gamma emissions were measured by using two Geiger counters, while five bubble detectors of the Defender type (by BTI-USA) were used for neutrons. The background level was measured at the beginning of the whole set of cavitation experiments. While the ionizing radiation measured by the Geiger counters was always comparable to the background level, integrated neutron intensity above background was obtained and higher values corresponded to higher Iron content.

Furthermore, two of the neutron detectors were separated from the samples by a wall made of a neutron moderator (Carbon powder) or a neutron adsorber (Boron powder): only the other three unscreened detectors registered neutron intensity higher than the background, thus indicating that neutrons came from the solutions.

These results are very intriguing, as Iron usually is the less indicated element for nuclear reactions. In fact, fusion reactions can occur with nuclei lighter than Iron while fission reactions with heavier. Iron is characterized by the highest binding energy per nucleon, thus making energetically expensive any nuclear reaction.

On the contrary, the presence of Iron is an enhancing factor in these reactions.

5. Nucleolysis and Nucleosynthesis

The above reported results indicate that a nontraditional type of nuclear reactions can occur. In particular, being the presence of Iron an enhancing factor, the observed effects are considered prohibited. In fact, in this case either heavier nuclei are formed from Iron, while usually nuclei heavier than Iron give rise to lighter nuclei by fission reactions, or lighter nuclei are formed, while usually nuclei lighter than Iron give rise to heavier nuclei by fusion reactions. In any case, the direction of the reaction is opposite to the traditional one.

Thus, names different from fission and fusion are used to distinguish these anomalous reactions: Nucleolysis indicates the formation of lighter nuclei from heavier ones and Nucleosynthesis the formation of heavier from lighter. These names are kept also when the direction is traditional, in order to indicate the different nature of the ***reaction***.

5.1. *Nuclear Metamorphosis in Mercury*

A confirmation of Nucleolysis and Nucleosynthesis reactions was obtained in a recent experiment with one mole of Mercury [9]. After a 180s treatment, solid material was obtained from liquid Mercury.

The composition of this material was determined by using different techniques in the framework of a collaboration among various Italian laboratories: ICP-OES (Inductively Coupled Plasma Optical Emission Spectroscopy- Perkin Elmer Optical emission spectrometer OPTIMA 8300); three different instruments for ICP-MS (Inductively Coupled Plasma Mass Spectroscopy - Thermo Fisher X series II, Perkin Elmer OPTIMA 2100 DV, Agilent 7005C Octopole Reaction System); four electron microscopes with EDS (Energy Dispersive Spectroscopy - FEI Quanta 200, LEO 1450 VP LAIKA Cambridge, FEI x 120 with EDAX ECON 4 EDS and SEM Cambridge Stereoscan 250 MK3); XRF (X-ray Fluorescence - Spectro x-Lab2000); INAA (Instrumental Neutron Activation Analysis - Gamma detector with High Purity Ge by ORTEC, HPGe ORTEC at Nuclear Reactror TRIGA Mark II- upgrade, ENEA-Casaccia, Italy).

Samples of non-treated Mercury taken from the original pool and parts of the experimental apparatus either in contact or close to the Mercury were also studied. All the elements found in these last samples were excluded from the list of reaction products, in order to evaluate a lower limit for the number of elements produced during the process. This cut is very drastic, as one of them is excluded on the basis of the atomic number Z, even if the isotope detected after transformation is different.

After this cut, the elements or isotopes identified by not less than two different techniques were considered as products of transformation: ^{7}Li, ^{47}Ti, ^{58}Ni, ^{69}Ga, ^{82}Se, ^{79}Br, ^{124}Sn, ^{177}Hf, ^{197}Au and ^{232}Th. The request of two different techniques is very severe since an element or isotope is excluded even if detected by two different instruments ruled by different teams in different laboratories if the same experimental technique was used. Particular relevance was also given to the presence of ^{238}U, although detected by one sole technique (ICP-OES).

These results are displayed in Fig. 1, where the mean energy per nucleon of the isotopes is reported as a function of their mass number. A rectangular framework marks the position of Mercury, the starting element of the nucleosynthesis and nucleolysis reactions in the present experiment, and three further isotopes, which are not reaction products but reference points from the energetic point of view: ^{56}Fe, the isotope with the highest binding energy, toward which the traditional fusion and fission reactions are directed but which is not an ending point for nucleosynthesis and nucleolysis reactions; ^{2}H, the nucleus with the lowest binding energy (no nucleons are bound together in the lighter ^{1}H); ^{4}He, which is characterised by a peculiar local maximum of energy (the so-called "^{4}He knee ").

From Fig. 1, it is evident that both heavier and lighter nuclei are produced from Mercury and that nucleolysis took place both toward nuclei lighter and heavier than Iron. These results were obtained reproducing in Mercury the conditions of **Lorentz symmetry breakdown**, already assumed to occur in a previous experiment [10-12].

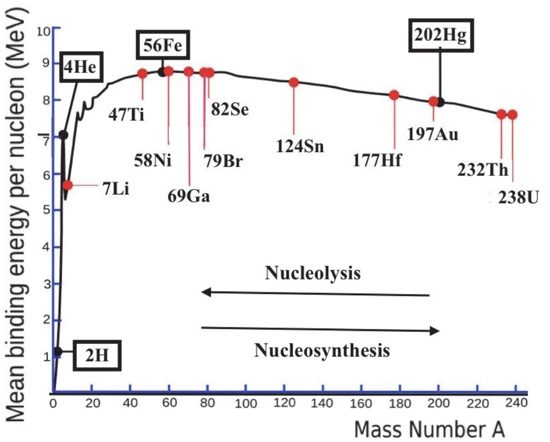

Figure 1. Isotopes considered as secure products of nuclear transformations. Rectangular frames: Mercury and three important energetic references: ^{2}H, ^{4}He and ^{56}Fe.

To this aim, the experimental conditions assumed to induce the breakdown were realised ten times during one year, between 2012 and 2013, by using a Startec patent; while three times non-optimized conditions were used. No visible amount of solid material was obtained in the three cases when the optimal conditions were not used.

Thus, the difference was attributed to attained or not attained conditions of Lorentz Symmetry Breakdown.

5.2. *Lorentz Symmetry Breakdown*

Lorentz Symmetry breakdown was observed in a previous experiment [10-12], where the spatial distribution of neutron energy, produced after ultrasound irradiation, was investigated. A steel bar, surrounded by 2 cm thick calorimeter made of Teflon (PTFE), was irradiated by 20 kHz ultrasounds. Sixteen neutron detectors of PADC Polycarbonate were set all around the PTFE casing.

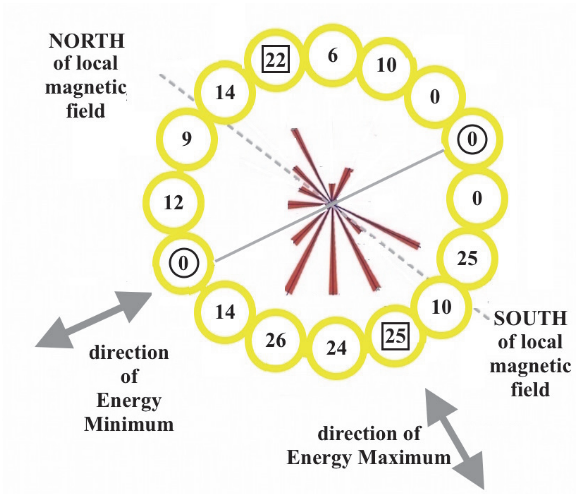

Figure 2. Neutron integrated intensity (in microSievert) registered by the sixteen PADC detectors around the Teflon calorimeter, which surrounded the ultrasound-irradiated bar of Steel.

After a 180s irradiation, the steel bar temperature raised from 20°C to 92°C. The energy so transferred to the bar was assessed to be about 6 kJ (its mass being 180g and the specific heat about 0.5 J °K/g). A much higher energy, about 60kJ, was deposited at higher temperature into the calorimeter. In fact, the PTFE melted and was locally carbonized.

Neutron emissions, as detected by PADC detectors, were assumed responsible of this latter energy transfer. In fact, no neutron intensity was measured in correspondence of the non-carbonized zones.

The spatial distribution of energy resulted quite asymmetric, as shown in Fig. 2. The direction of maximum emitted intensity (corresponding to 25+22 microSievert) is perpendicular to direction of the lowest intensity (corresponding to 0+0 microSievert). These two directions are put in comparison with that of Earth's magnetic field in Fig. 2.

6. The Deformed Space-Time Theory

The above reported phenomena can find an explanation in the framework of the Deformed Space-Time (DST) theory [13, 14]. It proposes that every interaction gets a local metric, which is different for the different interactions.

6.1. *Fundamentals of DST-Theory*

According to DST-theory, a 4D variety is attached at each point x $[x = (x_0,x_1,x_2,x_3) = (ct,x,y,z)]$ of the flat Minkowski 4D-Space Time. The corresponding generalized interval ds^2 is given by:

$$ds^2 \equiv b^2_0\, c^2\, dt^2 - b^2_1\, dx_1{}^2 - b^2_2\, dx_2{}^2 - b^2_3\, dx_3{}^2$$

The metric parameters b_i depend on the energy E of the process and on the interaction. Thus, the energy of a process, which in general is affected by the space-time characteristics, affects in turn the space-time properties. This fact can be considered a generalization of the Finzi Principle of Solidarity for General Relativity [15]: "Not only space-time properties affect phenomena, but also phenomena reciprocally affect space-time properties".

The metric parameters were obtained by fitting experimental data:

- lifetimes of the leptonic decay of the meson K_{0S} for weak interaction [16-17]:
. $b^2_0 = 1$;
. $b^2_1 = b^2_2 = b^2_3 = b^2(E)$, which is constant and equal to 1 if Energy is higher than threshold energy for weak interactions: $E_{0\,weak} = 80.4 \pm 0.2$ GeV; at lower energy, $b^2(E) = (E/\,E_{0\,weak})^{1/3}$
- pion pair production (UA1 at CERN-1984) for strong interaction [18]:
. $b^2_1 = 2/25$
. $b^2_2 = 4/25$
. $b^2_0 = b^2_3 = b^2(E)$, which is constant and equal to 1 if Energy is lower than threshold energy for strong interactions: $E_{0\,strong} = 367.5 \pm 0.4$ GeV; at higher energy, $b^2(E)= (E/\,E_{0\,strong})^2$
- Superluminar Electro-magnetic waves in conducting waveguides for electromagnetic interaction [19-23]:
. $b^2_0 = 1$;
. $b^2_1 = b^2_2 = b^2_3 = b^2(E)$, which is constant and equal to 1 if Energy is higher than threshold energy for electro-magnetic interactions: $E_{0\,em} = 4.5 \pm 0.2$ micro-elettronVolt; at lower energy, $b^2(E) = (E/\,E_{0em})^{1/3}$
- rate of clocks at different height for gravitational interaction [24]:
. b^2_0 (E) is constant and equal to 1 if Energy is lower than threshold energy for gravitational interactions: $E_{0\,grav} = 20.2 \pm 0.1$ microelettronVolt; at higher energy, $b^2_0 = [1+ (E/\,E_{0\,em})^2]\,/4$
. b^2_1, b^2_2 and b^2_3 could not be derived from the experimental data in this case.

6.2. *Thresholds*

The above reported energy thresholds were mainly obtained from data of elementary particles. The problem arose on their use in case of materials, which are more complex systems. In order to answer this question, it was noted [25] that both the cavities found in the solid ferrite bars, where nuclear reactions are supposed to occur (see §2), and the bubbles created during the cavitation phenomena in liquids, when neutron emission were observed (see §4), are characterized by a size of some microns. Thus, some microns were assumed as the typical size of micro-reactors where DST-reactions can take place and their volume was assumed to host the relevant energy. Following this way, thresholds of energy density, rather than thresholds of energy, were evaluated.

In the case of cyclically stressed AISI304 Steel samples (§3.2.), the average energy accumulated per unit volume when alpha emissions occurred were evaluated [25] and their values were close to the expected threshold density of weak and strong nuclear interactions: 1.0×10^8 J/m^3 and 4.7×10^8 J/m^3, respectively. In fact, values of 5.2×10^8, 1.5×10^9, 4.1×10^7, 3.1×10^8 and 3.3×10^8 were found in a sample and 6.3×10^8, 1.4×10^7, 6.4×10^7, 9.2×10^7, 4.6×10^7, 6.3×10^7, 8.6×10^6 and 2.8×10^8 in the other (± 20%). Thus, emissions can either be of leptonic or hadronic nature.

When a threshold is reached, energy is needed for the space–time deformation to occur. This energy is taken from the expected gamma radiation. Thus, no gamma emission is observed in correspondence of neutron (or alpha) emissions in DST-reactions. Once that Space-Time is locally deformed, other nuclear particles are free to escape, thus producing an intense burst, which is not isotropic due to the asymmetry of the microscopic region where the reaction occurs.

6.3. *DST-Theory as Unifying Vision of Non-Conventional Experimental Results*

In the last years, new names were coined to indicate that non-predicted nuclear reactions were observed: Cold

Fusion (CF), Low Energy Nuclear Reactions (LENRs), Condensed Matter Nuclear Science (CMNS), Piezo-Nuclear Reactions (PNR), Energy Catalyzer (E-cat).

In a recent paper [26], the DST-theory was proposed as a unifying vision to describe all these effects. In fact, they share the following characteristics, which are consequence of DST-reactions:

1. Presence of an energy threshold. In complex systems, it can be evaluated as a threshold in energy density. In practice, a delay time can be observed between the beginning of energy supply and the reaction start, so that a critical density is reached.
2. Change of atomic weight. Nuclei are found that were not present in the initial material.
3. No gamma radiation. The energy of gamma radiation is used to deform space-time.
4. Emission of nuclear particles in intense beams of very short life span.
5. Anisotropy. It is a consequence of the anisotropic metric governing the interaction.

In particular, the absence of gamma radiation is generally considered a symptom to state that no nuclear reaction occurred. On the contrary, according to DST-theory it can be a further confirmation that nuclear reactions occurred with energy lower than expected. On the other hand, absence of detected neutrons in the nuclear reactions of E-cat was considered a conceivable result for the authors [27]. On the contrary, intense, anisotropic and short living neutron beams could be emitted. In fact, intense beams of short duration are treated as impulsive noise by the whole detecting system. In particular, the neutron detectors are not usually optimised to record this kind of signal while the related electronics usually eliminate them as an undesirable noise: in order to overcome these problems, neutron detectors with a large volume and coated by boron were also used in DST experiments of §4.

Furthermore, traditional detectors lead to underestimate the neutron intensity, because one intense pulse is only recorded as one particle.

Finally, the anisotropic direction of the emitted beams can also play its role, when the receiving detectors are positioned in correspondence of some particular direction, where eventually beams are not directed.

6.4. *Next Steps*

In order to get a systematic use of DST-reactions, either a very large number of tests are performed starting from different elements and their isotopes, thus obtaining a complete map of all the reactions, or general laws are found able to predict the possible, or at least the most probable, reactions products. The latter way is more appealing, as it implies the knowledge of some underlying driving mechanism.

As a first attempt toward this direction, the reaction products obtained in carbon steel after ultrasound irradiation (see §2) were examined [28]. In particular, the concentration of elements having number Z less than 24 was found to increase close to the reaction zones. The number N of neutrons of these elements, considering the isotopes with higher natural concentration, was reported as a function of the number of protons Z.

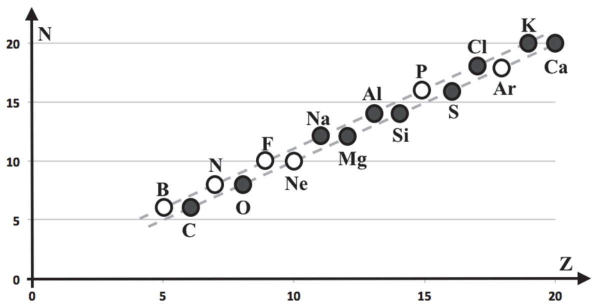

Figure 3. Reaction products in carbon steel after ultrasound irradiation: number N of neutrons vs. number Z of protons. Filled circle: Detected reaction products; Empty circle: Proposed reaction products.

A straight line then connected each atom of this plot to each other. The most of these segments had a slope of 45 degrees, corresponding to a difference of neutrons in the nucleus equal to the difference of protons, and they did not lie on the same straight line, but rather in two parallels. Other elements were proposed to be produced but not detected: Argon and Neon, being noble gas, could be formed but volatilized without interacting with other atoms; Nitrogen may have volatilized (differently from Oxygen, which may have formed oxides); Boron was present in the instrumentation and thus was excluded in the count. Only Phosphorus and Fluorine can be considered, at the moment, as missing elements with regard to the plot of Fig. 3.

The shorter distance between two nuclei of the same straight-line of Fig. 3 corresponds to two neutrons and two protons, i.e. to an alpha particle. It does not imply a sequence of alpha emissions but rather the occurrence of alpha clusters inside the parent nucleus, which breaks into smaller parts. This fact could give rise to the helium excess reported in many LENR experiments and related phenomenology, thus ruling out any hydrogen nuclear fusion at room temperature. At higher values of Z and N, an isotopic investigation could even give information on the nuclear structure and its clusters.

A further parameter introduced by DST-theory is Energy. Thus, a third axis (nucleus Energy per nucleon) can be introduced. The occurrence of energy thresholds in DST-reactions should correspond to particular surfaces in this three-dimensional plot and different nuclei can belong to the same surface with constant density of Energy. This way, transformation between nuclei that usually are forbidden can occur [29].

It is interesting to note that in any case we observe from a non-deformed (Minkoskyan) Space-Time, external to the nucleus, the consequences of reactions occurring in zones of deformed (non-Minkowskyan) space-time inside the nucleus.

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Considerations about Deformed Space-Time Neutron Spectra

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Within the general framework of studies and experiences regarding the neutrons produced under conditions of Deformed Space-time (DST), due to the violation of the Local Lorentz Invariance (LLI), some neutron energy spectra are investigated. DST-neutrons are produced by a mechanical process in which AISI 304 steel bars undergo a sonication using ultrasounds with 20 kHz and 330 W. The energy spectra of the DST-neutrons have been investigated both at low (less than 0.4 MeV) and at high (up to 4 MeV) energy by means of MICROSPEC2 Neutron Probe and also by means of a passive detector specifically designed for the detection of neutrons emitted in reactions related to the deformation of the space-time. The particular features of these DST - neutron spectra leads us to consider the hypothesis that DST- neutrons is not uniform also in energy, in a similar way already seen in time asynchrony and spatial asymmetry.

Keywords: Piezonuclear neutron detection, Neutron measurements, Neutron energy, Deformed Space-Time, Local Lorentz Invariance

1. Introduction

Emissions of nuclear particles, neutrons and alphas, from liquid or solid materials that have undergone changes in energy density induced by ultrasound pressure or other mechanical processes have been studied in past papers [1-6]. These emissions come from new nuclear reactions triggered by pressure named Piezonuclear reactions. The origin of these phenomenon may be explained and understood by means of the Deformed Space-Time (DST) theory [7-9]. The DST theory derives from the formalism of Deformed Special Relativity (DSR); a generalization of Special Relativity (SR) based on a "deformation" of Minkowski space-time, assumed to be endowed with a metric whose coefficients depend on the energy of the process considered [9]. The DSR formalism provides a geometrical description of the four fundamental interactions (electromagnetic, weak, strong and gravitational), in terms of phenomenological deformed metrics with a threshold behavior [9]. When interactions among nuclei occur in non-Minkowskian conditions the excess energy is partly absorbed by the hadronic space-time deformation, so that there is no emission of γ radiation, but, on the contrary, there is emission of neutrons characterized by a typical fingerprint [9]. Both Nuclear fission, either spontaneous or induced, and nuclear fusion, and collisions of ions (spallation neutron sources) produce both neutrons and gamma rays. These neutrons have in general very precise characteristics:

- isotropic emission, and well known directions of emission;

- known and stable fluxes and known and stable fluencies;

- known neutron spectra since the reactions that produce them are known, understood and modelled.

All of these features make them very easy to measure, by several passive and active techniques. As already stressed, several times, in our previous papers piezonuclear neutrons have not characteristics just mentioned [7, 8, 10]. In all the experiments that have been carried out so far, the neutrons detected have the following peculiar characteristics:

● they are emitted in bursts, not in a constant flux [4, 6, 9, 11];

● the directions of emission are not fixed but change in space and time;

● the neutron emission is asymmetrical, anisotropic and with no fixed timing [4, 6, 9, 11];

● no gamma emission accompanies the emission of these neutrons [4, 6, 9, 11].

The neutron spectrum presents some peaks in the epithermal region, but we suspect that each neutron burst may comprise neutrons at several energies [4]. Nothing so strange if we consider that piezonuclear neutrons are produced in connection with a new type of nuclear reaction in which the involved space-time is no flatter, as it usually is in the already well known nuclear reactions, but deformed [7, 8, 10]. A new phenomenon implies an open mind to look for the best ways to detect it and hence to describe it. Of course, we address ourselves with a critical sense and only after a thorough analysis our results are presented formally.

Moreover, it should be stressed that not always commercial available instrumentation (often designed for purposes of sanitary protection) corresponds to the need for a new experimentation. In some cases, it is useful, even necessary, to develop detection systems specially designed. These differences are integral part of our research. We do not walk along the main street that, historically speaking, stems from the discovery of natural radioactivity. Think how often, in the common talk, terms "radioactive" and "nuclear" are used almost interchangeably. On the contrary, starting from the deformation of space-time, we look at the ensuing reactions. As an example: let us envisage the whole energy available is spent in producing space-time deformation. We have no residual energy. Therefore, no gamma rays emission occurs. Let us note that all those characteristics of the neutron emission and the difficulty related to their detection and measurements were highlighted and discussed far from 1991 in the work:

"*On a possible method for measuring the energy of neutrons in short time emission*" [12]. In that work, for the first time, the neutron energy measurement problem during short duration piezonuclear emissions was faced up, and the term piezonuclear emissions was firstly introduced to indicate those emissions. Now, having in mind the theory of Deformed Space-time, that certainly helps us in understanding this new kind of phenomena, we prefer to use the term "*Deformed Space-time (DST) neutrons*". However, owing to very peculiar features presented above, it will be hard, or impossible, to detect these neutrons by usual techniques. We have stressed several times in our works that active neutron detectors are not the best choice to detect this type of neutron emission because of their complex and hence not immediate process to transduce neutrons into an electric signal, which is also often elaborated by software. Anyway, sometimes they are necessary, as in the case of spectral analysis, despite the difficulties encountered in past experiences in the use of this type of detectors. On the other hand, passive ones, like bubble detectors and track detectors (PADC), either screened or not by boron [13, 14] have higher detection efficiency; and being usually of small dimension, they can be arranged next to each other according to a suitable geometrical pattern in order to cover a solid angle, with the emitting apparatus at the centre, as close as possible to 4π steradians, and hence increase the geometrical. Moreover, their detecting mechanism is certainly more immediate than those of active detectors and more prompt to detect an emission that, far from being constant, has impulsive features in time, which are so far unpredictable. Conversely, as to the energy detection efficiency, also passive detectors are limited, like the active ones, by the energy dependence of the cross sections of the neutrons interaction with the nuclei of elements used in detection processes. In conclusion, we consider two main features: the energy threshold overcoming, $E > E_{0\ \mathrm{strong}}$, and the neutron emission in absence of γ radiation do provide the complete signature of reactions produced in non-Minkowskian conditions. Both these conditions are met in the experiments we carried up. In general, therefore, and in this work in particular, we put our attention on neutron energy, either directly by neutron spectrometry or by elaboration of data obtained from instruments designed to determine dose, fluence or other correlated quantities.

2. Experimental Part

The DST neutron emission of piezonuclear origin has been repeatedly observed and measured over several years and in numerous experiments [1, 4-7, 11, 15-17]. As for energy of the emitted neutrons we have to distinguish between two types of measures:

- measurements of dose (dose rate): these measurements can give only indirect indication of the neutron energy;

- real spectrometry of emitted neutrons.

This paper refers in particular to experiments connected to cavitation of AISI 304 steel, but a clearer understanding of emitted neutrons and their characteristics was realized for the first time sonicating 250 ml of a solution of Iron Chloride ($FeCl_3$) 1000 $\mu g \times ml^{-1}$ concentration (Boron Trifluoride Weldom Medical 2222A electronic detector and CR39 (PADC) detector) [6]. We obtained the result shown in Fig. 1.

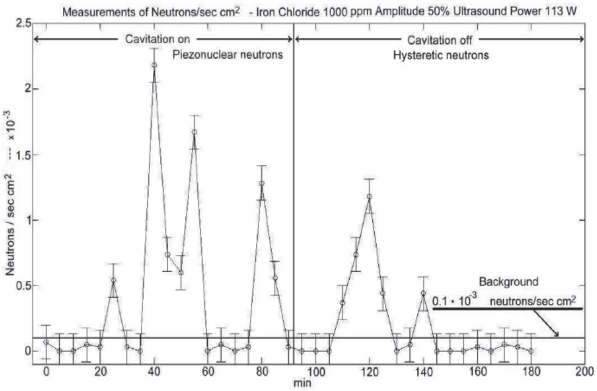

Figure 1. The graph shows the neutron pulses obtained during one of the sonication runs. Time in minutes is on the x-axis and neutron flux (neutrons/s cm^2) $\times 10^{-3}$ is on the y-axis. The error bars represent the sum of the pessimistic measured electronic noise of the whole measuring equipment and the pessimistic measured laboratory background flux i.e. 0.13×10^{-3} counts/(s cm^2).

The graphic is divided into two parts by a vertical line (at 92 minutes instead of 90 minutes for mere visual convenience). The left side from 0 to 90 minutes is the interval of time during which cavitation was on. The right side from 90 to 180 minutes is the interval of time during which cavitation was off but the neutron measurement went on. In both sides, some peaks stand well above the background level, pointing out that the emission of neutrons is not constant in time, but occurs in bursts of neutrons or *better* in pulses. In the left side of the Fig. 1, the first neutron pulse occurs after 40 minutes from the beginning of the cavitation and this circumstance was the same for all the cavitation runs that were carried out. More precisely, it turned out that in all cavitation runs the first neutron pulse appeared 40–50 minutes after switching on the ultrasounds. Although the cavitation was turned off and hence one would expect that the neutron pulses would stop along with it, there are two more peaks well above the background level [9]. These pulses, which were emitted in all cavitation runs after about 20 minutes the cavitation had been stopped, are a hysteretic behavior, and a possible candidate explanation to this fact is that some of the piezonuclear acoustic neutrons (*those neutrons* emitted during cavitation), had been absorbed most likely by the Carbon contained both in the steel sonotrode and in the materials of the supporting platforms and released after a latency (of about 20 minutes in our case) as it normally occurs in the graphite of nuclear reactors [9]. Neither coincidence nor correlation was found between neutron pulses and gamma equivalent dose rate and dose which were always compatible with the gamma background whose variations had been extensively studied all over the lab and were of the order of 0.14 ± 0.05 μSv/h (mean ± standard deviation) for equivalent dose rate and 0.22 ± 0.07 μSv (mean ± standard deviation) for equivalent dose [9]. The analysis of the neutron emission put in evidence in figure 1, along with the results of the preceding experiments, bring us to draw some conclusions: (a) neutron emission begins after a certain amount of energy is conveyed into the solution (in the case of solutions), or more generally into a material, after a certain time interval (energy density); (b) exist a threshold level, in power and energy, to have neutron production; (c) the bubble collapse, carefully described in many ours and other authors previous publications *[L. Crum and D. F. Gaitan, Observation of sonoluminescence from a single, stable cavitation bubble in a water glycerin mixture, Frontiers of Nonlinear Acoustic. 12 International Symposium of Nonlinear Acoustics, New York, Elsevier Applied Science (1990). C. E. Brennan, Cavitation and Bubble Dynamics, Oxford University Press (1995). M. Brenner, S. Hilgenfeldt, and D. Lohse, Rev. Mod. Phys. 74, 425 (2002). D. J. Flannigan and K. S. Suslick, Nature 434, 52 (2005). 9. F. Cardone, R. Mignani, and A. Petrucci, J. Adv. Phys. 1, 1 (2012). 7. F. Cardone and R. Mignani, Deformed Spacetime, Springer, Heidelberg (2007). 8. F. Cardone and R. Mignani, Energy and Geometry, World Scientific, Singapore (2004).]*, is the main microscopical mechanism to induce piezonuclear DST reactions, so the neutron emission does not take place as from a stable source, but it happens in bursts emitted in coincidence with the reaching of the well known bubbles collapse conditions. We hypothesized that a process similar to the bubble collapse in liquids might take place in solid bars of Iron alloys, due to the gas that they absorb during the casting process forming gas porosity [4]. A support to this conjecture was given by the experimental evidences previously obtained on Luzerna Granite during experiments performed at Turin Polytechnic [5]. So, as a subsequent phase of our experiments we subjected some cylindrical steel bars (AISI 304) with mass 180 g to sonication by ultrasounds with a frequency of 20 kHz and power of 330 W for a time interval of 180 sec [1, 16, 17]. We measured the DST-neutron emissions using a double system of detection. The first system was the PADC Polycarbonate-CR39 (Boric Acid) developed and calibrated at the ENEA Research Centre of Rome-Casaccia. It is an imaging system that gives a passive measurement of the integral neutron dose released in the device (the PADC plate) during the time interval of the sonication (and related emission) [14, 22]. The PADC-CR39 (AB) were placed around a sonicated steel bar to record the signature of the actual neutron emission for comparison with those produced during LLI violation [16]. The second system we used was the MICROSPEC2 Neutron Probe made by firm BTI, which was utilized to measure dose, fluence and spectrum of DST-neutrons coming from the sonicated steel bar [23, 24]. For a more comprehensive evaluation we also used a gamma-ray detector, the HDS 101 GN by MIRION, to confirm the absence of gammas above laboratory background during the DST-neutron emission, as recorded in all past experiments [25]. HDS-100 GN contains a n detector with LiI(Eu) scintillator and a γ detector and spectrometer with CsI(Tl) scintillator for low energy γ rays and a silicon diode for the high-energy ones. MICROSPEC2 is equipped with a He-3 counter for thermal neutrons up to 800 keV and a liquid scintillator NE-213 for neutrons from 500keV to 5 MeV, and with the multichannel analyzer MICROSPEC2.

The MICROSPEC2 Neutron Probe is a rather complex system. In particular, referring to methods to derive the neutron spectrum from the pulse height distribution we note that very poor explanation is offered by technical specifications. Therefore, the MICROSPEC2 was tested and its unique features reported by the firm-maker were verified during measurements performed at the ENEA Research Centre of Rome-Casaccia using three different neutron sources: Americium-Beryllium, nuclear reactor TRIGA and nuclear reactor TAPIRO (for information about these sources see e.g. [14, 22]). The Deformed Space-Time theory states that an interaction violating LLI in a nucleus results in a reaction characterized by emissions of nuclear particles in DST conditions [7, 25]. However, once produced, such particles travel in a flat Minkowskian space-time, which is also the space-time of the detector and of the physical processes giving rise to detection [7, 8]. Consequently, we positioned a PADC Polycarbonate-CR39 (AB) along the ideal segment connecting the neutron probe with the surface of the sonicated steel bar, at a distance between the neutron probe and the bar. In this way, the bar-PADC and bar-Neutron Probe distances were different but oriented along the same direction. We verified that the doses measured by the two systems had an inverse square relationship with the above defined distances [1, 17]. This was evidence that, after being emitted, the DST neutrons moved in a flat Minkowskian space-time and were detected by both detectors in flat Minkowskian conditions. Consequently, the requirements of the DST theory were fulfilled [7, 8, 16]. In fact, as reported above, this theory affirms that an interaction violating LLI in a nucleus brings about a reaction characterized by emissions of nuclear particles that takes places necessarily in DST conditions. However, once emitted, such particles travel in a flat Minkowskian space-time, which is also the space-time of the detector and of the physical processes giving rise to detection. Before and after the sonication tests we performed background measurement runs each lasting an interval time of 15 min, i.e. 5 times the time interval of sonication, using the MICROSPEC2 Neutron Probe. The background resulted quite negligible since in each run few neutrons (less than 10) have been registered only over the energy interval 0.5-2 MeV (see also [1, 16, 17]). This result was compatible with the electronic noise of the system used [23, 24]. For each spectrum, we performed a set of measurements which proved consistent, displaying neither significant nor substantial differences among them. In Fig. 2 we report the energy spectrum and the fluence. In Fig. 3 we show the high-energy spectrum, i.e. up to 4 MeV (Fig. 3A), obtained by the NE-213 scintillation counter, well suitable for this task due to its detection stability characteristics, and low energy spectrum, i.e. less than 0.4 MeV (Fig. 3B), obtained by the He-3 counter.

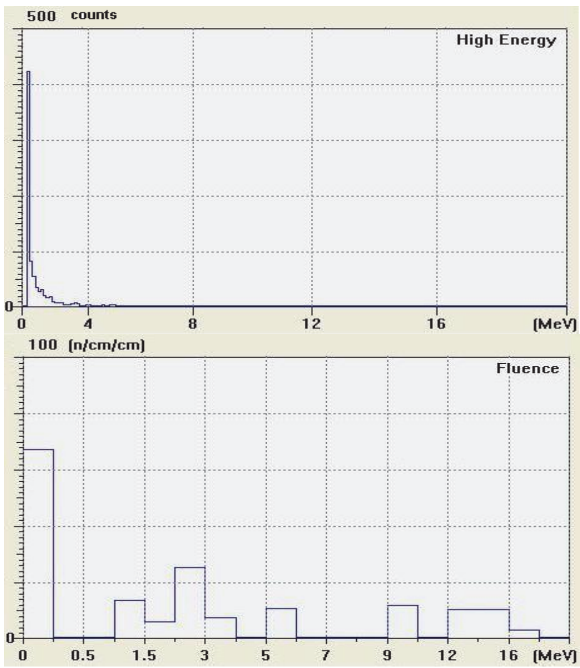

Figure 2. Deformed Space Time (DST) neutrons energy spectrum and related fluence.

The peculiar features of these spectra deserve further investigations. For now, we think that it is worth comparing these spectra with the graph in Fig. 1, showing the asynchrony of DST-neutron emissions, and the diagram in Fig. 4, showing the anisotropy of these neutron emissions. Collecting altogether the graphs of the spectra, the diagram of the spatial emission and the graph of the emission in time, we have the first complete, although preliminary, characterization of DST neutrons. This characterization shows that the DST-neutron emissions have peculiar features not just with respect to space and time, but also with respect to the energy spectrum of the emitted neutrons. These spectra have an irregular trend both at high and low energy. Conversely, the neutron spectra of known sources (Americium-Beryllium, fast neutron reactors, thermal neutrons), present single trend for the whole neutron spectrum (see, for instance, the spectra of these three mentioned sources in [14]).

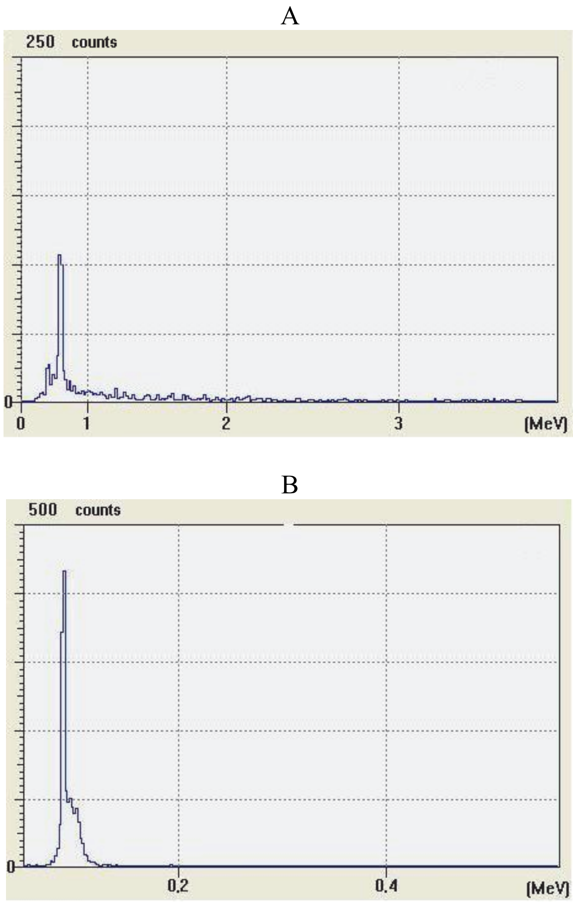

Figure 3. Deformed Space Time (DST) neutron spectrum in details: A) 0.3 – 3 MeV, NE-213 liquid scintillator detector, B) 0 – 0.3 MeV, Helium-3 gas detector.

Moreover, we have to keep in mind that the emission of nuclear particles in DST conditions was recorded to occur in bursts in all past experiments [3, 6, 7, 25-27]. Therefore, in accordance with the detectors used, a careful study of the energy, space and time effects (including dead-time effect) that these features (asynchrony, asymmetry, anisotropy, burst emission) have on the measurement efficiencies will be necessary in order to ensure improved accuracy of future measurements. In the past, a lot of experimental effort was directed at the discovery of new kinds of nuclear reactions and emissions related to them, with many experiments being performed [28]. Although a great deal of attention was paid to new types of nuclear reactions, less was given to the fact that they can give rise to particle emissions with new characteristics and peculiar features.

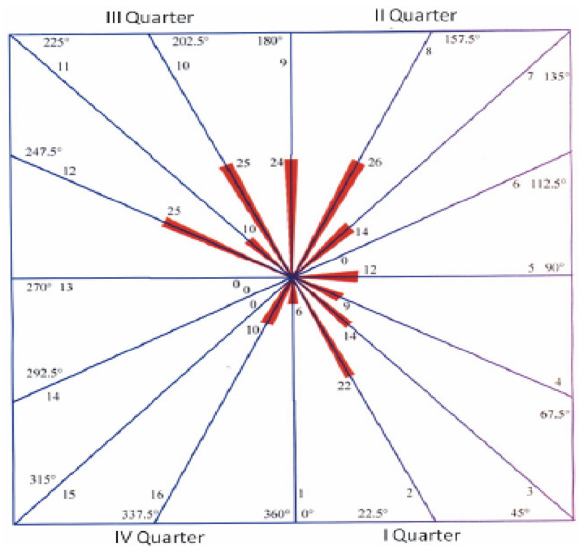

Figure 4. Spatial asymmetry of neutron bursts from DST emission [1, 16, 17].

Characteristics and features that current detectors, especially active detectors, and available techniques are incapable of adequately detecting, which limits our understanding of these new features, see e.g. [29, 30]. In the case of neutron emissions, some methods have been proposed, including neutron activation of Indium foils and the use of Uranium fission chambers [31]. It is evident that the DST-neutron spectra shown in Figs. 2 and 3 have a poor, or even no correlation with the behaviors shown in neutron cross-sections of Indium and Uranium, both U-235 and U-238. Moreover, it is our opinion that it will not be an easy task to plan and prepare an absorber and moderator for use in the detection of DST-neutrons due to the asynchrony, asymmetry and anisotropy of their emissions. Nevertheless, the measurement of these spectra can be an important step toward the exploitation of the energy produced in the DST-reactions [32-35].

3. Conclusions

In this work, a first attempt of DST neutrons energy measurements was successfully accomplished. From experimental data, it is possible to hypothesize that DST neutron production is not uniform in energy, similarly to its time asynchrony and spatial asymmetry, as already verified also by means of passive detector described above. However, we reported the difficulties encountered in measurement, having well in mind the significance of measurement as an integral part of the operational definition of a physical quantity. Since 1989 it was a debated question whether neutron emission from nuclear reactions of a new kind could even be fully measured or not. Now we think to have given a positive answer. But having now solved (at least in part) the problem of the detection of neutron energy [22] we do not believe to have fully accomplished the task of understanding the problem of their origin from DST reactions. Is this a signature of a new physics? We think that the data on the emission, the emission conditions, the dose and the neutron energy acquired by us put a series of questions and problems to the physics of the XXI century, as for as at the XX century beginning, the first scientific knowledge linked to the discovery of the radioactivity, put to the XIX century physics and to the classical physics more in general, questions to which they were not able to give a response.

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P, C and T: Different Properties on the Kinematical Level*

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We study the discrete symmetries (P,C and T) on the kinematical level within the extended Poincaré Group. On the basis of the Silagadze research, we investigate the question of the definitions of the discrete symmetry operators both on the classical level, and in the secondary-quantization scheme. We study the physical contents within several bases: light-front formulation, helicity basis, angular momentum basis, and so on, on several practical examples. We analize problems in construction of the neutral particles in the the $(1/2,0)+(0,1/2)$ representation, the $(1,0)+(0,1)$ and the $(1/2,1/2)$ representations of the Lorentz Group. As well known, the photon has the quantum numbers 1^-, so the $(1,0)+(0,1)$ representation of the Lorentz group is relevant to its description. We have ambiguities in the definitions of the corresponding operators P, C, T, which lead to different physical consequences. It appears that the answers are connected with the helicity basis properties, and commutations/anticommutations of the corresponding operators, P, C, T, and C^2, P^2, $(CP)^2$ properties.

Keywords: PCT, Lorentz Group, Discrete Symmetries

1. Introduction

In his paper of 1992 Silagadze claimed: "It is shown that the usual situation when boson and its antiparticle have the same internal parity, while, fermion and its antiparticle have opposite parities, assumes a kind of locality of the theory. In general, when a quantum-mechanical parity operator is defined by means of the group extension technique, the reversed situation is also possible", Ref.[1] Then, Ahluwalia *et al.* proposed[5] the Bargmann-Wightman-Wigner-type quantum field theory, where, as they claimed, the boson and the antiboson have oposite intrinsic parities (see also[6]). Actually, this type of theories has been first proposed by Gelfand and Tsetlin (1956), Ref.[7] In fact, it is based on the two-dimensional representation of the inversion group. They indicated applicability of this theory to the description of the system of K-mesons and the possible relations to the Lee-Yang paper. The (anti)comutativity of the discrete symmetry operations has also been investigated by Foldy and Nigam,[8] who claimed that it is related to the eigenvalues of the charge operator. The relations of the Gelfand-Tsetlin construct to the representations of the anti-de Sitter $SO(3,2)$ group and the general relativity theory have also been discussed in subsequent papers of Sokolik. E. Wigner[9] presented some relevant results at the Istanbul School on Theoretical Physics in 1962. Later, Fushchich *et al.* discussed the wave equations. Actually, the theory presented by Ahluwalia, Goldman and Johnson is the Dirac-like generalization of the Weinberg $2(2J+1)$-theory for the spin 1. The equations have also been presented in the Sankaranarayanan and Good paper of 1965, Ref.[10] In[11] the theory in the $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ representation based on the chiral helicity 4-eigenspinors was proposed. The corresponding equations have been obtained, e. g., in.[3] However, later we found the papers by Ziino and Barut[12] and the Markov papers,[13] which also have connections with the subject under consideration. The question of definitions of the discrete symmetries operators raised by Silagadze, has not yet been clarified in detail. Explicit examples are presented below and in the previous papers.[2–6,10–13]

2. Helicity Basis and Parity

The 4-spinors have been studied well when the basis has been chosen in such a way that they are eigenstates of the $\hat{\mathbf{S}}_3$ operator:

$$u_{\frac{1}{2},\frac{1}{2}} = N^+_{\frac{1}{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}, u_{\frac{1}{2},-\frac{1}{2}} = N^+_{-\frac{1}{2}}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}, \qquad (1)$$

$$v_{\frac{1}{2},\frac{1}{2}} = N^-_{\frac{1}{2}}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}, v_{\frac{1}{2},-\frac{1}{2}} = N^-_{-\frac{1}{2}}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}. \qquad (2)$$

And, oppositely, the helicity basis case has not been studied almost at all (see, however, Refs.[14,15]) Let me remind that the boosted 4-spinors $\Psi(\mathbf{p}) = column\,(\phi_R(\mathbf{p})\ \pm\phi_L(\mathbf{p}))$ in the common-used basis are the parity eigenstates with the eigenvalues of ± 1.

In the helicity spin basis the 2-eigenspinors of the helicity operator[17]

$$\frac{1}{2}\sigma\cdot\widehat{\mathbf{p}} = \frac{1}{2}\begin{pmatrix}\cos\theta & \sin\theta e^{-i\phi}\\ \sin\theta e^{+i\phi} & -\cos\theta\end{pmatrix}, \qquad (3)$$

θ, ϕ are the angles of the spherical coordinate system, can be defined as follows:[17,18]

$$\phi_{\frac{1}{2}\uparrow} \sim \begin{pmatrix}\cos\frac{\theta}{2}e^{-i\phi/2}\\ \sin\frac{\theta}{2}e^{+i\phi/2}\end{pmatrix}, \phi_{\frac{1}{2}\downarrow} \sim \begin{pmatrix}\sin\frac{\theta}{2}e^{-i\phi/2}\\ -\cos\frac{\theta}{2}e^{+i\phi/2}\end{pmatrix}, \qquad (4)$$

for $h = \pm 1/2 =\uparrow\downarrow$ eigenvalues, respectively.

We start from the Klein-Gordon equation, generalized for describing the spin-1/2 particles (i. e., two degrees of freedom), $c = \hbar = 1$:

$$(E + \sigma\cdot\mathbf{p})(E - \sigma\cdot\mathbf{p})\phi = m^2\phi\,. \qquad (5)$$

It can be re-written in the form of the system of two first-order equations for 2-spinors as in the Sakurai book.[16] At the same time, we observe that they may be chosen as the eigenstates of the helicity operator:

$$(E - (\sigma\cdot\mathbf{p}))\phi_\uparrow = (E - p)\phi_\uparrow = m\chi_\uparrow\,, \qquad (6)$$
$$(E + (\sigma\cdot\mathbf{p}))\chi_\uparrow = (E + p)\chi_\uparrow = m\phi_\uparrow\,, \qquad (7)$$
$$(E - (\sigma\cdot\mathbf{p}))\phi_\downarrow = (E + p)\phi_\downarrow = m\chi_\downarrow\,, \qquad (8)$$
$$(E + (\sigma\cdot\mathbf{p}))\chi_\downarrow = (E - p)\chi_\downarrow = m\phi_\downarrow\,. \qquad (9)$$

If the ϕ spinors are defined by the equation (4) then we can construct the corresponding $u-$ and $v-$ 4-spinors[b]

$$u_\uparrow = N_\uparrow^+\begin{pmatrix}\phi_\uparrow\\ \frac{E-p}{m}\phi_\uparrow\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{\frac{E+p}{m}}\phi_\uparrow\\ \sqrt{\frac{m}{E+p}}\phi_\uparrow\end{pmatrix}, \qquad (10)$$

$$u_\downarrow = N_\downarrow^+\begin{pmatrix}\phi_\downarrow\\ \frac{E+p}{m}\phi_\downarrow\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{\frac{m}{E+p}}\phi_\downarrow\\ \sqrt{\frac{E+p}{m}}\phi_\downarrow\end{pmatrix}, \qquad (11)$$

$$v_\uparrow = N_\uparrow^-\begin{pmatrix}\phi_\uparrow\\ -\frac{E-p}{m}\phi_\uparrow\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{\frac{E+p}{m}}\phi_\uparrow\\ -\sqrt{\frac{m}{E+p}}\phi_\uparrow\end{pmatrix}, \qquad (12)$$

$$v_\downarrow = N_\downarrow^-\begin{pmatrix}\phi_\downarrow\\ -\frac{E+p}{m}\phi_\downarrow\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{\frac{m}{E+p}}\phi_\downarrow\\ -\sqrt{\frac{E+p}{m}}\phi_\downarrow\end{pmatrix}, \qquad (13)$$

[b]One can also try to construct yet another theory differing from the ordinary Dirac theory. The 4-spinors may be *not* the eigenspinors of the helicity operator of the $(1/2, 0)\oplus(0, 1/2)$ representation space, cf.[11] They might be the eigenstates of the *chiral* helicity operator introduced in.[11]

where the normalization to the unit (± 1) was used. One can prove that the matrix $P = \gamma^0 = \begin{pmatrix}0 & I\\ I & 0\end{pmatrix}$ can be used in the parity operator as in the original Dirac basis. Indeed, the 4-spinors (10–13) satisfy the Dirac equation in the spinorial (Weyl) representation of the γ-matrices. Hence, the parity-transformed function $\Psi'(t, -\mathbf{x}) = P\Psi(t, \mathbf{x})$ must satisfy $[i\gamma^\mu\partial'_\mu - m]\Psi'(t, -\mathbf{x}) = 0$ with $\partial'_\mu = (\partial/\partial t, -\nabla_i)$. This is possible when $P^{-1}\gamma^0 P = \gamma^0$ and $P^{-1}\gamma^i P = -\gamma^i$. The P-matrix above satisfies these requirements, as in the textbook case.[19]

Next, it is easy to prove that one can form the projection operators

$$\mathcal{P}_+ = +\sum_h u_h(\mathbf{p})\bar{u}_h(\mathbf{p}) = \frac{p_\mu\gamma^\mu + m}{2m}, \qquad (14)$$

$$\mathcal{P}_- = -\sum_h v_h(\mathbf{p})\bar{v}_h(\mathbf{p}) = \frac{m - p_\mu\gamma^\mu}{2m}, \qquad (15)$$

with the properties $P_+ + P_- = 1$ and $P_\pm^2 = P_\pm$. This permits us to expand the 4-spinors defined in the basis (1, 2) in the linear superpositions of the helicity basis 4-spinors, and to find corresponding coefficients of the expansion:

$$u_\sigma(\mathbf{p}) = A_{\sigma h}u_h(\mathbf{p}) + B_{\sigma h}v_h(\mathbf{p}), \qquad (16)$$
$$v_\sigma(\mathbf{p}) = C_{\sigma h}u_h(\mathbf{p}) + D_{\sigma h}v_h(\mathbf{p}). \qquad (17)$$

Multiplying the above equations by $\bar{u}_{h'}$, $\bar{v}_{h'}$ respectively, and using the normalization conditions, we obtain $A_{\sigma h} = D_{\sigma h} = \bar{u}_h u_\sigma$, $B_{\sigma h} = C_{\sigma h} = -\bar{v}_h u_\sigma$. Thus, the transformation matrix from the common-used basis to the helicity basis is

$$\begin{pmatrix}u_\sigma\\ v_\sigma\end{pmatrix} = \mathcal{U}\begin{pmatrix}u_h\\ v_h\end{pmatrix}, \quad \mathcal{U} = \begin{pmatrix}A & B\\ B & A\end{pmatrix} \qquad (18)$$

Neither A nor B are unitary:

$$A = (a_{++} + a_{+-})(\sigma_\mu a^\mu) + (-a_{-+} + a_{--})(\sigma_\mu a^\mu)\sigma_3\,, \qquad (19)$$

$$B = (-a_{++} + a_{+-})(\sigma_\mu a^\mu) + (a_{-+} + a_{--})(\sigma_\mu a^\mu)\sigma_3\,, \qquad (20)$$

where

$$a^0 = -i\cos(\theta/2)\sin(\phi/2) \in \Im m\,, \qquad (21)$$
$$a^1 = \sin(\theta/2)\cos(\phi/2) \in \Re e\,, \qquad (22)$$
$$a^2 = \sin(\theta/2)\sin(\phi/2) \in \Re e\,, \qquad (23)$$
$$a^3 = \cos(\theta/2)\cos(\phi/2) \in \Re e\,, \qquad (24)$$

and

$$a_{++} = \frac{\sqrt{(E+m)(E+p)}}{2\sqrt{2}m}\,, a_{+-} = \frac{\sqrt{(E+m)(E-p)}}{2\sqrt{2}m}\,, \tag{25}$$

$$a_{-+} = \frac{\sqrt{(E-m)(E+p)}}{2\sqrt{2}m}\,, a_{--} = \frac{\sqrt{(E-m)(E-p)}}{2\sqrt{2}m}\,. \tag{26}$$

However, $A^\dagger A + B^\dagger B = I$, so the matrix $\mathcal{U}$ is unitary. Please note that this matrix acts on the *spin* indices (σ, h), and not on the spinorial indices; it is 4×4 matrix.

We now investigate the properties of the helicity-basis 4-spinors with respect to the discrete symmetry operations P, C and T. It is expected that $h \to -h$ under parity, as Berestetskiĭ, Lifshitz and Pitaevskiĭ claimed.[20] Indeed, if $\mathbf{x}\to -\mathbf{x}$, then the vector $\mathbf{p}\to -\mathbf{p}$, but the axial vector $\mathbf{S}\to\mathbf{S}$, that implies the above statement. The helicity 2-eigenspinors transform $\phi_{\uparrow\downarrow} \Rightarrow -i\phi_{\downarrow\uparrow}$ with respect to $\mathbf{p}\to -\mathbf{p}$, Ref.[18] Hence,

$$Pu_\uparrow(-\mathbf{p}) = -iu_\downarrow(\mathbf{p})\,, Pv_\uparrow(-\mathbf{p}) = +iv_\downarrow(\mathbf{p})\,, \tag{27}$$
$$Pu_\downarrow(-\mathbf{p}) = -iu_\uparrow(\mathbf{p})\,, Pv_\downarrow(-\mathbf{p}) = +iv_\uparrow(\mathbf{p})\,. \tag{28}$$

Thus, on the level of classical fields, we observe that the helicity 4-spinors transform to the 4-spinors of the opposite helicity.

Also,

$$Cu_\uparrow(\mathbf{p}) = -v_\downarrow(\mathbf{p})\,, Cv_\uparrow(\mathbf{p}) = +u_\downarrow(\mathbf{p})\,, \tag{29}$$
$$Cu_\downarrow(\mathbf{p}) = +v_\uparrow(\mathbf{p})\,, Cv_\downarrow(\mathbf{p}) = -u_\uparrow(\mathbf{p})\,. \tag{30}$$

due to the properties of the Wigner operator $\Theta\phi^*_\uparrow = -\phi_\downarrow$ and $\Theta\phi^*_\downarrow = +\phi_\uparrow$, $\Theta_{[1/2]} = -i\sigma_2$. Similar conclusions can be obtained in the Fock space.

We define the field operator as follows:

$$\Psi(x^\mu) = \sum_h \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{\sqrt{m}}{2E}\Big[u_h(\mathbf{p})a_h(\mathbf{p})e^{-ip_\mu x^\mu} + + v_h(\mathbf{p})b_h^\dagger(\mathbf{p})e^{+ip_\mu x^\mu}\Big]\,. \tag{31}$$

The commutation relations are assumed to be the standard ones[19,21–23] (compare with Refs.[3,11]). If one defines $U_P\Psi(x^\mu)U_P^{-1} = \gamma^0\Psi(x^{\mu'})$, $U_C\Psi(x^\mu)U_C^{-1} = C\Psi^\dagger(x^\mu)$ and the anti-unitary operator of time reversal $(V_T\Psi(x^\mu)V_T^{-1})^\dagger = T\Psi^\dagger(x^{\mu''})$, then it is easy to obtain the corresponding transformations of the creation/annihilation operators:

$$\begin{aligned} U_P a_h(\mathbf{p})U_P^{-1} &= -ia_{-h}(-\mathbf{p})\,,\\ U_P b_h(\mathbf{p})U_P^{-1} &= -ib_{-h}(-\mathbf{p})\,, \end{aligned}\tag{32}$$

$$\begin{aligned} U_C a_h(\mathbf{p})U_C^{-1} &= (-1)^{\frac12+h}b_{-h}(\mathbf{p})\,,\\ U_C b_h(\mathbf{p})U_C^{-1} &= (-1)^{\frac12-h}a_{-h}(-\mathbf{p})\,. \end{aligned}\tag{33}$$

As a consequence, we obtain (provided that $U_P|0>=|0>$, $U_C|0>=|0>$)

$$U_P a_h^\dagger(\mathbf{p})|0>= U_P a_h^\dagger U_P^{-1}|0>= ia^\dagger_{-h}(-\mathbf{p})|0>= = i|-\mathbf{p},-h>^+, \tag{34}$$
$$U_P b_h^\dagger(\mathbf{p})|0>= U_P b_h^\dagger U_P^{-1}|0>= ib^\dagger_{-h}(-\mathbf{p})|0>= = i|-\mathbf{p},-h>^-, \tag{35}$$

and

$$U_C a_h^\dagger(\mathbf{p})|0>= U_C a_h^\dagger U_C^{-1}|0>= (-1)^{\frac12+h}b^\dagger_{-h}(\mathbf{p})|0>= (-1)^{\frac12+h}|\mathbf{p},-h>^-\,, \tag{36}$$
$$U_C b_h^\dagger(\mathbf{p})|0>= U_C b_h^\dagger U_C^{-1}|0>= (-1)^{\frac12-h}a^\dagger_{-h}(\mathbf{p})|0>= (-1)^{\frac12-h}|\mathbf{p},-h>^+\,. \tag{37}$$

Finally, for the CP operation one should obtain:

$$\begin{aligned} &U_PU_C a_h^\dagger(\mathbf{p})|0>= -U_CU_P a_h^\dagger(\mathbf{p})|0>=\\ &= (-1)^{\frac12+h}U_P b^\dagger_{-h}(\mathbf{p})|0>=\\ &= i(-1)^{\frac12+h}b_h^\dagger(-\mathbf{p})|0>= i(-1)^{\frac12+h}|-\mathbf{p},h>^- \end{aligned}\tag{38}$$

$$\begin{aligned} &U_PU_C b_h^\dagger(\mathbf{p})|0>= -U_CU_P b_h^\dagger(\mathbf{p}) =\\ &= (-1)^{\frac12-h}U_P a^\dagger_{-h}(\mathbf{p})|0>=\\ &= i(-1)^{\frac12-h}a_h^\dagger(-\mathbf{p})|0>= i(-1)^{\frac12-h}|-\mathbf{p},h>^+ \end{aligned}\tag{39}$$

As in the classical case, the P and C operations anticommute in the $(\frac12,0)\oplus(0,\frac12)$ quantized case. This opposes to the theory based on the 4-spinor eigenstates of chiral helicity (cf.[3,8]). Since the V_T is an anti-unitary operator the T-problem must be solved after taking into account of the fact that the c-numbers should be put outside the hermitian conjugation *without* complex conjugation:

$$[V_ThAV_T^{-1}]^\dagger = [h^*V_TAV_T^{-1}]^\dagger = h[V_TA^\dagger V_T^{-1}]\,. \tag{40}$$

After applying this definition we obtain:[c]

$$V_Ta_h^\dagger(\mathbf{p})V_T^{-1} = +i(-1)^{\frac12-h}a_h^\dagger(-\mathbf{p})\,, \tag{41}$$
$$V_Tb_h(\mathbf{p})V_T^{-1} = +i(-1)^{\frac12-h}b_h(-\mathbf{p})\,. \tag{42}$$

[c] T should be chosen in such a way in order to fulfill $T^{-1}\gamma_0^TT = \gamma_0$, $T^{-1}\gamma_i^TT = \gamma_i$ and $T^T = -T$, as in Ref.[22]

Furthermore, we observed that the parity properties depend on the phase factor in the following definition:

$$U_P \Psi(t, \mathbf{x}) U_P^{-1} = e^{i\alpha} \gamma^0 \Psi(t, -\mathbf{x}) \,. \tag{43}$$

Indeed,

$$U_P a_h(\mathbf{p}) U_P^{-1} = -i e^{i\alpha} a_{-h}(-\mathbf{p}) \,, \tag{44}$$

$$U_P b_h^\dagger(\mathbf{p}) U_P^{-1} = +i e^{i\alpha} b_{-h}^\dagger(-\mathbf{p}) \,. \tag{45}$$

From this, if $\alpha = \pi/2$ we obtain *opposite* parity properties of the creation/annihilation operators for particles and anti-particles:

$$U_P a_h(\mathbf{p}) U_P^{-1} = +a_{-h}(-\mathbf{p}) \,, \tag{46}$$

$$U_P b_h(\mathbf{p}) U_P^{-1} = -b_{-h}(-\mathbf{p}) \,. \tag{47}$$

However, the difference with the Dirac case still preserves (h transforms to $-h$). We find similar situation with the question of constructing the neutrino field operator (cf. with the Goldhaber-Kayser creation phase factor).

Next, we find the explicit form of the parity operator U_P and prove that it commutes with the Hamiltonian operator. We prefer to use the method described in [21, §10.2–10.3]. It is based on the *anzatz* that $U_P = \exp[i\alpha\hat{A}]\exp[i\hat{B}]$ with $\hat{A} = \sum_s \int d^3\mathbf{p} [a^\dagger_{\mathbf{p},s} a_{-\mathbf{p}s} + b^\dagger_{\mathbf{p}s} b_{-\mathbf{p}s}]$ and $\hat{B} = \sum_s \int d^3\mathbf{p} [\beta a^\dagger_{\mathbf{p},s} a_{\mathbf{p}s} + \gamma b^\dagger_{\mathbf{p}s} b_{\mathbf{p}s}]$. On using the known operator identity

$$e^{\hat{A}} \hat{B} e^{-\hat{A}} = \hat{B} + [\hat{A}, \hat{B}]_- + \frac{1}{2!}[\hat{A}, [\hat{A}, \hat{B}]] + \dots \tag{48}$$

and $[\hat{A}, \hat{B}\hat{C}]_- = [\hat{A}, \hat{B}]_+ \hat{C} - \hat{B}[\hat{A}, \hat{C}]_+$ one can fix the parameters α, β, γ such that one satisfies the physical requirements that a Dirac particle and its antiparticle have opposite intrinsic parities.

In our case, we need to satisfy the requirement that the operator should invert not only the sign of the momentum, but the sign of the helicity too. We may achieve this goal by the analogous postulate $U_P = e^{i\alpha\hat{A}}$ with

$$\hat{A} = \sum_h \int \frac{d^3\mathbf{p}}{2E} \left[a_h^\dagger(\mathbf{p}) a_{-h}(-\mathbf{p}) + b_h^\dagger(\mathbf{p}) b_{-h}(-\mathbf{p}) \right] . \tag{49}$$

By direct verification, the requirement is satisfied provided that $\alpha = \pi/2$. You may compare this parity operator with that given in[19,21] for Dirac fields:[d]

$$U_P = \exp\Bigg[i\frac{\pi}{2} \int d^3\mathbf{p} \sum_s \big(a(\mathbf{p}, s)^\dagger a(\tilde{\mathbf{p}}, s) + \tag{50}$$

$$+ \; b(\mathbf{p}, s)^\dagger b(\tilde{\mathbf{p}}, s) -$$

$$- a(\mathbf{p}, s)^\dagger a(\mathbf{p}, s) + b(\mathbf{p}, s)^\dagger b(\mathbf{p}, s) \big) \Bigg] . \tag{51}$$

By direct verification one can also come to the conclusion that our new U_P commutes with the Hamiltonian:

$$\mathcal{H} = \int d^3\mathbf{x} \Theta^{00} = \int d^3\mathbf{k} \sum_h \left[a_h^\dagger(\mathbf{k}) a_h(\mathbf{k}) - b_h(\mathbf{k}) b_h^\dagger(\mathbf{k}) \right] , \tag{52}$$

i.e. $[U_P, \mathcal{H}]_- = 0$. Alternatively, we can try to choose other commutation relations[3,11] for the set of bi-orthonormal states. The formulas of the theory have been presented in the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representation based on the chiral helicity 4-eigenspinors, see below. Next, the theory, which is based on a system of 6-component Weinberg-like equations in the $(1, 0) \oplus (0, 1)$ representation, has also been constructed. The results are similar. The papers by Ziino and Barut[12] and the Markov papers[13] have connections with the subject under consideration.

3. Chiral Helicity Construct and the Different Definition of the Charge Conjugate Operator on the Secondary Quantization Level

In the chiral representation one can choose the spinorial basis (zero-momentum spinors) in the following way:

$$\lambda_\uparrow^S(\mathbf{0}) = \sqrt{\frac{m}{2}} \begin{pmatrix} 0 \\ i \\ 1 \\ 0 \end{pmatrix} , \lambda_\downarrow^S(\mathbf{0}) = \sqrt{\frac{m}{2}} \begin{pmatrix} -i \\ 0 \\ 0 \\ 1 \end{pmatrix} , \tag{53}$$

$$\lambda_\uparrow^A(\mathbf{0}) = \sqrt{\frac{m}{2}} \begin{pmatrix} 0 \\ -i \\ 1 \\ 0 \end{pmatrix} , \lambda_\downarrow^A(\mathbf{0}) = \sqrt{\frac{m}{2}} \begin{pmatrix} i \\ 0 \\ 0 \\ 1 \end{pmatrix} , \tag{54}$$

[d] Greiner used the following commutation relations $\left[a(\mathbf{p}, s), a^\dagger(\mathbf{p}', s')\right]_+ = \left[b(\mathbf{p}, s), b^\dagger(\mathbf{p}', s')\right]_+ = \delta^3(\mathbf{p} - \mathbf{p}')\delta_{ss'}$. One should also note that the Greiner form of the parity operator is not the unique one. Itzykson and Zuber[19] proposed another one differing by the phase factors from (10.69) of.[21] In order to find relations between those two forms of the parity operator one should apply additional rotation in the Fock space.

and

$$\rho^S_\uparrow(\mathbf{0}) = \sqrt{\frac{m}{2}}\begin{pmatrix}1\\0\\0\\-i\end{pmatrix}, \rho^S_\downarrow(\mathbf{0}) = \sqrt{\frac{m}{2}}\begin{pmatrix}0\\1\\i\\0\end{pmatrix}, \quad (55)$$

$$\rho^A_\uparrow(\mathbf{0}) = \sqrt{\frac{m}{2}}\begin{pmatrix}1\\0\\0\\i\end{pmatrix}, \rho^A_\downarrow(\mathbf{0}) = \sqrt{\frac{m}{2}}\begin{pmatrix}0\\1\\-i\\0\end{pmatrix}. \quad (56)$$

The indices $\uparrow\downarrow$ should be referred to the chiral helicity quantum number introduced in Ref. [11], $\eta = -h\gamma_5$. We use the notation of the previous papers on the subject. Ahluwalia and Grumiller used the helicity basis for the 2nd-type 4-spinors. The reader would immediately find the 4-spinors of the second kind $\lambda^{S,A}_{\uparrow\downarrow}(p^\mu)$ and $\rho^{S,A}_{\uparrow\downarrow}(p^\mu)$ in an arbitrary frame on using the boost operators:

$$\lambda^S_\uparrow(p^\mu) = \frac{1}{2\sqrt{E+m}}\begin{pmatrix}ip_l\\i(p^-+m)\\p^-+m\\-p_r\end{pmatrix}, \quad (57)$$

$$\lambda^S_\downarrow(p^\mu) = \frac{1}{2\sqrt{E+m}}\begin{pmatrix}-i(p^++m)\\-ip_r\\-p_l\\(p^++m)\end{pmatrix}, \quad (58)$$

$$\lambda^A_\uparrow(p^\mu) = \frac{1}{2\sqrt{E+m}}\begin{pmatrix}-ip_l\\-i(p^-+m)\\(p^-+m)\\-p_r\end{pmatrix}, \quad (59)$$

$$\lambda^A_\downarrow(p^\mu) = \frac{1}{2\sqrt{E+m}}\begin{pmatrix}i(p^++m)\\ip_r\\-p_l\\(p^++m)\end{pmatrix}, \quad (60)$$

and

$$\rho^S_\uparrow(p^\mu) = \frac{1}{2\sqrt{E+m}}\begin{pmatrix}p^++m\\p_r\\ip_l\\-i(p^++m)\end{pmatrix}, \quad (61)$$

$$\rho^S_\downarrow(p^\mu) = \frac{1}{2\sqrt{E+m}}\begin{pmatrix}p_l\\(p^-+m)\\i(p^-+m)\\-ip_r\end{pmatrix}, \quad (62)$$

$$\rho^A_\uparrow(p^\mu) = \frac{1}{2\sqrt{E+m}}\begin{pmatrix}p^++m\\p_r\\-ip_l\\i(p^++m)\end{pmatrix}, \quad (63)$$

$$\rho^A_\downarrow(p^\mu) = \frac{1}{2\sqrt{E+m}}\begin{pmatrix}p_l\\(p^-+m)\\-i(p^-+m)\\ip_r\end{pmatrix}. \quad (64)$$

Some of the 4-spinors are connected each other. The normalization of the spinors $\lambda^{S,A}_{\uparrow\downarrow}(p^\mu)$ and $\rho^{S,A}_{\uparrow\downarrow}(p^\mu)$ are the following ones:

$$\overline{\lambda}^S_\uparrow(p^\mu)\lambda^S_\downarrow(p^\mu) = -im, \overline{\lambda}^S_\downarrow(p^\mu)\lambda^S_\uparrow(p^\mu) = +im \quad (65)$$

$$\overline{\lambda}^A_\uparrow(p^\mu)\lambda^A_\downarrow(p^\mu) = +im, \overline{\lambda}^A_\downarrow(p^\mu)\lambda^A_\uparrow(p^\mu) = -im \quad (66)$$

$$\overline{\rho}^S_\uparrow(p^\mu)\rho^S_\downarrow(p^\mu) = +im \overline{\rho}^S_\downarrow(p^\mu)\rho^S_\uparrow(p^\mu) = -im \quad (67)$$

$$\overline{\rho}^A_\uparrow(p^\mu)\rho^A_\downarrow(p^\mu) = -im \overline{\rho}^A_\downarrow(p^\mu)\rho^A_\uparrow(p^\mu) = +im \quad (68)$$

All other conditions are equal to zero.

Implying that $\lambda^S(p^\mu)$ (and $\rho^A(p^\mu)$) answer for positive-frequency solutions; $\lambda^A(p^\mu)$ (and $\rho^S(p^\mu)$), for negative-frequency solutions, one can deduce the dynamical coordinate-space equations:[3]

$$i\gamma^\mu\partial_\mu\lambda^S(x) - m\rho^A(x) = 0\,, \quad (69)$$

$$i\gamma^\mu\partial_\mu\rho^A(x) - m\lambda^S(x) = 0\,, \quad (70)$$

$$i\gamma^\mu\partial_\mu\lambda^A(x) + m\rho^S(x) = 0\,, \quad (71)$$

$$i\gamma^\mu\partial_\mu\rho^S(x) + m\lambda^A(x) = 0\,. \quad (72)$$

They can be written in the 8-component form. This is just another representation of the Dirac-like equation in the appropriate Clifford Algebra. One can also re-write the equations into the two-component form, the Feynman-Gell-Mann equations. In the Fock space the operators of the charge conjugation and space inversions can be defined as in the previous Section. We imply the bi-orthonormal system of the anticommutation relations. As a result we have the following properties of the creation (annihilation) operators in the Fock space:

$$U^s_{[1/2]} a_\uparrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = -i a_\downarrow(-\mathbf{p}), \quad (73)$$
$$U^s_{[1/2]} a_\downarrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = +i a_\uparrow(-\mathbf{p}), \quad (74)$$
$$U^s_{[1/2]} b^\dagger_\uparrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = +i b^\dagger_\downarrow(-\mathbf{p}), \quad (75)$$
$$U^s_{[1/2]} b^\dagger_\downarrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = -i b_\uparrow(-\mathbf{p}), \quad (76)$$

that signifies that the states created by the operators $a^\dagger(\mathbf{p})$ and $b^\dagger(\mathbf{p})$ have different properties with respect to the space inversion operation, comparing with Dirac states (the case also regarded in[12]):

$$U^s_{[1/2]} |\mathbf{p}, \uparrow>^+ = +i|-\mathbf{p}, \downarrow>^+, \quad (77)$$
$$U^s_{[1/2]} |\mathbf{p}, \uparrow>^- = +i|-\mathbf{p}, \downarrow>^-, \quad (78)$$
$$U^s_{[1/2]} |\mathbf{p}, \downarrow>^+ = -i|-\mathbf{p}, \uparrow>^+, \quad (79)$$
$$U^s_{[1/2]} |\mathbf{p}, \downarrow>^- = -i|-\mathbf{p}, \uparrow>^- . \quad (80)$$

For the charge conjugation operation in the Fock space we have two physically different possibilities. The first one, *e.g.*,

$$U^c_{[1/2]} a_\uparrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = +b_\uparrow(\mathbf{p}), \quad (81)$$
$$U^c_{[1/2]} a_\downarrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = +b_\downarrow(\mathbf{p}), \quad (82)$$
$$U^c_{[1/2]} b^\dagger_\uparrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = -a^\dagger_\uparrow(\mathbf{p}), \quad (83)$$
$$U^c_{[1/2]} b^\dagger_\downarrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = -a^\dagger_\downarrow(\mathbf{p}), \quad (84)$$

in fact, has some similarities with the Dirac construct. The action of this operator on the physical states are

$$U^c_{[1/2]} |\mathbf{p}, \uparrow>^+ = +\,|\mathbf{p}, \uparrow>^- ,$$
$$U^c_{[1/2]} |\mathbf{p}, \downarrow>^+ = +\,|\mathbf{p}, \downarrow>^-, \quad (85)$$
$$U^c_{[1/2]} |\mathbf{p}, \uparrow>^- = -\,|\mathbf{p}, \uparrow>^+,$$
$$U^c_{[1/2]} |\mathbf{p}, \downarrow>^- = -\,|\mathbf{p}, \downarrow>^+ . \quad (86)$$

But, one can also construct the charge conjugation operator in the Fock space which acts, *e.g.*, in the following way:

$$\widetilde{U}^c_{[1/2]} a_\uparrow(\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = -b_\downarrow(\mathbf{p}), \quad (87)$$
$$\widetilde{U}^c_{[1/2]} a_\downarrow(\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = -b_\uparrow(\mathbf{p}), \quad (88)$$
$$\widetilde{U}^c_{[1/2]} b^\dagger_\uparrow(\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = +a^\dagger_\downarrow(\mathbf{p}), \quad (89)$$
$$\widetilde{U}^c_{[1/2]} b^\dagger_\downarrow(\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = +a^\dagger_\uparrow(\mathbf{p}). \quad (90)$$

Therefore,

$$\widetilde{U}^c_{[1/2]} |\mathbf{p}, \uparrow>^+ = -\,|\mathbf{p}, \downarrow>^-,$$
$$\widetilde{U}^c_{[1/2]} |\mathbf{p}, \downarrow>^+ = -\,|\mathbf{p}, \uparrow>^-, \quad (91)$$
$$\widetilde{U}^c_{[1/2]} |\mathbf{p}, \uparrow>^- = +\,|\mathbf{p}, \downarrow>^+,$$
$$\widetilde{U}^c_{[1/2]} |\mathbf{p}, \downarrow>^- = +\,|\mathbf{p}, \uparrow>^+ . \quad (92)$$

Next, by straightforward verification one can convince ourselves about correctness of the assertions made in [11b] (see also[8]) that it is possible to have a situation when the operators of the space inversion and the charge conjugation commute each other in the Fock space. For instance,

$$U^c_{[1/2]} U^s_{[1/2]} |\mathbf{p}, \uparrow>^+ = +i U^c_{[1/2]} |-\mathbf{p}, \downarrow>^+ = +i|-\mathbf{p}, \downarrow>^-, \quad (93)$$
$$U^s_{[1/2]} U^c_{[1/2]} |\mathbf{p}, \uparrow>^+ = U^s_{[1/2]} |\mathbf{p}, \uparrow>^- = +i|-\mathbf{p}, \downarrow>^- . \quad (94)$$

The second choice of the charge conjugation operator answers for the case when the $\widetilde{U}^c_{[1/2]}$ and $U^s_{[1/2]}$ operations anticommute:

$$\widetilde{U}^c_{[1/2]} U^s_{[1/2]} |\mathbf{p}, \uparrow>^+ = +i\widetilde{U}^c_{[1/2]} |-\mathbf{p}, \downarrow>^+ = -i\,|-\mathbf{p}, \uparrow>^-, \quad (95)$$
$$U^s_{[1/2]} \widetilde{U}^c_{[1/2]} |\mathbf{p}, \uparrow>^+ = -U^s_{[1/2]} |\mathbf{p}, \downarrow>^- = +i\,|-\mathbf{p}, \uparrow>^- . \quad (96)$$

Finally, the time reversal *anti-unitary* operator in the Fock space should be defined in such a way that the formalism to be compatible with the CPT theorem. If we wish the Dirac states to transform as $V(T)|\mathbf{p}, \pm 1/2 >= \pm\,|-\mathbf{p}, \mp 1/2>$ we have to choose (within a phase factor), Refs.:[19,22]

$$S(T) = \begin{pmatrix} \Theta_{[1/2]} & 0 \\ 0 & \Theta_{[1/2]} \end{pmatrix} \quad . \quad (97)$$

Thus, in the first relevant case we obtain for the $\Psi(x^\mu)$ field, composed of $\lambda^{S,A}$ or $\rho^{A,S}$ 4-spinors

$$V^T a^\dagger_\uparrow(\mathbf{p}) (V^T)^{-1} = a^\dagger_\downarrow(-\mathbf{p}),$$
$$V^T a^\dagger_\downarrow(\mathbf{p}) (V^T)^{-1} = -a^\dagger_\uparrow(-\mathbf{p}), \quad (98)$$
$$V^T b_\uparrow(\mathbf{p}) (V^T)^{-1} = b_\downarrow(-\mathbf{p}),$$
$$V^T b_\downarrow(\mathbf{p}) (V^T)^{-1} = -b_\uparrow(-\mathbf{p}), \quad (99)$$

that is not surprising.

4. The Conclusions

Thus, we proceeded as in the textbooks and defined the parity matrix as $P = \gamma_0$, because the helicity 4-spinors satisfy the Dirac equation, and the 2nd-type λ-spinors satisfy the coupled Dirac equations. Nevertheless, the properties of the corresponding spinors appear to be different with respect to the parity both on the first and the second quantization level. This result is compatible with the statement made by

Berestetskii, Lifshitz and Pitaevskii. We defined another charge conjugation operator in the Fock space, which also transforms the positive-energy 4-spinors to the negative-energy ones. In this case the operators P and C commute (instead of the anticommutation in the Dirac case), that is related to the eigenvalues of the charge operator, as in the Foldy and Nigam paper.

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Are Maxwell's Equations Fundamental?

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We explore the concepts of the manner in which Maxwell's equations and electrodynamic phenomena occupy a role in current standard model (SM) and grand unification theories (GUT). We process from the foundations of Hamilton-Jacobi (HJ) paired variable mechanics which underlies the quantum formalism in terms of phase space as (x,p) and (E,t) and also we have demonstrated that both the Poisson equation, the HJ theory are basic to the structure of general relativity. In this paper, we examine a unique approach to electromagnetism in terms of a paired variable or conically conjugate formalism. We have expanded this paired variable formalism to a model of the unification of the four force fields in a multidimensional geometry. This geometry is called the Descartes geometry in which we have formulated a group theoretical model. The electromagnetic paired variable formalism occupies a subset of our larger set of group theoretical operations in our attempt to formulate quantum gravity. Comparison is made between our group theoretical approach and the field theoretical quantum relativistic approach of quantum electrodynamics (QED). The drive for completing Einstein's vision of a unified field theory is basic to our approaches.

Keywords: Maxwell, Unified theory, Canonically conjugate variables

1. Introduction

Brian Josephson expressed the idea that he sees no reason that Maxwell's explanations are fundamental [1]. Are they emergent or basic? In the Unified Four Force field approach, of course, the electro-weak force combines the electromagnetic force, in which Maxwell's Equations combined electric and magnetic fields, which are then combined with the weak decay force [2, 3, 4]. What, of course, is interesting is that electromagnetism and gravity are long-range $1/r^2$ forces, where r is the distance between charges (EM) or matter bodies (Gravity), and also that of the strong and weak nuclear forces. The coupling constant strengths also vary so greatly between the strong force, α_s of almost 1 and the gravitational tension which has a force, $\alpha_g \sim 10^{-39}$.

The introduction of the quantification approach to EM theory yields quantum electrodynamics or QED. This theory is a relativistic form of Maxwell's Equation. The QED theory is extended to the strong force interactions, as quantum chromo-dynamics, QCD. This approach takes us to the possible development of a theory of Quantum Gravity. The fundamental difficulty to such an approach is the linear supposition of the quantum theory and the nonlinear form of the gravitational force. This is the fundamental problem of expanding the standard model to a Grand Unification Theory (GUT).

The superstring theory in M dimensions has four noncompactified (spacetime) and seven compactified dimensions for the four force formalism of strong, electroweak and gravitational force. The eleven dimensional supergravity involves the compactification such as or a Calabi-Yau manifold. The superstring in eleven dimensions, four of which are regular dimensions and seven compactified dimensions for unifying the strong, electroweak (electromagnetism and weak nuclear decay) and gravitational forces. We develop and present here a new multidimensional group theoretical approach in which we use extended dimensions. We start from an eleven-extended dimensional space called the Descartes space [4-17]. (See Tables 1 and 2.)

2. Multidimensional Geometries, A Different Approach to Unified Theory and the Role of Multidimensional Electromagnetism

We have developed group multiplication tables for an idealized universe [4, 11]. These relations lead to an extended set of canonically conjugate relations phase spaces and also a new set of Minkowski space-like and time-like relations. We apply this approach to expand and formulate a new relationship of classical gravitation and relativistic interpretation of physical variables, expressed in terms of Planck units [16] and in terms of universal constants. We assume the constancy of the universal constants throughout an idealized universe [17, 18]. The group theoretical approach of the canonically conjugate variables yield an interesting set of paired variable phase spaces which we describe with the context of the Hamilton-Jacobi classical mechanics, quantum mechanics and the special relativistic Minkowski spaces [9].

Through this approach, we formulate the basic structure of a multidimensional geometry, which demonstrates a possible fundamental relationship of the quantum theory and the relativistic Minkowski formalism. We generalize the quantum theory and relativity theory in the multidimensional Descartes space. We also expand our formalism to include electromagnetic Planck units, which we have developed [4, 7].

Fundamental to the structure of physics are canonically conjugate variables. We can represent the relations of the two variables in a phase space in which each variable is represented by each of two orthogonal axis. In the Hamilton-Jacobi mechanics formalism we can construct a phase space dimension from x (space) and p, momentum [18]. For x, p we have the well-known Heisenberg Uncertainty principle in quantum theory, $\Delta x \, \Delta p \geq \hbar$.

We can construct a phase space of E, energy and t, time giving us the form of the Heisenberg relation, $\Delta E \, \Delta t \geq \hbar$. This notation is short hand for $xp - px \geq \hbar$ forming a non-Abelian algebra where $xp - px \neq 0$ and $Et - tE \neq 0$. If two canonically conjugate variables A and B then if $AB - BA = 0$ then A and B comprise the elements of an Abelian algebra.

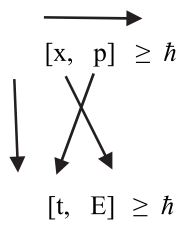
In the classical domain, our canonical conjugate variable form an Abelian algebra and associated group, where the elements of the algebra form E^{alg} = the group. Unlike some of our previous work [8] where we considered "rolled up" small dimensions in our multidimensional Descartes geometry, we coincide the use of an extended dimensional geometry [16].

In [6, 10] we utilize the Descartes geometry in general relativity to give closed cosmological solutions to Einstein's field equations. In this paper we apply our formalism to special relativity only. The key to our approach is to develop generalized canonically conjugate variables and generalized phase spaces in classical electromagnetic and quantum theory and special relativity. John A. Wheeler [19] attempted to geometerize electromagnetism in analogy to Einstein geometerization of gravity in general relativity. We expand on Wheeler's approach to derive a better more accurate formalism of the electromagnetic and gravitational geometerization with a more complete set of Planck units.

3. Generalized Canonically Conjugate Variables

We consider the pair relations as canonical pairs as elements of non-Abelian algebras. That is, the canonically conjugate pair of variables of physical variables such as x, p, E, t form commutation relations that are non-zero but are equal to simple sets of universal constants. The generalized set of commutation relations can be expressed in terms of a set of equivalent representations of universal constants as $\hbar$, c, G, Q, and k. For Planck's constant, $\hbar$ the velocity of light c the gravitational constant, G and the Bultzman constant K one charge Q which we can take as e, the charge on one elementary particle such as one electron. These constants represent a limit on and ultimate source of obtainable information of the physical world. The Heisenberg relations, in terms of E, t and x, p, represent a limit on observation of physical phenomena as relates to the relative magnitude of $\hbar$ in comparison to other universal constants where $\hbar$ is the "quantum of action" as termed by Planck.

We have the well-known relations, horizontal arrows. New non-Abelian algebras may be formed for vertical and diagonal arrows



(1)

In this section, we consider only those commutation relations that give $\hbar$, ℓ, ℓ' and $F = c^4/G$ where $\ell' = c\hbar$ and $\ell' = \hbar/c$.

Note x is the dimension of length or distance denoted as script ℓ, in our Tables. (See Tables 1 and 2.) This is done in order to not have confusion with our two new constants, ℓ' and ℓ. As discussed by Bohr and others the canonically conjugate pairs (x, p) and (t, E) relate in a quantum mechanical manner to $\hbar$ as a limit on the simultaneous observability of x and p and also E and t. These relations we call horizontal relations as the relations of x to p and t to E in Eq. (1). See the horizontal arrow. At the vertical relation between the four variables in Eq. (1) we see that t is the fourth component of E and t is the fourth component of p, in Einstein's four vector Minkowski light cone notation [4, 7, 8].

$$x' = (\underline{x}, t) \tag{2a}$$

and

$$P' = (\underline{p}, E) \tag{2b}$$

The first new relations are obtained paired relations between the diagonal pairs of quantum variables (indicated by the diagonal arrows in Eq. (1) in terms of the constants). These pairs are E, x and p, t and form the relations, in $\hbar$ and c as,

$$[E, x] \geq c\hbar \equiv \ell' \tag{3a}$$

$$[p, t] \geq \hbar/c \equiv \ell, \tag{3b}$$

The commutation relations between one space-like component of a four vector with the time-like component of a four vector is equal to ℓ or ℓ'. Let us examine the third possibility of the relation between pairs. We form the vertical canonical as pairs (x, t) and (p, E). As before, we are considering one component of x, is an isotopic space as $x_1 = x_2 = x_3$ or x, y, z and similarly for $p_1 = p_2 = p_3$ as p_x, p_y, p_z.

We consider the Poincare invariance as the homogenatic and isotropy of our new universal expanded dimensional space or Descartes space [14].

$$[x, t] = (\ell' \ \ell)/F = \hbar/F \ \ell'/cF \tag{4a}$$

and

$$[p, E] = = (\ell'\ell)^{1/2} F = \hbar/F = \ell' cF \tag{4b}$$

Note that P = cF is the variable power, [4] the force, F, is propionate in Einstein's field equation in the stress energy tensor $F = c^4/c$. So we have a generalized set of interrelated canonically conjugate relations or paired variables in terms of universal constants. We can also write [x, t] as $[x_1, x_4]$ and [p, E] and $[p_1, p_4]$. We have demonstrated four new cases of the generalized Heisenberg relations.

It should be stated that the present assumptions of isotropy represent a restricted case. Force causes non-uniformity or a stress in the spacetime manifold. In the absence of non-equilibrium forces there is a measure of the integratibility or homomorphisty or the space. We term the "almost proposition" or partial symmetries where commutation relations are not zero, for example, $\hbar$ would not exist in an isotopic space such as in classical mechanics. If all canonical commutation relations yield zero there would be no structure in spacetime and incidentally no quantum physics. The structure that does arise, quantum and classical physics relates to differentially or diffeomorphycity. The nonuniformity also gives an absolute orientation in space in a geometrical form and Mach's principle is a macroscopic description of an absolute reference frame from an orientation in space. Contrary to popular opinion, Einstein's Relativity Theory does not preclude an absolute non-inertial reference frame, it just does not speak of it. It deals with preferred, frames of reference [4] for the velocity of light c and G gravitational constant. The stress energy term is given by

$$\frac{8\pi G}{c^4} T_{\mu\nu} = \frac{8\pi}{F} T_{\mu\nu} \tag{5}$$

Now for the new canonically conjugate pairs for the vertical relations in Eq. (1).

We have,

$$[x, t] = (\ell'/F)^{1/2} (\ell \ /F)^{1/2} = (\ell'/cF) = \hbar/F \tag{6a}$$

for $\ell'/c = \hbar$ we define $\ell' \equiv \hbar/c$ as before.

This approach relates to the non-conservation of parity in weak interactions, and relates to the recent work on chiral symmetries.

We have as Planck units expressed in F [4, 6] from Table 3 [20].

$$x = (\ell'/F)^{1/2} \quad p = (\ell \ F)^{1/2} \qquad \text{(7a), (7b)}$$

$$t = (\ell \ F)^{1/2} \quad E = (\ell' F)^{1/2} \qquad \text{(7c), (7d)}$$

Then we have the non-Abelian commutation relations,

$$[x, p] = (\ell' \ \ell)^{1/2} = \hbar \tag{8a}$$

$$[t, E] = (\ell \ \ell')^{1/2} = \hbar \tag{8b}$$

From Eq. (1) we have for the horizontal arrows in which are the usual paired Heisenberg relations. We form new relations which we represented as the canonically conjugate paired variables of new Heisenberg relations. (See Eqs. (8a) and (8b))

The F/ℓ' has cosmological significance in Einstein's field equations. The term

$$\frac{8\pi G}{c^4} T_{\mu\nu} = \frac{8\pi}{F} \cdot \frac{F^2}{\ell'} = 8\pi\left(\frac{F}{\ell'}\right) \qquad (9)$$

In Figure 1, we have six canonically conjugate variables, four of which are new, represented by ε = 3, 4, 5 and 6 [4-6]. The usual Heisenberg relations are represented by t = 1 and 2. These six relations form a subject of a non-Abelian group theory. (See Tables 1 and 2.)

GENERALIZED HEISENBERG RELATIONS

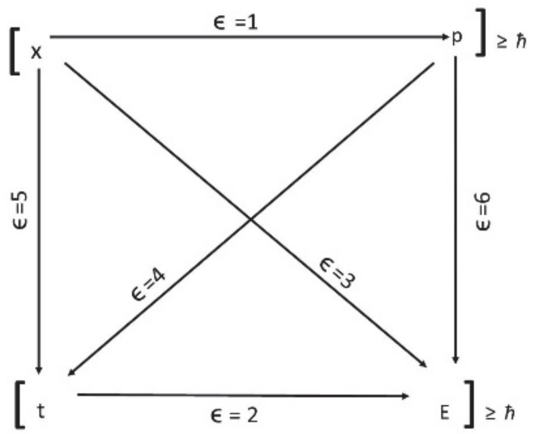


Figure 1. We present six canonically conjugate variable, two are the usual ones and as well as the standard Minkowski metric variables x, t.

QUANTUM INDETERMINISM

New ε = 3,4	New ε = 5,6
$[E, x] > c\hbar$	$[x, t] > \hbar/F$
$[p, t] > \hbar/c$	$[p, E] > \hbar f$

EINSTEIN FOUR VECTORS

$\underset{\sim}{X} = (\underline{x}, t)$

$\underset{\sim}{P} = (\underline{p}, E)$

We consider a fundamental frequency for $cF = \hbar\omega^2$ or $\hbar/F = c/\omega^2$ so that $\omega^2\hbar = cF$ quantization commutation relations of $(x, t) = \ell'/cF$ and the quantization relations $[x, t] = \ell'/\hbar\omega^2$. Also these relations can be obtained from the (p, E) commutations. In relating fundamental to usual quantization, we have $p = cF = \hbar\omega^2$ in Table 1. We can uniquely express the Universal Force in terms of the frequency as,

$$F = \left(\frac{\hbar\omega^2}{c2}\right) = \frac{\omega 2}{\ell} \qquad (10)$$

and,

$$\omega = \pm\left(\frac{cF}{\hbar}\right)^{1/2} = \pm(p\hbar)^{1/2} = \pm(F\ell) \qquad (11)$$

From Table 2, have $c = \omega x = 1.91 \times 10^{43}$ Hz x $1.60 \times 10^{-33\mathrm{cm}} = c = 3x/10^{10}$ cm/sec and $\omega x = c/x = 1/t$ where $t = 5.36 \times 10^{-44}$ sec.

We have

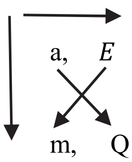
$$\omega = \frac{c}{x} \; or \; c = \frac{\omega}{x} \; and \; \ell' / \ell' = \frac{1}{c^2} \qquad (12)$$

The fundamental Force has an associated space frequency. An aspect of the existence of a rotational frequency or rate of rotation of the universe is expressed in terms of a geometrical form, π. The quantity π as a rotation through $2\pi R$ or 2πx introduces π into many physical expressions such as the $\frac{1}{2}\pi$ in the electric and magnetic energy and the 8π in the 8π G/c^4 $T_{\mu\nu}$ term in Einstein's field equations.

4. Electric and Magnetic Parried Variable Relations, Canonical Conjugate Phase Spaces and Metric Spaces

In order to "geometrize the electromagnetic field "in an extended Descartes," one must develop canonically conjugate phase spaces and a metric for electromagnetic field is what J.A. Wheeler attempted to do by introducing the variable $x = (G\ \hbar/c^3)^{1/2}$, termed the Wheeler "wormhole" as representing a discontinuous multi-connected structure in spacetime [19]. The geometry of spacetime is represented by a complete set of physical variables, which can be represented as a set of Descartes dimension as elements of groups.

Electromagnetic variables are also considered as representing dimensionality in the Descartes geometric interpretation. Therefore, another set of multi-component vectors such as the four vectors (x, t) and (p, E) exist involving electromagnetic and mechanical quantum variables do as given in Tables 1 and 2. As we stated before many such relations exist interconnecting paired variables geometrically in a metrical structure in the spacetime manifold. Let us examine the four relations as indicated by the oneness forming both canonically conjugate and also metrical variables.



(13)

Where a is the acceleration, a three vector, E is the electric field, a three vector, and m is the mass and Q is the charge which are scalars. Note that the "format" of Eq. (13) is similar to that of Eq. (1).

Again considering the generalized "Heisenberg" relation between quantities related by the vertical arrow we have,

$$[\text{a}, \text{m}] = \text{F} \tag{14}$$

the fundamental force and the other vertical relations is,

$$[E, \text{Q}] = \text{F} \tag{15}$$

as new "four vector" relations are formed with fundamental physical variables where the quantized electric field $E = \frac{c^4}{GQ} = \frac{F}{Q}$ and the quantized charge, Q = e, is considered to be a universal constant [21, 22].

Let us consider the horizontal relations (horizontal arrows) for (a, E) and (m, Q). We have,

$$[a, E] = \left(\frac{c^2F}{\ell}\right)^{1/2} \left(\frac{F}{Q}\right) = \left(\frac{c^2F^3}{\ell Q^2}\right)^{1/2} \tag{16a}$$

which can be expressed as

$$[a, E] = \left(\frac{\rho F}{\varepsilon}\right)^{1/2} \tag{16b}$$

For the quantal density, $\rho = \frac{c^5}{G^2\hbar} = \frac{F^2}{c^2}\ell$ and where ε is the permittivity in quantized variable it forms given as, $\varepsilon = \frac{Q^2}{c\hbar} = \frac{Q^2}{\ell'}$

$$[m, Q] = \frac{\ell \text{F}}{c^2} \tag{17a}$$

which can be expressed as,

$$[m, Q] = (\varepsilon F\ell)^{1/2} \tag{17b}$$

where ε is the of permittivity.

Lastly, let's consider the diagonal relations,

$$[a, \text{Q}] = Q\left(\frac{c^2F}{\ell}\right)^{1/2} = \left(\frac{c^2FQ^2}{\ell}\right)^{1/2} \tag{18}$$

where one component of a is considered, and for [E, m],

$$[\text{E}, \text{m}] = \frac{F}{Q}\left(\frac{\ell F}{\text{c}^2}\right)^{1/2} = \left(\frac{F^3\ell}{\text{Q}^2\text{c}^2}\right)^{1/2} \tag{19}$$

Where one component of E is considered. We can express [a, Q] as

$$[a, \text{Q}] = \left(\frac{EF}{\ell'}\right)^{1/2} \tag{20}$$

and [E, m] as

$$[E, \text{m}] = \left(\frac{\rho F}{\varepsilon\ell'}\right)^{1/2} \tag{21}$$

Again the quantized form of E is used, as $E = \frac{Q^2}{\ell'}$

Let us look at some invariant "light cone" relations corresponding to the electromagnetic Generalized Heisenberg Relations just presented.

We turn now to the invariant relations such as (vertical relations)

$$T_1{}^2 = a_x{}^2 + a_y{}^2 + a_z{}^2 - \left(\frac{c^4}{\ell^2}\right) m^2 \tag{22}$$

and

$$T_2{}^2 = E_x{}^2 + E_y{}^2 + E_z{}^2 - \left(\frac{Q^4}{\text{F}^2}\right) Q^2 \tag{23}$$

For the horizontal relations, we have for [a, E],

$$T_3{}^2 = a_x{}^2 + a_y{}^2 + a_z{}^2 - \left(\frac{Q^2c^2}{\ell \text{F}}\right)\left(E_x{}^2 + E_y{}^2 + E_z{}^2\right) \tag{24}$$

and for [m, Q]

$$T_4{}^2 = m2 + \left(\frac{c\hbar}{\text{GQ}^2}\right) Q^2 \tag{25}$$

For the diagonal relations [a, Q]

$$T_5{}^2 = a_x{}^2 + a_y{}^2 + a_z{}^2 - \left(\frac{c^2F}{\ell Q^2}\right) Q^2 \quad (26)$$

and for [E, m]

$$T_6{}^2 = E_x{}^2 + E_y{}^2 + E_z{}^2 - \left(\frac{c^2F}{\ell Q^2}\right) m^2 \quad (27)$$

Symmetries become apparent such as for [a, E] in Eq. (16a) and [E, m] in Eq. (19). Also [m, Q] in Eqs. (17a) and (17b) and [a, Q] in Eq. (18).

$$\left(\frac{c^2}{\ell}\right) [a, E] = [\underline{E}, m] \quad (28)$$

and

$$\left(\frac{\ell}{c^2}\right) [m, Q] = [\underline{a}, Q] \quad (29)$$

Also, Eq. (26) are related by the same set of constants,

$$\left(\frac{C^2F}{\ell Q^2}\right) = \frac{[(a,E)]^2}{F^2} \quad (30)$$

Wheeler [19] had attempted to geometrize the electromagnetic field and derive electric charge from his discontinuous topological structure describable in terms of the "wormhole" length, ℓ, where ℓ, represented the size of the diameter of the mouth of a wormhole structure in the spacetime manifold in terms of fine Planck Units. Electrical charge is then conjectured to be due to the emission of field lines from and entering into the mouths of a wormhole embedded in the space [23]. Many such structures would give rise to virtually produced pairs of oppositely charged particles [24]. This formulation of a geometric description of the electromagnetic field, briefly described above, is as developed in analogy to the geometric interpretation of Einstein's gravitational field, but not with the overwhelming success as in general relativity. It is suggested here that a first step in geometrizing the electromagnetic field is to assign a generalized metric in terms of many or all physical variables relevant to the particular force field to be considered. A method for constructing such a metric has been presented in in Ref. 9.

5. Conclusion

We have developed a unique, new multidimensional geometries for N > 4. We define a set of non-Abelian canonically conjugate physical variables expressed in terms of universal constants. This construct involves the extension of the Planck units. The approach to develop a quantum gravity model in terms of a ten and eleven dimensional Descartes space. We also construct a multidimensional space of metrical space, one subset space is the Minkowski four space.

We expand the Descartes space to include electromagnetic variables such as electric, magnetic, charge, etc. giving an expanded space of N ~ 20 dimension. Hence, we attempt to reconcile the nuclear and gravity strong coupling constant, 1 and weak coupling constant $\sim 10^{-14}$ Forces and gravity coupling constant $\sim 10^{-39}$ electromagnetic field, coupling constant $\sim 10^{-2}$. In the Standard Model, SM we consider the electroweak force. In our model and the SM Model we conclude that Maxwell's equations are part of our picture of fundamental physics. In Table 2, the universal constants we use are given in Ref. 23.

In the evaluation of the quantized variables in Table 2, we have used the values of $\ell = \hbar/c = 3.50 \times 10^{-38}$ gm-cm and $\ell' = c\hbar = 3.15 \times 10^{-17}$ erg-cm (or $\frac{gm-cm^3}{sec^2}$). Two cases are considered for Planck like units that are expressible in terms of charge, Q. We express them in terms of Q, where quantities are expressed in terms of Q = e = 4.80×10^{-10} $gm^{1/2}$ $cm^{3/2}$ /sec. This value by use of the fine structure constant relation for $\alpha = \frac{e^2}{\hbar c}$ = 1/137.04 gives us the value $e^2 = 2.30 \times 10^{-19}$ or $\frac{gm-cm^3}{sec^2}$.

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Appendices: Tables 1 to 3

TABLE 1. Quantities

	Planck like Units	Planck like units in terms of Force	Planck like units in terms ℓ, ℓ'
length	$\ell = \left(\frac{G\hbar}{c^3}\right)^{1/2}$	$\ell = \left(\frac{c\hbar}{F}\right)^{1/2}$	$\ell = \left(\frac{\ell'}{F}\right)^{1/2}$
time	$t = \left(\frac{G\hbar}{c^5}\right)^{1/2}$	$t = \left(\frac{\hbar}{cF}\right)^{1/2}$	$t = \left(\frac{\ell}{F}\right)^{\frac{1}{2}}$
Mass	$m = \left(\frac{c\hbar}{G}\right)^{1/2}$	$m = \left(\frac{\hbar F}{c^3}\right)^{1/2}$	$m = \left(\frac{\ell F}{c^2}\right)^{1/2}$
Energy	$\mathrm{E} = \left(\frac{c^5\hbar}{G}\right)^{1/2}$	$\mathrm{E} = (c\hbar F)^{1/2}$	$\mathrm{E} = (\ell' F)^{1/2}$
Momentum	$\mathrm{p} = \left(\frac{c^3\hbar}{G}\right)^{1/2}$	$\mathrm{p} = \left(\frac{\hbar F}{c}\right)^{1/2}$	$\mathrm{p} = E = \ell' F^{1/2}$
Angular momentum	$L = \hbar$	$\mathrm{L} = \hbar$	$\mathrm{L} = \hbar$
Force	$\mathrm{F} = \mathrm{c}^4/\mathrm{G}$	F = F	F = F
velocity	c = c	c = c	c = c
acceleration	$\mathrm{a} = \left(\frac{c^7}{G\hbar}\right)^{1/2}$	$\mathrm{a} = \left(\frac{c^3 F}{\hbar}\right)^{1/2}$	$\mathrm{a} = \left(\frac{c^2 F}{\ell}\right)^{1/2}$
power	$p = \frac{c^5}{G}$	$p = cF$	$p = cF$
pressure	$P = \frac{c^7}{G^2\hbar}$	$P = \frac{F^2}{c\hbar}$	$P = \frac{F^2}{\ell'}$
density	$\rho = \frac{c^5}{G^2\hbar}$	$\rho = \frac{F^2}{c^3\hbar}$	$\rho = \frac{F^2}{c^2\hbar}$
frequency	$\omega = \left(\frac{c^5}{G\hbar}\right)^{1/2}$	$\omega = \left(\frac{cF}{\hbar}\right)^{1/2}$	$\omega = \left(\frac{F}{\ell}\right)^{1/2}$
temperature	$T = \frac{1}{k}\left(\frac{c^5\hbar}{G}\right)^{1/2}$	$T = \frac{1}{k}(c\hbar F)^{1/2}$	$T = \frac{1}{k}(\ell F)^{1/2}$
entropy	S = k	S = k	S = k

Electric monopole	Q = Q	Q = Q	Q = Q
current	$J = Q\left(\frac{c^5}{\hbar G}\right)^{1/2}$	$J = Q\left(\frac{cF}{\hbar}\right)^{1/2}$	$J = Q\left(\frac{F}{\ell}\right)^{1/2}$
Electric field Strength	$E = \left(\frac{c^4}{G}\right)\frac{1}{Q}$	$E = \frac{F}{Q}$	$E = \frac{F}{Q}$
Permittivity	$\varepsilon = \frac{Q^2}{c\hbar}$	$\varepsilon = \frac{Q^2}{c\hbar}$	$\varepsilon = \frac{Q^2}{\ell'}$
Electric dipole	$\underline{P} = Q\left(\frac{G^3}{c\hbar}\right)^{1/2}$	$\underline{P} = Q\left(\frac{F}{c\hbar}\right)^{1/2}$	$\underline{P} = Q\left(\frac{F}{\ell'}\right)^{1/2}$
Electrical potential or emf	V_E (volts) $= \left(\frac{c^5\hbar}{G}\right)^{1/2}$	$V_E = (c\hbar F)^{1/2}$	$V_E = (\ell' F)^{1/2}$
Electric flux	$\Phi_E = c\hbar$	$\Phi_E = c\hbar$	$\Phi_E = \ell'$
Capacitance, farad	$C = Q^2\left(\frac{G}{\hbar c^3}\right)$	$C = Q^2\left(\frac{c}{\hbar F}\right)^{1/2}$	$C = Q^2\left(\frac{F}{\ell'}\right)^{1/2}$
Resistance, ohm	$R = \hbar/Q^2$	$R = \hbar/Q^2$	$R = \hbar/Q^2$
Magnetic monopole	$\mathcal{M} = \mathcal{M}$	$\mathcal{M} = \mathcal{M}$	$\mathcal{M} = \mathcal{M}$
Magnetic currents	$J_B = \mathcal{M}/t$	$J_B = \mathcal{M}\left(\frac{cF}{\hbar}\right)^{1/2}$	$J_B = \mathcal{M}\left(\frac{F}{\ell'}\right)^{1/2}$
Magnetic field Induction	$B = \frac{c^3}{G}\frac{1}{Q}$	$B = \frac{F}{cQ}$	$B = \frac{F}{cQ}$
Permeability	$\mu = \frac{\hbar}{c}\frac{1}{Q^2}$	$\mu = \frac{\hbar}{c}\frac{1}{Q^2}$	$\mu = \frac{\ell'}{Q^2}$
magnetic dipole moment	$u = \left(\frac{G\hbar}{c}\right)^{1/2} Q$	$u = \left(\frac{c^3\hbar}{F}\right)^{1/2} Q$	$u = \left(\frac{\ell' c^2}{F}\right)^{1/2} Q$
magnetic flux	$\varphi_B = \frac{\hbar}{Q}$	$\varphi_B = \frac{\hbar}{Q}$	$\varphi_B = \frac{\hbar}{Q}$
Induction, henry	$\mathcal{L} = \left(\frac{\hbar^3 G}{c^5}\right)^{1/2}\frac{1}{Q^2}$	$\mathcal{L} = \left(\frac{\hbar^3}{cF}\right)^{1/2}\frac{1}{Q^2}$	$\mathcal{L} = \left(\frac{c^2\ell^3}{F}\right)^{1/2}\frac{1}{Q^2}$

TABLE 2. Values

Planck like units	Planck like units in terms of Charge	Planck like units in terms ℓ, ℓ'
ℓ		1.60×10^{-33} cm
t		5.36×10^{-44} sec
m		2.22×10^{-5} gm
E		1.25×10^{16} ergs
P		$4.16 \times 10^{10}\ \frac{gm-cm}{sec}$
L		1.06×10^{-27} erg-sec
F		1.22×10^{49} dynes
a		$5.72 \times 10^{53}\ \frac{cm}{sec^2}$
p		$3.66 \times 10^{59}\ dyne\ \frac{cm}{sec}$
P		$4.75 \times 10^{114}\ \frac{dyne}{cm^2}$
ρ		$6.50 \times 10^{93}\ \frac{gm}{cm^3}$
ω		$1.91 \times 10^{43}\ \frac{cycles}{sec}$
T		3.60×10^{32} degrees
S		$1.34 \times 10^{-16}\ \frac{erg}{degree}$
Q = e	Q = e	$4.80 \times 10^{-10}\ \frac{gm^{1/2}\ cm^{3/2}}{sec}$
J	$1.87 \times 10^{43} \left(\frac{1}{sec}\right)$ Q	$9.00 \times 10^{33}\ \frac{gm\ cm^{3/2}}{sec^2}$
E	$1.22 \times 10^{49}\ (dynes)\frac{1}{Q}$	$2.55 \times 10^{58}\ \frac{gm^{1/2}}{cm^{1/2}\ sec}$
ε	$3.19 \times 10^{16} \left(\frac{1}{erg-cm}\right) Q^2$	$\frac{1}{137.04} = 7.3 \times 10^{-3}$
p	$6.20 \times 10^{32} \left(\frac{1}{cm-sec^{1/2}}\right)$	$2.98 \times 10^{23}\ \frac{gm^{\frac{1}{2}}\ cm^{\frac{1}{2}}}{sec^{\frac{3}{2}}}$
Φ_E		3.15×10^{-17} erg-cm
C	$3.48 \times 10^{43} \left(\frac{1}{sec}\right) Q^2$	$9.55 \times 10^{24}\ gm \left(\frac{cm}{sec}\right)^3$
R	$1.054 \times 10^{-27}\ (erg\ sec)\frac{1}{Q^2}$	$4.56 \times 10^{-9}\ \frac{sec}{cm}$
M = g	1.37×10^2	$6.60 \times 10^{-8}\ \frac{gm^{\frac{1}{2}}\ cm^{\frac{3}{2}}}{sec}$
ß	$4.05 \times 10^{38} \left(\frac{gm}{sec}\right)\frac{1}{Q}$	$8.48 \times 10^{47}\ gm \left(\frac{gm}{cm^3}\right)^{1/2}$
μ	$3.50 \times 10^{-38}\ (gm - cm)\frac{1}{Q_2}$	$1.52 \times 10^{-19} \left(\frac{sec}{cm}\right)^3$
u	$5.06 \times 10^{-23} \left(\frac{cm^5}{sec^3}\right)^{1/2}$ Q	$2.44 \times 10^{-32}\ gm^{1/2} \left(\frac{cm}{sec}\right)^4$
Φ_B	$1.05 \times 10^{-27}\ (erg - sec)\frac{1}{Q}$	$2.19 \times 10^{-38}\ \frac{(gm-cm)^{\frac{1}{2}}}{sec}$
$\mathcal{L}$	$1.78 \times 10^{74}\ (gm - cm^2)\frac{1}{Q^2}$	$7.70 \times 10^{92}\ \frac{sec^2}{cm}$

TABLE 3
Group Theoretical Canonical Conjugate Variables

Physical Variable	ℓ	t	m	E	p	F	L	c	a	p
ℓ	ℓ^2	$\hbar$/F	$\hbar$/c	c$\hbar$	$\hbar$	E	$\hbar\ell$	$\hbar$/m	c^2	cE
t	$\hbar$/F	t^2	$\hbar/c^2$	$\hbar$	$\hbar$/c	$(\hbar/c)^{1/2}$	$\frac{\hbar^3}{cF}$	ℓ	c	E
m	$\hbar$/c	$\hbar/c^2$	m^2	p^2	P^2/c	$\frac{p^3}{\hbar}$	$\frac{p^3}{F}$	$\frac{E}{c}$	F	Fp
E	c$\hbar$	$\hbar$	p^2	E^2	$\hbar$ F	$(c\hbar F^3)^{1/2}$	$(c\hbar F^3)^{1/2}$	$(c^3\hbar F)^{1/2}$	c^2F	$\frac{E^3}{\hbar}$
p	$\hbar$	$\hbar$/c	p^2/c	$\hbar$ F	p^2	$\frac{cp^3}{\hbar}$	$(\frac{\hbar^3F}{c})^{1/2}$	E	p	FE
F	E	$(\frac{\hbar^3}{c})^{1/2}$	$\frac{p^3}{\hbar}$	$(c^3\hbar F)^{1/2}$	$(\frac{F\hbar^3}{c})^{1/2}$	F^2	c$\hbar$	p	$(\frac{c^3F^3}{\hbar})^{1/2}$	cF^2
L	$\hbar\ell$	$(\frac{\hbar^3}{cF})^{1/2}$	$\frac{\hbar^2F}{c^2}$	$(c\hbar F)^{1/2}$	$(\frac{\hbar^3F}{c})^{1/2}$	$\hbar$ F	L^2	c$\hbar$	cE	E^2
c	$\hbar$ m	ℓ	E/c	$(c^3\hbar F)^{1/2}$	E	p	c$\hbar$	c^2	c^2/t	c^2F
a	c^2	c	F	c^2 F	p	$(\frac{c^3F}{\hbar})^{1/2}$	cE	c^2/t	a^2	$c\hbar t^3$
p	cE	E	Fp	$(c^2\hbar F)^{1/2}$	FE	cF^2	E^2	c^2F	$c\hbar t^3$	p^2

Quaternions and Elliptical Space
(*Quaternions et Espace Elliptique*[1])

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Translators Forward: This is generally a literal translation, terms such as *versor* and *parataxis* in use at the time are not updated to contemporary style, rather defined and briefly compared in an appendix. The decision to translate Lemaître's 1948 essay arose not merely because of personal interest in Lemaître's 'persona', Physicist - Priest, but from an ever increasing interest in Quaternions and more recent discovery of correspondence with Octonions in additional dimensional (XD) brane topological phase transitions (currently most active research arena in all physics), and to make this particular work available to readers interested in any posited historical value during the time quaternions were still considered a prize of some merit, which as well-known, were marginalized soon thereafter (beginning mid1880's) by the occluding dominance of the rise of vector algebra; indeed, Lemaître himself states in his introduction: "*Since elliptic space plays an increasingly important part ... I have thought that an exposition ... could present some utility even if the specialists ... must bear the judgment that it contains nothing really new*", but also because the author feels quaternions (likely in conjunction with octonions), extended into elliptical and hyperbolic XD spaces, especially in terms of the ease they provide in simplifying the Dirac equation, will be essential facilitators in ushering in post-standard model physics of unified field mechanics. The translator comes to realizes that 3rd regime Natural Science (Classical $\rightarrow$ Quantum $\rightarrow$ Unified Field Mechanics) will be described by a reformulated M-Theoretic topological field theory, details of which will be best described by Quaternion-Octonion correspondence.

QUATERNIONS AND ELLIPTICAL SPACE

The author applies the notion of quaternions, as practiced by Klein in the Erlangen program, to determine the fundamental properties of elliptical space[5].

Keywords: Elliptical space, Klein's Erlangen program, Quaternions

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[1] Original (*) note: Submitted February 8, 1948*, Pontificia Academia Scientiarum,* ACTA, Vol. XII, No. 8, pp. 57-80 [1, 2].
[2] Georges Lemaître [3].
[3] The Pontifical Academy of Sciences of Vatican City, established in 1936 by Pope Pius XI, promotes the progress of mathematical, physical, and natural sciences and the study of related epistemological problems.
[4] In the work of translation, effort was focused on *meaning*, and not maintaining more literal phraseology of 70 years ago, for example *certainly* might be substituted for *in all certainty.*
[5] Translation of Lemaître's original Latin abstract, termed *Summarium*.

1. Introduction

Quaternions were invented in 1843 by Sir William Rowan Hamilton. It is hard to imagine with what enthusiasm, and also with what confusion this awesome idea was developed by its author.

In the "Introduction to Quaternions" published in London (MacMillan, 1873) by P. Kelland and P.G. Tait, the first author declares: "*The first work of Sir Wm. Hamilton*" Lectures on Quaternions (1852), "*was very dimly and imperfectly understood by me and I dare say by others*". He added that the *Elements of Quaternions* (1865) [Hamilton] and even exposits that most of the work of his co-author P.G. Tait: *An Elementary Treatise on Quaternions* cannot be regarded as elementary.

The book itself in which these remarks were certainly taken in an elementary character, he even exaggerated in this direction, by presenting demonstrations of too familiar theorems for which the use of a new type of calculation does not seem to be justified.

However, the influence of Hamilton's discovery was very great. Not only did vector calculus, with its fecund notions of scalar product and vector product emerge, but also the development of elliptic geometry by Cayley, Clifford, etc., seems to have been strongly influenced by the new calculus, as shown by the title of one of these works: "Preliminary sketch on bi-quaternions" (1873).

I do not propose to disentangle the dense history of these discoveries, but by studying elliptic or spherical space, it appeared to me that quaternions provide extremely simple and elegant notations from which the properties of this space immediately flows.

Since elliptic space plays an increasingly important part in cosmogonic[6] theories, I have thought that an exposition which presupposes in the reader only elementary knowledge of analytic geometry could present some utility even if the specialists in the fields of algebra, geometry and history of science of the last century, must bear the judgment that it contains nothing really new.

For the history of the question, the reader may refer to treatises on geometry and particularly to the work of V. Blaschke, *Nicht Euklidische Geometrie und Mechanik* (Teubner 1942), which has more than one point in common with the present exposition but is addressed to a completely different category of readers.

[6] Cosmogony - theories of the origin of the universe. Cosmogony distinguishes itself from cosmology in that it allows theological argument.

2. Vectors

A vector will be represented by the geometric point of view and algebraic perspective.

In algebraic terms, the vector is obtained from the body of the real numbers, called a *scalar*, by introducing their new symbols not contained in the set of real numbers, and generally designated by the letters i, j, k.

Except for these three letters, whose employment is enshrined in use, we will assume that any Latin letter denotes a scalar, that is to say a real number.

A vector will be represented by

$$xi + yj + zk. \tag{1}$$

The addition of vectors and the multiplication by a scalar will be obtained by the ordinary rules of calculation as if i, j, k were numbers. The result of these operations will still be a vector.

Geometrically, the symbols i, j, k represents a basis, that is to say three vectors of unit length not situated in the same plane. We shall assume that this basis is orthogonal, that is, the three vectors i, j, k are mutually perpendicular.

Then, the three scalars x, y, z are the components, or orthogonal vector projections, onto the three vectors of the basis.

The components of the sum of two vectors are the sums of the components of these vectors.

3. Directions

It is customary to designate vectors by Greek letters. We will however deviate somewhat from this traditional notation by reserving Greek letters for the unitary vectors alone. That is to say to the vectors for which the sum of the squares of the components is equal to one.

We thought it necessary to introduce a shorter designation for the expression "unit vector". The term "direction" was deemed appropriate. Indeed, since a vector is a directed quantity, the unitary vector, whose magnitude is fixed once and for all, only indicates the direction thus the term direction is well suited to it.

4. Quaternions

The main idea of Hamilton has been to define the law of multiplying the symbols i, j, k in such a way that all the rules of the algebraic calculation remain valid except for one: The commutative property of multiplication. He thus founded the noncommutative algebra.

In this algebra, the value of a bridge product depends on the order of the factors.

Starting from the multiplication table of two of the symbols *i* and *j*,

$$i^2 = -1, \; j^2 = -1, \; ij = -ji = k \tag{2}$$

we can easily deduce from these formulas (by the application of ordinary rules of the calculus, taking care to respect the order in which the factors present themselves) that the analogous formulas obtained by circularly permuting the letters *i, j, k,*

$$k^2 = -1, \; jk = -kj = i, \; ki = -ik = j \tag{3}$$

are valid.

Applying these rules of computation to the product of a direction of components *x, y, z*, by another direction alpha of component *x', y', z'* we obtain

$$\begin{aligned}\alpha\alpha' = &-\left(xx' + yy' + zz'\right) \\ &+\left(yz' - zy'\right)i \\ &+\left(zx' - xz'\right)j \\ &+\left(xy' - yx'\right)k.\end{aligned} \tag{4}$$

This expression is formed with a scalar part and a vector part.

The usage has prevailed of calling the scalar product (dot product) the changed scalar part of sign, while the vector part is still what we call the vector product of the two vectors.

This aggregate of a scalar and a vector is called a quaternion.

5. Quaternion Conjugates

We can replace the three basis vectors *i, j, k* by another basis of opposite chirality, that is to say, presenting with the former the same relations as the right hand with the left hand.

Such a basis is

$$i' = -i, \; j' = -j, \; k' = -k \tag{5}$$

The relations that exist between the i', j', k' are analogous to those which exist between *i, j, k*. But the factors are transposed, that is to say, written in the reverse order.

For example,

$$k = ij = -ji \tag{6}$$

we deduce

$$k' = j'i' = -i'j'. \tag{7}$$

Suppressing the prime inflections as useless, would indicate that the quaternion conjugate is the same quaternion but referred to in the basis of opposite chirality. The conjugated quaternion will thus be obtained while retaining the scalar part and by changing the sign of the vector part or, if the quaternion is written as a product of quaternions, by multiplying the conjugates of the written factors in the reverse order.

6. Versors[7]

The product of a quaternion with a quaternion conjugate is a scalar, which is called the norm of the quaternion.

The norm of the product of two quaternions, Q and Q' is the product $\mathrm{QQ}'\mathcal{QQ}'$. But $\mathrm{Q}'\mathcal{Q}'$ the product of Q' by the quaternion conjugate $\mathcal{Q}'$ is the norm N' of Q', likewise $\mathrm{N} = \mathrm{Q}\mathcal{Q}$ is the norm of Q. The norm of the product of NN' is thus the product of the norm of the factors.

A quaternion whose norm is equal to one is called a 'versor'. The product of two versors is a versor.

One direction may be considered as a quaternion. It is a quaternion whose scalar part is zero.

Moreover, it is a versor. For if, in the formula of the product of two directions, we first make $\alpha' = \alpha$, this product is equal to minus one, so that the directions may be considered as roots of minus one.

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[7] *Versor*: - Rotations of Unit Quaternions. Term introduced by Hamilton in developing quaternions. Versor is sometimes used synonymously with "unit quaternion" with no reference to rotations. An algebraic parametrization of rotations. In classical quaternion theory, a versor is a quaternion of norm one (unit quaternion). Each versor has the form $q = \exp\left(ar\pi\right)$ $= \cos a + r \sin a, \; r^2 = -1, \; a \in \left[0, \pi\right]$, where the $r^2 = -1$ condition means that *r* is a 3D unit vector. In case $a = \pi / 2$, the versor is termed a right versor. The corresponding 3D rotation has the angle 2*a* about the axis *r* in axis–angle representation. The word is derived from Latin *versare* "to turn" or *versor* "the turner").

The conjugate of the vector is this vector of changed sign, the norm, product of the vector by the conjugate vector, is thus the square of the changed sign, that is to say, plus one. A direction is therefore a versor.

If u is the scalar and $v\gamma$ the magnitude v and direction γ the vector of a versor V, we will have

$$V = u + v\gamma \tag{8}$$

with

$$u^2 + v^2 = 1 \tag{9}$$

We can therefore write

$$u = \cos c, \quad v = \sin c \tag{10}$$

and with

$$V = \cos c + \gamma \sin c \tag{11}$$

If

$$V = \alpha\alpha' \tag{12}$$

- $\cos c$ is the scalar product of the two directions α and α' while the vector product is a vector of magnitude $\sin c$ and of direction γ.

We can thus interpret geometrically γ and c, saying that γ is a direction perpendicular to the plane of the two directions α and α', and that c is the supplement of the angle formed by these directions, that is, the external angle of these two directions.

Conversely, each versor is the product of two directions located in a plane perpendicular to the versor vector closing an angle, in the proper direction, equal to $\pi - c$.

The formulas of analytic geometry furnish an algebraic equivalent of these geometrical notions. They make it possible to establish the result which we have just obtained even if we take a purely algebraic point of view.

The product of two directions is a direction only when the product scalar is zero, that is to say when the two directions are perpendicular.

If α and β are perpendicular, that is to say if

$$\alpha\beta = -\beta\alpha \tag{13}$$

then this product is equal to a direction γ which is perpendicular to an α and a β.

7. Exponential Notation

It is very useful to represent a versor using the notation

$$V = e^{c\gamma} \tag{14}$$

which we will explain.

The exponential defines itself by its development in a series of powers

$$e^{c\gamma} = \sum_{n=0}^{\infty} \frac{c^n}{n!}\gamma^n \tag{15}$$

which can be decomposed into

$$\sum_{m=0}^{\infty} \frac{c^{2m}}{(2m)!}\lambda^{2m} + \sum_{m=0}^{\infty} \frac{c^{2m+1}}{(2m+1)!}\lambda^{2m+1} \tag{16}$$

As

$$\gamma^{2m} = (-1)^m \tag{17}$$

and

$$\gamma^{2m+1} = (-1)^m \gamma \tag{18}$$

and that the development of the cosine and sine are respectfully

$$\cos c = \sum_{m=0}^{\infty} \frac{(-)^m c^{2m}}{(2m)!} \tag{19}$$

and

$$\sin c = \sum_{m=0}^{\infty} \frac{(-)^m c^{2m+1}}{(2m+1)!} \tag{20}$$

one obtains

$$e^{c\gamma} = \cos c + \gamma \sin c \tag{21}$$

Clearly, until we have cause, the exponential continues in the same direction, that is to say for the same root of minus one, we can use the rules of the calculus of exponentials and in particular the law

$$e^{c\gamma}e^{c'\gamma}=e^{(c+c')\gamma} \tag{22}$$

Note also that if α is perpendicular to γ we have

$$\alpha e^{c\gamma}=e^{-c\gamma}\alpha \tag{23}$$

indeed, the first member is

$$\alpha\cos c+\alpha\gamma\sin c=\alpha\cos c-\gamma\alpha\sin c=\left(\cos c-\gamma\sin c\right)\alpha. \tag{24}$$

8. The Erlangen Program[8]

In our presentation of spherical and elliptical geometry, we will adopt the point of view proposed by Klein in the Erlangen program.

The geometry is then specified when we give, for every pair of points, a certain expression called the distance invariant. Two pairs of points for which the distance invariant has the same values are then considered as congruent or superimposable.

A transformation which transforms any pair of points into a pair of points invariant of distance is called a displacement and the study of groups of displacements is reduced to the study of groups of transformations which leave the invariant of distance invariant.

The distance itself must be a function of the distance invariant, such that the length of a line segment divided into two partial segments is the sum of the lengths of these segments.

The length of the segment is defined as the distance between its ends.

As for the straight line, we shall consider it as an axis of rotation, such as a locus of points left invariant by a displacement.

9. The Distance Invariant

We will assume that each point of the spherical space is specified by a versor V.

If V and V' are the versors representing two points we will define the distance invariant of this pair of points by the scalar

$$\mathrm{I}=\frac{1}{2}\left(\mathrm{V}V'+\mathrm{V}'V\right) \tag{25}$$

In this expression V and V' denote the conjugates of V and V′.

These definitions suffice to define the geometry in the sense of the Erlangen program.

Although this is not necessary for the rest of the exposition, we interspersed here some remarks which have no other purpose than to show how we were led to choose this point of departure.

If *u* is the scalar and *x, y, z* the components of the vector of the versor V, we have

$$x^2+y^2+z^2+u^2=1 \tag{26}$$

which can be considered as a hyper-sphere of radius one or spherical space. This shows how a pourer can characterize a point of the spherical space.

Similarly, if the primed letters designate the analogous quantities for the versor V′, the distance invariant is

$$\mathrm{I}=xx'+yy'+zz'+uu' \tag{27}$$

an expression which generalizes the expression of the cosine to four dimensions as a function of the angle of the direction of the cosine boundaries.

We can therefore predict that the distance invariant will be the cosine of the distance.

10. Parataxis[9]

A first group of displacements is obtained by multiplying the representative versor of the various points of the space by a fixed versor. We shall call these displacements of the parataxies, parataxies on the left if the multiplication is made on the left, parataxies on the right if it is made on the right.

Let us designate

[8] Erlangen Program - Method of characterizing geometries based on group theory and projective geometry introduced by Felix Klein in 1872 as *Vergleichende Betrachtungen über neuere geometrische Forschungen* (Comparative considerations on recent geometric researches [4]) named after the University Erlangen-Nürnberg, where Klein was given a professorship.

[9] Paratactic- A Parameter whose association/ arrangement/juxtaposition is only implied.

$$e^{a\alpha}, \quad e^{a'\alpha'} \tag{28}$$

two arbitrary points of space, Let $e^{c\gamma}$ be the fixed versor and $e^{b\beta}$ and $e^{b'\beta'}$ the two points in which the points $e^{a\alpha}$ and $e^{a'\alpha'}$ are transformed by a parataxis to the left, we shall have

$$e^{b\beta} = e^{c\gamma}e^{a\alpha} \qquad e^{b'\beta'} = e^{c\gamma}e^{a'\alpha'}. \tag{29}$$

If I' denotes the distance invariant after transformation, we have to verify that $\mathrm{I}' = \mathrm{I}$. It becomes

$$2\mathrm{I} = e^{b\beta}e^{-b'\beta'} + e^{b'\beta'}e^{-b\beta}. \tag{30}$$

For the conjugates

$$e^{-b\beta} = e^{-a\alpha}e^{-c\gamma} \tag{31}$$

and

$$e^{-b'\beta'} = e^{-a'\alpha'}e^{-c\gamma} \tag{32}$$

obtained by taking the product of the conjugates in the reverse order it becomes

$$\mathrm{I}' = e^{c\gamma}\mathrm{I}e^{-c\gamma} \tag{33}$$

which reduces to I since I is a scalar that can equally well be inscribed as head of the product.

From the fact that the product of two versors is a versor, the parataxis on the left form a group.

For the parataxis on the right, we will have the same

$$\begin{aligned} e^{b\beta} &= e^{a\alpha}e^{c\gamma} \\ e^{b'\beta'} &= e^{a'\alpha'}e^{c\gamma} \end{aligned} \tag{34}$$

and so

$$2I' = e^{a\alpha}e^{c\gamma}e^{-c\gamma}e^{-a'\alpha'} + e^{a'\alpha'}e^{c\gamma}e^{-c\gamma}e^{-a\alpha} = 2\mathrm{I}. \tag{35}$$

The parataxies with straight lines are thus also displacements and form a group of displacements.

11. Homogeneity of Space

A versor whose vector is zero reduces to the scalar one. We will call the corresponding point the origin.

Any point can be transformed at the origin by a right or left parataxy. It is enough to take for the symbol of the parataxis the versor conjugated to the symbol of the point to be transported at the origin. For $e^{c\gamma} = e^{-a\alpha}$ we have $e^{b\beta} = 1$.

It follows from this that the space considered is homogeneous since there are displacements which carry every point at the origin.

12. Rotation

If a parataxy is performed successively on a left and a right parataxis having as a symbol the versor conjugated to that of the parataxis on the left, evidently a displacement is obtained. That is to say a transformation which preserves the distance invariant. This Transformation transforms any point $e^{a\alpha}$ into a point $e^{b\beta}$ by the formula

$$e^{b\beta} = e^{c\gamma}e^{a\alpha}e^{-c\gamma} \tag{36}$$

If $e^{a\alpha}$ is the origin, $e^{b\beta}$ will also be at the origin. The transformation thus preserves the origin; we shall say that it is a rotation around the origin.

13. Straight Lines

This allows us to define a straight line as an axis of rotation.

The points that are retained by the rotation are included in the expression

$$e^{c'\gamma} \tag{37}$$

where c' can assume an arbitrary value.

This expression for the c' variable is the equation of a line passing through the origin.

By displacing the origin with a parataxis, we obtain the equation of a line passing through the point in which the parataxy has transformed the origin.

14. Straight Parataxies

The parataxies of the same species (i.e. all to the right or all to the left) of fixed direction γ, for any parameter *c* form a group, subgroup of the group of parataxies which results from

$$e^{c\gamma}e^{c'\gamma} = e^{(c+c')\gamma} \tag{38}$$

such that the two parataxies of parameter *c* and c' carried out successively in any order are equivalent to a single parataxy of parameter

$$c'' = c + c' \tag{39}$$

This particular group retains the straight line $e^{c\gamma}$ (*c* variable), that is to say, transforms the points of this line into points of the same line.

This group will retain (even if it is a left parataxy), all straight lines

$$e^{c\gamma}e^{x\xi} \tag{40}$$

(*c* variable, γ, x, ξ fixed).

For different values of *x* and ξ but the same value of γ, these lines are called parataxies (left).

Similarly, the straight lines

$$e^{x\zeta}e^{c\gamma} \tag{41}$$

(*c* single variable) are preserved by right parataxies and are right paratactic.

15. Distances

Consider two left parataxis of direction γ and parameters *c* and *c'*, carried out successively.

The first transforms the origin into the point $e^{c\lambda}$. The second transforms this point into a point in $e^{(c+c')\gamma}$.

When we have three points in a straight line, the length of the total segment must be the sum of the lengths of the partial segments. The length of a segment is the distance of the extremities, that is to say a function of the distance invariant for these two points. The invariant is $\cos c$ and $\cos c'$ for the partial segments and $\cos c''$ for the total segment.

c, c', c'' are functions of the distance invariants and since

$$c'' = c + c' \tag{42}$$

are additive functions.

For a suitable choice of the unit of length, c, c' and c'' are the distances themselves.

16. Perpendicular Lines

Consider two straight lines passing through the origin, as

$$e^{x\xi} \tag{43}$$

for the *x* variable and

$$e^{y\eta} \tag{44}$$

for the *y* variable.

Suppose further that the directions ξ and η are perpendicular to one another; we intend to show that the two straight lines are perpendicular.

This may seem obvious, but in reality, this must be demonstrated. Indeed, the directions have been introduced without reference to the spherical space and to its distance invariant.

We shall define the right angle, as in Euclid, by the condition that the angle is equal to the adjacent angle obtained by extending one of the sides. In other words, there must be a displacement (a rotation) which transforms the first right angle into the second and the second right angle into the angle opposite the first.

The calculation is very elementary, but we give it in detail by way of example of this type of calculation.

Since the directions ξ and η are perpendicular, there exists a direction ζ such that

$$\zeta = \xi\eta = -\eta\xi\,. \tag{45}$$

Consider then the rotation

$$e^{b\beta} = e^{\frac{\pi}{1}\zeta}e^{a\alpha}e^{-\frac{\pi}{1}\zeta} \tag{46}$$

which transforms any point $e^{a\alpha}$ into $e^{b\beta}$. We must show that if we set $e^{a\alpha} = e^{x\xi}$, we obtain $e^{b\beta} = e^{x\eta}$ and if we set $e^{a\alpha} = e^{y\eta}$, we obtain $e^{b\beta} = e^{-y\xi}$.

In the first case, we have

$$e^{b\beta} = \frac{1}{2}(1+\zeta)e^{x\xi}(1-\zeta) = \frac{1}{2}(1+\zeta)(1-\zeta)\cos x + \frac{1}{2}(1+\zeta)\xi(1-\zeta)\sin x \quad (47)$$

but

$$(1+\zeta)(1-\zeta) = 2 \quad (48)$$

and

$$(1+\zeta)\xi(1-\zeta) = (1+\zeta)^2\xi = 2\zeta\xi = 2\eta \quad (49)$$

it becomes therefore

$$e^{b\beta}\cos x + \eta\sin x = e^{x\eta}. \quad (50)$$

In the second case

$$e^{b\beta} = \frac{1}{2}(1+\zeta)e^{y\eta}(1-\zeta) = e^{-x\xi} \quad (51)$$

the calculation is the same, γ replacing ξ, but

$$\zeta\eta = -\xi. \quad (52)$$

17. Left Rectangles

Let $e^{c\gamma}$ be a fixed point; then for x and y variables with ξ perpendicular to η then

$$e^{c\gamma}e^{x\xi} \quad (53)$$

and

$$e^{c\gamma}e^{y\eta} \quad (54)$$

represent two straight lines perpendicular to one another.

In particular, if $\eta = \gamma$, the second

$$e^{(c+y)\gamma} \quad (55)$$

is a line which passes through the origin, it is the line joining the origin to the point $e^{c\gamma}$. The first line is the left parataxis to the line $e^{x\xi}$ passing through the fixed point, $e^{c\gamma}$.

As the line $e^{x\xi}$ is also perpendicular to the line $e^{(c+y)\gamma}$, we see that the two paratactics $e^{x\xi}$ and $e^{c\gamma}$ $e^{x\zeta}$ have a common perpendicular.

Let us make a right parataxis of symbol $e^{x'\xi}$ (x' fixed) the two lines $e^{x\xi}$ and $e^{c\gamma}$ $e^{x\xi}$, left parataxies, are each transformed into themselves and the common perpendicular moves while maintaining the same length.

The figure formed by the two parataxies and the two common perpendiculars is therefore a rectangle in the sense that it is a quadrilateral whose angles are right with the opposite sides equal to each other. But it is not a plane figure, but a left rectangle (in the English sense of "skew").

18. Clifford Surfaces

The Clifford surface is called, the place where the paratactic lines have the same straight line, called the axis of the surface, and such that the perpendicular common to the axis has the same length as the radius of the surface.

Let us first consider left parataxies; the surface points of the Clifford axis

$$e^{x\xi} \quad (56)$$

(x variable) are

$$e^{c\gamma}e^{x\xi} \quad (57)$$

where c is the radius of the surface and where x is variable as well as the direction γ which can represent all directions perpendicular to ξ.

For right parataxies, it would be the same

$$e^{x'\xi}e^{c\gamma'} \quad (58)$$

The two expressions are equal

$$x' = x,\quad xe^{c\gamma'} = e^{-x\xi}e^{c\gamma}e^{x\xi} \quad (59)$$

that is to say

$$\gamma' = e^{-x\xi}\gamma e^{x\xi}. \quad (60)$$

This shows that the place of the paratactic on the right is the same as that of the paratactic on the left.

The Clifford surface is the locus of points at constant distance *c* from the axis of the surface. It is a regulated surface which admits two systems of generators, the paratactic ones to the left and to the right of the axis of the area.

If one performs paratactic displacements which retain the axis, the Clifford surface transforms into itself, the generators of one system are transformed into themselves and the generators of the other system are interchanged.

Two pairs of generators of each of the two systems thus form parallelograms, the angles are equal or additional and the opposite sides are equal.

The angle of these parallelograms is easily calculated; in fact, the two generators passing through the point $e^{c\gamma}$ are

$$e^{c\gamma} e^{x\xi} \tag{61}$$

(x variable) and

$$e^{x'\xi} \;\; e^{c\gamma} \tag{62}$$

(x' variable). A parataxis with the left symbol $e^{-c\gamma}$ brings the vertex of the angle to the origin, the lines are transformed into

$$e^{x\xi} \tag{63}$$

and

$$e^{-c\gamma} \;\; e^{x'\xi} \;\; e^{c\gamma} \tag{64}$$

which are transformed one into the other by a rotation of the angle 2c.

Perhaps this last point is not perfectly clear, we shall return to it in an instant after having studied the plane.

19. Conjugate Lines

In the particular case where $c = \pi / 2$ we have

$$e^{c\gamma} = \gamma \tag{65}$$

and therefore, since γ and ξ are assumed to be perpendicular

$$e^{x'\xi}\gamma = \gamma e^{-x'\xi} \tag{66}$$

the two paratactic ones, the one on the right and the one on the left are therefore identical. Their points correspond to

$$x' = -x \tag{67}$$

Consider any point on the line

$$\mathrm{V}' = e^{x'\xi}\gamma \tag{68}$$

(that is to say, a particular value of the variable x') and any point on the line

$$\mathrm{V} = e^{x\xi} \tag{69}$$

These two straight lines are right parataxies for the exceptional case $c = \pi/2.$

We will show that these two points are the same distance $\pi/2$, that is to say that their distance invariant is zero. In fact

$$\begin{aligned} \mathrm{I} &= \frac{1}{2}\left(\mathrm{V}V' + \mathrm{V}'V\right) = \\ &\frac{1}{2}e^{x\xi}\left(-\gamma\right)e^{-x'\xi} + \frac{1}{2}e^{-x'\xi}\gamma e^{-x\xi} = . \\ &\frac{1}{2}\left[-e^{(x+x')\xi}\gamma + e^{(x'+x)\xi}\gamma\right] = 0 \end{aligned} \tag{70}$$

It would be easy to show that the straight line joining V and V', that is to say, any line intersecting the two straight lines v and v' (for x and x' variables) is perpendicular to these two straight lines. But no doubt we have given sufficient examples of these calculations.

The paratactic lines for $c = \pi/2$ are said to be conjugate or absolute polar.

20. The Plane

The plane can be defined as the locus of straight lines perpendicular to the same straight line

$$e^{c\gamma},\; e^{x\xi} \tag{71}$$

(*x* variable).

The points of the plane are thus represented by the versors

$$\mathrm{V} = e^{c\gamma} e^{y\eta} \tag{72}$$

y is arbitrary and η also but perpendicular to ξ.

We shall show that the plane is the locus of points situated at distance $\pi/2$ from a point

$$\mathrm{V}' = e^{c\gamma} \xi \tag{73}$$

called the center of the plane.

We must verify that the distance invariant of the two points V and V' is zero. We have

$$\begin{aligned} \mathrm{I} &= \frac{1}{2}\left(\mathrm{V}V' + \mathrm{V}'V\right) = \\ &\frac{1}{2} e^{c\gamma} e^{y\eta} \left(-\xi\right) e^{-c\gamma} + \frac{1}{2} e^{c\gamma} \xi e^{-y\eta} e^{-c\gamma} \end{aligned} \tag{74}$$

which is null, since for $\xi\eta = -\eta\xi$ we have

$$\xi e^{-y\eta} = e^{y\eta} \xi \tag{75}$$

Or we can put in the equation of the plane, the versor V' representing the center.

If

$$\zeta = \xi\eta \tag{76}$$

We have

$$\begin{aligned} \mathrm{V} &= \mathrm{V}'\left(-\xi\right) e^{y\eta} = \\ &\mathrm{V}'\left(-\xi \cos y - \zeta \sin y\right) = \mathrm{V}'\chi. \end{aligned} \tag{77}$$

It is easy to realize that χ is a direction that is arbitrary. Indeed, it is the direction whose orthogonal projections on the directions $-\xi$ and $-\zeta$ are respectively $\cos y$ and $\sin y$. χ is therefore in the plane of ξ and ζ forming an angle y with $-\xi$. But η is an arbitrary direction perpendicular to ξ and since y is arbitrary, χ is also arbitrary.

In particular, if the center is at the origin, we see that the directions represent the points of a plane, that is to say of a sphere of radius $\pi/2$ centered on the origin.

As the familiar theorems which show that the angles at the center are measured by the intercepted arc on the sphere apply without modification, it follows that the angle of two straight lines from the center $e^{x\xi}$ and $e^{y\eta}$ is the distance of the two directions ξ and η.

When the versors are reduced to directions, the distance is reduced to the scalar product of the two directions. The angle of the two straight lines is therefore the angle of the directions of these lines.

In particular, with the rotation

$$e^{b\beta} = e^{c\gamma} e^{a\alpha} e^{-c\gamma} \tag{78}$$

we have for $a = b = \pi/2$

$$\beta = e^{c\gamma} \alpha e^{-c\gamma} \tag{79}$$

and if α is perpendicular to γ

$$\beta = e^{2c\gamma} \alpha = \alpha \cos 2c + \gamma\alpha \sin 2c \tag{80}$$

β has thus rotated by an angle $2c$ in the plane perpendicular to γ.

This completes the justification of the end of section 18.

21. Antipodal Points

When x varies from zero to 2π, the expression

$$e^{x\xi} \tag{81}$$

represents successively the various points of a straight line, partly from the origin, and on returning there to traverse in the same order the points already traversed. In fact,

$$e^{(x+2\pi)\xi} = e^{x\xi}. \tag{82}$$

The line is therefore a closed line whose length is equal to 2π.

If we consider all the straight lines passing through the origin, that is, when we consider different values of the direction ξ, we see that for $x = \pi$ all these lines pass through the point -1.

This point is called the antipode of the origin.

If we consider similarly straight lines passing through a point $e^{c\gamma}$ we would see that all these lines pass through the point $-e^{c\gamma}$ the antipode of $e^{c\gamma}$.

The antipode points are thus represented by versors from opposite signs, every straight line passing through such a point also passes through the antipode of this point.

22. Elliptical Space

If, instead of the invariant of distance I, we had taken as invariant distance, I^2 or the absolute value of I, then two versors V and $\mathrm{V}' = -\mathrm{V}$ would have as distance invariant plus one. Instead of considering them as representing distinct points of space, the antipodes, they should be considered as two representations of one and the same point of space.

Apart from this circumstance concerning the disappearance of the antipodes, all the formulas established for the spherical space remain valid for the new space.

This is called the *elliptical* space.

Some authors nevertheless call it a simply elliptic space so as to leave to the term "*elliptical space*" a generic meaning which applies to both of the spaces considered as various "*form*s" of the elliptical space.

23. Representations of Elliptical Euclidean Space

First of all, we note that infinitely small figures of elliptical space can, in the limit, be considered as Euclidean figures.

This already appears in the fact that the angle of warping of a left rectangle is equal to the dimension; It therefore tends to zero if this side is infinitely small and then the rectangle becomes a plane and the geometry Euclidean.

We can also show that when x, y, z and x', y', z' are infinitely small, the invariant of distance I becomes, neglecting the quantities of order higher than the second

$$\mathrm{I} = 1 - \frac{1}{2}\left[\left(x-x'\right)^2 + \left(y-y'\right)^2 + \left(z-z'\right)^2\right] + \ldots \quad (83)$$

as I is the cosine of the distance *r*, it is at the same approximation equal to the Euclidean value

$$r^2 = \left(x-x'\right)^2 + \left(y-y'\right)^2 + \left(z-z'\right)^2 \quad (84)$$

We can use this remark, to represent the totality of the elliptical space, in a sphere of infinitely small radius ε. Let us note that by exception we use this Greek letter, in its traditional sense of an infinitely small scalar.

A point

$$e^{x\xi} \quad (85)$$

may be represented inside the sphere by the point

$$e^{x'\xi} = e^{\varepsilon x\xi} \quad (86)$$

or by neglecting the terms in ε^2 by the point

$$1 + \varepsilon x\xi \quad (87)$$

Since geometry can be considered as Euclidean in the sphere, we shall have, taking ε as units of Euclidean lengths, that a point of the elliptica space $e^{x\xi}$ is represented by a Euclidean vector of direction ξ and length *x*.

We obtain all the points on the line considering all the values of *x* from minus $\pi/2$ to plus $\pi/2$. The extreme points represented on the sphere of radius $(\pi/2)\varepsilon$ are the antipodal points of this sphere and would represent the antipodes of space if we consider the spherical space. As we consider the elliptical space these two points represent two representations of the same point of the elliptical space.

All the points of this space are thus represented inside our Euclidean sphere and the points situated on the frontier of the representation are represented there twice. It is therefore never difficult to follow the representation on a contour which reaches its edge, since all the points on the edge have two representations in such a way that, instead of leaving the sphere, it can always pass to the other representation of the same point and continue to walk towards the interior of the sphere.

24. Representations of Spherical Space

An analogous representation can be used for spherical space. It is now assumed that within the sphere there are two kinds of points. We will say the blue dots and the pink dots. The points of the frontier are not more of one species than the other. We will say that these are mauve points.

We shall suppose that we cannot pass from a pink point to a blue point than through a mauve dot.

In other words, there are, within the sphere, two distinct spaces, the blue space and the pink space, and these two spaces are connected by the purple border, the surface of the sphere.

This representation can be modified in a variety of ways with respect to the topology by making it resemble the projections of the sphere, such as the stereographic projection or the orthogonal projection. But these developments would lead us outside our subject.

* * *

Translators Appendices

Appendix A: Brief Definitions of Terms

1. Versor – An affinor (affine tensor) which effects a rotation of a vector through a right angle. For quaternions, the versor of an axis (or a vector) is a unit vector indicating its direction. In general, a versor defines all of the following: a directional axis; the plane normal to that axis; and an angle of rotation. For quaternions, the versor of an axis (or of a vector) is a unit vector indicating its direction [5].

Rotations of Unit Quaternions. Term introduced by Hamilton in developing quaternions. Versor is sometimes used synonymously with "unit quaternion" with no reference to rotations. An algebraic parametrization of rotations. In classical quaternion theory, a versor is a quaternion of norm one (unit quaternion). Each versor has the form $q = \exp(ar\pi)$ $= \cos a + r\sin a,\ r^2 = -1,\ a \in [0, \pi],$ where the $r^2 = -1$ condition means that r is a 3D unit vector. In case $a = \pi/2$, the versor is termed a right versor. The corresponding 3D rotation has the angle $2a$ about the axis r in axis–angle representation. The word is derived from Latin *versare* "to turn" or *versor* "the turner").

2. Parataxis – Corresponds to a hyperbolic displacement by a half line.

3. Paratactic lines – In elliptic geometry two oblique lines with an infinite set of common perpendiculars of the same length are called Clifford parallels, equidistant or paratactic lines in elliptic geometry if the perpendicular distance between them is constant from point to point.

The concept was first studied by William K. Clifford in elliptic space. Since parallel lines have the property of equidistance, the term *parallel* was taken from Euclidean geometry, but the lines of elliptic geometry are curves with finite length, unlike lines in Euclidean geometry. Quaternion algebra describes the geometry of elliptic space in which Clifford parallelism is made explicit.

4. Hyperbolic geometry – a Lobachevskian, or non-Euclidean geometry, where the Euclidean parallel postulate is replaced with: For any given line R and point P not on R, in the plane containing both line R and point P there are at least two distinct lines through P that do not intersect R.

5. Riemannian geometry – or elliptic geometry, is a non-Euclidean geometry regarding space as a sphere and a line like a great circle. Euclid's 5th postulate is rejected and his 2nd postulate modified. Simply, Euclid's 5th postulate states: through a point not on a given line there is only one line parallel to the given line. In Riemannian geometry, there are no lines parallel to the given line. Euclid's 2nd postulate is: a straight line of finite length can be extended continuously without bounds. In Riemannian geometry, a straight line of finite length can be extended continuously without bounds, but all straight lines are of the same length. However, Riemannian geometry allows the other three Euclidean postulates.

6. Elliptical space – Elliptic space can be constructed in a similar manner to the construction of 3D vector space: One uses directed arcs on great circles of the sphere. As directed line segments are equipollent[10] when they are parallel, of the same length, and similarly oriented, so directed arcs found on great circles are equipollent when they are of the same length, orientation, and great circle. These relations of equipollence produce 3D vector space and elliptic space, respectively. Access to elliptic space structure is provided through the vector algebra of William Rowan Hamilton: he envisioned a sphere as a domain of square roots of minus one. Then Euler's formula $e^{ix} = r(\cos\theta + i\sin\theta)$ where r is on the sphere, represents the great circle in the plane perpendicular to r. Opposite points r and $-r$ correspond to oppositely directed circles. In elliptic space, arc length is less than π, so arcs may be parametrized with θ in [0, π) or (–π/2, π/2]

[10] In Euclidean geometry, equipollence is a binary relation between directed line segments. A line segment *AB* from point *A* to point *B* has the opposite direction to line segment *BA*. Two directed line segments are equipollent when they have the same length and direction.
The concept of equipollent line segments originated with Giusto Bellavitis in 1835 [6]. Subsequently the term vector was adopted for a class of equipollent line segments.

7. Hyperbolic versor – Regarding versors, a parameter of rapidity specifying a reference frame change corresponding to the real variable in a one-parameter group of hyperbolic versors. With the further development of special relativity, the action of a hyperbolic versor is now called a Lorentz boost [7].

Appendix B: Erlangan Program

Method of characterizing geometries based on group theory and projective geometry as introduced by Felix Klein in 1872 in *Vergleichende Betrachtungen über neuere geometrische Forschungen* (Comparative considerations on recent geometric researches [4]) named after the University Erlangen-Nürnberg, where Klein was given a professorship.

Appendix C: Lemaître Biographical Note

Georges Henri Joseph Édouard Lemaître (July 1894–June 1966) was a Belgian Catholic priest, astronomer and professor of physics (rare mix in modern times) at the Catholic University of Leuven. As he was a secular priest, he was called *Abbé*, then, after being made a canon, *Monseigneur*.

He is best known for the discovery of the proposed expansion of the universe, still widely misattributed to Hubble. He was the first to derive what is now known as Hubble's law and made the first estimation of what is now called the Hubble constant, published in 1927, two years before Hubble's article; but since it was published in French it was unknown in the US for a time. Lemaître also proposed what is known as the Big Bang theory, which he called his "*hypothesis of the primeval atom*" or the "*Cosmic Egg*" [2].

References (Translators) Annotated / Notes

[1] Lemaître, G. (1948) *Quaternions et espace elliptique*, *Pontificia Academia Scientiarum,* (Acta Pontifical Academy of Sciences), ACTA, Vol. XII, No. 8, 12:57-80; http://www.casinapioiv.va/content/dam/accademia/ pdf/acta12/acta12-lemaitre2.pdf.

[2] Coxeter, H.S.M. (1950) also a contributor to elliptic geometry, summarized—Lemaître's paper, English synopsis of Lemaître, in Mathematical Reviews, MR0031739 (11,197f; in his review of Lemaître's paper, Coxeter comments on Lemaître's construction of the *versor* (quaternion of unit norm), as being expressible as $v = \cos c + \gamma \sin c = e^{c\gamma}$, where γ is the direction of a unit vector. A versor has four constituents interpreted as coordinates for a point in spherical 3-space, where the distance between points *u* and *v* is $\cos^{-1}\left(u\bar{v} + v\underline{\bar{u}}\right)/2 = c.$ Multiplying all versors on the left or right by a given versor $v = e^{c\gamma}$ is a left or right parataxy called a Clifford translation, which moves every point through a distance $\cos^{-1}\left(u + \bar{v}\right)/2 = c.$ The general displacement, $x \to uxv$ is obtained when the two types of Clifford translation are combined. If *u* and *v* are conjugate, point 1 is an invariant rotation.

For versors $e^{c\gamma}$, where angle *c* varies with direction γ remaining fixed, represent points on a line through point 1, which is the axis of the rotation $x \to e^{a\gamma} x e^{-a\gamma}$. Thus the line $e^{c\gamma}$ is described. The two Clifford parallels to it through any fixed-point *u* are $ue^{c\gamma}$ and $e^{c\gamma}u.$ When *u* varies along a line $e^{b\beta}$, the line $ue^{c\gamma}$ generates a Clifford surface, whose points are $e^{b\beta}e^{c\gamma}$ with β and γ fixed. Finally, elliptic space is derived from spherical space by identifying each pair of antipodal points $\pm v$. At the end of his note, Coxeter asks the reader to compare his monograph on 'non-Euclidean geometry' which predates Lemaître's paper by six years; Non-Euclidean geometry, H.S.M. Coxeter The University of Toronto Press, (1942).

[3] Short Lemaître bio, http://dictionary.sensagent.com/georges%20 lemaitre/en-en/.

[4] Felix Klein (1849-1925) published his inauguration paper (1872) *Vergleichende Betrachtungen über neuere geometrische Forschungen* for a professorship at the University of Erlangen (Bavaria, Germany). The paper acquired world-wide fame among mathemat-icians under the name of *Erlanger Program*; Original German: http://quod.lib.umich.edu/cgi/t/text/pagevieweridx?c=umhistmath;cc=umhistmath;idno=ABN7632. 0001.001;seq=1; Full English Translation: http://arxiv.org/abs/0807.3161.

[5] Encyc. Math: https://www.encyclopediaofmath.org/index.php/Main_Page.

[6] Crowe, M.J. (1967) A History of Vector Analysis, "Giusto Bellavitis and His Calculus of Equipollences", pp 52–4, University of Notre Dame Press.

[7] Robb, A.A. (1911) Optical Geometry of Motion: A New View, of the Theory of Relativity, Cambridge: Heffer & Sons.

Derivation of the Inertial Mass $m = E_o / c^2$ of an Electron Composed of a Circling Spin-½ Charged Photon

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Using Newton's second law of motion $\Sigma\vec{F} = d\vec{p}/dt = m\vec{a}$, the inertial mass $m = E_o / c^2$ is derived for a resting electron proposed to be composed of a circling spin-½ charged photon of energy E_o and circling momentum $p_o = E_o / c$. In this view, the inertia of a particle is not due to "vis inertiae" or an "inertial force" within matter that resists acceleration, as Newton proposed. Rather, the inertial mass of an elementary particle of matter is due to the momentum of a circling photon-like object composing the particle. The particle's inertial mass is calculated as the time rate of change of the momentum vector of a circling photon-like object composing the particle, divided by the associated centripetal acceleration of the circling photon-like object. A transluminal energy quantum model of the proposed spin-½ charged photon is introduced.

Keywords: Inertia, Mass, Inertial Mass, Energy, Momentum, Electron, Photon, Model, Transluminal, Newton's 2nd Law

1. Introduction

Matter has the interesting (and mysterious) physical property of inertia – the resistance to a change in its state of motion. The term "inertia" (Latin: lazy, idle) was first used in a physics context by Kepler in the sense taught by the Greek philosopher and naturalist Aristotle – that there are certain natural (unforced) motions and certain forced motions of physical objects. For Kepler, inertia was the presumed tendency of an object such as a planet in its elliptical orbit to be motionless unless it is moved in its orbit by an applied force. Galileo, a contemporary of Kepler, used the word "inertia" in a more modern sense. For Galileo, inertia was the tendency of an object like a ship or a polished metal ball on a horizontal surface to either remain motionless or to move horizontally with a constant speed in one direction unless acted on by an outside force such as friction or gravity. This tendency of matter as described by Galileo came to be called Galileo's "law of inertia".

Galileo's law of inertia was developed by Descartes and was later included in Newton's three laws of motion. In his *Principia*, Newton's[1] first law of motion states: "*Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed*." In modern terminology, Newton's first law has become: Unless acted upon by a net unbalanced force, an object will maintain a constant velocity.

No "*vis insita*" (inherent force) or as Newton also called it, "*vis inertiae*" (inertial force), was ever discovered to explain this inertial property of matter. In modern physics the unexplained inertial property of matter has been supplemented by the quantitative term "inertial mass". This is defined as the amount of resistance of an object to a change in its velocity, and is calculated with Newton's second law of motion: "A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed." In modern terms this has become $\Sigma\vec{F} = d\vec{p}/dt = m\vec{a}$. $\Sigma\vec{F}$ is the sum of forces (or net force) acting on an object, $d\vec{p}/dt$ is the time rate of change of the momentum $\vec{p}$ of the object, m is the inertial mass of the object and $\vec{a}$ is the acceleration of the object. Newton's second law states that the net force on an object is equal to the time rate of change of the object's momentum, which equals the inertial mass of the object times the acceleration of the object.

Newton originally defined the "quantity of matter" (or the mass) of an object as "a measure of matter that arises from its density and volume jointly." But since density is mass per unit volume, this definition of mass has been criticized as being circular. The mass of an object can be measured, relative to another object of some standard mass, by a) weighing the object compared to the standard mass object, b) physically colliding or interacting the object with the standard mass object and comparing the change of motions of the two objects during the collision or interaction, or c) comparing the accelerations of the two objects with the forces acting on them. The first method makes use of the fact that the masses of two objects are proportional to their weights (when the objects are weighed in the same physical location). The second and third methods make use of Newton's second law of motion. The second method also makes use of Newton's third law of motion: "To any action there is always an opposite and equal reaction." This means that when two objects interact, the change of momentum of the 1st object is equal and opposite to the change in momentum of the 2nd object.

2. Some Background About the Electron

Newton did not know about electrons, which are very small particles of electrically charged matter discovered in 1897. Electrons have mass and inertia. The mass *m* of an electron is 9.11×10^{-31} kilograms. Electrons also carry a negative electric charge $-e = -1.602 \times 10^{-19}$ Coulombs. Electrons have a characteristic spin component $S_z = \frac{1}{2}\hbar$ where $\hbar = h/2\pi$ and *h* is Planck's constant and equals 6.626×10^{-34} Joule seconds. This value of the spin component S_z is $S_z = 5.26 \times 10^{-35}$ Js for an electron. The electron's resting energy is $E_o = 0.511$ MeV, where MeV is one million electron volts, and one electron volt is equal to 1.602×10^{-19} Joules of energy. If a photon has the same energy $E_o = mc^2 = h\nu = hc/\lambda$ as a resting electron, this photon's wavelength λ is called the Compton wavelength $\lambda_{Compton}$ and is given by $\lambda_{Compton} = h/mc = 2.43 \times 10^{-12}$ meters. An electron also acts like a little magnet and has a property called its magnetic moment μ, which has been measured extremely precisely experimentally. This experimental value has also been predicted theoretically extremely precisely by the theory of quantum electrodynamics (QED).

3. $E_o = mc^2$ and Einstein's Theory of Relativity

It is commonly written in physics books for the general public and in some textbooks, and even by some well-known physicists, that the mass of an object increases with the object's speed as this speed approaches the speed of light. But this is not how mass is generally understood today by physicists. Nowadays the mass *m* of an object is defined as the object's invariant mass – the object's mass when it is at rest. This invariant mass *m* of an object is independent of the velocity of the object. An object having a mass *m* of one kilogram has the same invariant mass of one kilogram when it is moving at half the speed of light as when it is at rest. In Einstein's special theory of relativity, an object of invariant mass *m* has an associated energy $E_o = mc^2$ when the object is at rest. E_o is called the rest energy of the object. In the case of a resting electron, its inertial mass *m* is equal to the invariant mass $m = E_o / c^2$ of the electron.

The relation $E_o = mc^2$ for a stationary object of mass *m* indicates that this object contains energy $E_o = mc^2$. Under appropriate circumstances all or a portion of this energy can be released in the form of photons or other forms of energy such as kinetic energy. For example, an electron and its antiparticle the positron can mutually annihilate to create two or three photons whose total energy is equal to c^2 times the total mass of the electron and the positron before this annihilation. The idea that the loss of energy ΔE from an object is accompanied by a loss of mass $\Delta m = \Delta E / c^2$ by the object was introduced by Einstein[2] as a consequence of his special theory of relativity. Many attempts to theoretically derive $E_o = mc^2$ were made by Einstein and others. Though this mathematical formula is now well established experimentally, the road to its theoretical proof has been rocky, as described by Ohanian[3]. Okun[4,5] discusses the history, derivations and usage of the equation $E_o = mc^2$ as well as the more commonly known equation $E = mc^2$. This latter equation implies that the mass of an object is proportional to the object's total energy. Theoretical derivations of $E_o = mc^2$ for a particle have not explained the origin or nature of the inertial property of the mass *m* of the particle.

4. Justifications for Modeling Elementary Particles by a Circulating Photon-like Object

Several researchers such as Hestenes[6], Gauthier[7], Williamson and van der Mark[8], and Rivas[9] have modeled an electron as a circulating light-speed object moving in a circle or a helix. It may be argued that since photons don't move in a circle, how can a particle like an electron be composed of a circling photon? Also, since a photon has spin of $1\hbar$, or spin 1 in units of $\hbar$, how can a photon circulate to form an electron that has spin of $\hbar/2$, or spin ½ in units of $\hbar$? And how can a photon, which is uncharged, move in a circle to form an electrically charged electron?

One answer to these questions may be that there exists a previously unobserved variety of photon that is electrically charged and can circle in a double-loop to form a spin-½ electron or another electrically charged elementary particle. If this is the case, why have physicists never observed this variety of photon? It may be because a spin-½ charged photon is generally curled up and called an electron, or another name if it is another related particle.

A model of a relativistic electron composed of a spin-½ helically-moving charged photon was proposed by Gauthier[10], while a model of a relativistic electron composed of a helically-moving charged photon was earlier proposed by Gauthier[11]. The charged photon in both of these models would move in a circle in a resting electron. It is shown below that a model of a resting electron consisting of a circling charged photon-like object generates the electron's inertial mass.

One indirect source of support for the proposal of a spin-½ charged photon composing an electron comes from Dirac[12]. In his Nobel Prize lecture Dirac said in reference to the Dirac equation: "It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction that cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."

Dirac did not propose that the electron is a circling spin-½ charged photon. But the light-speed spin-½ electron that he describes as a solution to the Dirac equation sounds very much like the proposed circulating spin-½ charged-photon electron model. The spin-½ charged-photon electron model has, besides internal light-speed, three other properties of Dirac electron as mentioned in Barut and Bracken[13]: its internal small amplitude of oscillation $R_o = \hbar/2mc$, its internal high frequency $\nu_{zitt} = 2mc^2/h$ called the *zitterbewegung* frequency, and its spin $\frac{1}{2}\hbar$. The electron model also has one-half of the Dirac electron's magnetic moment $e\hbar/2m$. The electron model generates the relativistic electron's de Broglie wavelength $\lambda_{deBroglie} = h/\gamma mv$ for a moving electron.

It was recently found by Ballentine[14] that in certain experimental conditions involving two-dimensional motion, a photon can have spin ½ instead of the normal spin 1. Though this spin-½ photon is uncharged, it is still a surprising discovery about photons. It suggests that other surprising varieties of photon such as the spin-½ charged photon may also be discovered.

5. Derivation of the Resting Electron's Inertial Mass $m = E_o/c^2$

Let us first make a simple model of a resting elementary particle as composed of a circling photon-like object of energy E_o, momentum $p_o = E_o/c$ and speed *c*. The photon-like object moves in a circle of radius *R* with an angular velocity $\omega = c/R$, since $c = \omega R$. Newton's second law defines a force $\vec{F}$ as $\vec{F} = d\vec{p}/dt$, the time rate of change of the momentum of an object. For this photon-like object of momentum $\vec{p}_o$ moving in a circular orbit of radius *R*, the time rate of change of the photon-like object's momentum is given by $\vec{F} = d\vec{p}_o/dt = \omega p_o \hat{r}$ where $\hat{r}$ is a unit vector pointing towards the center of the circle. This force $\vec{F}$ on the circling photon-like object continually points towards the center of the circle as the photon-like object moves around the circle. There is also a centripetal acceleration $\vec{a}_c$ of the circling photon-like object, of magnitude $a_c = c^2/R = \omega^2 R$ since $c = \omega R$ for circular light speed motion. In vector terms, $\vec{a}_c = \omega^2 R\hat{r}$.

Starting with Newton's second law $\vec{F} = d\vec{p}/dt = m\vec{a}$, where *m* is the inertial mass of the circulating photon-like object that composes the particle and $\vec{a} = \vec{a}_c$ is the centripetal acceleration of the circulating photon-like object, we have

$$\begin{aligned} m &= \vec{F}/\vec{a}_c = (d\vec{p}_o/dt)/(\omega^2 R\hat{r}) \\ &= (\omega p_o \hat{r})/(\omega^2 R\hat{r}) = p_o/\omega R \\ &= p_o/c = (E_o/c)/c = E_o/c^2 \end{aligned} \tag{1}$$

This $m = E_o / c^2$ is the derived inertial mass of the circling photon-like object. Since the particle is proposed to be composed of this circling photon-like object, m is also the inertial mass of the particle. The above derivation of a particle's inertial mass applies to any particle composed of a circling photon-like object having a resting energy E_o (different for different types of particles) and momentum $p_o = E_o / c$.

While a circular orbit of the photon-like object was used for simplicity in the above inertial mass calculation, other smoothly curving trajectories of a photon-like object of energy E_o and momentum $p_o = E_o / c$ forming a resting particle would lead to the same result $m = E_o / c^2$. This is because at any point on the photon-like object's trajectory, there would be an instantaneous value of the angular velocity ω of the photon and an instantaneous value of R for the radius of curvature of the photon's curving trajectory, such that ωR equals c, the speed of the photon. This leads to the same result as above: the resting particle's inertial mass is $m = E_o / c^2$.

The above derivation doesn't require the trajectory of the elementary particle modeled by a circling photon-like object of energy E_o to have a particular radius. However, if an elementary particle such as an electron is to be modeled, the properties of the electron have to be taken into consideration in the modeling process, such as the electric charge, the spin and the magnetic moment of the electron. Gauthier[10] proposed a new variety of photon that carries the electron's negative electric charge –e and its spin ½ $\hbar$. Like uncharged spin-1 photons, this proposed spin-½ charged photon has energy $E = h\nu$ and momentum $p = E / c = h\nu / c$. If a resting electron of mass $m = 9.11 \times 10^{-31}$ kg is modeled by a circling spin-½ electrically-charged photon of rest energy $E_o = 0.511 MeV$, then the light frequency ν of the circling charged photon is found from $E_o = h\nu = mc^2$. Applying the light wave formula $c = \nu\lambda$, the circling spin-½ charged photon's wavelength λ is then found from $E_o = h\nu = hc / \lambda = mc^2$. This gives $\lambda = h / mc$, the Compton wavelength that equals 2.43×10^{-12} m. To get the correct spin-½ for the electron model, the spin-½ charged photon is formed into a double loop of total length one Compton wavelength. This gives the radius R_o of the double-looped spin-½ charged photon model of the electron to be $R_o = \hbar / 2mc = 1.93 \times 10^{-13}$ m.

6. Calculated Magnitudes of the Internal Angular Frequency, Internal Momentum, Internal Centripetal Acceleration, and Internal Radial Force in a Spin-½ Charged-Photon Model of a Resting Electron

We have derived the inertial mass $m = E_o / c^2$ of a resting electron, modeled as a circling spin-½ charged photon-like object, without mentioning the magnitudes of a) the internal angular frequency ω of rotation, b) the circulating internal momentum $p_o = mc$, c) the internal centripetal acceleration $a_c = \omega^2 R_o$, and d) the internal force $\vec{F} = d\vec{p}_o / dt = \omega p_o \hat{r}$ required to rotate the internal momentum $p_o = mc$ of the charged photon at this internal angular frequency ω. These quantities are calculated below.

6.1. *The Internal Angular Frequency*

The internal *zitterbewegung* ("jittery motion") angular frequency $\omega = \omega_{zitt}$ in the resting electron model is given by $\hbar\omega_{zitt} = 2mc^2$, or $\omega_{zitt} = 2mc^2 / \hbar$. The *zitterbewegung* angular frequency corresponds to the *zitterbewegung* frequency $\nu_{zitt} = 2mc^2 / h$ found for the internal frequency of the Dirac electron. So

$$\begin{aligned} \omega_{zitt} &= 2mc^2 / \hbar \\ &= 2 \times (9.11 \times 10^{-31})(3.00 \times 10^8)^2 / (6.63 \times 10^{-34} / 2\pi) \\ &= 1.55 \times 10^{21} \text{ rad/s} \end{aligned} \tag{2}$$

6.2. *The Internal Momentum*

The value of $p_o = mc$ is given by

$$\begin{aligned} p_o = mc &= 9.11 \times 10^{-31} \text{ kg} \times 3.00 \times 10^8 \text{ m/s} \\ &= 2.73 \times 10^{-22} \text{ kg m/s} \end{aligned} \tag{3}$$

6.3. *The Internal Centripetal Acceleration*

This is given by

$$\begin{aligned} a_c &= \omega_{zitt}{}^2 R_o \\ &= (2mc^2 / \hbar)^2 \times (\hbar / 2mc) \\ &= 2mc^3 / \hbar \\ &= 2 \times (9.11 \times 10^{-31}) \times (3.00 \times 10^8)^3 / (6.63 \times 10^{-34} / 2\pi) \\ &= 4.66 \times 10^{29} \text{ m/s}^2 \end{aligned} \tag{4}$$

where $R_o = \hbar / 2mc = 1.93 \times 10^{-13}$ m is the radius of the double-looping charged-photon model of the electron.

6.4. *The Internal Radial Force*

This is given by

$$\begin{aligned}\vec{F} &= d\vec{p}_o / dt = \omega_{zitt} p_o \hat{r} \\ &= (1.55\times10^{21}\,\text{rad/s})(2.73\times10^{-22}\,\text{kg m/s})\,\hat{r} \\ &= 0.424\text{N}\,\hat{r}\end{aligned} \tag{5}$$

The source of this remarkably large force $F = 0.424\text{N}$ and correspondingly large centripetal acceleration $a_c = 4.66\times10^{29}\,\text{m/s}^2$ within the spin-½ charged photon model of the electron model is unknown. An accelerated electric charge normally loses energy through radiation, according to standard electromagnetic theory. But the circulating spin-½ charged photon apparently does not lose energy due to this large centripetal acceleration. In quantum mechanics, an electron in the ground state of the hydrogen atom does not radiate energy in spite of its internal motion. Something similar may be going on with the proposed centripetally-accelerated charged photon-like object within an individual electron.

7. The Equations for the Transluminal Energy Quantum Spin-1 and Spin-½ Photon Models

The proposal here is that a circling spin-½ charged photon may compose an electron. The equations for a proposed transluminal energy quantum spin-½ charged photon are presented below. First the equations proposed by Gauthier[15] for a translumial energy quantum model of a spin-1 photon are presented for comparison. It is shown below also that the *z*-component of the spin of the spin-1 and spin-½ photons are calculated to be $\hbar$ and $\hbar/2$ respectively. The speed of the helically moving transluminal energy quantum is calculated for both models to be $c\sqrt{2}$. The forward helical angle of the helical trajectory of the transluminal energy quantum is found to be 45 degrees in both of these models. The spin-1 photon transluminal energy quantum's helical trajectory makes one full helical turn per longitudinal photon wavelength λ and has a helical radius of $\lambda/2\pi$. The spin-½ charged photon transluminal energy quantum's helical trajectory makes two full helical turns per longitudinal photon wavelength λ and has a helical radius of $\lambda/4\pi$.

For a right-handed spin-1 photon model traveling in the $+z$ direction with energy $E = \hbar\omega$, angular frequency ω and wavelength $\lambda = c/\nu = 2\pi c/\omega$, the equations for the trajectory of the transluminal energy quantum (neglecting a possible phase factor) are:

$$\begin{aligned}x(t) &= \frac{\lambda}{2\pi}\cos(\omega t), \\ y(t) &= \frac{\lambda}{2\pi}\sin(\omega t), \\ z(t) &= ct\end{aligned} \tag{6}$$

for the components of the circulating transluminal energy quantum's position with time, and

$$\begin{aligned}p_x(t) &= -\frac{h}{\lambda}\sin(\omega t), \\ p_y(t) &= \frac{h}{\lambda}\cos(\omega t), \\ p_z(t) &= \frac{h}{\lambda}\end{aligned} \tag{7}$$

for the components of the circulating transluminal quantum's momentum with time.

The *z*-component of spin of the spin-1 model above is calculated from its equations as

$$\begin{aligned}S_z &= (\vec{R}\times\vec{p})_z = x(t)\times p_y(t) - y(t)p_x(t) \\ &= \frac{\lambda}{2\pi}\times\frac{h}{\lambda}[\cos^2(\omega t)+\sin^2(\omega t)] \\ &= \frac{h}{2\pi} = \hbar\end{aligned} \tag{8}$$

which is the spin of a spin-1 photon.

The speed $v(t)$ of the transluminal energy quantum for the spin-1 model is calculated from the velocity components of the transluminal energy quantum, which are derived by differentiating the position components for the spin-1 model in the equations above as

$$\begin{aligned}v_x(t) &= dx(t)/dt = -\frac{\lambda\omega}{2\pi}\sin(\omega t) \\ v_y(t) &= dy(t)/dt = \frac{\lambda\omega}{2\pi}\cos(\omega t) \\ v_z(t) &= dz(t)/dt = c\end{aligned} \tag{9}$$

and

$$\begin{aligned} v(t)^2 &= v_x(t)^2 + v_y(t)^2 + v_z(t)^2 \\ &= [-\frac{\lambda\omega}{2\pi}\sin(\omega t)]^2 + [\frac{\lambda\omega}{2\pi}\cos(\omega t)]^2 + c^2 \\ &= c^2[\sin^2(\omega t) + \cos^2(\omega t)] + c^2 \qquad (10) \\ &= c^2 + c^2 \\ &= 2c^2 \end{aligned}$$

As a result, $v(t) = \sqrt{2c^2} = c\sqrt{2}$ for the speed of the transluminal energy quantum of the spin-1 photon model.

For a right-handed spin-½ charged photon with energy $E = \hbar\omega$, angular frequency ω and wavelength $\lambda = 2\pi c/\omega$, traveling in the +*z* direction, the equations for the trajectory of the transluminal quantum (again neglecting a possible phase factor) that makes two helical turns per photon wavelength λ are:

$$\begin{aligned} x(t) &= \frac{\lambda}{4\pi}\cos(2\omega t), \\ y(t) &= \frac{\lambda}{4\pi}\sin(2\omega t), \qquad (11) \\ z(t) &= ct \end{aligned}$$

for the components of the circulating transluminal energy quantum's position with time, and

$$\begin{aligned} p_x(t) &= -\frac{h}{\lambda}\sin(2\omega t), \\ p_y(t) &= \frac{h}{\lambda}\cos(2\omega t), \qquad (12) \\ p_z(t) &= \frac{h}{\lambda} \end{aligned}$$

for the components of the circulating transluminal energy quantum's momentum with time.

The *z*-component of spin of the spin-½ charged photon model above is calculated from its equations as

$$\begin{aligned} S_z &= (\vec{R} \times \vec{p})_z = x(t) \times p_y(t) - y(t)p_x(t) \\ &= \frac{\lambda}{4\pi} \times \frac{h}{\lambda}[\cos^2(2\omega t) + \sin^2(2\omega t)] \qquad (13) \\ &= \frac{h}{4\pi} = \hbar/2 \end{aligned}$$

which is the spin of a spin-½ photon.

The corresponding calculation for the speed of the transluminal energy quantum of the spin-½ charged photon model also gives $v(t) = c\sqrt{2}$.

8. Discussion

The proposal here is that the inertial mass *m* of a resting electron is derived from the circling momentum $p_o = E_o/c$ of a circling spin-½ charged-photon of energy E_o that is proposed to compose the resting electron. This short derivation was discovered after Gauthier[9] developed a model of the relativistic electron as composed of a spin-½ charged-photon. This relativistic electron model was developed from an internally superluminal model of a resting electron developed earlier by Gauthier[13].

The formula $E_o = mc^2$, relating the energy E_o of a resting particle to its inertial mass *m* is well-grounded in experimental evidence. What is being proposed here is that an elementary particle does not get its inertial mass directly from its resting energy E_o. Rather, an elementary particle derives its inertial mass from the circling momentum E_o/c of a photon-like object proposed to compose this elementary particle.

One can object that, although the derivation of a particle's inertial mass *m* from the circulating momentum of a photon or photon-like object using Newton's 2nd law may be correct, there is no experimental evidence that a photon or photon-like object can actually move in a circle or helix to form an elementary particle such as an electron. A normal spin-1 uncharged photon is not known to be able to move in a circle small enough to form an elementary particle.

The same may not be true for a hypothesized new variety of photon that *can* move in a circle to form a resting electron and in a helical trajectory to form a relativistic electron. An electron has spin-½ and carries a negative electrical charge. Gauthier[9] proposed an electron model that is composed of a spin-½ negatively charged photon that moves along a circular trajectory (for a stationary electron) or along a helical trajectory (for a moving electron). In this model the proposed spin-½ charged photon composing the electron moves along its circular or helical trajectory at the speed of light, and moves in the forward or longitudinal direction at the electron's observed speed, which is less than the speed of light.

So far, no one to my knowledge has shown that this proposed spin-½ charged photon cannot exist or cannot move in a circular or helical trajectory to form an electron. In fact the electric charge of the proposed spin-½ charged photon may be what causes the charged photon to move in a circular or helical trajectory. It remains for experiment to test this spin-½ charged photon hypothesis. If a spin-½ charged photon does exist, it may not have been recognized yet partly because it has already been named an electron or some other known particle with mass, and therefore has not been looked for experimentally as a spin-½ charged photon.

It can also be objected that even if such a spin-½ charged photon does exist and composes an electron, it is not really a photon because it does not have spin 1 and has electric charge, unlike a normal photon. However, the proposed spin-½ charged photon has other properties of a photon. It obeys the well-known wave formula $\lambda\nu = c$ (where λ is the charged photon's wavelength and ν is its frequency). It also obeys the well-known formulas for a photon's energy $E = h\nu$ (where h is Planck's constant) and momentum $p = h\nu / c$. The author prefers to call this proposed spin-½ charged light-speed object a new variety of photon rather than giving it a completely different particle name.

Another possible objection to a circling-photon-like-object model of a fundamental particle is that a single photon-like object would seem to violate the law of conservation of momentum by traveling in a circular trajectory instead of a straight trajectory. However, the proposed spin-½ charged-photon model assumes that the circling spin-½ charged photon composing a resting electron is acted on by a central force $\vec{F} = d\vec{p}_o / dt$, which changes the momentum $p_o = E_o / c$ of the spin-½ charged photon so that it moves in a circle instead of a straight line. This central force $\vec{F}$, when divided by the circling charged photon's centripetal acceleration $a_c = \omega^2 R_o$ as described above, yields the inertial mass m of the circling spin-½ charged photon and therefore the inertial mass m of the particle composed of the circling spin-½ charged photon.

The nature of this central force proposed to act on the circling photon-like object composing an elementary particle is currently unknown. Still, its value can be easily calculated for any particular circling-photon-like-object model of a particle. For example, in the spin-½ charged-photon model of the electron, the value of the central force $\vec{F}$ acting on the double-looping circling charged photon is calculated above to be 0.424 N, or 0.095 pounds. This is a remarkably strong, presumably non-nuclear force that is proposed to be related to a single electron.

The spin or angular momentum of an elementary particle is currently unexplained. Spin is considered to be an "intrinsic" property of an elementary particle like an electron. The spin of a particle like an electron can be explained by the internal circulation of a single photon-like object composing the electron, but not without the circling charged photon appearing to violate the law of conservation of momentum. In the spin-½ charged-photon model of an electron, the electron model's spin component S_z is calculated by multiplying the circling charged photon's momentum $p_o = E_o / c = mc$ by the double-looping circle's radius $R_o = h / 4\pi mc$, giving $S_z = h / 4\pi = \hbar / 2$. This is the exact experimental value of the spin of an electron. If the spin-½ charged photon did not appear to violate the conservation of linear momentum, it could not move in a double-looping circular trajectory to give the electron particle model the experimentally correct electron spin.

More generally, other non-light-speed fundamental particles with inertial mass such as quarks, neutrinos, W particles, Z particles, the Higgs boson and even some proposed dark matter particles, may each be composed of an internally-circulating light-speed photon-like object. These other particles could also derive their inertial masses from their circulating internal momenta. In this view, the inertia of a particle is not a property of matter due to an "inherent force" or "inertial force" in matter that resists acceleration, as Newton proposed but never explained. Rather, the inertial mass of an elementary particle of matter is calculated from the rate of change of the circling momentum vector of a photon-like object composing the particle. When this rate of change of circling momentum is combined with Newton's 2nd law of motion $F = ma$ and the circulating particle's centripetal acceleration, the result is the property of matter called inertial mass $m = E_o / c^2$. This analysis of a particle's inertial mass m supports the idea that ordinary spin-1 photons carry inertial mass $m = E / c^2 = h\nu / c^2$. But it is only when the energy and momentum of a photon becomes localized in a particle with mass by circulating, as in the case of the proposed spin-½ charged photon, that a photon's inertial mass becomes a particle's rest mass or invariant mass.

Usually Einstein's formula is written $E = mc^2$. This better-known formula is less precise than $E_o = mc^2$, because the total energy E of a moving particle increases with the particle's speed, while the moving particle's "invariant mass" or "rest mass" m is independent of the particle's speed and is always proportional to E_o. But why should the term c^2 even occur in the formula for the energy contained in a particle at rest like an electron that has inertial mass m? The simplest answer is that there is something moving at light speed c inside the particle or composing the particle.

As mentioned above, several researchers have proposed that the electron is composed of something moving internally at light speed *c*. To the author's knowledge a derivation of a particle's inertial mass $m = E_o / c^2$ using Newton's second law of motion $\Sigma\vec{F} = d\vec{p}/dt = m\vec{a}$ on the rotating momentum vector $p_o = E_o / c$ of a circling photon-like object of resting energy E_o proposed to compose the particle, along with the centripetal acceleration a_c of the circling photon-like object, has not previously been presented by another researcher.

9. Conclusions

A model of a resting elementary particle composed of a circling photon-like object of resting energy E_o and circling momentum $p_o = E_o / c$ is used to provide a short, non-relativistic derivation of the particle's inertial mass $m = E_o / c^2$, or $E_o = mc^2$. To obtain this result, Newton's second law $\Sigma\vec{F} = d\vec{p}/dt = m\vec{a}$, or $m = (d\vec{p}/dt)/\vec{a}$, which defines inertial mass *m,* is applied to the circulating photon-like object's changing vector momentum $\vec{p}_o$ and its centripetal acceleration $a_c = c^2 / R$. Spin-1 photons are uncharged and do not move in a circular trajectory. An internally-transluminal spin-½ charged-photon model is proposed that can move in a double-looping circular trajectory to give the resting electron model a spin of ½. At highly relativistic velocities, as shown in Gauthier[10], the spin-½ charged photon model of the electron internally moves along a helical trajectory at light speed to give the sub-light-speed electron model a spin of ½ as well.

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Kinematic Solutions to the Twin Paradox in Special Relativity

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This paper deals with the twin paradox within special relativity. The paper reveals the cause of time dilation of the inertial stay-at-home twin occurring, as believed by the non-inertial travelling twin, throughout his motion except for a short-time turn, though it is the stay-at-home twin who by the time of the return of the traveler has aged more. This cause is the absolutization of the individual observer's state of rest. Certain kinematic solutions to the paradox are given without resorting to a non-inertial reference frame. The existence of such solutions is shown both in special relativity and in Lorentz ether theory.

Keywords: Twin paradox, Clock paradox, Einstein, Lorentz, Ether theory

1. Introduction

The source of the twin paradox is an assertion that when there is relative motion of a set of twins – a stay-at-home twin and a travelling twin – they experience reciprocal time dilation (slowing of time relative to each other). The time dilation of the stay-at-home twin, occurring as seen by the travelling twin throughout his journey except for a short-time turn, seems paradoxical, as by the time of return of the traveler the stay-at-home twin has aged more.

As one of the solutions to the twin paradox, an argument is often put forward that the use of inertial reference frames cannot be extended to a twin who if only for a brief period becomes non-inertial. Another solution proceeds from an assumption that time flow rates are different at the points of a non-inertial during the turn traveler-related reference frame [1-2]. The present paper offers two of such solutions. One of them will be dealt with within the kinematics of special relativity, whereas the other will be discussed going back to Lorentz' ether theory. In so doing, we are going to demonstrate that the ether-related solution is not only free from incongruity but is particularly simple and illustrative.

2. The Ambiguity of the Notion of Relative Motion of Two Point Objects

Imagine two observers, observer A who is at rest at point a of a certain inertial reference frame, and observer B moving at some moment through point b of this reference frame at a distance l_0 from observer A. The moving observer B, for a short time finding himself at a given moment at point b (flying through it), has velocity v directed perpendicularly to segment ab, as shown in the Figure 1.

Let each of the observers have an observation tube, a clock with a green face and a source of equidirectional monochromatic green colored radiation, and let each observer register a change of the wavelength of another observer's emitter.

At first glance, each of the observers, owing to the transverse Doppler effect, having directed the tube along the line segment ab, will register a reddening of radiation from another observer's emitter. After all, it seems apparent that if observer B is moving relative to observer A, then observer A is moving relative to observer B. In fact, the answer is not that simple. The above statement of the problem does not maintain that the observers are moving "relative to each other". It is only the speed v of observer B that is spoken about. With this problem formulation, the observers may, for example, stand on a rotating disk – observer A in the centre of the disk at point a, and observer B on its edge. In so doing the non-inertial observer B, circling in certain reference frame at velocity v, has this velocity v

at any point on the circumference, among them at point b, in which he finds himself at a certain moment in time, as shown in Figure 2.

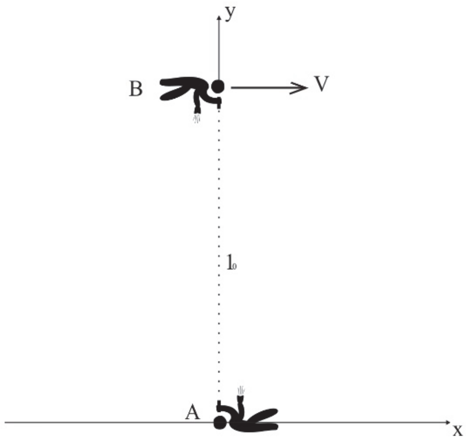

Figure 1. Observer A is at rest at point a of an inertial reference frame. The velocity of observer B located at point b is equal to v. The distance ab between the observers is equal to l_0.

Regardless of observer B's motion, we cannot say he is moving relative to observer A, or that observers A and B are moving relative to each other since the distance between the observers remains unchanged.

Observer B is moving in a circle, however, not relative to observer A, but in the inertial system K (or relative to it), wherein observer A is at rest. In this case, it does not matter whether the observers find themselves on a rotating disk or observer B is moving in a circle around its center in any other way.

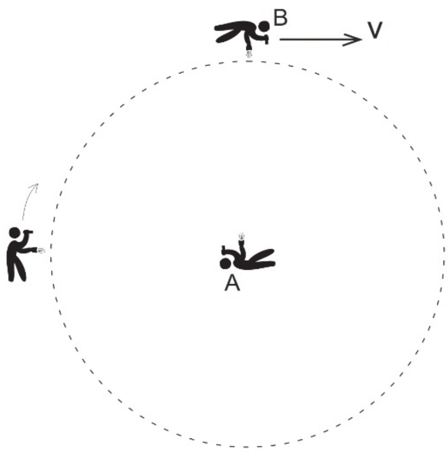

Figure 2. Observer B is moving in a circle at velocity v around observer A, which is at rest in the inertial reference frame K.

3. The Observer's Circular Motion

If observer A, located in the centre of a circle at point a of the reference frame K, tilts the tube along the line segment ab to point b through which at some moment observer B has flown at a certain velocity v, then he will see the source of a "warmer", for example yellow, color. This wavelength shift is equivalent to the slowing down of the clock of the moving observer B. If the face of the clock is colored green, then it will be perceived by the observer A as yellow.

Moving along a circular path crosswise relative to the flux of monochromatic emission, observer B will perceive the color of the source and the clock's face in the centre of the circle as blue and not yellow. Due to aberration, a flow of light falls on the observer, not at a right angle relative to its direction of motion, but at an angle θ', acute to this direction. This results in a shift of the emission spectral line to shorter wavelengths.

In order to see the observer in the centre, the observer on the circle has to look at an angle, acute to the direction of his motion (or align the tube at this angle). In the reference frame K', the tube tilt angle θ' to the direction of motion of observer B is equal to arccos (v/c).

Formally, this fact can be expressed using the known ratio

$$f' = \frac{f_0\sqrt{1-v^2/c^2}}{1-(v/c)\cos\theta'}, \qquad (1)$$

linking frequency f' of the received light signal and proper frequency f_0 of the emitted signal at an angle θ' between the tube tilt angle and the velocity vector. Since the tube tilt angle θ' to the direction of motion of observer B is equal to arccos (v/c), then after substituting the angle θ' with arccos (v/c) in formula (1) the mentioned ratio becomes

$$f' = f_0\Big/\sqrt{1-v^2/c^2} \qquad (2)$$

The frequency of the source emission is proportional to the clock rate, and the proper frequency of the green sources is the same for both observers. Hence, if at the initial moment of time the readings t' of observer B's clock and the readings t of observer A's clock were zero, then by analogy with formula (2) we can write

$$t = t'\Big/\sqrt{1-v^2/c^2} \qquad (3)$$

To explain this fact without going beyond kinematics, observer B should acknowledge the fact of his motion relative to a reference frame conventionally at rest. Moving crosswise to a beam of light, he has to consider light aberration and tilt his tube to see the light. Because of the slowness of his clock and of the rate of his time flow, he fixes the seemingly accelerated rate of observer's *A* clock, which is actually not present in the reference frame *K*.

Observer *B*, on the circle for obvious reasons, cannot regard himself at rest in an inertial frame and cannot observe the effects of the slowness of the clock and the yellowing of observer's *A* source. He likewise has no right to explain the blue color of the source in terms of the longitudinal component of the Doppler effect caused by the central observer's motion. It is he who is moving (in the reference frame *K*), crosswise to the beam, and not the central observer. He can account for the blue color by the fact that, moving crosswise to the green beam, he experiences time dilation and, being in delayed time conditions, perceives green light as blue.

If a great number of observers with green light sources are moving in a circle and one of them has emitted a light pulse, which, having passed the center of the circle, hits the diametrically opposite point on the circle, where at this time another observer is moving, the latter will perceive the received pulse as green. This will take place despite the fact that each of the observers, having exchanged the pulse, is moving in the reference frame *K* with velocity *v* in opposing motion. The value often called "relative velocity" of observers, according to the rule of velocity addition is equal to $2v/(1+v^2/c^2)$. The received pulse will evidently be green, given the fact that the pulse sent to a diametrically opposite point passes through the center and acquires the yellow color for the central observer. On the other hand, the frequency of the yellow pulse emitted from the center of the circle, which has reached the circling observer, increases, according to the latter, $1/\sqrt{1-(v/c)^2}$ times, as it follows from formula (2), and the pulse becomes green.

If each of the observers, having exchanged the light pulse, acknowledges the fact of his motion, then the observers will attribute the invariance of the pulse colour to their identical time dilation due to the same speed in the reference frame *K*. The pulse from the green source of one observer, which has turned yellow in the reference frame *K* due to time dilation, is perceived by another observer as green again due to his time dilation.

4. The Rectilinear Motion of the Observer

Now consider a case when both observers are inertial.

Let observer *C′* move tangentially to the circle through point *b* at a constant speed *v*. At first, the inertial observers, *A* and *C′* approach each other, then, as observer C′ appears at point *b,* for a short time they find themselves at a distance l_0, which is minimal from each other throughout the whole motion period, whereupon they begin to move away from each other.

As distinct from observer *B*, observer *C′* is capable of viewing both the slowing down and the acceleration of the central observer's clock rate, as well as to see the central observer's source and clock face in different colors, specifically, in yellow and in blue. The color of the viewed source and clock face depends on the direction of the observer's gaze (or on the direction of the tube).

An ability of observer *C′* to see different colors of the source belonging to observer *A* in physical terms is explained by the fact that, unlike observer *B,* observer *C′* accepts from the central source not only the emission that comes into the tube in the circle but also that outside it. Observer *A* is also capable of recording different colors of light coming from observer *C′*, which is also due to his ability to receive light emitted by observer *C′*s source outside the circle.

The question arises: how are observers *A* and *C′* supposed to carry out their observations if they can see whatever they want?

First, they must choose a reference frame. If they are not interested in the interrelationship of the results of physical observations, then each of them can choose their own system. This is exactly what observers in special relativity do. None of the Einsteinian observers acknowledges the fact of their motion relative to other observers, and it leads to time dilation symmetry like $\Delta t' < \Delta t$ while $\Delta t' > \Delta t$. However, if observers *A* and *C′* wish to obtain interrelated results of their observations, they must choose one, and only one, inertial frame, and all further actions are to be carried out given this choice. For example, they can choose the reference frame *K* wherein observer *A* is at rest and observer *C′* is in motion.

If observer *C′* as well as observer *B* who is moving in a circle, would assume that he, finding himself at point *b*, is moving in the reference frame *K* at velocity *v* crosswise to the beam, and would vector the tube in the same direction in which observer *B* stationed on the circle is looking, then, as is the case with the latter, he will see the blue source and the blue clock face in the hands of observer *A*.

Assume now that observer *C′* is flying at velocity *v* parallel to the *X*-axis of the reference frame *K* at some distance from it and monitoring the flow of time of the system *K* on the clocks placed along the *X*-axis.

If observer *C′* will look in one direction at the clock in the stationary reference frame *K* he is flying by, then, consecutively fixing the readings of a great number of clocks passing the point on which his eyes are focused, he will fix an accelerated flow of time T_K as a sequence of the set of the readings of the clocks in the reference frame *K*, relative to which he is moving. The rate of the flow of time will not depend on the direction of his gaze, although the color of the clock faces depends on it. With the initial readings of the clocks of the reference system *K* and of observer C′ equal to zero, the time acceleration is expressed (within an accuracy up to the propagation time of light from the clock to the observer) as a ratio $T_K = t' \big/ \sqrt{1-(v/c)^2}$, where *t′* is the time shown by the flying observer's C′ clock. Such accelerated time flow in the reference frame *K* is not consistent with the behavior of each individual clock or each individual emitter in the reference frame *K*, if the observer C′, believing that he is at rest and the reference frame *K* is moving relative to him, will tilt the tube to be perpendicular to its motion. Given this orientation of the tube, while tracking the behavior of the source emitting, for example, the green light, he will detect a shift of the wavelength of the emitter according to the transverse Doppler effect to the long-wave region and turning of the green beam to a yellow one. The shift of the wavelength will correspond to the slowness of the rate of each individual clock flying past observer *C′*. This inconsistency in the rate of an individual clock and of a great number of clocks is no less paradoxical than the twin paradox itself. How does time flow in the reference frame *K*, if, according to successive readings of a great number of clocks which observer C′ is flying by in the reference frame *K*, the time in the reference frame *K* passes faster, whereas according to the time of each of the clocks flying past observer *C′*, the time in the system *K* passes slower?

The solution to the paradox of inconsistent clocks is the right selection of the tilt angle of the tube moving in the system *K* by observer *C′*. The paradox occurs when observer *C′* forgets that he himself is moving, and referring the state of rest to himself, counts each clock of the system *K* as moving relative to him. If the moving observer *C′* remembers that there are not the clocks of the reference frame *K* moving past him, but he himself is moving past the clocks, then he will tilt the tube at a needed angle and find that the light has become blue, and the emission frequency has exceeded that of the original green source $1 \big/ \sqrt{1-(v/c)^2}$ times. It means that in this position of the tube the pace of change of a great number of clock readings and the pace of a single clock prove consistent. It is just this position of the tube that corresponds to speed *v* of observer *C′,* relative to the reference frame *K*. The tube tilt angle θ' to the direction of motion of observer *C′* is equal to arccos (*v/c*).

Due to the consistency of the pace of an individual clock and of a great number of clocks of the stationary reference frame *K,* we can talk about the acceleration of the rates of the clocks in the reference frame *K* as a whole, relative to the clock rate of observer *C′*. This acceleration also relates to the clock of observer *A*, which is an element of the reference frame *K*.

5. Regarding the Relative Motion of Intrinsic Inertial Systems

Ascribing a state of rest to all reference frames in motion relative to one another leads to inconsistency of physical quantities like *A*>*B*, whereas *A*<*B*.

This inconsistency may be eliminated, for example, by way of arbitrary selection of a cardinal stationary reference frame in relation to which all other systems acquire certain velocities. The assignment of the state of rest to a cardinal inertial reference frame can be done by way of the following definition: "The stationary inertial reference system is such a system in which for any pair of points belonging to this system the light propagation time from one point to another is equal to the light propagation time between these points in the opposite direction."

Under this definition, the difference in the velocities of light in opposite directions in the moving reference systems becomes dependent on the direction and on their speed relative to the stationary reference system, while the average value of the speed of light in a closed path is maintained.

If observer *C′*, who has acknowledged the fact of his motion in the above example, will rigidly fix himself to the reference frame K'_{mov}, which is moving together with him in relation to the reference system *K*, then all the observers who have acknowledged their state of motion in the reference frame K'_{mov} will register an accelerated time flow in the stationary reference frame *K*. Moreover, if synchronization of clocks in a moving reference frame has been performed taking into account its own motion relative to the system *K* and compliance to a uniform simultaneity in the systems *K* and K'_{mov}, then the accelerated flow of time in the stationary system *K* can be detected not only by visual observations. This acceleration may as well be detected by way of comparison of the movement of each individual clock in the reference frame *K* with a pair of clocks in the system K'_{mov}. In turn, if each of the observers in the reference frame *K*, for example, observer *A*, looks in one direction at a clock of the reference system K'_{mov}, moving past him, then, consecutively fixing readings of a great number of clocks passing the point on which his eyes are focused, he will fix not the acceleration but the slowness of the flow of time in the reference frame K'_{mov}.

The consistency of the speeds of time flow in different reference systems results from the unity of simultaneity. Note that we are talking about the unity, and not about the absoluteness of simultaneity. The choice of a cardinal stationary reference system is an arbitrary and conditional action, and when changing the cardinal system, the nature of simultaneity changes because of the relativity of the latter; the simultaneity becomes uniform again, though for other events. Only in case there existed an absolute reference frame or the stationary ether, with a cardinal reference frame rigidly fixed to it, could we talk about absolute simultaneity, and the physical quantities would acquire absolute character. Nevertheless, even in this case, the possibility of invariant recording of the laws of nature would remain because special relativity and Lorentz ether theory are mathematically and experimentally equivalent, differing only in philosophical content.

6. A Solution to the Twin Paradox in Special Relativity

Now we turn to the twin paradox.

Imagine twin *B′* escaping point *g* (with twin *A* remaining) to point *h* of the inertial system *K*, who after a brief turn at point *h*, has made a reverse journey and returned to twin *A* at point *g*. Assume now that twin *B′* is flying at velocity *v*, parallel to the *X*-axis of the reference frame *K*, at some distance from it and monitoring the flow of time of the system *K* on the clocks placed along the *X*-axis.

If twin *B′* acknowledges the fact of his motion at velocity *v* in the stationary reference frame *K* and directs his look or tilt the tube at an angle arccos (*v/c*), then he will fix the acceleration of time *t* in the reference frame relative to which he is moving. The acceleration is expressed as a ratio $\Delta t/\Delta t' = 1\big/\sqrt{1-(v/c)^2}$, where Δt the time interval that has elapsed in the reference frame *K* during some time $\Delta t'$, which has elapsed for the flying twin according to his own clock. Twin *B′* will find the same acceleration if he compares the pace of his clock with the flow of readings consecutively taken by him from the clock's past which he is flying, belonging to the reference frame *K*.

Thus, the travelling twin, who has acknowledged the fact of his movement in the reference frame *K*, detects acceleration of the time flow of his inertial twin brother remaining at rest during the whole period of separation of the twins.

The ratio $\Delta t/\Delta t' = 1\big/\sqrt{1-(v/c)^2}$ can be viewed as a consequence of the inverse Lorentz transformation $t = (t'+vx'/c^2)\big/\sqrt{1-(v/c)^2}$. Whatever point *x′* of the reference system *K′*, which is in relative motion with the reference system *K*, the twin *B′* would mentally bind himself to, the time interval Δt at this point is equal to the quantity $\Delta t'\big/\sqrt{1-(v/c)^2}$.

7. A Solution to the Twin Paradox in the Ether Theory

There is actually no paradox related to the ether theory. If one of two twins who are at rest in the ether at one point flies at speed *v* to a distant point and then after a while returns to twin *A* remaining at rest, then for the twin flying in the ether his "local time" characterizing the rate of physical processes in his body and the pace of the movement of his clock on both segments of his flight (there and back) slows down due to interaction with the ether. For this reason, the lapse of his "local time" will be $1\big/\sqrt{1-(v/c)^2}$ times less than for the twin at rest in the ether, and the "travelling" twin will get less "old".

Moreover, what will happen if the two twins are flying side by side in the ether at speed *v* – with their "local time" passing slower – then one of them stops, staying at rest in the ether for some time, then catching up with the travelling twin? The twin who continued his flight in the ether with no information about the fact of his motion in the ether perceives this maneuver of his brother as a round trip to a distant point.

Let at the time of stop of one of the twins in the ether, the clocks of the parting twins show zeros. Suppose that after making a stop for some time the twin who has lagged behind, at the moment t_1 of the ether time when his clock (because of the stop) was showing this time, left at speed *u*, such that $v<u<c$, follows his brother flying away from him. The distance between the twins at the start of the twin who was left behind is equal to vt_1. Setting out, the twin left behind will catch up with the twin flying at a constant speed *v* at the point in time t_2, having spent the time equal to $vt_1/(u-v)$. During this period, by the clock of the twin following the flying away brother at speed *u*, there will be a lapse of proper time, which is $1\big/\sqrt{1-(v/c)^2}$ times less than the ether time and equals $vt_1\sqrt{1-(u/c)^2}\big/(u-v)$. Let us assume the velocity *u* such that the proper time t'_2 - t'_1 of the catching up twin is numerically equal to the time t_1 of his stay at rest relative to the ether, i.e. $t'_2 - t'_1 = t_1$ or

$$t_1 = vt_1\sqrt{1-(u/c)^2}\big/(u-v) \qquad (4)$$

This equality meets the condition under which the twin spends the same proper time on a trip to a distant point and back. By elementary transformations of the equality (4), we can obtain the value of velocity *u*, which is equal to $2v/(1+{}^2v/c^2)$. Substituting this value in the expression for the time $vt_1/(u\text{-}v)$ required for the return of the twin, and summing the time $vt_1/(u\text{-}v)$ and the time t_1, we obtain the ether time spent by the twin lagging behind on the stop and return to the flying twin. This time is equal to $2t_1/(1-v^2/c^2)$. Since the clock of the inertial twin flying at a speed v go $1/\sqrt{1-(v/c)^2}$ times slower than the clock at rest in the ether, the flying twin will determine the time spent by the lagging twin on the stop and return to the flying twin as a quantity meeting the equality:

$$t'_2 = 2t_1/\sqrt{1-(v/c)^2} \qquad (5)$$

Since the time elapsed for the non-inertial twin by the moment of his return is numerically equal to $2t_1$, and the time of the inertial twin is numerically equal to $2t_1/\sqrt{1-(v/c)^2}$, then the lapse of time for the non-inertial twin is $1/\sqrt{1-(v/c)^2}$ times shorter, and he has aged less than the inertial twin has.

8. Conclusion

One of the reasons for the paradoxical effects of special relativity is the unconditional approach to the state of proper rest by an observer moving relative to some reference frame. Declaring the relativity of states of motion and rest, Einsteinian observers always assign the state of rest to themselves and to their reference frames and never do so for the state of motion. The introduction to the relativistic theory of observers who recognize the state of proper motion relative to third party reference frames allows for the solution of the twin paradox to be confined to the consistent kinematics of their motion.

The ether theory does not need any tricks to account for the age difference between an inertial and a non-inertial twin who have met each other after parting. The fact that the inertial twin always turns out to have aged more than the non-inertial one at their meeting after parting, in the ether theory, is an elementary consequence of the slowing down of the rate of processes in bodies moving in the ether.

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The Non-Relativistic Models of the Relativistic Bell's Paradox

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Here the Bell's accelerating rockets paradox is examined. The non-relativistic models of Bell's effect are presented where, like the theory of special relativity, the proper distance between two rockets following one another is increased, the two rockets being accelerated on identical programmes. It is shown that the proper distance increase is determined by Einstein's simultaneity of the moments of the start of the programmes' execution on the rockets. It is also shown that Einstein's relative simultaneity does not ensure reversibility of the proper distance between the rockets upon their joint return to their initial state. The reversibility is only achieved by the introduction of the assigned reference frame (not necessarily absolute!) and of the universal time in all inertial reference frames.

Keywords: Relativistic Bell's paradox, Einstein, Lorentz, Ether theory

1. Introduction

The relativistic Bell's paradox or, in other words, the accelerating rockets paradox refers to the solution of the following problem.

In a certain inertial reference frame, K two identical rockets at rest with absolutely identical engines are considered. A thin non-stretch, delicate stiff string connects the tail of one rocket to the nose of the other rocket. At a certain moment in time $t=0$, the engines are simultaneously ignited on identical programmes, and the rockets start accelerating, following one another along a straight line on which the string connecting the rockets lies. Due to space homogeneity, the rockets at each moment in time t moving with identical acceleration $g(t)$ and identical velocity $v(t)$ travel in strict synchronism along the straight line due to mutual synchronism of travel, staying an invariable distance apart in the frame K. The question arises as to what will happen to the string when the rockets continue to accelerate indefinitely long.

According to the solution given by Bell, the string will break, which is often presented as a paradoxical effect [1-6]. Bell's solution has been disputed by a number of physicists. At the same time, there is no paradox related to the string break whatsoever, and the discussions only once again confirm that there is often no understanding of the basic effects of the theory of special relativity even among high-ranking physicists. It is only this lack of understanding that can explain an appearance of a whole series of discussion papers [1-5] on Bell's paradox in the American Journal of Physics.

2. The Break of the String and the Essence of the Relativistic Bell's Paradox

Within the framework of the theory of special relativity, the Bell's paradox has a simple explanation. Skobeltsyn in his book "The Twin Paradox in Relativity" gave a sufficiently complete explanation of the effect related to the break of a string connecting synchronously accelerating rockets [6].

The longitudinal length L of an object moving at velocity v is connected with proper length L_0 of this same object by the Lorentz contraction formula $L = L_0\sqrt{1-(v/c)^2}$, where c is the speed of light. According to the formula, proper length at all times exceeds the length of a moving object.

At all times! If, after the acceleration of an object, its proper length stays invariable, then the length of an object moving after acceleration decreases. However, if in the process of and after acceleration of an object the length constancy of a moving object is forced, then its proper length increases.

Let us in what follows assume that an object has accelerated to a velocity v, whereupon the acceleration stops, and the object becomes inertial. For the sake of convenience, let us assume that a velocity v is such that

$\sqrt{1-(v/c)^2}$ equals ½. Then, if within the reference frame *K* an elastic rod has been accelerated to a velocity *v*, retaining at its ends an invariable (within this reference frame) distance apart (synchronously accelerating them) and preventing the rod from contracting, then the length *L* of the rod moving within the frame *K* at a velocity *v* stays invariable and numerically equal to the value L_0. The proper length of the rod L'_0, i.e. the length of the rod within the inertial system *K′*, where it is at rest after the acceleration has been discontinued, becomes equal to 2*L*, i.e. the relation $L'_0 = 2L$ becomes valid, now as $L = L_0$, then as $L'_0 = 2L_0$.

In the case of two rockets, the distance *L* between them within the reference frame *K* during a synchronous acceleration in this system remains numerically equal to the proper distance L_0 separating the rockets at their start. At the same time, the distance between the accelerating rockers as registered by inside observers increases, i.e. according to the observers the rockets are moving away from one another. The engines having stopped, the proper distance L'_0 between the rockets in the inertial reference frame *K′* where the rockets are at rest becomes equal to 2*L* or, which is the same, to $2L_0$. It is apparent that if the rockets are connected by a thin fragile string, the latter will break.

The string between the rockets is present in Bell's problem for better clarity and intrigue. The essence of Bell's paradox is not in the break of the string but in the increase of proper distance between the rockets, resulting in the break. It is namely on this effect and not on the break of the string that we are going to focus our attention in what follows.

3. The Conditions of Reversibility of the Proper Distance Between the Rockets Upon Return to Their Initial State

Let us consider a somewhat extended two-stage modification of Bell's thought experiment.

Let us assume that the rockets, having moved apart during the first stage of the thought experiment and finding themselves at rest in the reference frame *K,* after the engines have stopped turning 180 degrees, and then this same rocket acceleration experiment is repeated under identical initial conditions. To do this, some time elapsing after the turn of the rockets, the engines are started on the same programme used for the first stage. Then the rockets at the second stage of the experiment are accelerated to a velocity *v* within the reference frame *K′* (they are slowed down accordingly to zero velocity within the reference frame *K*). It is clear that following this each rocket will return to a state of rest within the initial reference frame *K*. The question consists in the following: what proper distance will separate the rockets after their stop within the reference frame *K*?

Let us note that during the first stage after the stop of both engines the clocks mounted on the rockets will become misaligned within the inertial reference frame *K′* [7]. In order to meet the above-mentioned initial conditions and to "correctly" replicate the experiment in the backward direction, having simultaneously started the engines, the clocks have to be resynchronised, i.e. to be synchronised anew using Einstein's method, meeting the condition of the equality of the speed of light in opposite directions. As regards Bell, it is not the clocks that are synchronised but the moments of the start of the engines. To this purpose, a light signal emitted from the point equally remote from the rockets at rest is used; that, however, does not affect the heart of the problem because such a start is equivalent to the one on the clock moving synchronously in the Einstein's sense.

During the second stage of the experiment on the return of the rockets to the state of rest within the system *K*, the proper distance between the rockets will not contract to the initial proper distance L_0, but it will again increase two-fold, giving $L_1 = 2L'$ or $L_1 = 2L'_0$ within the reference frame *K*. This is quite clear because the inertial systems *K* and *K′* are equal and the acceleration results within them should be identical. Considering that $L'_0 = 2L_0$, the proper distance L_1 between the rockets will be equal to $4L_0$, i.e. four times more than the initial proper distance L_0.

What will happen if we do not resynchronise the clocks?

If the clocks are not resynchronised and the engines are started under different initial conditions, i.e. according to the "incorrectly" going within the reference frame K' non-resynchronised clocks, then the rockets upon their return to a state of rest within the reference frame *K* will stay the initial distance L_0 apart. This can be verified having examined the acceleration of a pair of rockets, their stop and slowdown from the reference frame *K*. In this system, all actions performed by the rockets with missing resynchronization of the clocks will become synchronous, and the distance *L* between them will prove invariable at both stages of the experiment.

The proper distance between the rockets, which increased during the first stage, will decrease two-fold during the second stage (from the value L'_0 to the value L_0).

We can make this experiment more sophisticated if we demand the return of the rockets, not simply to a state of rest within the initial reference frame *K*, but to the points within the system *K*, from which the rockets started at the beginning of the experiment.

The behaviour of each of the rockets in this case is clear.

If one of the rockets – Rocket *A* – from point *a* within the reference frame *K* is accelerated within this system to a velocity *v*, and then, elapsing some time after the stop of the engine, turns 180 degrees and, having started the engine again, slows it down within this same reference frame to zero velocity, then the rocket will return to its initial state of rest within the reference frame *K*, even though it will find itself in point *c*, remote from point *a*. Now, if the rocket is subjected to exactly the same actions that were performed with it when moving it from point *a* to point *c* on an identical programme, but in the opposite direction, then, fuel consumption neglected, the rocket will return from point *c* to point *a*.

Rocket *B* would behave in just the same way, and, should its flight be performed independent of the flight of Rocket *A*, then, having departed from point *b* and having completed the programmes of accelerations, braking and inertial flight without touching the clock, it would have returned to point b. Thus, a pair of rockets, each of them behaving independently, with the inside observers not intervening in the movement of the clocks, would return to their initial points. But a pair of rockets, if the start of the engines is performed synchronously within inertial systems, in which they for some time find themselves at rest, will not return to their starting points because, in order to ensure the synchronism of the clocks, one has to intervene in their natural movement. At best, one of the rockets, for example, Rocket *B*, may return to the initial point *b*, whereas the other one will find itself not in point *a*, but in point *d*, which is remote from point *b*. It is easy to understand that if the hands of the clock on rocket *B* are not touched, and the synchronism of the clock within each of the inertial reference frames (where the rockets find themselves at rest) is ensured by moving the hands of the clock on Rocket *A*, it is Rocket *B* that will return to point *b*, whereas Rocket *A* will not return to point *a* but will find itself in point *d*.

Therefore, we have a rather strange situation, in which the rockets to do not return to their initial position provided the experiment is performed under the "correct" synchronisation, and do return there if the moments of the start of the engines are synchronised "incorrectly". Under the "correct" performance of the multi-stage experiment and with multiple accelerations of a pair of rockets in the forward and backward directions, the proper distance between the rockets constantly increases. At the same time, we call an experiment "correct" in which our subjective deductions and a forced movement of the clock hands are needed, and "incorrect" in which it is performed without our artificial manipulations with the clocks. Thus, the principal condition of the reversibility of the proper distance between the rockets upon their return to the initial state of rest within the initial reference system *K* is the absence of clock resynchronization in the course of the experiment.

4. The Relativist Concept in Bell's Problem Solution and Modifications Thereof

Why do the rockets after their acceleration to a great velocity on identical programmes turn out to move away from each other within their proper reference frame? One would think that the reciprocal separation of the rockets could be explained by a metric reason, which is that the rockets and the onboard meter rulers accelerated to a velocity *v* within the reference frame *K* become shorter. This contraction, with a distance *L* between the rockets retained, must increase the ratio of the distance *L* to the lengths of the rocket bodies or the contracted meter rulers. It is clear that the shortening of the bodies and of the contents of the rockets, including the rulers and with the distance between the rockets remaining invariable, must be expressed by the increase in the numerical value of the distance. Indeed, it is exactly what is happening within the reference frame *K*. However, with regard to the reference frame *K′*, such an explanation within special relativity is unacceptable because a state of rest is assigned to each of the proper reference frames and observers of different reference frames do not recognize their proper motion and do not accept the fact of their proper contraction within the reference frame relative to which they are in motion themselves. If we concede the motion of a proper reference frame, for example, the system *K′*, then we have also to concede the possibility of the clock synchronization within its proper reference frame on the basis of inequality of the time of the motion of the signal between fixed points in the space of this system in its forward and backward motion.

5. Solutions to Bell's Problem Within the Framework of the Ether Concept

At first sight, Lorentz ether theory is not compatible with the solution to Bell's problem as given by Bell and Skobeltsyn because in ether at rest and in the reference frame moving relative to it a pair of rockets must behave differently. Indeed, the Lorentz theory does not only allow to mentally reproduce the behaviour of two rockets in ether according to a scenario described by Skobeltsyn [7] within the framework of Einstein's etherless theory but makes it possible to explain the reason for such behaviour within the framework of the ether concept.

Let us assume that the initial reference system *K* considered above finds itself at rest in relation to ether. At a certain moment in time, the engines of the rockets are simultaneously started, and they start acceleration in the ether.

During acceleration of a pair of identically pre-programmed rockets, the inside observers according to the formula of Lorentz contraction must register separation of the rockets. What is the cause of this separation? Indeed, in the ether the distance between the rockets owing to the synchrony of their motion stays invariable.

As part of Lorentz worldview, separation of the rockets registered by inside observers can be easily accounted for by actual contraction of the rockets and their contents moving through the ether.

As the rockets and all their contents travelling in the ether are actually contracting in the forward direction, the distance between the moving rockets measured with measurement rods found inside the rockets is perceived as increased (due to shortening of the rods).

And what would happen if the rockets uniformly moving in the ether after stopping their engines make a 180-degree turn and, some time having elapsed (on the universal ether time), having simultaneously started the engine in the direction opposite to the motion of the rockets, begin to synchronously decelerate to a complete stop in the ether?

As part of the absolute universal ether time, the actual simultaneity is absolute, so resynchronization of the rocket clocks is not required. It is clear that the lengths of the rockets and measurement rods when braking in the ether will start increasing as their velocities decrease in relation to the ether and, having come to a state of rest in relation to the ether, the rockets will obtain their initial lengths. The distance between the rockets as fixed by observers will thus decrease to the initial one. It will occur thanks to an actual increase in lengths of bodies and measurement rods to their initial size.

Moreover, what about a repeated separation of rockets, which is observed in the theory of special relativity after the rockets return to the initial reference frame? Is it possible to observe it in the ether?

Yes, indeed.

One should not forget that, in Lorentz ether theory, besides an absolute real time, there is a local fictitious time. This fictitious local time ensures artificial equality of speeds of light in opposite directions.

If the observers in the rockets refuse to use uniform ether simultaneity and resort to resynchronization of the clocks by the Einstein method, then the simultaneous start of engines on a local time turns out not to be simultaneous on an absolute ether time. The engine on the rear rocket in the direction of motion – let it be rocket B – starts earlier on real ether time, so deceleration of the rocket begins earlier. It is easy to show (see the appendix) that after the stop of the engines the rockets return to a state of rest in the ether and find themselves separated by an actual distance L_1, which is four times more than the actual distance L separating the rockets at the first stage of the experiment. In the process of deceleration in the ether, the lengths of the rockets and their contents increase. By this moment the rockets have moved apart and the actual distance between them has increased four times, i.e. the actual quadruple separation of rockets is twice bigger than the seeming double reduction of the distance between them. For this reason, at the second stage of the experiment, the observers of rockets register a double increase in the distance. The main conclusion of the ether theory related to behaviour of accelerating rockets is that when using Einstein simultaneity in the ether at rest and within a reference frame moving in the ether, the results of acceleration of a pair of rockets are perceived by observers as identical.

6. The Circular Model of Bell's Effect

An entertaining version of Bell's effect may be exampled by a circular model which, due to lack of full equality of the rotary and inertial motion, may be considered as non-relativistic.

Let us consider two identical rockets resting on a circle of big diameter in the inertial system *K*. The distance L_0 between the rockets is much less than the diameter of the circle and is practically equal to the length of the arch connecting the rockets. The length of each rocket is equal to l_0.

Let us place a pulse light source in the centre of the circle. At a given moment in time, the source emits a light pulse, which upon reaching the rockets, starts their engines for some time. If the programmes that guide the engines of the rockets are absolutely identical, then the rockets are synchronously accelerated along the circle, remaining at the identical distance of $L = L_0$ within the inertial system *K*, rigidly bound to the circle. Let us imagine that after the stop of the engines the linear speed v of the circling rockets is such that the lengths l of the bodies of the rockets in motion owing to Lorentz contraction become twice less than their initial length at rest l_0. As $l = ½l_0$, and the distance L stays invariable and equal to L_0, then the ratio L/l becomes equal to $2L_0/l_0$. As the lengths of the rocket bodies and their contents are perceived by observers as invariable, they register the seeming double increase in distance between the rockets to the value L'_0, numerically equal to $2L_0$.

Let us note that an identical result would be received if the rocket engines were started, not by a pulse from the central source, but by a light pulse from the source located in the centre of the arch connecting the rockets.

Now, if the circling spaceships are turned 180 degrees and on an impulse from the central light emitter the engines are simultaneously started, then after synchronous deceleration on the same programmes that guided acceleration, the rockets will find themselves at rest.

In the course of deceleration, an actual elongation of the bodies of rockets occurs, perceived by inside observers as a reduction of the distance between the rockets. After the stop of the engines and return of the rockets to a state of rest, the inside observers discover that the distance between the rockets becomes equal to the initial value L_0.

Now let us imagine that in order to start the engines in the deceleration mode the inside observers use, not a pulse from the central source, but a pulse from the source at the time of emission located in the centre of the arch connecting the circling rockets. In the inertial reference system K, the rockets are moving, and the light omnidirectionally propagating in it at an identical velocity reaches the rear rocket before the front one. A delay in the start of the front rocket engine will result in a larger than L_0 value of the distance between the rockets after the completion of operation of the decelerating engines. As well as in the previous model, this larger value will become equal to $4L_0$ (see the appendix).

The reference system K is assigned within a circular model in a sense that, unlike quasi-inertial systems connected with the rockets, it is inertial indeed and, unlike other inertial systems, it is only within this one that all rockets circle at identical speeds.

7. The Simulation of Bell's Paradox in Aqueous Medium

In recent years, we have presented the kinematic model of the theory of special relativity [8-9]. Taking barges moving in still water and high-speed boats as an example, all known relativistic kinematic phenomena and paradoxes including that of Bell's are simulated.

A rigid (solid) body, which can also be represented by an imaginary spaceship or rocket, within this model is simulated as a group of barges found on a surface of a flat-bottom reservoir. Each barge is equipped with a pendulum clock. A high-speed shuttle performs the role of the pendulum: it is taking the shortest path between the barge and the bottom at a speed V. Here, the speed V is the usual "earthly" speed equal, for example, to 100 km/h. The frequency of the pendulum of the barge at rest on a water surface is equal to $V/2h$, where h is the depth of this flat-bottom reservoir. If the barge is floating at a speed v, where $v<V$, then the vertical component of speed V (the speed of floating up and sinking) of the shuttle is equal to $V\sqrt{1-v/V}$, while the pendulum frequency is equal to $V\sqrt{1-v/V}/2h$. Thus, the clock on the moving barge is $1/\sqrt{1-(v/V)^2}$ times slower than the one on the barge at rest. By combining barges in groups that are at rest and those in motion, it is possible to fully simulate the kinematic phenomena of the theory of special relativity.

The constancy of distance between the barges in the group simulating a solid body is maintained on a "pseudolocation" principle. The role of a "pseudolocation" signal in realizing this method is performed by high-speed boats running to and fro between the barges at a speed V. The principle of a "pseudolocation" is as follows.

A high-speed boat regularly starts from each of the barges to the next barge, and upon reaching it, starts its way back. The instruments on the barges by means of simulated clocks measure the travel time of the boat to the next barge and back; if needed, they move the next barge nearer or farther so as to retain this time and the invariance of the "pseudolocation" distance "seemingly" sensed by the instruments on the barges.

The rigidity of a simulated body is understood as the constancy of the "pseudolocation" distance between the barges. If under external actions the "pseudolocation" distance between the barges can be changed, then such a group cannot be viewed as rigid.

With the acceleration of a group of barges to a speed of v and with the constancy of the "pseudolocation" distance, the actual longitudinal dimensions of a group of barges in motion contract for the following reason.

In order to cover the distance l between the barges, one needs the time Δt_1, equal to $l/(V-v)$ for the forward motion of the group of barges, and the time Δt_2, equal to $l/(V+v)$ for its backward motion. The overall time $\Delta t_1 + \Delta t_2$ of the travel there and back between the barges makes $2lV/(V^2-v^2)$ or $2l/V(1-v^2/V^2)$. If $v=0$, then this time is equal to $2l/V$, if $v \neq 0$, then the time $\Delta t_1 + \Delta t_2$ is $1/(1-v^2/V^2)$ times more than $2l/V$. As the clocks on the barges in motion go $1/\sqrt{1-(v/V)^2}$ times slower than on those at rest, the instruments, through pseudolocation monitoring of the distance between the barges and retaining it constant, reduce the actual longitudinal distance between the barges, not $1/(1-v^2/V^2)$ times, but $1/\sqrt{1-(v/V)^2}$ times.

Let us simulate Bell's paradox with a string in the aqueous medium. For this purpose, we will consider two groups of barges at rest, simulating rockets. The groups find themselves some distance L_0 apart, and they are linked by a thin chain of barges, simulating a string. After the start the actual longitudinal distances between the barges within each of the two groups moving one after another as well as in the chain of barges connecting these groups will start to decrease. If the breaking chain of barges is simulated, then at least in one link of the chain the forced increase in "pseudolocation" distance leads to its break.

If elapsing some time, the moving groups of barges begin to perform the back action and simultaneously on the clock or on a signal from external observers start deceleration, then the inverse process will proceed in such a way that the groups after the completion of breaking and the stop will be expanded again and will return to an initial state. Owing to the expansion of the groups, the instruments on the barges will record distance reduction between the groups in units of length of their groups up to the initial (starting) value. That will occur at an invariable actual distance between the groups.

However, if the groups of barges in motion start deceleration not on an external signal, but on a clock pre-synchronized by means of a high-speed boat, the result may be twofold.

If at synchronization of the clock one considers the fact that in relation to the groups the high-speed boat moves at a speed of $V - v$ in the direction of the forward motion of the groups and at a speed of $V + v$ in the direction of their backward motion, then the prior distance between the groups will remain, and the pseudolocation distance between them will decrease to the initial one. However, if at synchronization we make a false assumption on the equality of the speed of the boat relative to the barges in their forward and backward motion, then the distance between the groups will increase. It occurs because of a time delay in the start of deceleration of the front rocket in a row of those moving one after another. As well as in the cases described above and clarified in the appendix, such a delay will lead to the fact that groups of barges after their stop will find themselves at an actual distance $1/(1\text{-}v^2/V^2)$ times larger than they were before the start of deceleration. Such an increase in the distance between the groups exceeds the reduction of the pseudolocation distance between them $1/\sqrt{1-(v/V)^2}$ times, which is why the instruments on the groups of barges will register an increase in location distance between the groups after the stop of deceleration, not $1/(1\text{-}v^2/V^2)$ times, but $1/\sqrt{1-(v/V)^2}$ times.

8. Conclusion

The behaviour of accelerating rockets in ether and circular models treated above as well as its simulation in aqueous medium physically differs from their behaviour on the theory of special relativity. In all the three models, there is an assigned reference system K at rest, and there are inertial or, as is the case of the circular model, pseudo-inertial systems moving relative to the assigned reference system K. The rockets of these models, moving at a speed v within the dedicated reference frame K and possessing actual length $L_0\sqrt{1-(v/c)^2}$, become actually shorter at a further increase in speed within the reference frame K, and they are actually expanded to the maximum value of rest, with the speed decreasing. During the further run of the engines, the rockets having slowed down to zero velocity start gaining speed again within the reference frame K at rest, though in an opposite direction, which accounts for the shortening of the rockets again. When applying uniform simultaneity of an assigned reference frame in all reference systems, an increase in distance between the rockets, seemingly perceived by observers, is coherent with the actual shortening of the rockets.

Therefore, the seeming distance between the rockets increases, provided they accelerate within the reference frame K, and decreases up to the value equal to L_0 if they are slowed down to a state of rest (in system K). During the further run of the engines and the acceleration of the rockets, the seeming distance between the rockets increases.

Such a specific behaviour of rockets in models with an assigned reference frame would seem essentially incompatible with their behaviour in the ether-less world. However, upon close examination of the model of the ether-less universe and the non-relativistic models, one may notice that the behaviour of the rockets is determined, not by our ideas of assigned reference frames, but by synchronization of the clocks. The representations regarding the assignment of a reference frame are used only for the justification of one or the other synchronization. Going beyond such justifications and applying an identical synchronization in different models, it is possible to obtain identical behaviour of rockets in different models.

If in the ether-less model for purely practical purposes we conditionally introduced the assigned inertial reference system and applied universal (not absolute, but, explicitly, conditionally universal!) time and uniform scales of physical quantities in other reference frames, then such a model would mathematically describe the behaviour of the material world, as though in the ether model this assigned reference frame were rigidly fixed to the ether, while the other systems were moving in relation to it. Thus, an invariance of mathematical notation of physical laws is broken, but there emerges an invariance of physical quantities in different reference frames. For example, the longitudinal length of a rod moving relative to an assigned reference frame and which has shrunk Lorentz-wise will be identical in all reference frames.

On the other hand, if in the ether model, having introduced an artificial requirement of equality of the velocity of light in opposite directions, one should refuse an assigned reference frame and equalize all reference systems, there will emerge an invariance of mathematical notation of physical laws and the imaginary relativity of physical quantities (Lorentz ether theory with Poincare-Lorentz transformations). All this could be understood, having analysed the results of the simulation of kinematics related to the theory of special relativity stated in the works [8-9].

Appendix

Let us consider two rockets, rocket *A* and rocket *B*, moving one after another in a straight line at a velocity v, within the inertial reference frame *K* at rest. Under an inertial reference frame at rest, we understand a reference system, which is conditionally or conceptually assigned a state of rest. The distance between the rockets moving forward, the front rocket *A* and the rear rocket *B*, is equal to *L*. At a certain time period from the central point located between the rockets at a distance ½*L* from each of them, an omnidirectional signal is emitted, propagating in the reference frame *K* at a speed c (in the case of simulation in aqueous medium the role of speed c is played by speed V). As the rear rocket *B* is moving in the reference system *K* towards the signal at a speed v, the signal overtakes the rocket *B* after a time period $½L/(c+v)$. The signal travelling to the front rocket *A*, which is moving away from it, needs a longer time period, equal to $½L/(c-v)$. Therefore, rocket *A* begins deceleration later than rocket *B*. The difference Δt of the times upon which the signals reach the rockets *A* and *B* is equal to $½L/(c-v) - ½L/(c+v)$, i.e. $vL/(c^2-v^2)$.

If the rockets started simultaneous acceleration within the reference frame *K*, then owing to synchronism of deceleration within this reference frame they would find themselves after the stop of the engines the identical distance *L* apart. But the front rocket *A* started deceleration a time period Δt later than rocket *B*, during this time period having travelled an extra run Δx, equal to $v\Delta t$ or $v^2L/(c^2-v^2)$. For this reason the distance $L + \Delta x$ between the starting point of rocket B and the starting point of rocket *A* is equal to $L+v^2L/(c^2-v^2)$, which after transformation can be written as $L/(1-v^2/c^2)$. Owing to full identity of the rockets and of the programmes operating their engines, the distance that each of them will travel from a braking point to a point of arrival at a state of rest will be identical. For this reason, the rockets, having started deceleration at different time moments from the points that are a distance $L/(1-v^2/c^2)$ apart, will likewise finish deceleration at different times at the points staying the identical distance apart.

If $\sqrt{1-(v/c)^2}$ is equal to ½, then $1-v^2/c^2$ is equal to ¼ and $L/(1-v^2/c^2) = 4L$.

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Quantum Gravitational Applications of Nuclear, Atomic and Astrophysical Phenomena

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By following the old concept of "gravity is having a strong coupling at nuclear scale" and considering the 'reduced Planck's constant' as a characteristic quantum gravitational constant, in this letter we suggest that: 1) There exists a gravitational constant associated with strong interaction, $G_s \sim 3.32956087 \times 10^{28}$ m^3/kg/sec^2. 2) There also exists a gravitational constant associated with electromagnetic interaction, $G_e \sim 2.374335472 \times 10^{37}$ m^3/kg/sec^2. Based on these two assumptions, in a quantum gravitational approach, an attempt is made to understand the basics of final unification with various semi empirical applications like melting points of elementary particles, strong coupling constant, proton-electron mass ratio, proton-neutron stability, nuclear binding energy, neutron star's mass and radius, Newtonian gravitational constant and Avogadro number. With further research and investigation, a practical model of 'quantum gravitational string theory' can be developed.

Keywords: Quantum gravity, Strong interaction, Electromagnetic interaction, Newtonian Gravitational constant, Schwarzschild interaction strength, Neutron star

1. Introduction

Even though 'String theory' and 'Quantum gravity' models [1, 2] are having a strong mathematical back ground and sound physical basis, both the models are failing in understanding the role of the Newtonian gravitational constant [3-8] in atomic and nuclear physics and thus seem to fail in developing a 'workable' model of final unification.

W. Lerche says [1]: "*The most dramatic extension of the Standard Model of particle physics that has been proposed so far is string theory. However, as we will discuss in more detail below, string theory too does not provide very concrete answers to the questions posed above. But what string theory does is to provide a resolution of conceptual problems that are on a far deeper level than these "practical" problems. One of the most important problems in modern theoretical physics is the apparent mutual incompatibility of quantum mechanics and general relativity (the theory of gravity) – one theory describing well the world at very short, the other at long distances. Certainly a truly satisfying unified theory should incorporate the gravitational interaction as well, even though traditionally it is not considered as belonging to particle physics*".

Juan M. Maldacena says [2]: "*We now have a theory, called string theory (or M-theory), which has been able already to provide a solution to the first two challenges. Unfortunately, we do not know yet how to solve the third challenge. May be string theory is the solution and we just have to understand it better or maybe we have to modify it in some way. String theory is a theory under construction. We know several limits and aspects of the theory, but we still do not know the fundamental axioms of the theory that would enable us to approach the third challenge*" (To Explain the Big Bang and the parameters of the Standard Model).

According to Roberto Onofrio [9, 10], weak interactions are peculiar manifestations of quantum gravity at the Fermi scale, and that the Fermi coupling constant is related to the Newtonian constant of gravitation. In his opinion, at atto-meter scale, Newtonian gravitational constant seems to reach a magnitude of $8.205\times10^{22}\text{m}^3\text{kg}^{-1}\text{sec}^{-2}$. In this context, one can see plenty of papers on 'strong gravity' in physics literature [11-28]. It may be noted that, till date, 'strong gravity' is a non-mainstream theoretical approach to Color confinement/particle confinement having both a cosmological scale and a particle scale gravity. In between ~(1960 to 2000), it was taken up as an alternative to the then young QCD theory by several theorists, including Abdus Salam [11]. Very interesting point to be noted is that, Abdus Salam showed that the 'particle level gravity approach' can produce confinement and asymptotic freedom while not requiring a force behavior differing from an inverse-square law, as does QCD.

In pursuit of bridging the gap in between 'General theory of relativity' and 'Quantum field theory' - in the earlier publications [29-37], the authors proposed three basic assumptions. The authors strongly encourage the readers to go through the above cited references. It may be noted that, in the earlier publications, the authors suggested and validated the role of two gravitational constants associated with strong and electromagnetic interactions. In an integrated approach the authors also showed that, 'quantum of angular momentum' is a characteristic result of the combined effects of gravitational constants associated with proton and electron. In this letter the authors compiled important characteristic relations for good understanding, better accuracy and best presentation**.** Each relation seems to have its own characteristic inner meaning.

2. Two Basic Assumptions of Final Unification

In the earlier publications [29-37] the authors proposed and established the following two assumptions.

Assumption-1: Magnitude of the gravitational constant associated with the electromagnetic interaction is, $G_e \cong 2.374335472\times10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$.

Assumption-2: Magnitude of the gravitational constant associated with the strong interaction is, $G_s \cong 3.32956087\times10^{28}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$.

Note: It may be noted that, with reference to the operating force magnitudes, protons and electrons cannot be considered as 'black holes'. But electrons and protons can be assumed to follow the relations that black holes generally believed to follow. Clearly speaking, in the study of black holes, Newtonian gravitational constant G_N plays a major role, whereas in the study of elementary particles, G_s and G_e play the key role. For detailed information, see the following sub section.

2.1. *Key Points to be Noted*

1) If it is true that c and G_N are fundamental physical constants, then $\left(c^4/G_N\right)$ can be considered as a fundamental compound constant related to a characteristic limiting force [38-41].
2) Black holes are the ultimate state of matter's geometric structure.
3) Magnitude of the operating force at the black hole surface is of the order of $\left(c^4/G_N\right)$.
4) Gravitational interaction taking place at black holes can be called as 'Schwarzschild interaction'.
5) Strength of 'Schwarzschild interaction' can be assumed to be unity.
6) Strength of any other interaction can be defined as the ratio of operating force magnitude and the classical or astrophysical force magnitude $\left(c^4/G_N\right)$.
7) If one is willing to represent the magnitude of the operating force as a fraction of $\left(c^4/G_N\right)$ i.e. X times of $\left(c^4/G_N\right)$, where $X \ll 1$, then

$$\frac{X \text{ times of } \left(c^4/G_N\right)}{\left(c^4/G_N\right)} \cong X \to \text{Effective } G \Rightarrow \frac{G_N}{X} \tag{1}$$

If X is very small, $\frac{1}{X}$ becomes very large. In this way, X can be called as the strength of interaction. Clearly speaking, strength of any interaction is $\frac{1}{X}$ times less than the 'Schwarzschild interaction' and effective G becomes $\frac{G}{X}$.

8) With reference to Schwarzschild interaction, for electromagnetic interaction, $X \approx 2.811\times10^{-48}$ and for strong interaction, $X \cong 2.0\times10^{-39}$.
9) Characteristic operating force corresponding to electromagnetic interaction is $\left(c^4/G_e\right) \approx 3.4\times10^{-4}$ N and characteristic operating force corresponding to strong interaction is $\left(c^4/G_s\right) \approx 242600$ N.

10) Characteristic operating power corresponding to electromagnetic interaction is $\left(c^5/G_e\right) \approx 10990$ J/sec and characteristic operating power corresponding to strong interaction is $\left(c^5/G_s\right) \approx 7.27 \times 10^{13}$ J/sec .

11) Based on these concepts, it is possible to assume that,

$$\frac{\left(m_e c^2\right)^{\frac{3}{2}} \left(m_p c^2\right)^{\frac{1}{2}}}{\sqrt{\left(c^4/G_e\right)\left(c^4/G_s\right)}} \cong \hbar c \tag{2}$$

$$\frac{\left(m_e c^2\right)^{\frac{3}{2}} \left(m_p c^2\right)^{\frac{1}{2}}}{\sqrt{\left(c^5/G_e\right)\left(c^5/G_s\right)}} \cong \hbar \tag{3}$$

12) As $\left[\left(c^4/G_e\right),\left(c^4/G_s\right)\right] << \left(c^4/G_N\right)$ and $\left[\left(c^5/G_e\right),\left(c^5/G_s\right)\right] << \left(c^5/G_N\right)$, protons and electrons cannot be considered as 'black holes', but may be assumed to follow similar relations that black holes generally believed to follow.

13) According to S.W. Hawking [42], temperature of black hole takes the following expression.

$$T_B \cong \frac{\hbar c^3}{8\pi G_N k_B M_B} \tag{4}$$

where M_B and T_B represent the mass and temperature of a black hole respectively.

According to Abhas Mithra [43, 44], currently believed 'black holes' are a kind of "Eternally Collapsing Objects". The so-called massive Black Hole Candidates (BHCs) must be quasi-black holes rather than exact black holes and during preceding gravitational collapse, entire mass energy and angular momentum of the collapsing objects must be radiated away before formation of exact mathematical black holes. Abhas Mitra's peer reviewed papers describe why continued physical gravitational collapse should lead to formation of ECOs rather than true black holes, and the mathematical "black hole" states can be achieved only asymptotically. An ECO is essentially a quasi-stable ultra-compact ball of fire (plasma) which is so hot due to preceding gravitational contraction that its outward radiation pressure balances its inward pull of gravity. Some astrophysicists claimed to have verified this prediction that astrophysical Black Hole Candidates are actually ECOs rather than true mathematical black holes. One can find relevant information at http://www.cv.nrao.edu/tuna/past/2006/NEW_QSO_STRUCTURE_FOUND.pdf.

By considering these two views and by considering the proposed views, melting temperature of elementary particles can be estimated very easily.

3. Role of the Newtonian Gravitational Constant in Nuclear Physics

A) To Understand the Proton Rest Mass

$$m_p \cong \left(\frac{G_N}{G_e}\right)^{\frac{1}{6}} \sqrt{M_{pl} m_e} \tag{5}$$

where, $M_{pl} \cong \sqrt{\frac{\hbar c}{G_N}}$

B) To Understand the Excited Levels of Proton

$$m_x \cong \left(\frac{G_N}{G_e}\right)^{\frac{1}{6}} \left(\frac{(n\hbar) c m_e^2}{G_N}\right)^{\frac{1}{4}} \cong (n)^{\frac{1}{4}} \times m_p \tag{6}$$

where, $n = 1, 2, 3, ..$

For, $n=1$, $m_x \cong 938.3$ MeV

For, $n=2$, $m_x \cong 1115.8$ MeV

For, $n=3$, $m_x \cong 1234.8$ MeV

For, $n=4$, $m_x \cong 1326.8$ MeV

For, $n=5$, $m_x \cong 1403.3$ MeV

For, $n=6$, $m_x \cong 1468.5$ MeV

For, $n=7$, $m_x \cong 1526.1$ MeV

For, $n=8$, $m_x \cong 1578.0$ MeV

For, $n=9$, $m_x \cong 1625.2$ MeV

For, $n=10$, $m_x \cong 1668.6$ MeV

These estimated levels assumed to be associated with proton can be compared with currently believed nucleon resonances up to some extent [45]. Extending this idea, other baryonic masses can also be estimated and the authors are working on this.

Based on relation (6), G_N can be estimated with the following relation.

$$G_N \cong \left\{ \left(\frac{m_e}{m_p} \right)^{12} \left(\frac{G_s m_p^2}{\hbar c} \right) \right\} G_s \qquad (7)$$

C) To Understand the Strong Coupling Constant

$$\alpha_s \cong \left(\frac{G_e}{G_N} \right)^{\frac{1}{6}} \left(\frac{G_s}{G_e} \right) \cong \frac{G_s}{G_N^{1/6} G_e^{5/6}} \qquad (8)$$

4. To Estimate the Gravitational Assumed to be Connected with Proton

A) Nuclear Unit Charge Radius: It can be understood as follows [46, 47, 48].

$$R_0 \cong \frac{2 G_s m_p}{c^2} \qquad (9)$$

Based on relation (9), G_s can be estimated with the following relation.

$$G_s \cong \frac{c^2 R_0}{2 m_p} \qquad (10)$$

5. To Estimate the Gravitational Assumed to be Connected with Electron

A) Ratio of Rest Mass of Proton and Electron: It can be understood as follows.

$$\left(\frac{m_p}{m_e} \right) \cong \left(\frac{G_e m_e^2}{\hbar c} \right) \left(\frac{G_s m_p^2}{\hbar c} \right) \qquad (11)$$

Thus, based on relation (11), G_e can be estimated with the following relation.

$$G_e \cong \left(\frac{\hbar^2 c^2}{G_s m_e^3 m_p} \right) G_s \cong \frac{2\hbar^2}{m_e^3 R_0} \qquad (12)$$

6. To Estimate the Magnitudes of $\left(G_s, G_e, G_N, \alpha_s\right)$

Based on the reference [45, 49], let,

$$\begin{cases} e \cong 1.602\ 176\ 565(35) \times 10^{-19}\ \text{C}, \\ \varepsilon_0 \cong 8.854187817 \times 10^{-19}\ \text{F/m} \\ m_n \cong 1.674\ 927\ 471(21) \times 10^{-27}\ \text{kg}, \\ m_p \cong 1.672\ 621\ 777(74) \times 10^{-27}\ \text{kg} \\ m_e \cong 9.109\ 382\ 91(40) \times 10^{-31}\ \text{kg}, \\ \hbar \cong 1.054\ 571\ 726(47) \times 10^{-34}\ \text{J.sec} \end{cases}$$

$$\left. \begin{array}{l} \text{If } R_0 \cong \left(1.23\underline{84} \text{ to } 1.23\underline{88}\right) \text{fm}, \\ G_s \cong \left(3.32\underline{7167052} \text{ to } 3.32\underline{8241718}\right) \times 10^{28}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\ G_e \cong \left(2.37\underline{5276497} \text{ to } 2.37\underline{6043705}\right) \times 10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\ G_N \cong \left(6.67\underline{0254668} \text{ to } 6.67\underline{456314}\right) \times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\ \alpha_s \cong \left(0.1179\underline{231391} \text{ to } 0.1179\underline{29483}\right) \cong 0.1185 \pm 0.0006 \end{array} \right\}$$

7. Characteristic Atomic and Nuclear Applications

A) Fermi's Weak Coupling Constant: It can be understood as follows [45, 49].

$$F_W \cong \left(\frac{m_e}{m_p} \right)^2 \hbar c R_0^2 \cong \frac{4 G_s^2 m_e^2 \hbar}{c^3} \qquad (13)$$

B) Root Mean Square Radius of Proton: It can be understood as follows [45, 49, 50].

$$R_p \cong \frac{\sqrt{2} G_s m_p}{c^2} \qquad (14)$$

C) Bohr Radius of Electron: It can be understood as follows.

$$a_0 \cong \left(\frac{4\pi\varepsilon_0 G_e m_e^2}{e_e^2} \right) \left(\frac{G_s m_p}{c^2} \right) \qquad (15)$$

D) Proton-Neutron Beta Stability Line: It can be understood as follows.

$$\begin{cases} \text{Let,} \\ k \cong \left(\dfrac{G_s m_p m_e}{\hbar c}\right) \cong \left(\dfrac{\hbar c}{G_e m_e^2}\right) \cong 1.6\times 10^{-3} \end{cases}$$

$$A_s \cong 2Z + k\left(2Z\right)^2 \tag{16}$$

E) Nuclear Binding Energy at Stable Atomic Nuclides: It can be understood as follows [51, 52].

For $\left(Z \geq 5\right)$,

$$BE \cong -\left(Z-2+\sqrt{\frac{Z}{30}}\right)\sqrt{\left(\frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R_p}\right)\left(\frac{3}{5}\frac{G_s m_p^2}{R_p}\right)}$$

$$\cong -\left(Z-2+\sqrt{\frac{Z}{30}}\right)\times 19.8 \text{ MeV} \tag{17}$$

8. Characteristic Sub-Nuclear Applications

RHIC have tentatively claimed to have created a quark–gluon plasma with an approximate temperature of **4 trillion-degree Kelvin** [53-56]. A new record breaking temperature was set by ALICE at CERN on August, 2012 in the ranges of **5.5 trillion-degree Kelvin.** In June 2015, an international team of physicists have produced quark-gluon plasma at the Large Hadron Collider by colliding protons with lead nuclei at high energy inside the supercollider's Compact Muon Solenoid detector at a temperature of **4 trillion-degree Kelvin**. With reference to the recommended up, down and strange quark masses, estimated geometric mean melting point is **14 trillion-degree K** and can be compared with the experimental results.

A) Melting Point of Proton: It can be understood as follows.

$$T_{proton} \cong \frac{\hbar c^3}{8\pi k_B\left(G_s m_p\right)} \cong 0.147 \text{ Trillion K} \tag{18}$$

B) Melting Point of Electron: It can be understood as follows.

$$T_{electron} \cong \frac{\hbar c^3}{8\pi k_B\left(G_e m_e\right)} \cong 5670 \text{ Trillion K} \tag{19}$$

It may be noted that, as electron is a weakly interacting particle, its melting temperature seems to be 38580 times higher than melting temperature of proton.

9. Characteristic Astrophysical Applications

A) Mass of Neutron Star: It can be understood as follows [57-62].

$$\frac{G_N M_N m_n}{\hbar c} \cong \sqrt{\frac{G_s}{G_N}} \text{ and}$$

$$M_N \cong \sqrt{\frac{G_s}{G_N}}\left(\frac{\hbar c}{G_N m_n}\right) \tag{20}$$

$$\cong 3.17\times \text{ Solar mass}$$

where $\left(M_N, m_n\right)$ represent masses of neutron star and neutron respectively.

B) Radius of neutron star: It can be understood as follows [63].

Let $\left(R_N, R_n\right)$ represent the radii of neutron star and neutron respectively.

$$\frac{R_N}{\left(R_0/2\right)} \cong \sqrt{\frac{G_s}{G_N}}$$

$$\rightarrow R_N \cong \sqrt{\frac{G_s}{G_N}}\left(\frac{R_0}{2}\right) \cong \sqrt{\frac{G_s}{G_N}}\left(\frac{G_s m_n}{c^2}\right) \cong 13.8 \text{ km} \tag{21}$$

where $R_n \cong \dfrac{G_s m_n}{c^2} \cong 0.62$ fm

10. About Avogadro's Number

It is noticed that,

$$\sqrt{\frac{G_e}{G_N}} \approx 5.96\times 10^{23} \tag{22}$$

In this context the authors could publish interesting contributions in Indian DAE-BRNS conference proceedings and International Intradisciplinary Conference on the Frontiers of Crystallography [33-36]. Even though, this is a semi empirical procedure, Avogadro number seems to be strongly connected with crystal structures as well as unification of fundamental forces. With this unified semi empirical procedure, it is possible to increase the scope and applicability of Avogadro number and with further research, independent of the 'gram mole' concept, absolute procedure for estimating the value of the Avogadro number can be developed.

11. Discussion

It may be noted that,

1. Relations (5), (6), (7) and (22) clearly suggest the possible role of (G_N, G_e) in nuclear and atomic physics.
2. Relations (9), (13), (14). (17) and (18) clearly suggest the possible role of G_s in nuclear physics.
3. Relations (11) and (15) clearly suggest the combined role of (G_s, G_e) in nuclear and atomic physics.
4. Relations (16) clearly suggests the possible role of (G_s, G_e) in understanding the proton-neutron stability.
5. Relations (20), and (21) clearly suggest the possible combined role of (G_s, G_N) in astrophysics.
6. Relations (19) clearly suggests the possible role of G_e in sub-nuclear physics.

The authors would like to stress the fact that, with currently believed unified (main stream) physics models it is impossible to discover/fit/derive such relations. If one is willing to consider this fact as a real inadequacy of current unified physics models, the proposed two gravitational constants can be recommended for in-depth study at fundamental level.

From unification point of view, one can find many critical reviews on the foundations, predictions, current status and success of string theory in physics literature [64-66]. Reiner Hedrich says [67]: "String theory is at the moment the only advanced approach to a unification of all interactions, including gravity. But, in spite of the more than thirty years of its existence, it does not make any empirically testable predictions, and it is completely unknown which physically interpretable principles could form the basis of string theory. At the moment, "string theory" is no theory at all, but rather a labyrinthic structure of mathematical procedures and intuitions. The only motivations for string theory consist in the mutual incompatibility of the standard model of quantum field theory and of general relativity as well as in the metaphysics of the unification program of physics, aimed at a final unified theory of all interactions, including gravity".

Edward Witten says [68]: "Even though we do not really understand it, quantum gravity is supposed to be some sort of theory in which, at least from a macroscopic point of view, we average, in a quantum mechanical sense, over all possible spacetime geometries. (We do not know to what extent this description is valid microscopically.)"

In this context, it is very clear to say that, when a well believed theoretical model is failing in addressing the basic and practical problems connected with unification of general theory of relativity and quantum mechanics, first of all, it must be reviewed at fundamental level to have a well-defined set of physical quantities and physical constants to proceed further for testable predictions at observable energy scales associated with elementary particles physics and astrophysics.

12. Conclusion

Proposed relations (5 to 22) clearly demonstrate the role of proposed gravitational constants assumed to be associated with proton and electron. At first sight, their physical existence appears to be ad-hoc, but by seeing the applications one may be forced to say that, there is 'some new physics' behind their assumed 'presence'. Along with the proposed assumptions, key points and semi empirical relations, if one is willing to recall the old concepts which broadly falls in the category of 'strong gravity' as suggested by Abdus Salam, C. Civaram, K.P. Sinha, E. Racami, K. Tennakone, Usha Raut, V. De Sabbatta and Roberto Onofrio, everyone will be forced to consider the above relations for in-depth analysis at fundamental level.

Proceeding further, if one is willing to explore the possibility of incorporating the proposed assumptions either in String theory models or in Quantum gravity models, certainly, back ground physics assumed to be connected with proposed semi empirical relations can be understood and a 'practical' model of "everything" can be developed.

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The Velocity Addition Formula According to Special Relativity — The Most Unsustainable Formula in All Physics

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Two aspects of the velocity addition formula for special relativity will be reviewed. The first is its derivation and the second its functionality and consistency. We will bring two arguments that prove the unsustainable derivation of velocity addition formula and three other arguments that will show dysfunction of this formula. The conclusion of this review is that the velocity addition formula according to the special theory of relativity is nothing more than rhetorical tautology.

Keywords: Velocity addition formula, Special relativity, Lorentz transformations

1. Introduction

The Galileo principle and the special theory of relativity (STR) relate only to Galilean relative motion. This motion is defined with two inertial systems; one system (K) is at rest, and the other (K') moves with velocity v relative to system K; and a particle (body) moves with velocity u' relative to the system K' (Fig. 1).

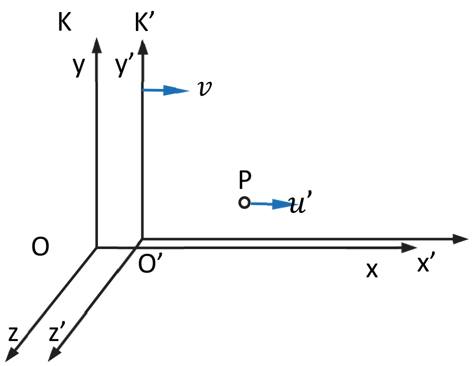

Fig. 1. Galilean relative motion.

It is considered that the Galilean and Lorentz transformations refer to the same motion. To show that the Lorentz transformations (LT) hold true in this, as well as any other case, for example in deriving the velocity addition formula (VAF) we must clearly define the quantities and the meaning of those quantities which are part of this relative motion. Even though this might seem very trivial, here we will once again define the quantities which describe relative motion in Fig. 1 according to the STR; the systems of reference and their relationships are set out above: the distance between systems K and K' is vt; the distance between particle (body) P and origin O' is written as x'; the velocity of the particle (body) P in relation to origin O' is $u' = x'/t'$; simultaneously, the velocity of the particle (body) P, which represents the event in system K', without which there would be no relative motion, just as without the motion of system K' there would be no relative motion [1]; time t' is the time which flows in system K'; the velocity of the particle (body) P in relation to origin O of system K is $u = x/t$; the time t is the time which flows in system K. The event in system K' can have a random velocity of u'; while, in the case when in system K' the motion of light is observed, it is clear that $u' = c$.

2. The Derivation of The Velocity Addition Formula

For the first time the VAF according to the STR we find to Poincare's works [2], [3], but he has not commented this formula. There are three key steps for the derivation the VAF according to the STR. The system of equations:

$$\left.\begin{aligned} x &= \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}} \\ t &= \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned}\right\} \qquad (1)$$

represents the LT. If we divide the equations (1) we obtain:

$$\frac{x}{t} = \frac{x' + vt'}{t' + \frac{vx'}{c^2}}. \quad (2)$$

Now if we use these equations:

$$\left.\begin{aligned} u &= \frac{x}{t} \\ u' &= \frac{x'}{t'} \end{aligned}\right\} \quad (3)$$

from the equation (3) we obtain VAF:

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}. \quad (4)$$

The equations (1), (3) and (4) correspond to the Fig. 1, if instead the particle (body) P we have photon. From the first sight, the VAF (4) is consistent with the definition of u and u' in introduction of this paper. But if we look deeper, there are two problems that shake the equation (4): the first, the LT is obtained for the light as event in system of reference K' (Fig. 1), thus $u' = c$ since the start of LT derivation procedure; and the second, the STR uses these equations:

$$\left.\begin{aligned} c &= \frac{x}{t} \\ c &= \frac{x'}{t'} \end{aligned}\right\} \quad (5)$$

as one of conditions to obtain the LT, and simultaneously represent the second postulate of STR [4]. These two problems show that in equation (2) cannot be substituted equations (3), because there are no physical basis for such substitution. In equation (2) eventually can be substituted equations (5) and then would be obtained: $\frac{x}{t} = \frac{x'}{t'}$. One more argument that can't be obtained something else from equation (2), is the fact that two equations of system (1) are linearly dependent; and they differ from each other exactly just for a constant (c). As illustration of this, we can see an Einstein calculation. Einstein for arguing "magic" of equation (4) in [4] says: *"Aided by the following illustration, we can readily see that, in accordance with the Lorentz transformation, the law of the transmission of light in vacuo is satisfied both for the reference-body K and for the reference-body K'. A light-signal is sent along the positive x-axis, and this light-stimulus advances in accordance with the equation x = ct, i.e. with the velocity c. According to the equations of the Lorentz transformation, this simple relation between x and t involves a relation between x' and t'. In point of fact, if we substitute for x the value ct in the first and fourth equations of the Lorentz transformation, we obtain:*

$$x' = \frac{(c - v)t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t' = \frac{\left(1 - \frac{v}{c}\right)t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

from which, by division, the expression x' = ct' immediately follows."

However, Einstein's operation shows no magic, nor any physical properties of the light or relative motion; this operation is simple, substitution of initial conditions in an obtained equation (2), and as result we have the return at the beginning: $\frac{x}{t} = \frac{x'}{t'}$.

All of this can be verified in another way - by deriving LT according to Max Born. After a very tiring reasoning about the physics of the problem Born manages to set out this system of equations:

$$\left.\begin{aligned} \alpha x &= x' + vt' \\ \alpha x' &= x - vt \end{aligned}\right\} \quad (6)$$

where α is a proportional parameter that must be found [5]. The equations of system (6) according to Born represent the relative motion of light in two frame of reference with relative motion v between them. We can note that basis of system of equations (6) is Galilean transformation; but let us see, how much worthy is this arbitrary intrusion of the "proportional parameter α" in the equations of system (6). To find the "proportional parameter α" Born substitutes the system of equations (5) in (6), then multiplies the equations of system (6), and obtains the parameter: $\alpha = \sqrt{1 - \frac{v^2}{c^2}}$, and finally, from (6) obtains the standard LT for length and time (1). However, solving the system of equation (6) by substitution method and using system of equations (5) we obtain:

$$\left.\begin{aligned} x &= x'\sqrt{\frac{c+v}{c-v}} \\ t &= t'\sqrt{\frac{c+v}{c-v}} \end{aligned}\right\}. \qquad (7)$$

Thus:

- if in the system of equations (6) is acceptable the "proportional parameter α";

- if in the system of equations (6) is acceptable the time t' in first equation; and

- if for transformation of the system of equations (6) we use the system of equations (5); then, it is an undeniable fact that the system of equations (7) is equivalent to the system of equations (1); it is undeniable that the system equations (7) also represent the "Lorentz transformation".

Finally, can easily be seen that by dividing the equations of system (7) we don't obtain the VAF according to the STR (4), but what we obtain is what must be obtained – the condition of system: $\frac{x}{t} = \frac{x'}{t'}$.

Therefore, we can conclude that VAF according to the STR has no physical, nor algebraic basis.

3. The Functional Aspect of The Velocity Addition Formula

In his book on the theory of relativity [4], Einstein, like many great physicists in later works, was astonished to derive $u = c$ from equation (4), having substituted $u' = c$. This was applauded – and still is, by relativists. However, there is nothing to applaud in this; it should rather concern us because the fact that velocity v does not enter to the addition with u', but is eliminated (simplified) after the substitution $u' = c$ demands a physical explanation rather than applause. Moreover, an explanation is demanded for the fact that the result $u = c$ for $u' = c$ is valid for every value of the velocity v.

Formula (3) cannot be called a velocity addition formula when in fact it does not add velocities. In order to prove that this fault is real and insurmountable we will take the following example.

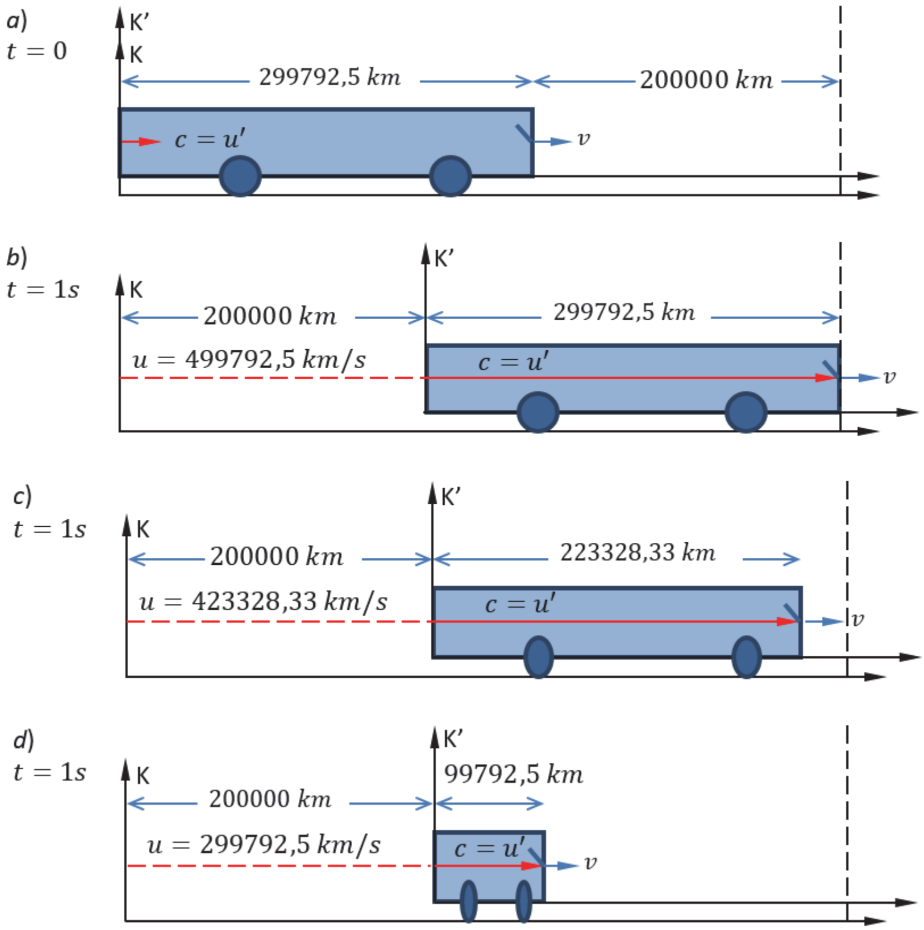

Fig. 2. The water tap example.

An attempt to measure a train's velocity with the velocity addition formula. A passenger sitting on a train asks himself, 'would I be able to find the velocity v of the train, which is constant, if a colleague from the station from which the train set out tells me the value of u, if u' is c, thus if I emit a light signal in the direction of the train's motion?' Before asking the colleague for value of u, the passenger solves equation (3) by v and finds:

$$v = \frac{u - u\prime}{1 - \frac{uu\prime}{c^2}} \qquad (8)$$

and when he substitutes $u' = c$, he sees that there is no need to phone his friend at the station at all, because the value of u is not important since it is eliminated (simplified) from equation (8) and it is calculated that $v = -c$.

Thus, since velocity v is not required for the addition (subtraction) of other velocities, it can now be seen from equation (8) that the same thing occurs with velocity u, i.e. that it is not added to (subtracted from) other velocities. This can be explained only by the fact that equation (4) is only a rhetorical tautology.

The example with inequalities. As is seen in Fig. 1, for the observer at the origin of system K, system K' moves away with velocity v for which we can write:

$$v = \frac{x_0}{t} \qquad (9)$$

where x_0 represents the distance between the origins of the two systems, which is usually written as vt. Meanwhile, velocity u is the velocity of particle (body) P relative to the observer at the origin of system K, for which the following equation can be written:

$$u = \frac{x}{t} \qquad (10)$$

From the two last expressions, the unarguable conclusion can be reached: since in the two last expressions, time t is the same and since for relative motion (in the case where particle (body) P sets off from the origin and moves in a positive direction on the x axis, x' respectively) the following inequality always holds true:

$$x > x_0 \qquad (11)$$

then from equations (9) and (10) it results that this inequality is also always true:

$$u > v. \qquad (12)$$

This means that in the case where $v = c$ then from inequality (11) we obtain:

$$u > c. \qquad (13)$$

This conclusion disagrees with the STR.

The water tap example. Let's assume a closed coach which is *299792,5 km* long. The coach stops at a station. At the front of the coach is an observer, and outside the coach is another observer. The end of the coach is fitted with a water tap with a photo-element. At the moment when the coach starts moving with a constant velocity $v = 200000\ km/s$, the observer in the coach emits a light signal in the direction of the tap (Fig. 2*a*). Will the water flow after $t = 1\ s$? At what kilometer along the tracks (i.e. at what kilometer of system K) will the water flow? Three figures are set out as possible answer. Fig. 2*b* shows the answer derived from a basis on the Galileo principle for relativity. The two answers in Fig. 2*c* and Fig. 2*d* are based on special relativity. To the question of whether water will flow at time $t = 1\ s$ for the observer outside the coach, all of these figures give the response 'yes', because otherwise there would be a violation of two postulates of STR. All that remains, therefore, is the second question (at what kilometer does the water flow) so that it can be seen whether the velocity addition formula for special relativity is valid. In Fig. 2*c* it is calculated that the coach is shortened according to STR, time dilates according to STR but that the water flows at kilometer 423328,5 of the tracks, and this is not in accordance with STR, because in this variant $u = 423328{,}5\ km/s$. So let us look at the other answer. The answer in Fig. 2*d* fulfills the requirements of the postulate that u should be equal to c but this solution has two serious faults. The first is that this solution requires that the coach does not move, despite the fact that its velocity is not zero; the second fault is that for the coach to be shortened as much as is shown in Fig. 2*d* it would have to move with a velocity much greater than the velocity given for v. Thus from this example, too, it can be concluded that equation (4) is not physically consistent.

4. Conclusions

When the velocity addition formula for special relativity is reviewed from the perspective of its derivation, it is concluded that it is rhetorical tautology. It is rhetorical tautology also within the theory of special relativity itself which means that even if this theory pertains, the velocity addition formula according this theory cannot pertain. This conclusion is supported with an analysis of its functional aspect and it can be seen that it does not hold true either algebraically or physically at the level of the velocity addition formula in relative motion.

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Newton-Einstein G-Duality and Dirac-Majorana Fusion Modeling as Mediated by Ontological-Phase Topological Field Theory

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Ontological-Phase Topological Field Theory (OPTFT) under seminal development to formally describe 3rd regime Unified Field Mechanics (UFM) (classical-Quantum-UFM) is extended to relate the duality of Newton-Einstein gravitation theory by added degrees of freedom in a semi-quantum limit enabling insight into topological Dirac-Majorana doublet fusion supervening the uncertainty principle.

Keywords: Gauge theory, Geometrodynamics, Ontological-phase, Topological field theory, Quasar luminosity, Yang-Mills Kaluza-Klein correspondence

1. Introduction

Newton claimed instantaneous G-influence; Einstein insisted no influence propagated faster than *c*. Quantum Mechanics (QM) the so-called *basement of reality*, posits a Quantum Gravity, for which no *a priori* science exists. We propose a *paradigm shift* with duality between a semi-quantum Standard Model (SM) limit and Large-Scale Additional Dimensionality (LSXD) [1] in an M-Theoretic Unified Field (UF) brane arena as the regime of integration described by an Ontological-Phase Topological Field Theory (OPTFT) requiring fundamental changes in the concept of dimensionality and matter. Two processes emerge for creating XDs: 1) duality, with Ds of fundamentally different character, and 2) anticommutativity, where Ds are fundamentally the same [2]. Yang-Mills (YM) Kaluza-Klein (KK) correspondence can drive Physics beyond the SM. Horizontal and vertical subspaces in the tangent bundle of M (M = X x G) defined by YM connections are orthogonal with respect to a KK metric suggesting orthogonal extension to XD beyond the 4D limit of the SM. CERN LHC research seeks KK XD beyond the SM. Current thinking posits XD as $\hbar$-scale since they are not observed; however, this is not the only interpretation. A LSXD alternative hidden by subtractive interferometry is proposed [3-5]. Albeit, our OPTFT iteration of M-Theory is based on radical extensions of the original hadronic string theory because of inherent key elements: virtual tachyon/tardyon interactions and a variable concept of string tension, $T_S = T_0 + \Delta\hbar$ [3, 6]. A and B-type topological string theories, and a related Topological M-Theory with mirror symmetry, while interesting, since they allow sufficient dimensionality by Calabi-Yau mirror symmetry perceived essential for developing UFM; a distinction between these theories causes our model to diverge. A key parameter is *topological charge* in brane dynamics which by definition makes correspondence to a de-Broglie-Bohm super-quantum potential synonymous with an ontological *Force of Coherence*, an inherent aspect of UF dynamics [3-5]. Thus, UFM predicts no phenomenal graviton (perceived artifact of incompleteness of Gauge Theory, i.e. Gauge Theory is approximate suggesting new physics).

The difference between 4D quantum field phenomenology and LSXD topological field ontology is the *energyless* exchange process. Information (Shannon related) is transferred ontologically by the dynamics of *topological switching* in M-Theoretic branes carrying topological charge [3-5]. Completing Geometrodynamics inherently includes Newton/Einstein duality [5]; evidenced by interpreting quasar luminosity as *G*-shock waves [3] countering Big Bang interpretations of large redshift, Z based on Doppler recession. Instead redshift results from periodic photon mass-anisotropy by coupling to a covariant polarized Dirac vacuum [3]. A further conundrum exists by defining a Manifold of Uncertainty (MOU) of finite dimensional radius, allowing a wave-particle-like duality with a quantal-like virtual graviton in the *semi-quantum limit* – an intermediary between *field phenomenology* and *topological ontology*. This has increasing importance for the new field of Relativistic Information Processing (RIP) which introduces G-effects in *parallel transport* of brane topological switching [3-5]. From the broad context above our central theme is the introduction of a topological formalism for a new set of UF transformations beyond the Galilean, Lorentz-Poincairé. An empirical protocol falsifying the model is developed [3-5]; which if successful has far reaching consequences for experimentally validating XD, M-Theory and leads to obsolescence of usual TeV$\rightarrow$PeV supercollider particle sprays, of the successful 100-year history of high energy collision physics, by providing a new form of table-top low-energy UFM XD brane collision (LSXD topological fusion) *cross section* alternatives for *viewing* putative SUSY partners in a trans-D *slice* rather than standard cross section collision physics [7]. This is a seminal introduction to a paradigm shift (OPTFT) and thus fraught with a plethora of detail.

2. Cosmology of G-Shock Waves, and Newton-Einstein G-Duality

Conflicts within the SM call into question the fundamental interpretation of the Doppler component of the putative Hubble Expansion Law and the nature of events in spacetime associated with conventional coordinates of the line element as attached to the physical basis of the observer. Also of paramount importance is that Einstein's Geometrodynamics is not a complete theory of gravity as stated by Einstein himself. We postulate nonlinear effects associated with the propagation of light in an intense G-field produces shock waves creating *light-booms* along boundary conditions at cosmological distances approaching the limit of observation, that if correct would explain Quasi-Stellar Object (QSO) luminosity. These G-shock waves are considered observationally manifest in the spectrum of QSOs and Supernova as a continuous front of *light booms* produced by superluminal boosts associated with continuous coordinate transformations relative to a distant observer, suggesting that QSOs are a form of Seifert spiral galaxy with Active Galactic Nuclei (AGN) in the vicinity of the putative observational limit of the Hubble radius, H_R, creating an issue of fundamental basis of Geometrodynamics. Newton's formulation of the G-force law requires each particle to respond instantaneously to every other massive particle regardless of the distance between them which he *proved*; but the proof is only valid in Euclidian space. Today this would be described by the Poisson equation,

$$\left(\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2\right)\varphi(x,y,z)= f(x,y,z) \tag{1}$$

according to which, when the mass distribution of a system changes, its G-field instantaneously adjusts. Therefore, theory requires the speed of G to be infinite. Einstein's Geometrodynamics $G_{\mu\nu}+\Lambda g_{\mu\nu}=\left(8\pi G/c^4\right)T_{\mu\nu}$. is a classical extension of Newtonian-G and therefore incomplete. Physical theory incorporates an upper limit on the propagation speed of an interaction, maintaining that instantaneous action-at-a-distance is impossible. However, quantum entanglement between separated particles enables instantaneous EPR correlations which led to the puzzle as to whether causality or locality must be abandoned.

In summarizing the *Cosmological Principle* (universe homogeneous and isotropic) [8] events are idealized spacetime instants defined by arbitrary time and position coordinates *t, x, y, z*, written collectively as x^i with *i*, 0 to 3. The standard line element is

$$ds^2=\sum_{ij} g_{ij}dx^i dx^j = g_{ij}dx^i dx^j, \tag{2}$$

where the metric tensor $g_{ij}(x)=g_{ji}(x)$ is symmetric [8]. In local Minkowski form all first derivatives of g_{ij} vanish at the event taking the form

$$ds^2=dt^2-dx^2-dy^2-dz^2. \tag{3}$$

The Cosmological Principle generally suggests that the clocks of all observers are synchronized throughout all space because of the inherent homogeneity and isotropy. Because of this synchronization of clocks for the same world time *t*, for commoving observers the line element becomes,

$$ds^2=dt^2+g_{\alpha\beta}dx^\alpha dx^\beta=dt^2-dl^2, \tag{4}$$

where dl^2 represents spatial separation of events at the same world time, *t*. This spatial component of event dl^2

can be represented as an Einstein 3-sphere (compatible with dual 6D Calabi-Yau 3-tori)

$$dl^2 = dx^2 + dy^2 + dz^2 + dw^2 \quad (5)$$

which is represented by the set of points (x, y, z, w) at a fixed distance R from the origin: $R^2 = x^2 + y^2 + z^2 + w^2$ where

$$w^2 = R^2 - r^2 \text{ and } r^2 = x^2 + y^2 + z^2 , \quad (6)$$

so finally, we may write the line element of the Einstein 3-sphere as

$$dl^2 = dx^2 + dy^2 + dz^2 + r^2 dr^2 / R^2 - r^2 \text{ [8].} \quad (7)$$

By imbedding an Einstein 3-sphere in a flat HD space, specifically as a subspace manifold of a new 3rd regime UFM complex 12D superspace, [3, 4, 9] new theoretical interpretations of standard cosmological principles are feasible. This is the line element most compatible with the oscillatory spacetime boundary parameters required by our model of G-shock waves in QSO luminosity [3].

According to MTW [10] junction conditions may act as generators of G-shocks; the dynamics of spacetime geometry for a 3-surface, Σ which includes *intrinsic* Riemann scalar curvature invariants, *R*, also includes an *extrinsic* curvature tensor, K_{ij}. When imbedded in an enveloping 4-geometry hypersurface it can change (shrinkage and deformation) in vector, *n* parallel transported as junction conditions applicable to the G-field (spacetime curvature) and the stress-energy generating it. A discontinuity in K_{ij} across a null surface without stress-energy producing it is a geometric manifestation of a G-shock-wave generated by a different embedding in spacetime *above* Σ than *below* Σ [3, 10].

Dray and 't Hooft [11] found conditions for introducing G-shock waves in a class of vacuum solutions to Einstein's equations by coordinate shift. Their model generalizes G-shock waves for a massless particle moving in flat Minkowski space formulated as two Schwarzschild black holes of _equal_ masses glued together at the horizon. For a spherical shell of _unequal_ masses moving along $u = u_0 \neq 0$; their solution [12] represents two Schwarzschild black holes glued together at $u = u_0$. By infinitely boosting the Dray -'t Hooft solutions various forms of G-shock waves have been found [13, 14]. Sfetsos [15] extends these results to the case with matter fields and a non-vanishing cosmological constant. Using the d-D spacetime metric

$$ds^2 = 2A(u,v)\,dudv + g(u,v)\,h_{ij}(x)\,dx^i dx^j , \quad (8)$$

with $(i, j = 1, 2, ..., d-2)$ he uses a string based dilatonic black hole G-solution [16] from the perspective of a conformal background field theory of coset $SL(2,\mathbb{R})/\mathbb{R}\otimes\mathbb{R}^2$ to achieve a differential shift factor

$$\begin{aligned}&\left(d^2/d\rho^2 + (1/\rho)(d/d\rho) - \varepsilon\right) f(\rho) = \\ &-16\varepsilon\rho(1/p)\delta(\rho)\end{aligned} \quad (9)$$

where $\rho^2 = x^2 + y^2$ and for a black hole singularity case with $\varepsilon = 1$, (9) is a modified Bessel equation [15]. Spitkovsky [17] simulates a relativistic Fermi emission shock process that could provide an alternative to, or component process for our G-shock work. His simulations on relativistic collisionless shocks propagating in initially unmagnetized electron-positron pair plasmas showed natural production of accelerated particles as part of shock evolution. He studied the mechanism that populates the suprathermal tail for particles gaining the most energy. The simulation showed the main acceleration occurs near the shock where for each reflection these particles gain energy, $\Delta E \sim E$ as is expected in relativistic shocks [18].

Newton's G required instantaneous action at a distance or the conservation of angular momentum would be violated; but for Einstein's GR an instantaneous influence would violate causality and SR and so must be mediated by a field. This is the dual nature of gravity that we have put as the basis for our model. We have tried to show that it is possible with further study to relate shock phenomena to G-waves especially for narrow axis massive cosmological objects such as AGN QSOs that readily lend themselves to *light-boom* effects that could therefore be used to explain QSO luminosity as further evidence of the insurmountable shortcomings of Big Bang cosmology. Our model works best contrasting both modes of the intrinsic dual nature of G because nonlinear jumps in flow occur with discontinuity. From the 2nd Law of Thermo-dynamics entropy only increases when particles cross a shock. The duality of the propagation of the G-influence is evident in Birkhoff's theorem [3, 9] in that a spherically symmetric G-field is produced by a massive object such as a QSO at the origin; if there were another concentration of mass-energy elsewhere, this would disturb spherical symmetry. This effect could occur if interference occurs between the usual modes of the G-influence by shock parameters.

3. From Geometric Phase to Ontological Phase

Ontological-phase topological field theory (OPTFT) introduces fundamental 3rd regime postulates: 1) A semi-quantum mirror symmetric Calabi-Yau finite radius manifold of uncertainty, 2) with a 4D Minkowski-Riemann subspace, and 3) cyclical duality of *phenomenological* (quantal) field mediation and an *ontological charge* (energyless) topological switching unified field. As initial simplistic modeling of Ontological-phase we adapt the phasor or phase vector concept as a precursor to ontological topological phase. In general, a phasor is a complex number for a sinusoidal (π rotation) function with amplitude *A*, angular frequency ω and initial phase θ, with all time invariant. The complex constant is the phasor [4]. Euler's formula can represent sinusoids as the sum of two complex-valued functions:

$$A\cdot\cos\left(\omega t+\theta\right)=A\cdot\left(e^{i(\omega t+\theta)}+e^{-i(\omega t+\theta)}/2\right), \quad (10)$$

or as the real part of function:

$$A\cdot\cos\left(\omega t+\theta\right)=\mathrm{Re}\left\{A\cdot e^{i(\omega t+\theta)}\right\}$$

$$=\mathrm{Re}\left\{Ae^{i\theta}\cdot e^{i\omega t}\right\}. \quad (11)$$

The function, $A\cdot e^{i(\omega t+\theta)}$ is the analytic representation of $A\cdot\cos\left(\omega t+\theta\right)$. Multiplication of the phasor, $Ae^{i\theta}e^{i\omega t}$ by a complex constant, $Be^{i\phi}$, produces another phasor changing the amplitude and phase of the underlying sinusoid**:**

$$\mathrm{Re}\left\{\left(Ae^{i\theta}\cdot Be^{i\phi}\right)\cdot e^{i\omega t}\right\}=\mathrm{Re}\left\{\left(ABe^{i(\theta+\phi)}\right)\cdot e^{i\omega t}\right\}$$

$$=AB\cos\left(\omega t+\left(\theta+\phi\right)\right). \quad (12)$$

If function, $A\cdot e^{i(\omega t+\theta)}$ is depicted in a complex plane, the vector formed by imaginary and real parts rotates around the origin. *A* is magnitude, *i* is the imaginary unit, $i^2=-1$; one cycle is completed every $2\pi/\omega$ seconds, and θ is the angle formed with a real axis at $t=n\cdot 2\pi/\omega$, for integer values of *n* [4].

This type of addition (Fig. 1a) occurs when sinusoids interfere constructively or destructively. Three identical sinusoids with a specific phase difference may perfectly cancel. To illustrate, we place three equal length vectors matching up head to tail to form an equilateral triangle with a 120 (2π/3 radians) angle between each phasor of 1/3 wavelength, $\lambda/3$, so the phase difference between each wave is 120°, $\cos(\omega t)+\cos(\omega t+2\pi/3)$ $+\cos(\omega t-2\pi/3)=0$.

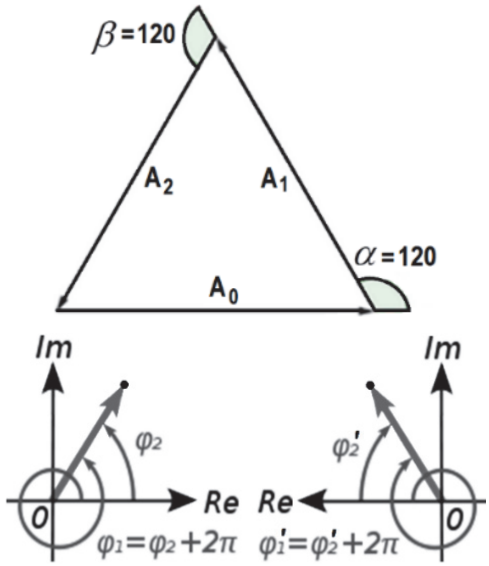

Fig. 1. a) Phasor diagram of three waves in perfect destructive interference. b) Left-Right phase argument, prep for phase transition channels in Dirac-Majorana duality.

In the three waves example, the phase difference between 1st and last waves is 240°. In the many waves limit, phasors must form a circle for destructive interference, so that the 1st phasor is nearly parallel with the last. Thus, for many sources, destructive interference happens when the 1st and last wave differ by 360°, a full wavelength, λ [4]. For any complex number in polar form, such as $re^{i\theta}$, the phase factor is the complex exponential factor, $e^{i\theta}$. As such, *phase factor* relates more generally to term phasor, which may have any magnitude (i.e., need not be part of circle group). A phase factor is a unit complex number of absolute value 1 commonly used in quantum mechanics (QM). The variable θ is referred to as the phase. Multiplying the equation for a plane wave $A^{ei(k\cdot r-\omega t)}$ by a phase factor shifts the phase of the wave by

$$\theta\text{:}\; e^{i\theta}Ae^{i(k\cdot r-\omega t)}=Ae^{i(k\cdot r-\omega t+\theta)}. \quad (13)$$

In QM, a phase factor is a complex coefficient $e^{i\theta}$ that multiplies a ket $|\psi\rangle$ or bra $\langle\phi|$, not, in itself, having any physical meaning in standard QM, since introducing a phase factor does not change the expectation values of a Hermitian operator. That is, the values of $\langle\phi|A|\phi\rangle$ and $\langle\phi|e^{-i\theta}Ae^{i\theta}|\phi\rangle$ are the same [4]. However, *differences* in phase factors between two interacting quantum states can be measurable under certain conditions such as in Berry phase, which has important consequences [4]. The argument for a complex number $z = x + iy$, denoted arg z, is defined as:

- Geometrically, in the complex plane, as the angle φ from the positive real axis to the vector representing z. The numeric value given by the angle in radians is positive if measured counterclockwise (Fig. 1b).
- Algebraically, the argument is defined as any real quantity, φ such that $z = r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$ for some positive real r (Euler's formula). The quantity r is the *modulus* of z, as $|z|$: $r = \sqrt{x^2 + y^2}$.

Use of the terms *amplitude* for the modulus and *phase* for the argument are often used equivalently; by both definitions, the argument of any (non-zero) complex number has many possible values: firstly, as a geometrical angle, whole circle rotations do not change the point, so angles differing by an integer multiple of 2π radians are the same. Similarly, from the periodicity of sin and cos, the 2nd definition also has this property. An N-particle system can be represented in non-relativistic QM by a wavefunction, $\psi(x_1, x_2, \ldots x_n)$, where each x_i is a point in 3D space. A classical phase-space contains a real-valued function in 6N Ds (each particle contributes 3-spatial coordinates and 3-momenta. Quantum phase-space involves a complex-valued function on a 3N dimensional space. Position and momenta are represented by non-commuting operators, and ψ lives in the math structure of a Hilbert space. Aside from these differences, the analogy holds. In physics, this addition occurs with constructively or destructively interfering sinusoids. The static vector concept provides useful insight into questions like: What phase difference is required for three identical sinusoids to perfectly cancel (again Fig. 1a)? Waves are characterized by amplitude and phase, and both may vary as a function of those parameters. According to Berry [19], if parameters of the Hamiltonian of quantum system undergoes adiabatic changes, cyclically returning to original values, the wave function can acquire geometrical and dynamical phase. This additional *Berry phase* is $\neq 0$ when the trajectory in parameter space is near a point of degenerate states. Berry assumed the Hamiltonian is Hermitian (linear) in deviations of parameters from a point. He considered such points to be *monopole-like* when calculating geometrical phase. Thus, such points *generate* a field coinciding in monopole-like form, and the flux of Berry's field through a contour gives the geometrical phase of the system. Berry phase occurs in Aharonov–Bohm effects, where the adiabatic parameter is the magnetic field enclosed by two cyclical interference paths forming a loop and conical intersections (adiabatic parameters are molecular coordinates) of two potential energy surfaces, a set of geometrical points where the two potential energy surfaces are degenerate (intersect) and the non-adiabatic couplings between these two states are non-vanishing. Generally, geometric phase occurs whenever at least two wave parameters in the vicinity of a singularity/hole in the topology; two are required because either the set of nonsingular states will not be simply connected (shrink closed cure to point), or there will be nonzero holonomy. A Berry phase difference is acquired over the course of a cycle, when a system is subjected to cyclic adiabatic processes resulting from the geometrical properties of the parameter space of the Hamiltonian [4, 19]. In addition to QM it can occur whenever there are at least two parameters describing a wave in the vicinity of a singularity or topological hole.

In a quantum system at the n^{th} eigenstate, if adiabatic (adapts to gradually changing external conditions; but for rapidly varying conditions there is insufficient time, so the spatial probability density remains unchanged) evolution of the Hamiltonian evolves the system such that it remains in the n^{th} eigenstate, while also obtaining a phase factor. The phase obtained has a contribution from the state's time evolution and another from the variation of the eigenstate with the changing Hamiltonian. The 2nd term, Berry phase, which for non-cyclical variations of the Hamiltonian can be made to vanish by different choices of phase associated with eigenstates of the Hamiltonian at each point in the evolution. But if variation is cyclical, Berry phase cannot be cancelled, as it is invariant and becomes an observable property of the system. From the Schrödinger equation, the Berry phase

$$\gamma \text{ is: } \gamma[C] = i\oint_C \langle n,t|\left(\nabla_R |n,t\rangle\right) dR, \quad (14)$$

where R parametrizes the cyclic adiabatic process. It follows a closed path C in the appropriate parameter space. Geometric phase along the closed path C can also be calculated by integrating the Berry curvature over surface enclosed by C [4]. The Foucault pendulum is a simple example of geometric phase. The pendulum precess when it is taken around a general path C. For transport along the equator, the pendulum does not precess. But if C is made up of geodesic segments, precession arises from the angles where the segments of the geodesics meet; the total precession is equal to the net deficit angle, which equals the solid angle enclosed by C modulo 2π. We can approximate any loop by a sequence of geodesic segments, from which the most general result is that the net precession is equal to the enclosed solid angle. Since there are no inertial forces on the pendulum precess, precession, relative to the direction of motion along the path, is entirely due to the turning of the path. Thus, the orientation of the pendulum undergoes parallel transport [4].

4. Tight-Bound States and New Spectral Lines

Topological quantum field theories (TQFT) were created to avoid infinities in quantum field theory. In topological field theory, the concern is topological invariants, objects computed from a topological space (smooth manifold) without any metric. Topological invariance is invariance under the diffeomorphism group of the manifold. TQFT flourished through the work of Witten and Atiyah [4]. To experimentally move from SM Hilbert space to UFM ontological-phase space we must define topological switching [3-5]. We begin looking at the ambiguous Necker cube [4] where the central vertices *switch* ontologically (energyless) by *topological charge*. Recently, Tight Bound States (TBS) due to em-interactions at small distances below the lowest Bohr orbit have been postulated for the Hydrogen atom [20, 21]. Summarizing this seminal work, in the usual atomic physics spin-orbit and spin-spin coupling perturbations give rise to only small corrections in classic Bohr energy levels. However, with distances in the $1/r^3$ and $1/r^4$ range these interaction terms, until now overlooked, can be much higher than the Coulomb term at distances ≪ than the Bohr radius - predicting new physics [20]. In a further development, Corben noticed motion of a point charge in a magnetic dipole field at rest is highly relativistic with orbits of nuclear dimensions. Extending the Pauli equation [20, 21], to a two-body system defined by the Hamiltonian,

$$H = \frac{1}{2m_1}\left(P_1 - e_1 A(r_1)\right)^2 + \frac{1}{2m_2}\left(P_2 - e_2 A(r_2)\right)^2 + \frac{1}{4\pi\varepsilon_0}\frac{e_1 e_2}{|r_1 - r_2|} + V_{dd} \tag{15}$$

with, m_1 mass, P_1 momentum, e_1 charge, r_1 position of the particles (i = 1,2), A is electromagnetic vector potential and V_{dd}, the dipole-dipole interaction term:

$$V_{dd} = -\left(\frac{\mu_0}{4\pi}\right)\mu_1\mu_2\delta\left(r_1 - r_2\right) + \left(\frac{\mu_0}{4\pi}\right) \left[\frac{\mu_1\mu_2}{|r_1 - r_2|^3} - \frac{3\left[\mu_1\left(r_1 - r_2\right)\right]\cdot\left[\mu_2\left(r_1 - r_2\right)\right]}{|r_1 - r_2|^5}\right]. \tag{16}$$

In a center-of-mass frame with normal magnetic moment, $\mu = (e/m)s$ Hamiltonian, H above is:

$$\begin{aligned} H = {} & \frac{1}{2m_1}p^2 - \left(\frac{\mu_0}{4\pi}\right)\frac{e_1e_2}{m_1m_2}\frac{SL}{r^3} + \\ & \left(\frac{\mu_0}{4\pi}\right)^2 \frac{e_1^2e_2^2\hbar^2}{4m_1m_2m}\frac{1}{r^4} + \frac{1}{4\pi\varepsilon_0}\frac{e_1e_2}{r} - \\ & \left(\frac{\mu_0}{4\pi}\right)\frac{e_1e_2}{m_1m_2}s_1s_2\delta(r) + \\ & \left(\frac{\mu_0}{4\pi}\right)\frac{e_1e_2}{m_1m_2}\left[\frac{s_1s_2}{r^3} - \frac{3\left(s_1r\right)\cdot\left(s_2r\right)}{r^5}\right]. \end{aligned} \tag{17}$$

The possibility of TBS physics as derived from Hamiltonian (17) is shown in simplified form when limited to spherically symmetric terms by the radial Schrödinger equation [20]:

$$\frac{d^2X}{dr^2} + \frac{2m}{\hbar^2}\left[E - V\left(r\right)\right]X = 0 \tag{18}$$

and contains a form for the effective potential in the inverse power law:

$$V\left(r\right) = \frac{A}{r^4} + \frac{B}{r^3} + \frac{C}{r^2} + \frac{D}{r}. \tag{19}$$

At large distances this potential is an attractive Coulomb tail with a repulsive core at small distances due to the A/r^4 term [20]. For proper values of potential V its coefficients could have another potential well in addition to the one at distances of the order of the Bohr radius (location of new physics). Additional theoretical details on the seminal development of TBS by Vigier can be found in [4, 5, 21].

5. Additional Dimensionality and Topological Transformation

Idealization of SM elementary particles as 0D points/charge in coordinate context with no known composite subparticles, arose because *size* is considered irrelevant. Paraphrasing Rowlands: *fundamental physics reduces to explaining the structures and interactions of fermions. Fermions appear as singularities not extended objects, with no obvious way of creating such structures within 3D observation space. But, the Dirac equation suggests fermions require a double, rather than single, vector space, confirmed by the double rotation of spin ½ objects, and associated zitterbewegung and Berry phase shift. The 2nd 'space' reveals that it is an 'antispace', with the same information as real space but in less accessible form. The two spaces cancel forming a norm 0 (nilpotent) object with the exact mathematical structure required for a fermionic singularity* [5].

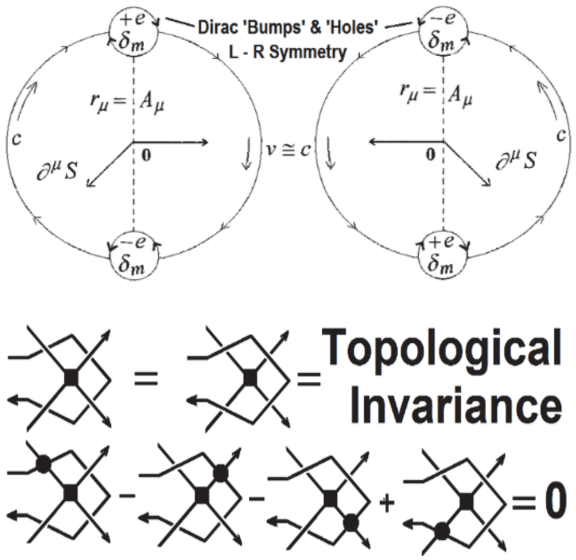

Fig. 2. a) Oppositely charged sub-elements rotating at v ≅ *c* around center 0 behaving like dipole bumps and holes on the topological surface of a covariant polarized Dirac vacuum, b) Topological Invariance must be included in any phase labeling.

He further notes that fermions as singularities exist in a multiply-connected space requiring double rotations to return to starting position. Fermions also undergo *zitterbewegung* continually switching between real space and complex vacuum space. The double circuit in real space is required because a fermion only exists in this space for half its existence. It is not coincidental that fermion algebra (gamma matrices) requires a commutative combination of two vector spaces for full representation; thus, obviously constructing a *singularity* requires a dual space [5, 22]. The nilpotent space-antispace model extends understanding of a singularity in terms of the SM, but quaternionic algebra is not a penultimate description of nature; Rowlands' model, *avant garde* to the SM is not sufficiently radical to satisfy the needs of UFM [4-7]; but inspires basis for correspondence to LSXD UFM.

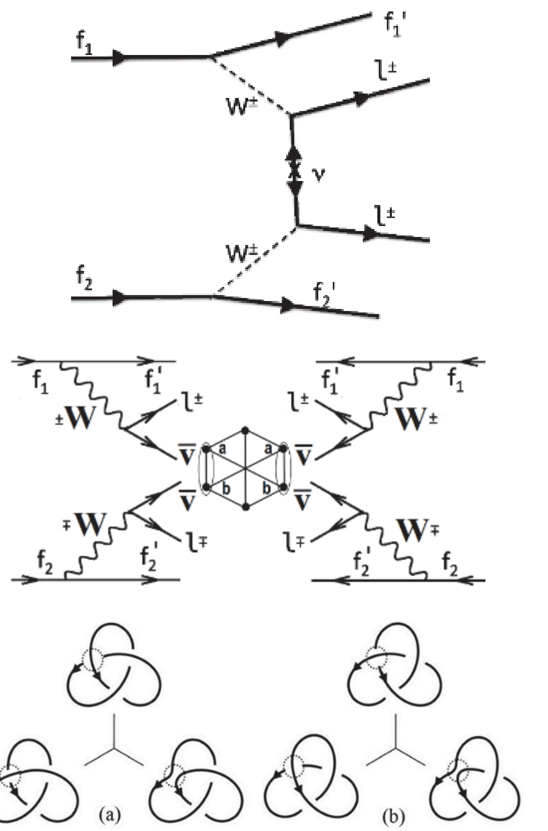

Fig. 3. a) Fundamental diagram changing lepton number transitions by two units, generalized for Majorana modes, b) with a-b Berry phase cycles in graphene, fusion channels can appear, c) Reduction schemes for L & R-handed trefoil knots adding degrees of freedom sufficient for applied fusion of Dirac-Majorana doublets supervening uncertainty/topological protection.

The Fano snowflake configuration (Fig. 7) involutes to form a 2D hexagon (graphene) or vertices of a Euclidean ambiguous Necker 3-cube used to explore possible topological moves for fusion of ontological-phase transitions. In the context of graphene, Berry phase is the phase an eigenstate acquires after *p* is forced to evolve a full circle at constant energy around a Dirac vortex point. When parallel transport creates a deficit angle in brane raising and lowering dynamics, in addition to Reidemeister moves, rotations, reflections and any other topological moves, other types of phase transition with lattice charge in anyon braid fusion channels apply. Half of the leptons are neutrinos, but unknown if they are Dirac or Majorana; finding neutrinoless double β-decay would demonstrate existence of the Majorana nature of neutrinos. Neutrinoless double β-decay occurs when two neutrons in a nucleus decay simultaneously, a fundamental diagram changing lepton number by two units. We begin to explore a plethora of crossover links and moves cataloging various transformations applicable to anyon fusion channels studied to supervene the inaccessibility of topological braiding, $a \times b = \sum_c N_{a,b}^c C$, where $a \& b \rightarrow c$ [23, 24]. We wish to illustrate fusion-duality as a *Principle*, by taking the more simplistic case of de Broglie *fusion*, coordinates x_1, y_1, z_1 and x_2, y_2, z_2 become

$$X = x_1 + x_2 / 2, Y = y_1 + y_2 / 2, Z = z_1 + z_2 / 2. \quad (20)$$

Then for identical particles of mass, m without distinguishing coordinates, the Schrödinger equation (center of mass) is

$$-i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2M} \Delta \psi, M = 2m \quad (21)$$

Eq. (21) corresponds to the present, Eq. (22a) the advanced wave and (22b) the retarded wave [3].

$$-i\hbar \frac{\partial \phi}{\partial t} = \frac{1}{2M} \Delta \phi, -i\hbar \frac{\partial \varphi}{\partial t} = \frac{1}{2M} \Delta \varphi. \quad (22)$$

Extending Rauscher's concept for a complex 8-space differential line element

$$dS^2 = \eta_{\mu\upsilon} dZ^\mu dZ^{*\nu}, \quad (23)$$

where indices run 1 to 4, $\eta_{\mu\nu}$ is the complex 8-space metric, Z^μ the complex 8-space variable and

$$Z^\mu = X_{\text{Re}}^\mu + iX_{\text{Im}}^\mu \text{ and } Z^{*\nu} \quad (24)$$

is the complex conjugate, to 12D continuous-state HAM spacetime; we write just the dimensions for simplicity and space constraints

$$x_{\text{Re}}, y_{\text{Re}}, z_{\text{Re}}, t_{\text{Re}}, \pm x_{\text{Im}}, \pm y_{\text{Im}}, \pm z_{\text{Im}}, \pm t_{\text{Im}}, \quad (25)$$

where $\pm$ signifies Wheeler-Feynman/Cramer type future-past/retarded-advanced dimensions. This XD provides a framework for applying hierarchical harmonic oscillator parameters [3, 9].

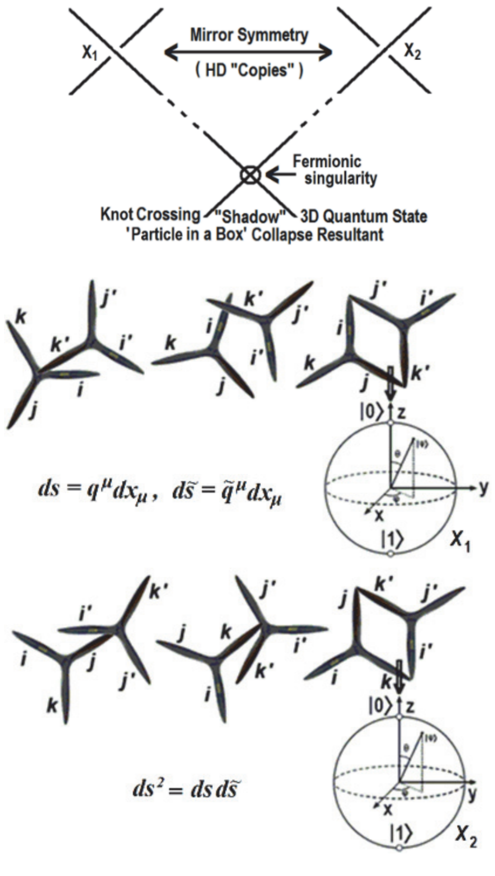

Fig. 4. a) Bottom, uncertainty principle causes a knot shadow in 3-space of XD topological degrees of freedom. b) SM line element, X_1, X_2, semi-classical Riemann Bloch 2-spheres, *basement of reality*; Top, 1st step to UFM. Relativistic space-antispace mirror symmetric quaternionic vertices cycle from QM chaos to topological order as faces of 3-cube.

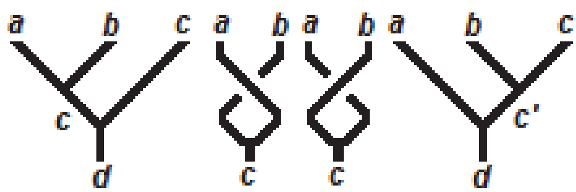

Fig. 5. Knot crossover links for anyon topolog-ically protected fusion channels.

An important feature of TQFTs is they do not presume fixed topology for space/spacetime; in dealing with an n-D TQFT, one is free to choose any $(n-1)$-

D manifold to represent space at a given time. Given two such manifolds, S and S', one is free to choose any nD-manifold M to represent the spacetime between S and S'. Mathematicians call M a *cobordism* from S to S'. We write $M : S \to S'$, because M can be the process of time from moment S to moment S'. For example, Fig. 6b depicts a 2D manifold M going from a 1D manifold S (pair of circles) to a 1D manifold S' (single circle). Crudely, M represents two separate spaces colliding to form a single one! Seemingly *outré*, but physicists are willing to speculate about processes in which the topology of space changes with time [25].

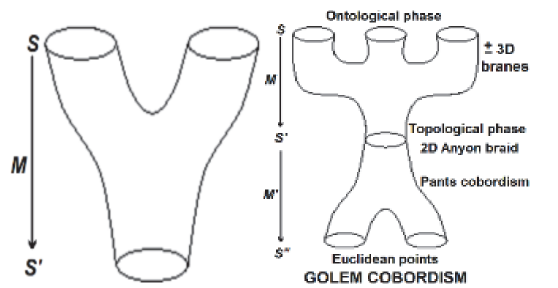

Fig. 6. a)Anyon topologically protected fusion channels, b) basic pants cobordism, c) Golem, composition of cobordisms designed to handle ontological-phase fusion transformations.

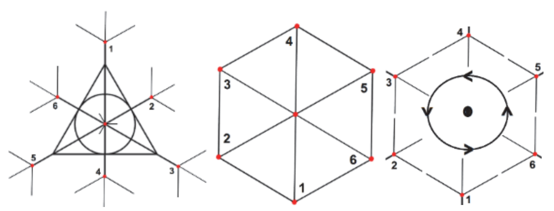

Fig. 7. a) *Antennas* (snowflakes) on a Fano plane represent b) vertices on the circumference of a hexagon/cube, c) center rotates unconnected so position 1 or 2 can create the front/rear vertices of an ambiguous Necker cube. b) Antennas 1-6 combine to form the outer vertices of a cube/hexagon depending on what dimensional phase the state is in.

Various operations can be performed on cobordisms; we describe two. 1) *Compose* two cobordisms $M : S \to S'$ and $M' : S' \to S''$, obtaining cobordism $M'M : S \to S''$, Fig. 6c. The idea is that the passage of time corresponding to M followed by the time corresponding to M' equals the time corresponding to $M'M$. This is analogous to the idea that waiting t seconds followed by waiting t' seconds is the same as waiting $t + t'$ seconds. The difference in TQFT is we cannot measure time in seconds, because no background metric exists to let us count the passage of time! We track topology change. Just as ordinary addition is associative, so is the composition of cobordisms:

$$(M''M')M = M''(M'M). \tag{26}$$

But, cobordism composition is not commutative - The famous noncommutativity of observables in QT [25]. 2) Any (n–1)D manifold S representing space, there is a cobordism $1_S : S \to S$ called the *identity* cobordism, representing passage of time without topological change. For example, if S is a circle, the identity cobordism 1_S is a cylinder. In general, the identity cobordism 1_S has the property that for any cobordism $M : S' \to S$ we have $1_S M = M$, while for any cobordism $M : S \to S'$ we have $M1_S = M$ [25]. These properties say that an identity cobordism is analogous to waiting 0 seconds: if you wait 0 seconds and then wait t more seconds, or wait t seconds and then wait 0 more seconds, this is the same as waiting t seconds. These operations just formalize of the notion of 'the passage of time' in a context where the topology of spacetime is arbitrary and there is no background metric. Atiyah's axioms relate this notion to QT as follows: 1) a TQFT must assign a Hilbert space $Z(S)$ to each (n – 1)D manifold S. Vectors in this Hilbert space represent possible states of the universe given that space is the manifold S. 2) the TQFT must assign a linear operator

$$Z(M) : Z(S) \to Z(S') \tag{27}$$

to each nD cobordism $M : S \to S'$. This operator describes how states change given that the portion of spacetime between S and S' is the manifold M: If space is initially manifold S and the state of the universe is ψ, with the passage of time corresponding to M, the state of the universe is $Z(M)\psi$ [25]. TQFTs must satisfy a list of properties. Two are: 1) A TQFT preserves composition: given cobordisms $M : S \to S'$ and $M' : S' \to S''$, we must have

$$Z(M'M) = Z(M')Z(M), \tag{28}$$

where the right-hand side denotes the composite of the operators $Z(M)$ and $Z(M')$. 2) It must preserve identities; given any manifold S representing space, we must have $Z(1_S) = 1_{Z(S)}$, where the right-hand side denotes the identity operator on the Hilbert space $Z(S)$ [25]. These axioms are not unreasonable if one ponders them a bit. The first says that the passage of time corresponding to the cobordism M followed by the passage of time corresponding to M' has the same effect on a state as the combined passage of time corresponding to $M'M$. The second says that a passage of time in which no topology change occurs has no effect at all on the state of the universe. This seems paradoxical at first, since it seems we regularly observe things happening even in the absence of topology change. However, this paradox is easily resolved: a TQFT describes a world quite unlike ours, one without local degrees of freedom. In such a world, nothing local happens, so the state of the universe can only change when the topology of space itself changes.

Loosely speaking, they all say that a TQFT maps structures in differential topology (study of manifolds) to corresponding structures in quantum theory. Atiyah took advantage of power between differential topology and quantum theory [25]. This analogy between differential topology and QT is the sort of clue we should pursue for a deeper understanding of quantum gravity. At first glance, GR and QT look very different mathematically: one deals with space and spacetime, the other with Hilbert spaces and operators, not easy to combine; but TQFT suggests they are not so different. Quantum topology is quite technical, but it should be obvious that differential topology and QT must merge in order to understand background-free QFT. Physics ignoring GR, treats space as a background for displaying world states. Similarly, spacetime is treated as a background for the process of change; these idealizations must be overcome in a background-free theory. In fact, concepts of *space* and *state* are two aspects of a unified whole, as likewise the concepts of *spacetime* and *process* [25]. In an alternative derivation of string tension, T_S we met this challenge by finding a unique background independent M-Theory [3], that after another decade led to OPTFT as the putative 3rd regime integrating GR and UFM [4].

6. Toward Experimental Design - Empirical Tests

A photon, 2-component, 2D traveling plane *wave* projecting at right angles to the direction of propagation has a *particulate radius* not able to pass a slit $> \lambda$. We propose that behind the inherent backcloth of cyclic bumps and holes in the polarized Dirac vacuum (Fig. 2a) [4], the uncertainty principle is hiding the XD topology of the MOU (Fig. 4), which is not singular as in the SM because cyclic boost-compactification occurs continuously from asymptotic virtual $\hbar$ (shadow of uncertainty, Fig. 2), to the Larmor radius of the hydrogen atom, making correspondence to dynamical Type-II M-theoretic Calabi-Yau florets (multiply-connected Kahler manifold) undergoing translation, rotation, reflection as part of the process. Spectral lines characterize atoms by, $E = \hbar\nu = \hbar c / \lambda$ or wave number, $\sigma \equiv 1/\lambda = E/\hbar c$ by discrete wavelengths confirmed by monochromatic x-ray bombardment. Excited states, E_2 decay to lower states, E_1 by emission of photon energy, E_2 - E_1 frequency, ν, wavelength, λ and wave number,

$$E_2 - E_1 = \hbar\nu = \hbar c / \lambda = \hbar c \sigma. \tag{29}$$

By conditions hinted at in Fig. 4 we propose new spectral lines *below* the lowest (ground state) Bohr orbit. Kowalski's interpretation from laser experiments [26] shows that emission and absorption between Bohr states takes place within a time interval equal to one period of the emitted-absorbed photon wave, the corresponding transition time is the time needed for the orbiting electron to travel one full orbit around the nucleus. We note that the same Lorentz conditions denoted in our tachyon measurement experiment apply directly to the TBS experiment with slight phase control alterations in the Cramer-like standing-wave oscillation of the HD Calabi-Yau mirror symmetries [6]. Standard Hypervolume values for increasing n-dimensionality and radius, r of a unit sphere or n-ball equal to 1 can be used to initially predict two TBS spectral lines hidden within the 6D Calabi-Yau dual 3-torus, the putative wavelengths of can be calculated from the general hyperspherical n-volume equation, of $\left(1/2\pi^2\right)$, 4.9346 units for 4D, and $\left(8/15\pi^2\right)$, 5.2638 units for 5D. If Randall-Sundrum are correct, the 6D cavity will be degenerate, and the signal escape to infinity. We postulate a Manifold of Uncertainty (MOU) with a finite dimensional radius corresponding to what string theory calls T-Duality [3-5]. For preliminarily predictions we could calculate hyperspherical volume or surface area of 2D-5D MOU. The general n-volume equation is

$$V(n,r) = \pi^{\frac{n}{2}} r^n / \Gamma\left(\tfrac{n}{2}+1\right), \tag{30}$$

where $V_{n,r}$ is volume per number of dimensions, n of radius r and Γ a factorial constant. These n-volume equations relate to volumetric properties of the MOU for calculating an HD C-QED volume hierarchy for predicting new Tight-Bound State (TBS) spectral lines in hydrogen [4, 21]. If LSXD exist, degeneracy would occur at the limit of *r* discovered in the same manner the outermost energy level of an atom is detected when an outer electron acquires sufficient energy to escape to infinity.

7. Overview - Shortcomings

Certainly, efforts have been insufficient, especially in latter half of this essay; in initial sections, we tried to show inadequacy of Big Bang cosmology in explaining QSO luminosity, with *Z*s beyond which inflation could account for giving a critique of Hubble's Law as applied to Doppler expansion.

Redshift refers generally to motion of a source relative to an observer; with blueshift for motion toward the observer, $Z < 0$ and redshift for velocity away from the observer, $Z > 0$ for an object not in the line of sight the relativistic form of the Doppler effect is

$$1 + Z = \frac{1 + v\cos(\theta)/c}{\sqrt{1 - v^2/c^2}}. \tag{31}$$

When the motion of the source is in the line of sight, $\theta = 0$ the equation reduces to the general formula

$$1 + Z = \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{32}$$

where one can tabulate *Z*:

V	Z
.5c	.73
~.6c	1
.75c	1.64
.8c	2.00
.85c	2.51
.95c	5.24
.96c	~6
.99c	13.11

Table 1. Tabulation of *Z* compared to velocity approaching *c*.

The largest *Z* currently known is for the most distant QSO CFHQS J2329-0301 with Z $\simeq$ 6.43 [27]. A QSO with Z > 10 has been observed but is still unconfirmed. Hubble's redshift law is considered quite variable; and interpretation depends on a number of factors like the specific cosmological model utilized or if Λ is 0, + or -. The best indirect evidence supporting our thesis is that QSO's are the most luminous objects in the known universe and that an object, especially one as massive as a QSO is supposed to be, receding at ~c would indicate ~ infinite mass.

Next, we suggested that in multiverse cosmology QSO shock waves were suggestive of a duality between Newton and Einstein G-models, leading to the proposal that duality needed to be more broadly elevated to a principle of physics and cosmology; and here, where the main theme of the paper inadequately kicks in.

Classical/quantum mechanics are insufficient to the task of rigorously describing this *Principle of Duality*, which requires delineation of a 3rd regime Unified Field Mechanics (UFM), we feel is best described by introducing an OPTFT making correspondence to a semi-quantum limit with an inherent *manifold of uncertainty* of finite radius [4, 5, 21] beyond which a new set of noetic transformations beyond the current SM Galilean, Lorentz-Poincairé are needed.

We then suggested that the most likely avenue of discovery for duality would occur in tests for Dirac-Majorana fusion. Anyon quasiparticle braiding is topologically protected (inaccessible) (Fig. 3) [28], however application of nonlocal ontological *topological switching* (energyless information exchange) by the parameters of OPTFT [4] allows the central pillars of quantum field theory, its basic assumptions of *locality* and *unitarity,* to be supervened as the uncertainty principle is surmounted when passing from QM to the 3rd regime of UFM [29].

Gravity appears in this scenario also in terms of the new field of Relativistic Information Processing (RIP) [4]. This entails a new set of UFM transformations beyond the Galilean, Lorentz-Poincairé for which the final derivation is incomplete at the time of writing; and without which the experimental tests cannot be performed.

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On the Test of Newton's Inverse Square Law and Unification of Gravitation and Electromagnetism

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Newton's inverse-square law of gravitation is the oldest standing mathematical description of a fundamental interaction. However, both Newton and Einstein failed to explain the Anomaly of the Space-Probes and flybys. Moreover, they also failed to explain experiments on the weight reductions of a charged metal ball, a charged capacitor and heated-up metal. These show that $E = mc^2$ is not valid. To show that the weight reduction is not due to mass reduction, one can measure the acceleration of a neutral object in a free fall. We shall show that the accurate test of Newton's law is related to the unification of gravitation and electromagnetism that includes the charge-mass interaction and the current-mass interaction. Thus, to have an accurate test of this law, we must understand Einstein's unification, and exactly how temperature affects the measurement of weight. However, Einstein and his followers failed to show the need of unification due to inadequacy in non-linear mathematics and physics. Also, we point out that the accuracy of the J. Luo Newtonian gravitational constant, needs more work since temperature dependence of gravitation has been verified.

Keywords: Anti-gravity coupling, Gravitational radiation, Repulsive gravitation, Principle of causality
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1. Introduction

Newton's inverse-square law of gravitation is the oldest standing mathematical description of a fundamental interaction. Experimental tests of gravity's distance-dependence define a frontier between our understanding of gravity and many proposed forms of new physics. As gravity is ~ 10^{40} times weaker that electromagnetism, gravity remains hidden by experimental backgrounds at distances smaller than the diameter of a fine human hair. The recent invited talk of Charles Hagedorn [1] by APS surveys the past, present, and near-future of the experimental field, with substantial emphasis on precision sub-millimeter laboratory experiments. However, Hagedorn did not know as most of the APS members, the crucial fact that the measured weight of testing matter actually depends on its temperature [2] (see also Appendix A).1)

Although Faller [3] is aware that error budgets in the measurements of the Newtonian coupling constant are fundamentally flawed because they cannot make allowances for error sources that have not been thought of. However, he also did not know that the current measurements to obtain the Big G could not be accurate due to ignorance on the influence of heat to weight [2].

Thus, the most accurate Newtonian coupling constant obtained by J. Luo (罗俊) is questionable [4].

We shall explain why an experimental measurement of gravity is actually temperature dependent.

Thus, in principle, the temperature dependence of measurements must be understood before an accurate test of Newton's inverse square law.

2. Limitations of Newton and Einstein

The Newtonian inverse square gravitational law is supported by observations of the planets, and it is the theory that we relying on to go to the moon. However, Einstein found that Newtonian theory is unsatisfactory because its instantaneous interaction is against special relativity. Then, Einstein proposed general relativity with a field equation [5]

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - \kappa T_{\mu\nu} \qquad (1)$$

where $T_{\mu\nu}$ is the energy-stress tensor, and κ is the coupling constant. This static equation is confirmed by the observed bending of light [5, 6]. However, for the dynamic cases, there are obstacles that prevent the progress, due to errors in mathematics and in physics.

2.1. *Errors Due to Inadequacy in Mathematics*

Einstein thought that his theory is valid to calculate the remaining perihelion of Mercury [5]. In fact, this is why he has confidence on his general relativity [7]. However, Gullstrand [8], the chairman (1922-1929) of the Nobel Prize Committee for Physics, pointed out that Einstein's calculation cannot be derived from the approach of a many-body problem. Hence, it is also questionable whether the Einstein equation has a bounded dynamic solution.

Moreover, in 1937, Einstein and Rosen [9] also questioned the existence of a gravitational wave solution. Nevertheless, in 1973 Misner, Thorne, & Wheeler [10] incorrectly claimed to obtain a bounded wave solution[2)] as follows:

$$ds^2 = c^2dt^2 - dx^2 - L^2\left(e^{2\beta}dy^2 + e^{-2\beta}dz^2\right) \qquad (2)$$

where $L = L(u), \beta = \beta(u)$, $u = ct - x$, and c is the light speed. Then, the Einstein equation $G_{\mu\nu} = 0$ becomes

$$\frac{d^2L}{du^2} + L\left(\frac{d\beta}{du}\right)^2 = 0 . \qquad (3)$$

In 1984, Wald [11] also claimed he can have 2nd order dynamic solutions, although he never provided one.

In 1993 Christodoulou and Klainerman [12] claimed that they have constructed dynamic solutions. Apparently, their book and the work of Witten [13] convinced the 1993 Nobel Committee for Physics to change their mind and claimed that the Einstein equation has bounded dynamic solutions. Then, the 1993 Nobel Prize for Physics was awarded to Hulse and Taylor for their work on the gravitational radiation with erroneous announcements [14].[3)] Thus, it seems that Einstein had won.

However, in 1995 it is found that Gullstrand is correct and the Einstein equation actually does not have any dynamic solution [15]. In other words, the 1993 Nobel Committee for Physics is wrong. This paper was published in Astrophysical Journal by the Editor-in-Chief Chandrasekhar, a Nobel Laureate and an expert on general relativity.[4)]

Moreover, the claim of Misner et al., is actually due to mathematical errors at the undergraduate level [16].[5)] In fact, it is not possible to have an approximation of metric (2) of the following form,

$$ds^2 = c^2dt^2 - dx^2 - (1+2\phi)dy^2 - (1-2\phi)dz^2 \qquad (4)$$

where ϕ is a bounded function of u $(= ct - x)$. Note that metric (4) is the linearization of metric (1) if $\phi = \beta(u)$. Thus, the waves illustrate that the linearization is not valid for the dynamic case when gravitational waves are involved.[6)]

Another clear evidence that eq. (1) has no bounded dynamic solution is, as shown by Hu, Zhang, & Ding [17], that the calculated gravitational radiation depends on the perturbation approach used.

Thus, it is necessary to examine the book of Christodoulou and Klainerman [12] carefully. It is found that they actually have not completed their construction [18]. Their so-called "dynamic" solutions are merely constructed from their presumed strong asymptotically flat (S.A.F.) "initial data sets" without showing the physical relevance. In fact, they have not shown the existence of a case other than the static solutions. Thus, their claim of the existence of a dynamic solution is invalid. Moreover, a S.A.F. initial data set is incompatible with the Maxwell-Newton approximation, the linear field equation for weak gravity. It is concluded that the only valid S.A.F. initial data sets are the static solutions [18, 19].

Perlick [20], who wrote a review that appeared in ZFM in 1996 and republished in the journal of GRG in 2000 [21] complained, "What makes the proof involved and difficult to follow is that the authors introduce many special mathematical constructions, involving long calculations, without giving a clear idea of how these building-blocks will go together to eventually prove the theorem. The introduction, almost 30 pages long, is of little help in this respect. Whereas giving a good idea of the problems to be faced and of the basic tools necessary to overcome each problem, the introduction sheds no light on the line of thought along which the proof will proceed for mathematical details without seeing the thread of the story. This is exactly what happened to the reviewer."

Moreover, Perlick [20] also pointed out "Before this book appeared in 1993 its content was already circulating in the relativity community in the form of a preprint that gained some notoriety for being extremely voluminous and extremely hard to read. Unfortunately, any hope that the final version would be easier to digest is now disappointing. Nonetheless, it is to be emphasized that the result presented in this book is very important. Therefore, any one interested in relativity and/or in nonlinear partial differential equations is recommended to read at least the introduction." (Note that Christodoulou was a Ph. D. student of Wheeler.) In fact, their claim on "dynamic" solutions was met with wide spread skepticism [20], and their errors can be identified in the introduction.[7)]

However, a modified Einstein equation with an additional gravitational energy-stress tensor with an anti-gravity coupling, does have a bounded dynamic solution [21, 22]. The modified Lorentz-Levi-Einstein equation is as follows:

$$G_{\mu\nu} \equiv R_{\mu\nu} - (1/2)g_{\mu\nu}R = - \kappa [T_{\mu\nu} - t(g)_{\mu\nu}], \quad (5)$$

where $t(g)_{\mu\nu}$ is the gravitational energy-stress tensor. Note that it is eq. (5) not the Einstein equation (1) is consistent with the linearized equation for the massive case, and can do an approximate calculation for the gravitational radiation.[8)] Moreover, the space-time singularity theorems of Penrose and Hocking are actually irrelevant to physics [23] because the crucial assumption cannot be satisfied n physics.

Historically, eq. (5) was first proposed by Lorentz [24] and one year later it was also proposed by Levi-Civita [25] as $Kt(g)_{ab} = G_{ab} + KT_{ab}$. However, Einstein [26] objected to eq. (5) on the grounds that his equation (1) implies $t(g)_{\mu\nu} = 0$. Now, Einstein is clearly wrong since his equation is proven invalid for the dynamic case [27].

Note that there is an intrinsic conflict between Einstein's formula $E = mc^2$ and his equation. According to Eq. (1), an electromagnetic energy-stress tensor cannot affect the curvature $R = Kg^{\mu\nu}T_{\mu\nu}$, but a massive energy-stress tensor does. Moreover, the existence of the anti-gravity coupling implies that the $E = mc^2$ may not be valid. In fact, this formula is actually only a speculation, because Einstein had failed [28] to prove it after many years of efforts (1905-1909).

2.2. *Inadequacy in Physics*

Moreover, there are also errors in physics. For instance, it has been found [29, 30] that for an electromagnetic wave, there is no valid solution unless a photonic energy-stress tensor with an anti-gravitational coupling is added. i.e.,

$$G_{ab} = \kappa [T(E)_{ab} - T(p)_{ab}], \quad (6)$$

where $T(E)_{ab}$ and $T(P)_{ab}$ are the energy-stress tensors for the electromagnetic wave and the related photons. (However, Einstein [31] incorrectly claimed that there is no antigravity coupling for this case.) Thus, the photonic energy includes the energy for its gravitational wave component. This solves the puzzle that the electromagnetic energy-stress tensor is traceless, but the photonic energy can be equivalent to mass. Experimentally, a π_0 meson can be decayed into two photons i.e., $\pi_0 \to \gamma + \gamma$, in addition to Einstein's incomplete proof for the equivalence of mass and photonic energy [5].

The necessary existence of the anti-gravity coupling implies that the energy conditions in the space-time singularity theorems of Hawking and Penrose cannot be satisfied (see Appendix B) and thus their theorems are irrelevant to physics.

These errors of Hawking and Penrose have three sources namely: 1) the acceptance of Einstein's speculation of general validity of $E = mc^2$; 2) the inadequate understanding of the non-linear mathematics; 3) inadequate understanding on the principle of causality (see Appendix C). Inadequate understanding of non-linear mathematics leads to the false belief [32] that the linearization always provides a valid approximation for the non-linear Einstein equation. Penrose [16, 33] being only a mathematician, does not understand the principle of causality, and thus accepts unbounded solution as valid.

One might expect that mathematicians could help the problems in physics. However, those who do not understand physics, could provide misleading information. An example is the positive mass theorem of Yau and Schoen [34], who use the formulas for energy and momentum given by Arnowitt, Deser, and Misner (ADM). However, ADM does not understand that the Einstein equation has no dynamic solution, and thus their formula is not applicable to the dynamic case. The theorem of Yau and Schoen is misleading because their theorem implicitly uses the invalid physical assumption, the unique sign for all the couplings. Thus, Yau actually created more problems. The fact that Witten [13] adapted Yau's invalid view, may give some insight on why there is little progress in string theory.[9)] Their theorems were cited as achievements by Fields Medal because mathematicians such as Atiyah[10)] and Faddeev[11)] do not understand general relativity [35].

Another obvious problem is the absence of the gravitational radiation reaction force.[12)] A serious problem of $E = mc^2$ is that gravity is mistakenly considered as the effect of mass only.

3. The Repulsive Gravitation and Necessary Extension of General Relativity

Currently, the gravitational effects of the electromagnetic energy are not understood [2]. Consequently, Einstein failed to prove his conjecture of unification between electromagnetism and gravitation.

3.1. *The Reissner-Nordstrom Metric and the Repulsive Gravitation*

Due to the existence of many intrinsic errors, essentially nothing has been done on the energy of electromagnetism until 1997 [36]. Now, let us reexamine again the Reissner-Nordstrom metric [10] (with c =1) as follows:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \qquad (7)$$

where q and M are the charge and mass of a particle, and r is the radial distance from the particle center. In metric (7), the gravitational components generated by electricity have not only a very different radial coordinate dependence but also a different sign that makes it a new repulsive gravity in general relativity [37].

However, Einstein did not accept this and theorists such as Herrera, Santos, & Skea [38] argued that M in (7) involves the electric energy. Then they obtained a metric that would imply a charged ball would increase its weight as the charge Q increased [37]. However, this is in disagreement with experiments [39]. Nevertheless, they are not alone. For instance, Nobel Laureates 't Hooft [40] and Wilczek [41] also have mistaken that m = E/c^2 was universally true.[13)]

On the other hand, if the mass M is the inertial mass of the particle, the weight of a charged metal ball can be reduced [42]. Thus, as Lo expected [36], experiments on two exactly the same metal balls [39] supports that the charged ball has a reduced weight. This is an experimental direct proof that the electric energy is not equivalent to mass. According to metric (7), a particle with charge q, the repulsive force to a particle of mass m at a distance r is approximately mq^2/r^3. It will be shown that such a force leads to the necessity to extend the theoretical framework of general relativity.

3.2. *The Extension of General Relativity and Einstein's Unification*

To show the static repulsive effect, one needs to consider only g_{tt} in metric (7). According to Einstein [5, 6], the equation of motion is the geodesic equation

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0, \text{ where}$$
$$\Gamma^\mu{}_{\alpha\beta} = (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta})g^{\mu\nu}/2 \qquad (8)$$

and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Note that the gauge affects only the second order approximation of g_{tt} [43].

Let us consider only the static case. For a particle *P* with mass m at **r**, the force on *P* is

$$-m\frac{M}{r^2} + m\frac{q^2}{r^3} \qquad (9)$$

in the first order approximation because $g^{rr} \cong -1$. Thus, the second term is a repulsive force.

If the particles are at rest, then the force acting on the charged particle *Q* has the same magnitude, i.e.,

$$\left(m\frac{M}{r^2} - m\frac{q^2}{r^3}\right)\hat{r}, \text{ where } \hat{r} \text{ is a unit vector} \qquad (10)$$

because the action and reaction forces are equal and in the opposite directions. However, for the motion of the charged particle with mass M, if one calculates the metric according to the particle *P* of mass m, only the first term is obtained.

Then, it is necessary to have a repulsive force with the coupling q^2 to the charged particle *Q* in a gravitational field generated by masses. Thus, force (10) to particle *Q* is beyond current theoretical framework of gravitation + electromagnetism. As predicted by Lo, Goldstein, & Napier [44], general relativity leads to a realization of its inadequacy.

The repulsive force in (7) comes from the electric energy [37]. An immediate question would be whether such a charge-mass repulsive force mq^2/r^3 is subjected to electromagnetic screening. Physically, this force, being independent of a charge sign, should not be subjected to such a screening. However, it would be according to general relativity.

Note that this force can be considered as a result of q^2 interacting with a field created by the mass m. Thus such a field is independent of electromagnetism and is beyond general relativity, and the need of unification is established.[14)] To test such a possibility, one can measure whether there is such a repulsive force outside a charged capacitor.

3.3. *The Attractive Current-Mass Interaction*

While the electric energy leads to a repulsive force from a charge to a mass, the magnetic energy would lead to an attractive force from a current toward a mass [45]. Also, if a non-charged capacitor has no reduction of weight, it is necessary to have the current-mass interaction to cancel out the charge-mass interaction. In a normal situation, theorists such as Galileo, Newton and Einstein actually assume implicitly that the charge-mass repulsive force would be cancelled by the current-mass force This general force is related to the static charge-mass repulsive force similar to the Lorentz force is related to the Coulomb force.

The existence of such a current-mass attractive force has been verified by Martin Tajmar and Clovis de Matos [46]. It is found that a spinning ring of superconducting material increases its weight much more than expected. According to quantum theory, spinning super-conductors should produce a weak magnetic field. Thus, they are measuring also the interaction between an electric current and the earth. The current-mass interaction would generate a force which is perpendicular to the current. Moreover, such interaction could be identified as the cause for the anomaly of flybys.[15)]

One may ask what the formula for the current-mass force is. However, unlike the static charge-mass repulsive force, this general force would be beyond general relativity since a current-mass interaction would involve the acceleration of a charge that would generate electromagnetic radiation. Then, the electromagnetic radiation reaction force and the variable of the fifth dimension must be considered [44]. Thus, we are not yet ready to derive this force.

Nevertheless, we may assume that, for a charged capacitor, the resulting force is the interaction of net macroscopic charges with the mass. This current-mass interaction also explains the phenomenon that it takes time for a capacitor to recover its weight after being discharged. This was observed by Liu because his rolled-up capacitors keep heat better [2]. A discharged capacitor needs time to dissipate the heat, and the motion of its charges would accordingly recover to normal.

In short, there are three factors that determine the weight of matter. They are; 1) the mass of the matter; 2) the charge-mass repulsive force; and 3) the attractive current-mass force. For a piece of a heated-up metal, the current-mass attractive force due to orbital electrons is reduced, but the charge-mass repulsive force would increase. Therefore, a net result is a reduction of weight [2] instead of increased weight as Einstein predicted [47].

Thus, to test the inverse square law accurately, one must know exactly how temperature affects the weight.

4. Einstein's Theory of Unification and the Five-dimensional Relativity

The coupling with q^2 leads to a five-dimensional space of Lo et al. [44] since such a coupling does not exist in a four-dimensional theory, the five dimensional theories of Kaluza [48] or Einstein & Pauli [49].

Now, in the five-dimensional relativity. the five-dimensional geodesic of a particle is

$$\frac{d}{ds}\left(g_{ik}\frac{dx^k}{ds}\right)=\frac{1}{2}\frac{\partial g_{kl}}{\partial x^i}\frac{dx^k}{ds}\frac{dx^l}{ds}+\left(\frac{\partial g_{5k}}{\partial x^i}-\frac{\partial g_{5i}}{\partial x^k}\right)\times\frac{dx^5}{ds}\frac{dx^k}{ds}-\Gamma_{i,55}\frac{dx^5}{ds}\frac{dx^5}{ds}-g_{i5}\frac{d^2x^5}{ds^2} \tag{11a}$$

$$\frac{d}{ds}\left(g_{5k}\frac{dx^k}{ds}+\frac{1}{2}g_{55}\frac{dx^5}{ds}\right)=\Gamma_{k,55}\frac{dx^5}{ds}\frac{dx^k}{ds}-\frac{1}{2}g_{55}\frac{d^2x^5}{ds^2}+\frac{1}{2}\frac{\partial g_{kl}}{\partial x^5}\frac{dx^l}{ds}\frac{dx^k}{ds}, \tag{11b}$$

where $ds^2=g_{\mu\nu}dx^\mu dx^\nu$, μ, ν = 0, 1, 2, 3, 5 ($d\tau^2=g_{kl}dx^k dx^l$; *k, l* = 0, 1, 2, 3).

If instead of *ds*, $d\tau$ is used in (11), for a particle with charge q and mass M, the Lorentz force suggests

$$\frac{q}{Mc^2}\left(\frac{\partial A_i}{\partial x^k}-\frac{\partial A_k}{\partial x^i}\right)=\left(\frac{\partial g_{i5}}{\partial x^k}-\frac{\partial g_{k5}}{\partial x^i}\right)\frac{dx^5}{d\tau}. \tag{12a}$$

Thus,

$$\frac{dx^5}{d\tau}=\frac{q}{Mc^2}\frac{1}{K},\quad \frac{d^2x^5}{d\tau^2}=0$$

and
$$K\left(\frac{\partial A_i}{\partial x^k}-\frac{\partial A_k}{\partial x^i}\right)=\left(\frac{\partial g_{i5}}{\partial x^k}-\frac{\partial g_{k5}}{\partial x^i}\right) \tag{12b}$$

where *K* is a constant. It thus follows that (11) is reduced to

$$\frac{d}{d\tau}\left(g_{ik}\frac{dx^k}{d\tau}\right)=\frac{1}{2}\frac{\partial g_{kl}}{\partial x^i}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}+\left(\frac{\partial A_k}{\partial x^i}-\frac{\partial A_i}{\partial x^k}\right)\times\frac{q}{Mc^2}\frac{dx^k}{d\tau}-\Gamma_{i,55}\left(\frac{q}{Mc^2}\right)^2\frac{1}{K^2}, \tag{13a}$$

$$\frac{d}{d\tau}\left(g_{5k}\frac{dx^k}{d\tau}+\frac{1}{2}g_{55}\frac{q}{KMc^2}\right)=\Gamma_{k,55}\frac{q}{KMc^2}\frac{dx^k}{d\tau}+\frac{1}{2}\frac{\partial g_{kl}}{\partial x^5}\frac{dx^l}{d\tau}\frac{dx^k}{d\tau}. \tag{13b}$$

One may ask what the physical meaning of the fifth dimension is. Our position is that the physical meaning of the fifth dimension is not yet very clear [44], except some physical meaning is given in the equation, $dx^5/d\tau = q/Mc^2K$ where M and q are respectively the mass and charge of a test particle, and K is a constant. We shall denote the fifth axis as the w-axis. Our approach is to find out the full physical meaning of the w-axis as our understanding gets deeper.

For a static case, we have the forces on the charged particle Q in the ρ -direction

$$-\frac{mM}{\rho^2} \approx \frac{Mc^2}{2} \frac{\partial g_{tt}}{\partial \rho} \frac{dct}{d\tau} \frac{dct}{d\tau} g^{\rho\rho}, \text{ and}$$

$$\frac{mq^2}{\rho^3} \approx -\Gamma_{\rho,55} \frac{1}{K^2} \frac{q^2}{Mc^2}\, g^{\rho\rho} \tag{14a}$$

and $\Gamma_{k,55} \frac{q}{KMc^2} \frac{dx^k}{d\tau} = 0$, where

$$\Gamma_{k,55} \equiv \frac{\partial g_{k5}}{\partial x^5} - \frac{1}{2}\frac{\partial g_{55}}{\partial x^k} = -\frac{1}{2}\frac{\partial g_{55}}{\partial x^k} \tag{14b}$$

in the (*-r)*-direction. The meaning of (14b) is the energy momentum conservation. Thus, $g_{tt} = 1 - \frac{2m}{\rho c^2}$, and

$$g_{55} = \frac{mMc^2}{\rho^2} K^2 + \text{constant.} \tag{15}$$

In other words, g_{55} is a repulsive potential. Because g_{55} depends on M, it is a function of local property, and thus is difficult to calculate. This is different from the metric element g_{tt} that depends on a distant source of mass m.

On the other hand, because g_{55} is independent of q, this force would penetrate electromagnetic screening. From the above, it is also possible that a charge-mass repulsive potential would exist for a metric based on mass M of charged particle Q. However, because P is neutral, there is no charge-mass repulsion force (from $\Gamma_{k,55}$) on P.

Thus, general relativity must be extended to accommodate the charge-mass interaction. For this, a five-dimensional relativity is a natural candidate. According to Lo et al. [44], the charge-mass interaction would penetrate a charged capacitor. To verify the five-dimensional theory, one can simply test the repulsive force on a charged capacitor. This has been experimentally confirmed [37, 50, 51]. On the other hand, from four-dimensional theory, we would not get any repulsive force acting on a test massive particle outside a capacitor since the electromagnetic field would be screened out. Thus, one may expect that there are surprises in 5D theory.

However, journals such as the Physical Review and Proceedings of the Royal Society A, still have not recognized these important experiments due to inadequacy in nonlinear mathematics and still have blind faith toward Einstein. In addition to the temperature and the composition of the test particle, the gravity also has some problems with the sun [37].

5. Applications of the Charge-Mass Repulsive Force and Anomaly of the Space Probes

The Reissner-Nordstrom metric was first published in 1916, the same year that first paper on general relativity was published. Thus, the repulsive charge-mass interaction should have been discovered shortly afterward. However, this was not recognized until 1997 [36], because Einstein and his followers believed in his invalid speculation of $E = mc^2$. The existence of such repulsive gravitation was inadvertently verified by the charged metal ball experiment [39] in 2005.

Note that, the calculation of (10) is essentially based on general relativity. The five-dimensional theory is invoked only to justify that the new force is not subjected to electromagnetic screening. However, it is theoretically crucial to establish a charge-mass repulsive force, which is independent of electromagnetism.

Then, the charge-mass repulsive force between a point charge q and a point mass m is

$$F = \frac{q^2 m}{r^3} \tag{16}$$

in the r-direction. The five-dimensional theory supports that it is not subjected to electromagnetic screening, and this is supported by the experiment of weighing charged capacitors [52] because a concentration of static charges would provide such repulsion. Since this repulsive force is inversely proportional to the square of the distance r. it would become weaker faster than gravity at long distance. Hence a capacitor lifter would hover on earth only in a limited height [50].

The space probes also give a good opportunity to check the mass-charge interaction. If the repulsive force comes from the sun, then m in (16) would be m_p the mass of the pioneer, and distance r would be R the distance between the sun and the space probe. However, the charge term is not clear since for the sun we do not know what the non-linear term q^2 should be.

Nevertheless, since such forces act essentially in the same direction, we could use a parameter P_s to represent the collective effect of the charges. Then, the effective repulsive force F_p would be

$$F_p = \frac{P_s m_p}{R^3} . \tag{17}$$

Since the neutral sun emits light and is in an excited state, the sun has many locally charged particles, and P_s is not negligible. If the data fits well with an appropriate parameter P_s, then this would be a confirmation of the charge-mass interaction.

Since this force is much smaller than the gravitational force from the sun, in practice the existence of such a repulsive force would result in a very slightly smaller mass M_{ss} for the sun of mass M_s, i.e.

$$F = \frac{M_s m_p}{R^2} - \frac{P_s m_p}{R^3} \tag{18a}$$

and

$$\frac{M_s m_p}{R_0{}^2} - \frac{P_s m_p}{R_0{}^3} = \frac{M_{ss} m_p}{R_0^2} \tag{18b}$$

where R_0 is the distance from earth to the sun. Then, we have

$$F = \frac{M_{ss} m_p}{R^2} + \frac{P_s m_p}{R^2} \left(\frac{1}{R_0} - \frac{1}{R}\right) \tag{19}$$

Thus, it appears that there is an additional attractive force for $R > R_0$.

Moreover, such a force would not be noticeable from a closed orbit since the variation of the distance from the sun is small. However, for open orbits of the pioneers, there are great variations. When the distance is very large, the repulsive force becomes negligible, and thus an additional attractive force would appear as the anomaly. Such a force would appear as a constant over some distance. Thus, the repulsive fifth force satisfies the overall requirements of the data [53].

When the four planetary probes experienced unaccountable changes in velocity as they passed Earth, they experienced an additional repulsive force from the Earth because the core of the globe has charged currents. Moreover, depending on the way of approaching the globe, a planetary probe would also experience an additional attractive force due to current-mass interaction. Thus, a planetary probe would experience an additional acceleration or de-acceleration.[16)]

However, this problem does not affect the gravity of the moon, thus the orbits around the moon are reliable.

6. Conclusions and Discussions

Clearly, it is not possible to test the inverse square law of Newton accurately due to the existence of the charge-mass interaction. To have an accurate test of Newton's law, we must understand how temperature would affect the charge-mass interaction and the current-mass interaction. However, this has little effect on the gravity related to the moon.

Newton's inverse-square law of gravitation is the oldest standing mathematical description of a fundamental interaction. Moreover, in most applications to the moon, one can ignore the effect of the temperature. Now, we see that the accurate test of this law is intimately related to the unification of gravitation and electromagnetism. Thus, to have an accurate test of this law, we must first understand Einstein's unification of gravitation and electromagnetism [2].

A major error of Einstein was that he failed to recognize that the Einstein equation does not have any dynamic solution [15] as Gullstrand [8] suspected. (A deeper reason is, however, that Einstein failed to recognize the repulsive gravitation.) Although Hu, Zhang, & Ding [17] show that the calculated gravitational radiation depends on the perturbation approach used, many still failed to see the non-existence of bounded dynamic solution [19, 27]. Then, general relativity was considered as superseding Newtonian gravity because it has been mistaken that the two-body problem has a bounded solution [54]. Moreover, general relativity is not complete because of the absence of the gravitational radiation reaction force [19]. Note that Einstein and Rosen [9] are the first who discovered there is no gravitational wave solution for the Einstein equation.

Moreover, the principle of causality implies that boundedness of a solution is crucial for its being valid in physics. However, many believed to allow acceptance of unbounded "time-dependent" solutions as physical waves [16]. Nevertheless, Hawking is supported by mathematicians such a S. T. Yau. and E. Witten. This is due to that Yau and Witten also used the same invalid assumption to prove his misleading positive mass theorem [13, 34].

In general relativity, famous institutes such as Harvard, Princeton Advanced Studies,[7)] and the Royal Society, are the sources of errors [16, 19]. The fact that so many theorists believed in and awarded the errors of Christodoulou testified that many theorists still do not understand the non-linear mathematics and general relativity [32]. Moreover, in 2016 Witten was inappropriately awarded the APS Medal for Exceptional Achievement in Research without any experimental supports because APS also failed to know his errors in general relativity [55] because no editor has adequate background in pure mathematics.[17)]

Einstein failed to see that the mass and electromagnetic energy are intrinsically different. Thus, he failed to see that his field equation is in conflict with $E = mc^2$. Since he had proposed inadequately that the photons would consist of only electromagnetic energy, Einstein had mistaken that the equivalence of mass and photonic energy was a proof for the equivalence of mass and electromagnetic energy. Thus, he did not see that the existence of photons is a consequence of general relativity [29, 30], and that gravity is the connection between relativity and quantum mechanics.

This error leads to the spacetime singularity theorems of Hawking and Penrose [23]. Not only their Theorems have no experimental supports, but also are actually irrelevant to physics [23]. These theorems are the starting points for the notion of black holes and the assumption of an expanding universe. Now, one must find new justifications for these theories.

The invalid formula $E = mc^2$ leads to negligence of the gravity generated by non-massive energy-stress tensor. This is why the Reissner-Nordstrom metric was not investigated for a long time, and the charge-mass interaction was overlooked. The charge-mass repulsive force not only shows the non-equivalence between mass and electromagnetic energy, but also is crucial for the unification between electromagnetism and gravitation [37]. A consequence is that a charged capacitor would fall slower [56].[18)] This will show that the weight reduction of a charged capacitor is not due to a loss of mass. Recently, the repulsive gravitation has been explicitly observed with the torsion balance scale [57]. Thus, APS is clearly behind in the field of gravitation because of the past burden of errors and some of Einstein's views have been out-dated.

General relativity was incorrectly believed as effective only for large scale problems. Thus, the study for the applications of general relativity on earth and understanding material structure is neglected or ignored. For example, there are numerous experiments on the weight reduction of a charged capacitor [50, 51]. However, these experiments were incorrectly regarded as due to errors or simply ignored by many journals. These experiments are important because they support the charge-mass interaction that is crucial for Einstein's unification [37].

In general relativity, the charge-mass interaction would be subjected to electromagnetic screening. However, it is unnatural that a neutral force could be screened. From the viewpoint of the five-dimensional theory, however, the charge-mass repulsive force would be understood as that the charge interacts with a new field created by a mass. Thus, the repulsive force would not be subjected to such screening. Such force is a test for the existence of a five-dimensional space.

Einstein believed that the increment of energy would increase the gravitational attraction [47]. However, experimentally, a charged capacitor is lighter. In a charged capacitor, both the positive and the negative charges are concentrated, and thus an effect of the repulsive force would be observed as a lighter weight for the charged capacitor [50]. The cases of heated-up metals [2] are also examples that can show experimentally the invalidity of Einstein's prediction [56]. This reveals that *a major error of Einstein's theory is due to over-looking the repulsive gravitation* [37].

Gravitation was considered by Newton and Einstein as producing only attractive force, and all the coupling constants were assumed to have the same sign. Recently, it is proven that for the gravitational radiation of binary pulsars the coupling constants must have different signs [15, 23]. Since the electromagnetic energy is not equivalent to mass, the picture provided by Newton is just too simple for a phenomenon as complicated as gravity that relates to everything.[19)]

Einstein and his followers failed because of over confidence due to that not only they do not understand non-linear mathematics, but also ignored on some experiments. Another problem is their inadequacy in mathematics. They make mathematical errors without knowing them because they do not have enough background in pure mathematics. Thus, Einstein's followers often advocate their own errors, but do not want to read papers beyond their circle. Physically, Einstein failed unification because he did not understand as Maxwell did that unification requires new interaction.[20)]

Physicists had incorrectly believed that pure mathematics are useless because they did not find it directly useful. Now, the usefulness of pure mathematics is clearly shown in general relativity. Moreover, it is the famous institutes that made the mathematical errors.

The discovery of the repulsive gravitation is important because it would solve a puzzle as to why we have never seen a black hole. If gravity is always attractive to mass, simulation convinces Wheeler that a black hole must be formed [45]. Another problem is that because of the existence of repulsive gravitation, it is not yet possible, in principle, to have a accurate measurement of the Newtonian gravitational constant. Although J. Luo is an excellent experimentalist on gravitational measurements, he is not a theorist and thus may not see that his measurements are questionable.

Moreover, it is urgent that the American Physical Society should made an effort to correct the errors by improving the pure mathematical ability of physicists.

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Appendix A: Influence of the Temperature of a Body on its Weight

A. L. Dmitriev, E. M. Nikushchenko, and V. S. Snegov [58] show that brass metal rods heated by ultrasound confirms a dependence of the weight of the rods on their temperature. Their results can be shown by the following figure.

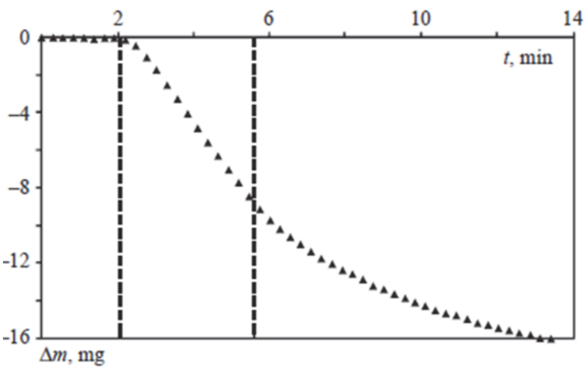

Fig. A1. Change in mass of a brass rod mounted in the closed doer vessel. Ultrasound frequency 131.27 kHz. The dashed lines indicate the moments when the ultrasound was switched on and off.

Dmitriev et al. [58] observed the temperature dependence of the weight reduction. A problem is that they have not been able to correctly identify the cause of the weight reduction.

Appendix B: The Space-Time Singularity Theorems and the Unique Sign of Couplings

Let us examine the energy conditions in the singularity theorems [11], listed as the following:

Theorem 1. Let (M, g_{ab}) be a globally hyperbolic spacetime with $R_{ab}\xi^a\xi^b \geq 0$ for all timelike ξ^a, which will be the case if the Einstein equation is satisfied with the strong energy condition holding for matter. Suppose there exists a smooth (or at least C^2) spacelike Cauchy surface Σ for which the trace of the extrinsic curvature (for the past directed normal geodesic congruence) satisfies $K \leq C < 0$ everywhere C is a constant. Then no past directed timelike curve from Σ can have length greater than $3/|C|$. In particular, all past directed timelike geodesics are incomplete.

Theorem 2. Let (M, g_{ab}) be a strongly causal spacetime with $R_{ab}\xi^a\xi^b \geq 0$ for all timelike ξ^a, as will be the case if the Einstein equation is satisfied with the strong energy condition holding for matter. Suppose there exists a compact, edgeless, achronal smooth spacelike hypersurface S such that for the past directed normal geodesic congruence form S we have $K < 0$ everywhere on S. Let C denote the maximum value for K, so $K \leq C < 0$ everywhere on S. Then at least one inextendible past directed timelike geodesic from S has length no greater that $3/|C|$.

Theorem 3. Let (M, g_{ab}) be a connected, globally hyperbolic spacetime with a noncompact Cauchy surface Σ. Suppose $R_{ab}k^ak^b \geq 0$ for all null k^a, as will be the case if (M, g_{ab}) is a solution of Einstein's equation with matter satisfying the weak or strong energy condition. Suppose, further, that M contains a trapped surface T. Let $\theta_0 < 0$ denote the maximum value of θ for both sets of orthogonal geodesic on T. Then at least one inextendible future directed orthogonal null geodesic from T has affine length no greater than $2/|\theta_0|$.

Theorem 4. Suppose a spacetime (M, g_{ab}) satisfies the following four conditions. (1) $R_{ab}v^av^b \geq 0$ for all timelike and null v^a, as will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. (2) The timelike and null generic conditions are satisfied. (3) No closed timelike curve exists. (4) At least one of the three properties holds: (a) (M, g_{ab}) posses a compact achronal set without edge [i.e., (M, g_{ab}) is a closed universe], (b) (M, g_{ab}) possesses a trapped surface, or (c) there exists a point $p \in M$ such that the expansion of the future (or past) directed null geodesics emanating from p becomes negative along each geodesic in this congruence. Then (M, g_{ab}) must contain at least one incomplete timelike or null geodesic.

The energy condition is related to the energy-momentum tensor T_{ab}. According to the Einstein equation [5]

$$G_{ab} \equiv R_{ab} - (1/2)\, g_{ab}\, R = 8\pi T_{ab}, \quad \text{(B1)}$$

one would have

$$R_{ab} = 8\pi\, [T_{ab} - (1/2) g_{ab}\, T] \text{ where } T = g^{ab}T_{ab} \quad \text{(B2)}$$

Then,

$$R_{ab}\,\xi^a\xi^b = 8\pi\,[T_{ab} - (1/2)g_{ab}\,T]\,\xi^a\xi^b =$$

$$8\pi[T_{ab}\xi^a\xi^b+(1/2)T], \text{ for a unit timelike } \xi^a \qquad (B3)$$

It is believed that for all physically reasonable classical matter the energy condition is non-negative, i.e.,

$$T_{ab}\,\xi^a\xi^b \geq 0 \qquad (B4)$$

for all timelike ξ^a. This is known as the weak energy condition. However, it also seems physically reasonable that the stress of matter will not become so large and negative as to make the right-hand side of eq. (B3) negative. This assumption,

$$T_{ab}\,\xi^a\xi^b \geq -(1/2)T \qquad (B5)$$

for all unit timelike unit vector ξ^a, is known as the strong energy condition. An implicit assumption of these energy-conditions (B3)-(B5) is that all the coupling constants have the same sign.

To illustrate this, consider the Bondi, Pirani, & Robinson metric [59] as follows:

$$ds^2 = e^{2\varphi}\left(d\tau^2 - d\xi^2\right) - u^2\begin{bmatrix}\cosh 2\beta\left(d\eta^2 + d\varsigma^2\right)\\ +\sinh 2\beta\cos 2\theta\left(d\eta^2 - d\varsigma^2\right)\\ -2\sinh 2\beta\sin 2\theta\, d\eta\, d\varsigma\end{bmatrix} \qquad (B6a)$$

where φ, β and θ are functions of u (τ - ξ). It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u\left(\beta'^2 + \theta'^2 \sinh^2 2\beta\right) \qquad (B6b)$$

which is a special cases of $G_{\mu\nu} = 0$. They claimed this is a wave from a distant source and weak gravity invalid. The metric is irreducibly unbounded because of the factor u^2. And linearization of (B6b) does not make sense since u is not bounded.

Moreover, when gravity is absent, it is necessary to have ϕ = sinh 2β = sin 2θ = 0. These would reduce (B6a) to

$$ds^2 = (d\tau^2 - d\xi^2) - u^2(d\eta^2 + d\zeta^2) \qquad (B6c)$$

However, this metric is not equivalent to the flat metric. Thus, metric (B6c) violates the principle of causality.

Appendix C: The Principle of Causality and the Physics of Plane-Waves

There are two aspects in causality: its relevance and its time ordering. In time ordering, a cause event must happen before its effects. This is further restricted by relativistic causality that no cause event can propagate faster than the light speed in vacuum. The time-tested assumption that phenomena can be explained in terms of identifiable causes will be called the principle of causality. This is the basis of relevance for all scientific investigations. Thus, the principle of causality implies that any parameter in a solution for physics must be related to some physical causes. Moreover, the principle of causality implies a weak source would produce a weak gravity. Here this principle will be elucidated first in connection with symmetries of a field, the boundedness of a field solution, and consequently in the validity of a field equation in physics.

In practice, when the considered field is absent, physical properties are ascribed to the space-time as in a "normal" state. For example, the electromagnetic field is zero in a normal state. Then, any deviation from the normal state must have physically identifiable causes. Thus, the principle of causality implies that the symmetry must be preserved if no cause breaks it. The implication of causality to symmetry has been used in deriving the inverse square law from Gauss's law. The normal state of a space-time metric is the flat metric in special relativity. Thus, if a metric does not possess a symmetry, then there must be physical cause(s) which has broken such a symmetry. For a spherically symmetric mass, causality requires that the metric is spherically symmetric and asymptotically flat. Also, a weak cause can lead to only weak gravity. Therefore, Einstein's notion of weak gravity is a consequence of the principle of causality.

However, the physical cause(s) should not be confused with the mathematical source term in the field equation. In general relativity, the cause of gravity is the physical matter itself, but not its energy tensors in the source term of Einstein's field equation. The energy-stress tensors (for example the perfect fluid model) may explicitly depend on the metric. Since nothing should be a cause of itself, such a source tensor does not represent the cause of a metric. For the accompanying gravitational wave of an electromagnetic wave, the physical cause is the electromagnetic wave. Thus, one should not infer the symmetries of the metric based on the source term (instead of its causes) although their symmetries are not unrelated.

Moreover, inferences based on the source term can be misleading. The source term may have higher symmetries than those of the cause and the metric. For instance, a transverse electromagnetic plane-wave (1) is not rotationally invariant with respect to the z-direction of propagation. But the related electromagnetic energy-stress tensor component $T(E)_{tt}$ for a circularly polarized wave is rotationally invariant. This assumption violates causality and results in theoretical difficulties.

Classical electrodynamics implies that the flat metric is an accurate approximation, caused by the presence of weak electromagnetic waves. This physical requirement is supported by the principle of causality which implies such a metric to be a bounded periodic function. However, this required boundedness is not satisfied by solutions in the literature [32, 60, 61]. These solutions also violate causality directly since they involve parameters without any physical cause [32]. They also do not satisfy the equivalence principle [62, 63] although they are Lorentz manifolds.

A necessary and sufficient condition for satisfying the equivalence principle is that a time-like geodesic represents a physical free falling; but this does not mean the existence of Minkowski spaces in a neighborhood. Another problem in general relativity is that many theorists and journals do not understand physics, such as the principle of causality adequately.

Endnotes

1) There are three types of experiments that show the formula $E = mc^2$ is invalid. They are: 1) the weight reduction of a charged metal ball [39]; 2) the weight reduction of a charged capacitor [52]; 3) the weight reduction of a piece of heated-up metal [2]. In the April APS Meeting (2015), I have called the attention on this matter for officials and editors of the APS. Tsipenyuk and Andreev [39] discovered the reduction of weight of a charged metal ball, but do not know that his can be related to general relativity [42]. The weight reduction of a charged capacitor has a valid explanation [51] only after the five-dimensional theory was used. In 2003, Dmitriev et al. [5658] has observed the weight reduction of heated-up brass and Fan et al. [6364] verified the weight reduction due to heated-up for six kinds of metal in 1910. Recently, the existence of repulsive gravitation has been directly observed [57] From this experiment, the existence of repulsive gravitation is no longer questionable although the details of such a repulsion force due to heat is not yet clear [2]. Now, it is clear that, there are incorrect papers on general relativity published in the Proceeding of the Royal Society A, Classical and Quantum Gravity, General Relativity and Gravitation, and the Annals of Physics, in addition to the Physical Review. These journals all accepted that the Einstein equation has dynamic solutions because they have mistaken that linearization of the Einstein equation would generally produce approximate solutions [32].

2) The 1993 Nobel Committee for physics adapted the invalid view of Wald [11] on the equivalence principle. They were unaware of that Einstein's equivalence principle has been verified [16].

3) S. Chandrasekhar, Nobel Laureate and expert in general relativity, approved Lo's paper in 1995, after the 1993 Nobel Prize for Hulse and Taylor. Thus, Chandrasekhar also objected to the errors of 1993 Nobel Committee. Also, Morrison of MIT had gone to Princeton University to question J. A. Taylor on their justification in calculating the gravitational radiation of the binary pulsars. However, Taylor was unable to give a justification [65].

4) The editors of General Relativity and Gravitation considers the claims of the Wheeler School as "well-established science", but were unable to provide supporting evidence [March 8, 2012]. Note that since there is no bounded dynamic solution for the Einstein equation [16], the thesis of A. Ashtekar (editor-in-chief), "*Asymptotic Structure of the Gravitational Field at Spatial Infinity*", just inherits the errors of Wald [11]. Moreover, he failed to see that the photons must include gravitational energy [29, 30]. C. M. Will, editor-in-chief of Classical and Quantum Gravity, continues to ignore the errors of the Wheeler School [16]. In fact, due to inadequacy in pure mathematics, like Pauli [66] the Wheeler School misinterpreted Einstein's equivalence principle [16]. and were unable to rectify their local time shown in their eq. (40. 14). The misinterpretation of Misner et al. [10] creates the so-called Lorentz invariance.

5) Due to inadequacy in non-linear mathematics, physicists (including the editors of journals such as the Physical Review and Proceeding of the Royal Society A) were unable to see that, for the dynamic case, the linearized equation does not provide an approximate solution for the non-linear equation [32]. From Dr. Daniel T. Kulp, Editorial Director of American Physical Society, I find out most of the editors in physics have background only in applied mathematics, but not pure mathematics. However, to be a competent editor for physics, one must know pure mathematics.

6) Bertschinger [67] did not know that, for the dynamic case, the linearized equation and the non-linear Einstein are actually independent equations since the non-linear equation has no bounded dynamic solutions [32].

7) To see the inadequacy of mathematics of the Wheeler School, one can simply find their errors at the undergraduate level from their analysis of eq. (3) in their book [10]. However, many failed this because they have the same problem.

8) The Wheeler School [10] even mistaken Einstein's 1911 assumption as Einstein's equivalence principle. Note that it is well-known that the 1911 assumption has been proven incorrect after the 1919 British expeditions [5].

9) That Witten [13] adapted Yau's invalid view shows that Witten does not understand the physics of general relativity. However, Witten is widely regarded as a leader in string theory, but he actually does not understand physics. He was asked once what is most important in physics, his answer was self-consistency. The correct answer was in agreement with experiment because in physics we often have inconsistency. It is well-known that quantum theory is often inconsistent with classical theory. Witten's undergraduate education is in history, and thus his understanding of pure mathematics is at most half-bake. For instance, he does not know that the Einstein equation does not have any dynamic solution. He also does not understand Einstein's equivalence principle because he agrees with the misinterpretation of Wheeler [10]. It would be very surprising that the string theory under his leadership would achieve a lot.

10) Michael Francis Atiyah has been leader of the Royal Society (1990-1995), master of Trinity College, Cambridge (1990-1997), chancellor of the University of Leicester (1995-2005), and President of the Royal Society of Edinburgh (2005-2008). Since 1997, he has been an honorary professor at the University of Edinburgh (Wikipedia). Apparently, Atiyah does not understand the physics and the non-existence of a dynamic solution for the Einstein equation [35].

11) Ludwig D. Faddeev, the Chairman of the Fields Medal Committee, wrote ("On the work of Edward Witten"): "Now I turn to another beautiful result of Witten – proof of positivity of energy in Einstein's theory of gravitation. Hamiltonian approach to this theory proposed by Dirac in the beginning of the fifties and developed further by many people has led to the natural definition of energy. In this approach a metric γ and external curvature h on a space-like initial surface $S^{(3)}$ embedded in space-time $M^{(4)}$ are used as parameters in the corresponding phase space. These data are not independent. They satisfy Gauss-Codazzi constraints – highly non-linear PDE. The energy H in the asymptotically flat case is given as an integral of indefinite quadratic form of $\nabla \gamma$ and h. Thus, it is not manifestly positive. The important statement that it is nevertheless positive may be proved only by taking into the account the constraints – a formidable problem solved by Yau and Schoen in the late seventy as Atiyah mentions, 'leading in part to Yau's Fields Medal at the Warsaw Congress'." The error is that the so-called 'natural definition of energy' is invalid in physics because it excludes the dynamic cases by assuming implicitly all the coupling constants have the same sign [35].

12) This will not affect the inverse square law, which deals with only static gravitational force. Also, the invalidity of Einstein's covariance principle has been found through explicit examples [68] just as P. Y. Zhou [69] claimed.

13) Almost all Noble Prize winners make the same mistake. Apparently, nobody checks this formula $E = mc^2$ adequately.

14) The weight reduction of a charged capacitor has been known since the last century [50, 51]. However, such observed facts are rejected or ignored by theorists because they cannot explain such phenomena in terms of their theory.

15) The cause of anomaly in flybys is a puzzle that cannot be explained with Newtonian gravitation.

16) It was claimed that the Pioneer Space-Probe Anomaly has been resolved by a heat-radiation model. However, a discoverer of the anomaly, Erik Anderson (April 1, 2011 at 12:57) commented, " … Science will have suffered the worst sort of dysfunction if the Pioneer Anomaly gets swept under the convenient rug of 'the plausible.' Even so, we will still have the Earth flyby anomalies and the so-called 'A.U.' anomaly left uncovered. All three anomalies seem to be manifestations of a singular phenomenon — the latter two cannot be dismissed as heat radiation. Heat-radiation models, like string theory, can be customized to fit any set of observational parameters. There is no limit on sophistication. We should not be so easily impressed. Nothing has been resolved."

17) The errors of Witten were still not recognized by the American Physical Society since in the announcement of awarding him the 2016 APS Medal for mathematics [55], no mention was made on his mathematical errors. This is, in part, due to that none of the APS editors has an adequate background in pure mathematics, according

to Dr. D. Kulp, Interim Editor in Chief at American Physical Society, who also has a Ph. D. degree in applied mathematics only.

18) Some [58, 64] have incorrectly interpreted the weight reduction as due to a mass reduction. However, if they measured the periods of a pendulum made of heated-up metal, he will see that it is only the weight that has been reduced [56].

19) The theory of general relativity provides the best opportunity to discover the repulsive gravitation. However, due to the general incompetence of mathematics among physicists, such an opportunity was over-looked for a long time.

20) Because Einstein was unable to recognize the limitations and errors of his earlier work, he failed to make progress in relativity after he arrived in the US. Understandably, Einstein refused to extend his life by available medicine [70] by claiming "It is tasteless to prolong life artificially, I have done my share, it is time to go. I will do it elegantly." Had Einstein known that he was very close to his unification, would he still be that willing to go?

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Unified Discrete Mechanics: Bifurcation of Hyperincursive Discrete Harmonic Oscillator, Schrödinger's Quantum Oscillator, Klein-Gordon's Equation and Dirac's Quantum Relativist Equations

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After the recall of the theory of incursive discrete harmonic oscillator, it is shown the hyperincursive discrete harmonic oscillator is separable into two incursive oscillators. It is shown that any differential continuous derivative bifurcates into two difference discrete derivatives. For second order differential equations, a generalized discrete derivative is presented, that can become complex, defining so a complex velocity. The hyperincursive discrete time equation of the Schrödinger quantum oscillator is recalled, to show that the hyperincursive discrete equations contribute to a unification of classical and quantum mechanics. Finally, we develop the theoretical presentation of the hyperincursive discrete equation of the Klein-Gordon differential second order equation which bifurcates to 4 first order discrete equations that gives the original Dirac quantum relativist equation. This is a remarkable result because the Dirac equation is rediscovered from this new method based on the hyperincursive discrete second order equation which bifurcates to first order equations.

Keywords: Discrete mechanics, Harmonic oscillator, Incursive oscillator, Hyperincursive oscillator, Quantum mechanics, Schrödinger equation, Klein-Gordon equation, Dirac equation

1. Introduction

This paper is the continuation of my paper on Hyperincursive Algorithms of Classical Harmonic Oscillator Applied to Quantum Harmonic Oscillator Separable into Incursive Oscillators (Dubois, 2016).

Section 2 recalls the theory of incursive discrete harmonic oscillator.

Section 3 deals with a complete study of the hyperincursive discrete harmonic oscillator separable into two incursive oscillators.

In section 4, it will be shown that any differential continuous derivative bifurcates into two difference discrete derivatives. For second order differential equations, a generalized discrete derivative is presented depending on a weight. This weight can become complex, defining so a complex velocity.

Then, section 4 shows that two quantum harmonic oscillators are similar to the two discrete incursive oscillators. The purpose is to show that two harmonic oscillators are linked to an invariant given by the Planck constant, similarly to the two discrete incursive oscillators that are related to the discrete time.

Section 5 presents the time and space solutions of the Schrödinger quantum harmonic oscillator, as an introduction to the next section 6 on the hyperincursive discrete time Schrödinger quantum oscillator.

In section 7, one recalls the hyperincursive discrete time equation of the Schrödinger quantum oscillator. This is useful to see that the hyperincursive discrete equations contributes to a unification of classical and quantum mechanics.

Finally, section 8 deals with the survey of the Klein-Gordon differential second order equation for bosons and the Dirac quantum relativist equations for fermions.

And the last section 9 develops the theoretical presentation of the hyperincursive discrete equation of the Klein-Gordon differential second order equation which bifurcates to 4 first order discrete equations which are the discrete representation of the 4 first order Dirac differential equations. When the interval of time and space tend to zero in these discrete representation, it is demonstrated that the original Dirac quantum relativist equation. This is a remarkable result because the Dirac equation is rediscovered from this new method based on the hyperincursive discrete second order equation which bifurcates to first order equations.

2. Incursive Discrete Harmonic Oscillators

Let us consider the harmonic oscillator with m the oscillating mass and k the spring constant, represented by the ordinary differential equations:

$$dx(t) = v(t)$$

$$dv(t)/dt = -\omega^2 x(t) \qquad (2.1a\text{-}b)$$

where x(t) is the position and v(t) the velocity as functions of the time t, and where the pulsation ω is related to k and m by $\omega^2 = k/m$.

The solution is given by

$$x(t) = x(0)\cos(\omega t) + [v(0)/\omega]\sin(\omega t)$$

$$v(t) = -\omega\, x(0)\sin(\omega t) + v(0)\cos(\omega t) \qquad (2.2a\text{-}b)$$

with the initial conditions x(0) and v(0). In the phase space, given by (x(t), v(t)), the solutions are given by closed curves (orbital stability). The period of oscillations is given by $T = 2\pi/\omega$. The energy e(t) of the harmonic oscillator is constant and is given by

$$e(t) = k\, x^2(t) / 2 + m\, v^2(t) /2$$

$$= k\, x^2(0) / 2 + m\, v^2(0) /2 = e(0) = e_0 \qquad (2.3)$$

The harmonic oscillator is computable by recursive functions from the discretized differential equations.

The differential equations of the harmonic oscillator depend on the current time. In the discretized form, there are the current time t and the interval of time $\Delta t = h$. The discrete time is defined as: $t_k = t_0 + kh$ with k = 0,1,2,... where t_0 is the initial value of the time and k is the counter of the number of interval of time h.

The discrete variables are defined as $x_k = x(t_k)$ and $y_k = y(t_k)$.

The discrete equations consist in computing firstly, the first equation to obtain, x_{k+1}, and then computing the second equation in using the just computed x_{k+1}

$$x_{k+1} = x_k + h\, v_k$$

$$v_{k+1} = v_k - h\, \omega^2 x_{k+1} \qquad (2.4a\text{-}b)$$

I called such a system, an incursive system, for inclusive or implicit recursive system (Dubois, 1998).

A second possibility occurs if the second equation is firstly computed, and then the first equation is computed in using the just computed v_{k+1}, as follows

$$v_{k+1} = v_k - h\, \omega^2 x_k$$

$$x_{k+1} = x_k + h\, v_{k+1} \qquad (2.5a\text{-}b)$$

An important difference between the incursive and the recursive discrete systems is the fact that in the incursive system, the order in which the computations are made is important: this is a sequential computation of equations.

The two incursive harmonic oscillators are numerically stable, contrary to the classical recursive algorithms like Euler and Runge-Kutta (Dubois and Kalisz, 2004).

In the recursive systems, the order in which the computations are made is without importance: this is a parallel computation of equations.

In my paper (Dubois, 1995), I defined a generalized forward-backward discrete derivative

$$D(w) = w\, D_f + (1 - w)\, D_b \qquad (2.6)$$

where w is a weight taking the values between 0 and 1, and where the discrete forward and backward derivatives on a function f are defined by

$$D_f(f) = \Delta^+ f / \Delta t = [\, f_{k+1} - f_k\,] / h$$

$$D_b(f) = \Delta^- f / \Delta t = [\, f_k - f_{k-1}\,] / h \qquad (2.7a\text{-}b)$$

The generalized incursive discrete harmonic oscillator is given by (Dubois, 1995) as:

$$(1-w)\, x_{k+1} + (2w-1)\, x_k - w\, x_{k-1} = hv_k$$

$$w\, v_{k+1} + (1-2w)v_k + (w-1)v_{k-1} = -h\omega^2 x_k \qquad (2.8a\text{-}b)$$

When w = 0, $D(0) = D_b$, this gives the first incursive equations:

$$x_{k+1} - x_k = h\, v_k$$

$$v_k - v_{k-1} = -h\, \omega^2 x_k \qquad (2.9a\text{-}b)$$

When w = 1, $D(1) = D_f$, this gives the second incursive equations:

$$x_k - x_{k-1} = h\, v_k$$

$$v_{k+1} - v_k = -h\, \omega^2 x_k \qquad (2.10a\text{-}b)$$

When w = 1/2, $D(1/2) = [D_f + D_b]/2$, this gives the averaged (hyperincursive) equations:

$$x_{k+1} - x_{k-1} = 2\, h\, v_k$$

$$v_{k+1} - v_{k-1} = -2\, h\, \omega^2 x_k \qquad (2.11a\text{-}b)$$

These eqs. (2.11a-b) integrate the two incursive equations (Dubois, 1998, 1999, 2000). In putting v_k of the first equation to the second one, one obtains the hyperincursive discrete harmonic oscillator,

$$x_{k+2} - 2x_k + x_{k-2} = -4 h^2 \omega^2 x_k \quad (2.12)$$

with the velocity

$$v_k = (x_{k+1} - x_{k-1})/2h \quad (2.13)$$

corresponding to the second order differential equations of the harmonic oscillator

$$d^2x(t)/dt^2 + \omega^2 x(t) = 0 \quad (2.14)$$

A complete mathematical development of incursive and hyperincursive systems was presented in a series of papers by Adel F. Antippa and Daniel M. Dubois (2002, 2004, 2006a-b, 2007, 2008a-b, 2010a-c) on, firstly, anticipation, orbital stability and energy conservation in discrete harmonic oscillators, secondly, on the dual incursive system, thirdly, on the superposed hyperincursive system, fourthly, on synchronous discrete harmonic oscillator, and fifthly. The paper (Antippa and Dubois, 2010b) deals with a deduction of this forward-backward discrete derivative, with the deduction of this time-symmetric discretization of the harmonic oscillator.

3. Hyperincursive Discrete Harmonic Oscillator Separable into Two Incursive Oscillators

For the discrete harmonic oscillator, let us use the dimensionless variables X, V and H (cfr Antippa and Dubois, 2010a), for the variables, x, v and h:

$$X(k) = \sqrt{[k/2]}\, x_k,$$
$$V(k) = \sqrt{[m/2]}\, v_k, \quad (3.1a\text{-}b)$$

$$\tau = \omega t \text{ with } \omega = \sqrt{[k/m]}, \text{ and}$$
$$\Delta\tau = \omega \Delta t = \omega h = H \quad (3.2a\text{-}b)$$

So, the two incursive dimensionless harmonic oscillators are given by the following 4 first order discrete equations:

First Incursive Oscillator

$$X_1(k+1) = X_1(k) + H V_1(k)$$
$$V_1(k+1) = V_1(k) - H X_1(k+1) \quad (3.4a\text{-}b)$$

Second Incursive Oscillator:

$$V_2(k+1) = V_2(k) - H X_2(k)$$
$$X_2(k+1) = X_2(k) + H V_2(k+1) \quad (3.5a\text{-}b)$$

These incursive discrete oscillators are non-recursive computing anticipatory systems. Indeed, in eq. (3.4b) of the first incursive oscillator, the velocity $V_1(k+1)$ at future next time step, k + 1, is computed from the velocity $V_1(k)$ at current time step, k, and the position $X_1(k+1)$ at the future next time step, k + 1, which represents an anticipatory system represented by an anticipation of one time step, k.

Similarly, in eq. (3.5b) of the second incursive oscillator, the position $X_2(k+1)$ at future next time step, k + 1, is computed from the position $X_2(k)$ at current time step, k, and the velocity $V_2(k+1)$ at the future next time step, k + 1, which represents an anticipatory system represented by an anticipation of one time step, k.

With the dimensionless variables the, hyperincursive dimensionless harmonic oscillator is given by the Hyperincursive Time-Symmetric Discrete Oscillator

$$X(k+1) = X(k-1) + 2 H V(k)$$
$$V(k+1) = V(k-1) - 2 H X(k) \quad (3.6a\text{-}b)$$

In putting V(k) of eq. (3.6a) to eq. (3.6b), this gives the Hyperincursive Second order Discrete Oscillator

$$X(k+2) - 2X(k) + X(k-2) = -4 H^2 X(k) \quad (3.7)$$

with the time-symmetric discrete velocity

$$V(k) = [X(k+1) - X(k-1)]/2H \quad (3.8)$$

This hyperincursive second order discrete oscillator is a recursive computing system.

With the dimensionless variables, the dimensionless energy is given by

$$E(k) = X^2(k) + V^2(k) \quad (3.9)$$

TABLE 1A

Hyperincursive harmonic oscillator, separable into two incursive harmonic oscillators (see table 1B).

	HYPERINCURSIVE HARMONIC OSCILLATOR	
	$X(k+1) =$ $X(k-1) + 2 H V(k)$	$V(k+1) =$ $V(k-1) - 2 H X(k)$
	Boundary conditions:	
	$\mathbf{X(0) = C_1, V(1) = C_2,}$	$V(0) = C_3, X(1) = C_4$
k	**Iterations**	
1	$\mathbf{X(2) = X(0) + 2 H V(1)}$	$V(2) = V(0) - 2 H X(1)$
2	$X(3) = X(1) + 2 H V(2)$	$\mathbf{V(3) = V(1) - 2 H X(2)}$

3	**X(4) = X(2) + 2 H V(3)**	V(4) = V(2) – 2 H X(3)
4	X(5) = X(3) + 2 H V(4)	**V(5) = V(3) – 2 H X(4)**
5	**X(6) = X(4) + 2 H V(5)**	V(6) = V(4) – 2 H X(5)
6	X(7) = X(5) + 2 H V(6)	**V(7) = V(5) – 2 H X(6)**
…	---	---

Indeed, let us demonstrate that the hyperincursive discrete harmonic oscillator, given by the eqs. (3.6a-b), is separable into two independent incursive harmonic oscillators, as shown in table 1A and table 1B.

TABLE 1B

Separation of the hyperincursive harmonic oscillator (see table 1A) into two independent incursive oscillators, with different boundary conditions.

	FIRST INCURSIVE HARMONIC OSCILLATOR	**SECOND INCURSIVE HARMONIC OSCILLATOR**
	Boundary conditions: **X(0) = C₁, V(1) = C₂**	Boundaryconditions: V(0) = C₃, X(1) = C₄
k	**Iterations**	**Iterations**
1	**X(2) = X(0) + 2 H V(1)**	V(2) = V(0) – 2 H X(1)
2	**V(3) = V(1) – 2 H X(2)**	X(3) = X(1) + 2 H V(2)
3	**X(4) = X(2) + 2 H V(3)**	V(4) = V(2) – 2 H X(3)
4	**V(5) = V(3) – 2 H X(4)**	X(5) = X(3) + 2 H V(4)
5	**X(6) = X(4) + 2 H V(5)**	V(6) = V(4) – 2 H X(5)
6	**V(7) = V(5) – 2 H X(6)**	X(7) = X(5) + 2 H V(6)
…	---	---

The first Incursive Harmonic Oscillator with boundary conditions, X(0), V(1), is given by

$$\mathbf{X(2k)} = X(2k-2) + 2\,H\,V(2k-1)$$

$$V(2k+1) = V(2k-1) - 2\,H\,\mathbf{X(2k)} \quad (3.10a\text{-}b)$$

and the second Incursive Harmonic Oscillator with boundary conditions, V(0), X(1), is given by

$$\mathbf{V(2k)} = V(2k-2) - 2\,H\,X(2k-1)$$

$$X(2k+1) = X(2k-1) + 2\,H\,\mathbf{V(2k)} \quad (3.11a\text{-}b)$$

for k = 1, 2, 3, …

Let us remark that these two incursive discrete oscillators (3.10a-b) and (3.11a-b) are identical to the two incursive discrete oscillators (3.4a-b) and (3.5a-b), as we will explain.

These incursive oscillators are incursive, that means implicit non-recursive, because the order in which the iterations are made is important.

Indeed, for the first incursive oscillator, (3.10a-b), the position X(k+1) is initially computed and then the velocity V(k+2) is sequentially computed, in taking into account the computed value of X(k+1).

And for the second incursive oscillator, (3.11a-b), the velocity V(k+1) is initially computed and then the position X(k+2) is sequentially computed, in taking into account the computed value of V(k+1).

Table 1A gives the iterations of the hyperincursive harmonic oscillator given by eqs. (2.16a-b).

The difference between the two incursive oscillators, given by eqs. (3.4a-b), (3.5a-b) and (3.10a-b), (3.11a-b), holds in the labelling of the successive time steps. In the incursive oscillators, (3.4a-b), (3.5a-b), the position and velocity are computed at the same time step while in the incursive oscillators, (3.10a-b), (3.11a-b), the position and the velocity are computed at successive time steps, but the numerical simulations of both give the same values. Each incursive oscillator is the dicrete time inverse, $+\Delta t \rightarrow -\Delta t$ and $-\Delta t \rightarrow +\Delta t$ of the other incursive oscillator, defined by time forward and time backward derivatives. So each of the two incursive oscillators is not reversible. But the superposition of the two incursive oscillators given by the hyperincursive oscillator is reversible.

The discrete position and the discrete velocity of the dimensionless discrete harmonic oscillator are given by the following analytical solution

$$X_k = \cos(2k\pi/N) \quad (3.12a)$$

$$V_k = -\sin(2k\pi/N) \quad (3.12b)$$

where N is the number of iterates for a cycle of the oscillator, with the index of iterations k = 0, 1, 2, 3 ....

The table 2 shows the numerical simulation of the hyperincursive oscillator.

The interval of discrete time H depends of N

$$H = \sin(2\pi/N) \quad (3.13a)$$

When N is large,

$$H = \sin(2\pi/N) \approx 2\pi/N = \omega\,\Delta t = 2\pi\Delta t/T \quad (3.13b)$$

So the period T of the harmonic oscillator is

$$T = N\,\Delta t \quad (3.14)$$

The number of iterates is given by N = 12.

For the simulations, the values of the boundary conditions are given by:

$$X(0) = C_1 = 1 \quad (3.15a)$$

$$V(0) = C_3 = 0, \quad (3.15b)$$

So, the values for the two other boundary conditions are given by

$$X(1) = C4 = \cos(\pi/6) = 0.8660,$$

$$V(1) = -\sin(\pi/6) = -0.5 \quad (3.16)$$

Thus, the dimensionless energy, eq. 2.9, is given,

$$E(0) = X^2(0) + V^2(0) = E_0 = 1 \quad (3.17)$$

TABLE 2

The simulation of the hyperincursive harmonic oscillator (eqs. (3.6a-b)) gives exactly the theoretical values (eqs. (3.12a-b)) that represent alternatively the values of the two incursive harmonic oscillators, given at tables 3A and 3B. There is the conservation of energy, E(k)=1.

HYPERINCURSIVE OSCILLATOR					
N	**H**	**k**	**X(k)**	**V(k)**	**E(k)**
12	0.5	0	**1.0000**	0.0000	1
		1	0.8660	**–0.5000**	1
		2	**0.5000**	–0.8660	1
		3	0.0000	**–1.0000**	1
		4	**–0.5000**	–0.8660	1
		5	–0.8660	**–0.5000**	1
		6	**–1.0000**	0.0000	1
		7	–0.8660	**0.5000**	1
		8	**–0.5000**	0.8660	1
		9	0.0000	**1.0000**	1
		10	**0.5000**	0.8660	1
		11	0.8660	**0.5000**	1
		12	**1.0000**	0.0000	1
		13	0.8660	**–0.5000**	1

TABLE 3A

Simulation of the first incursive oscillator (eqs. (3.4a-b)). There is no conservation of energy, $E_1(k)$, but averaged energy on half a cycle is constant, $[E_1(k-1)+E_1(k)+E_1(k+1)]/(N/2) = E_0 = 1.0$. There is a conservation of FORWARD ENERGY, $E_F(k) = E_{F0} = 0.75$ (see eqs. (3.18a-b-c)).

FIRST INCURSIVE HARMONIC OSCILLATOR							
N	**H**	**k**	**X_1(k)**	**V_1(k)**	**E_1(k)**	**FE**	**E_F**
6	1	0	1.000	–0.500	1.25	–0.50	0.75
		1	0.500	–1.000	1.25	–0.50	0.75
		2	–0.500	–0.500	0.50	0.25	0.75
		3	–1.000	0.500	1.25	–0.50	0.75
		4	–0.500	1.000	1.25	–0.50	0.75
		5	0.500	0.500	0.50	0.25	0.75
		6	1.000	–0.500	1.25	–0.50	0.75
		7	0.500	–1.000	1.25	–0.50	0.75

For the first incursive oscillator (3.4a-b), I introduced the concept of FORWARD ENERGY (Dubois, 2014), $E_F(k)$ for k = 0, 1, 2, 3, …, given by the sum of the energy, $E_1(k)$, and the forward H-dependent energy, $FE_1(k)$, as

$$E_F(k) = E_1(k) + FE_1(k) = E_{F0}$$

$$E_1(k) = X_1^2(k) + V_1^2(k)$$

$$FE_1(k) = + H\, X_1(k)V_1(k) \quad (3.18a\text{-}b\text{-}c)$$

and for the second incursive oscillator (3.5a-b), the concept of BACKWARD ENERGY (Dubois, 2014), $E_B(k)$ for k = 0, 1, 2, 3, …, given by the sum of the energy, $E_2(k)$, and the backward H-dependent energy, $BE_2(k)$, as

$$E_B(k) = E_2(k) + BE_2(k) = E_{B0},$$

$$E_2(k) = X_2^2(k) + V_2^2(k)$$

$$BE_2(k) = - H\, X_2(k)V_2(k) \quad (3.19a\text{-}b\text{-}c)$$

TABLE 3B

Simulation of the second incursive oscillators (eqs. (3.5a-b)). There is no conservation of energy, $E_2(k)$, but averaged energy on half a cycle is constant, $[E_2(k-1)+E_2(k)+E_2(k+1)]/(N/2) = E_0 = 1.0$. Moreover, there is a conservation of BACKWARD ENERGY, $EB(k) = E_{B0} = 0.75$ (see eqs. (3.19a-b-c)).

SECOND INCURSIVE HARMONIC OSCILLATOR							
N	**H**	**k**	**X_2(k)**	**V_2(k)**	**E_2(k)**	**BE_2**	**E_B**
6	1	0	0.866	0.000	0.75	0.00	0.75
		1	0.000	–0.866	0.75	0.00	0.75
		2	–0.866	–0.866	1.50	–0.75	0.75
		3	–0.866	0.000	0.75	0.00	0.75
		4	0.000	0.866	0.75	0.00	0.75
		5	0.866	0.866	1.50	–0.75	0.75
		6	0.866	0.000	0.75	0.00	0.75
		7	0.866	0.000	0.75	0.00	0.75

Let us remark that in the expression of the H-dependent energy, the interval of time, H, is positive for the forward oscillator and negative for the backward oscillator, that are in opposition of phase.

Let us notice that the averaged energies on the two incursive oscillators give the constant energy,

$$[E_1(k) + E_2(k)]/2 = \text{constant} = E0 = 1.0 \quad (3.20)$$

There is the conservation of this remarkable uncertainty relation, depending on discrete time, H,

$$BFE(k) = [- BE_2(k) - FE_1(k)]/2 =$$

$$H\, [X_2(k)V_2(k) - X_1(k)V_1(k)]/2 =$$

$$\text{constant} = 0.25 \quad (3.21)$$

Moreover, there is a conservation of FORWARD and BACKWARD ENERGIES,

$$E_F(k) = E_{F0} = \text{constant} = 0.75$$

$$E_B(k) = E_{B0} = \text{constant} = 0.75 \qquad (3.22a\text{-}b)$$

These functions are constants of motion for the two incursive discrete harmonic oscillators.

The two incursive oscillators are independent of each other and are link by their boundary conditions.

Figures 1 to 6 give the simulation of the hyperincursive discrete harmonic oscillator from eqs. (3.6a-b), with N = 3, 4, 6, 12, 24 and 48 time steps (from Dubois, 2014).

The figures of the simulations of the hyperincursive discrete harmonic oscillator sow the stability and the precision of the algorithm for values of time steps N = 3, 4, 6, 12, 24 and 48.

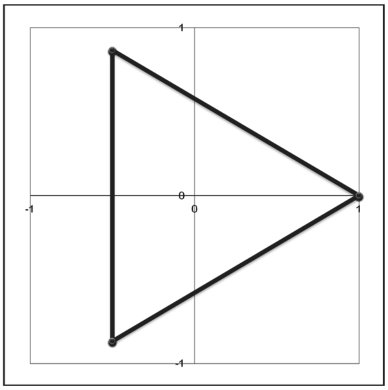

Figure 1. Figure of the simulation of the hyperincursive discrete harmonic oscillator with eqs. (2.16a-b), with N = 3 time steps. The horizontal axis represents the position X(k), and the vertical axis represents the velocity V(k) of the oscillator.

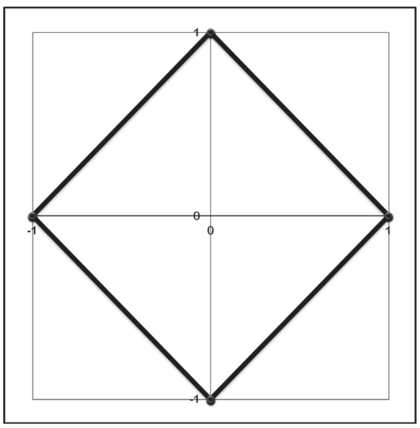

Figure 2. continuation of Figure 1 with N = 4 time steps.

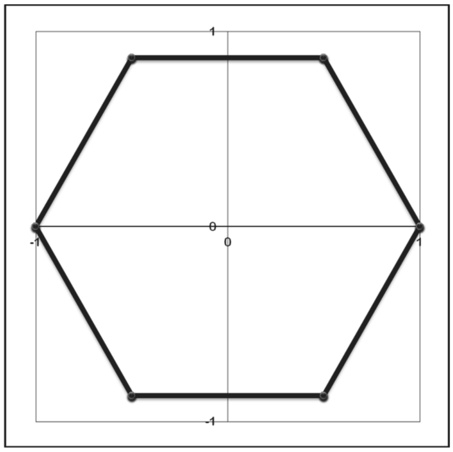

Figure 3. continuation of Fig. 2 with N = 6 time steps.

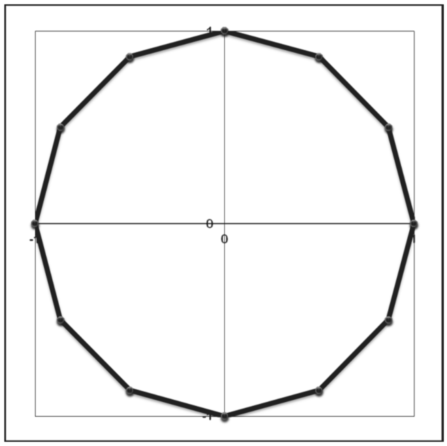

Figure 4. continuation of Fig. 3 with N = 12 time steps, and this case corresponds to the numerical values given in Table 2.

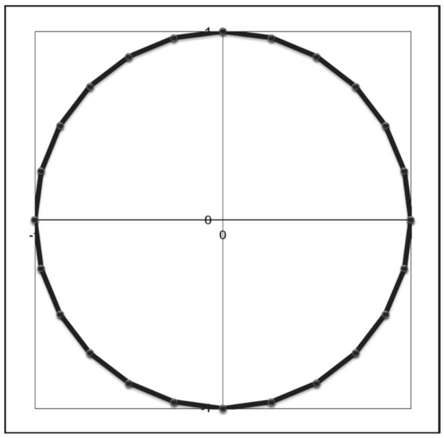

Figure 5. continuation of Fig. 4 with N = 24 time steps.

The representation of the harmonic oscillator tends to a circle when the number of time steps increases.

So, this confirms that the incursive and hyperincursive algorithms are totally numerically stable with the conservation of energy.

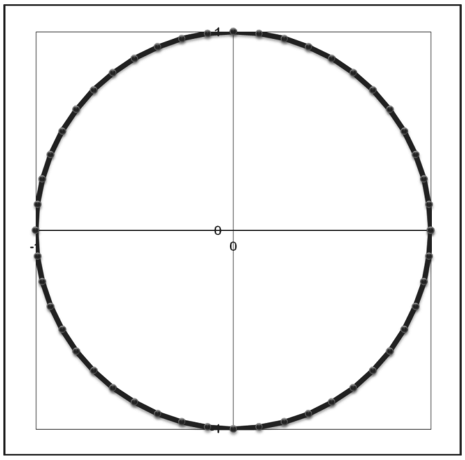

Figure 6. continuation of Fig. 5 with N = 48 time steps.

In conclusion, these incursive and hyperincursive algorithms are actually the best ones for making computer simulations of systems with a high degree of performance.

4. Bifurcation of Differential Continuous Derivative into Two Difference Discrete Derivatives

The differential continuous time derivative of the function f(t), df/dt bifurcates to two difference discrete derivatives $\Delta^+f/\Delta t$ and $\Delta^-f/\Delta t$.

This is also true for the space.

The differential continuous space derivative of the space function g(x), dg/dx bifurcates to two difference discrete derivatives $\Delta^+g/\Delta x$ and $\Delta^-g/\Delta x$. This can be generalized to the three spaces x, y and z components.

This section recalls the demonstration of the generalized complex discrete derivative in my paper (Dubois, 1998).

In fact two derivatives can be defined for a discrete variable x(t):

$$\Delta_f x / \Delta t = (x(t + \Delta t) - x(t))/\Delta t$$

$$\Delta_b x / \Delta t = (x(t) - x(t - \Delta t))/\Delta t \qquad (4.1a\text{-}b)$$

The forward derivative (4.1a) and the backward derivative (4.1b) are not always equal (only at the limit for $\Delta t = 0$ for continuous derivable equations); for non-derivable continuous equations like in fractal equations systems, two derivatives must be defined.

Let us remark that when Δt is replaced by $-\Delta t$, the forward and backward derivatives become the backward and forward ones.

Moreover, the successive application of the forward derivative to the backward derivative, or the inverse, gives the second order derivative, which is interval of time Δt invertible:

$$\Delta^2 x / \Delta t^2 = [x(t + \Delta t) - 2x(t) + x(t - \Delta t)] / \Delta t^2 \quad (4.2)$$

Let us define a generalized discrete derivative by a weighted sum of these derivatives as follows (Dubois, 1995):

$$\Delta_w x/\Delta t = w.\Delta_f x /\Delta t + (1 - w).\Delta_b x/\Delta t = [w.x(t + \Delta t) +$$

$$1 - 2.w).x(t) + (w - 1).x(t-\Delta t)]/\Delta t \qquad (4.3)$$

where the weight w is defined in the interval [0,1].

For w = 1, the forward derivative (4.1a) is obtained, $\Delta_f x / \Delta t = (x(t + \Delta t) - x(t))/\Delta t$ and for w = 0, the backward derivative (4.1b) $\Delta_b x / \Delta t = (x(t) - x(t - \Delta t))/\Delta t$.

For w = 1/2, derivative 18 becomes

$$\Delta_{1/2} x/\Delta t = (x(t + \Delta t) - x(t - \Delta t))/2\Delta t =$$

$$[\Delta_f x/\Delta t + \Delta_b x/\Delta t]/2 \qquad (4.4)$$

which is an average derivative of backward and forward derivatives.

With this generalized derivative (4.3), the discrete harmonic oscillator equations system can be defined by

$$(1 - w).q(t + \Delta t) + (2w - 1).q(t) - w.q(t-\Delta t) =$$

$$\Delta t.p(t)/m$$

$$w.p(t + \Delta t) + (1 - 2w).p(t) + (w - 1).p(t - \Delta t) =$$

$$- \Delta t.\omega^2.m.q(t) \qquad (4.5a\text{-}b)$$

From eq. (4.3) of the generalized discrete derivative, the second order derivative is given by the successive application of eq. (4.3) for w and (1−w), or the inverse:

$$\Delta_w\Delta_{1-w}x/\Delta t^2 = [x(t + \Delta t) - 2x(t) + x(t - \Delta t)]/\Delta t^2 +$$

$$w(1 - w) [x(t + 2\Delta t) - 4x(t + \Delta t) + 6x(t) - 4x(t - \Delta t) +$$

$$(t - 2\Delta t)]/\Delta t^2 = \Delta_{1-w}\Delta_w x/\Delta t^2 \qquad (4.6)$$

which is the sum of the classical discrete second order derivative and a factor, weighted by w(1 − w), which is similar to a fourth order discrete derivative (multiplied by Δt^2).

For w = 0 and w = 1

$$w(1 - w) = 0 \qquad (4.7a)$$

the classical second order derivative is obtained.

For w = 1/2,

$$w(1 - w) = 1/4 \quad (4.7b)$$

the second order derivative is also obtained but with a double time interval 2 Δt.

In choosing the value of the w(1 – w) equal to 1/2, we obtain weights w, solutions of the second order equation of second order

$$w^2 - w + 1/2 = 0 \quad (4.7c)$$

which are given by the complex numbers

$$w = 1 / 2 \pm i /2 \quad (4.8)$$

and

$$1 - w = 1 / 2 \pm (- i / 2) = w^* \quad (4.9)$$

where w^* is the complex conjugate of w.

This is remarkable result shows the origin of the bifurcation of the discrete equations by the apparition of the complex number i. Moreover, the complex number i appears with two signs + and –, as ± i, which means that the two solutions must be taken together. This is not a choice. So this is really a bifurcation, and the solutions of the discrete equations are given by vectors of solutions.

So eq. (4.3) of the generalized discrete derivative can be rewritten as

$$\Delta_w z /\Delta t = w.\Delta_f z/\Delta t + w{*}.\Delta_b z /\Delta t =$$

$$[w.z(t + \Delta t) + (w^* - w).z(t) + w^*.z(t-\Delta t)] /\Delta t \quad (4.10)$$

where the generalized complex derivative can be applied to a complex variable

$$z = x + i y \quad (4.11)$$

The second order derivative is given by the successive application of eq. (4.11) for w and w^*, or the inverse:

$$\Delta_w \Delta_{w*} z / \Delta t^2 = [z(t + \Delta t) - 2z(t) + z(t - \Delta t)] / \Delta t^2 +$$

$$w w^* [z(t + 2\Delta t) - 4z(t + \Delta t)+6z(t)-4z(t-\Delta t) +$$

$$z(t - 2\Delta t)] / \Delta t^2 = \Delta_{w*} \Delta_w z / \Delta t^2 \quad (4.12)$$

which is the sum of the classical discrete second order derivative and a factor, weighted by the real number ww^*, which is similar to a fourth order discrete derivative (multiplied by Δt^2).

With the complex weight w given by eq. (4.8), the first derivative of the position x (eq. (4.3)) gives rise to the complex velocity v

$$v = [x(t + \Delta t) - x(t - \Delta t)]/2\Delta t$$

$$\pm i [x(t + \Delta t) - 2x(t) + x(t - \Delta t)]/2\Delta t \quad (4.13)$$

In defining a forward velocity

$$v_f = [x(t + \Delta t) - x(t)]/\Delta t \quad (4.14a)$$

and a backward velocity,

$$v_b = [x(t) - x(t - \Delta t)]/\Delta t \quad (4.14b)$$

the complex velocity (4.13) is given by

$$v = [v_f + v_b]/2 \pm i [v_f - v_b]/2 \quad (4.15a)$$

Let us remark that the complex velocity given by eq. (4.15a) is similar to the complex velocity proposed by L Nottale (1993), where only a negative imaginary part is present.

With the notation $v_+ = v_f$ and $v_- = v_b$

$$v = [v_+ + v_-]/2 \pm i [v_+ - v_-]/2 \quad (4.15b)$$

where the + and – refer to +Δt and –Δt.

The velocity is then given by

$$\mathbf{v} = \begin{pmatrix} \frac{1}{2}+\frac{i}{2} & \frac{1}{2}-\frac{i}{2} \\ \frac{1}{2}-\frac{i}{2} & \frac{1}{2}+\frac{i}{2} \end{pmatrix} \begin{pmatrix} v_+ \\ v_- \end{pmatrix} \quad (4.16)$$

The real part of the velocity is the average of the forward and backward velocities and the imaginary part is the difference of these forward and backward derivatives.

In considering the inverse time interval in replacing + Δt by – Δt, the forward and backward derivatives become the backward and forward derivatives. So, if the velocity is time invertible, the plus and minus signs correspond to Δt ≤ 0 and Δt ≥ 0.

In the continuous limit Δt tending to zero, $v_f = v_b$, and the classical Newtonian velocity is rediscovered and the imaginary part tends to zero.

The kinetic energy is a real number given by

$$vv^*/2 = [v_f^2 + v_b^2]/4 \quad (4.17)$$

Let us remark also that the acceleration given by the second derivative of the position x is a real variable, because w(1 – w) = 1/2 is real in eq. (4.12).

The result given in this section is very important because this is the justification of the matrices of Dirac (see section 9 on The Hyperincursive Discrete Klein-Gordon Equation Bifurcates to the Dirac 4 Solutions).

Indeed, with the bifurcation of the velocity into a vector with two discrete velocities v_+ and v_- by the discrete times $\pm \Delta t$, there are the bifurcation by the three discrete space $\pm \Delta x$, $\pm \Delta y$, and $\pm \Delta z$. So the number of variables is $2 \times 2 \times 2 \times 2 = 16$, which corresponds to the Dirac 4×4 matrices.

5. Two Quantum Harmonic Oscillators Similar to the Two Discrete Incursive Oscillators

The purpose of this section is to show that two harmonic oscillators are linked to an invariant given by the Planck constant, similarly to the two discrete incursive oscillators that are related to the discrete time.

This section is based on my paper (Dubois, 2008) that deals with a quantum harmonic oscillator showing a pulsating wave packet, $\Phi(x, t)$, which is the solution of the Schrödinger equation

$$i\hbar\, \partial\, \Phi(x, t)/\partial t =$$

$$-(\hbar^2/2m)\, \partial^2\, \Phi(x, t)/\partial x^2 + V(x)\, \Phi(x, t) \qquad (5\text{-}1)$$

for the harmonic oscillator potential

$$V(x) = m\omega^2 x^2/2 \qquad (5\text{-}2)$$

The case where the wave packet pulsates without oscillation will be considered.

Let us consider the following Gaussian quantum wave function for the harmonic oscillator V(x)

$$\Phi(x, t) = \pi^{-1/4}\, q(t)^{-1/2} \exp[\, i\, (mx^2/2\hbar)\, d \ln q(t)/dt \qquad (5\text{-}3)$$

where q(t) is a complex solution

$$q(t) = q_1(t) + iq_2(t) \qquad (5\text{-}4)$$

of a classical harmonic oscillator

$$d^2q(t)/dt^2 + \omega^2 q(t) = 0 \qquad (5\text{-}5)$$

A similar formalism was proposed for the construction of pulsating Gaussian wave packets from the solutions of a complex harmonic oscillator (Arnaud, 2000).

To simplify the notations, q and p will be used for q(t) and p(t).

With this notation, the wave function (5-3) becomes:

$$\Phi(x, t) = \pi^{-1/4}\, q^{-1/2} \exp[\, i\, (mx^2/2\hbar)\, d \ln q/dt\,] \qquad (5\text{-}6)$$

From the complex solution q

$$q = q_1 + iq_2 \qquad (5\text{-}7)$$

the modulus qq* is obtained

$$qq^* = q_1^2 + q_2^2 \qquad (5\text{-}8)$$

and the logarithmic term is given by

$$d \ln q/dt = q^{-}1\, dq/dt = (1/(q_1 + iq_2))\, d(q_1 + iq_2)/dt \qquad (5\text{-}9)$$

with the momentum p

$$m\, dq/dt = m\, dq_1/dt + i\, m\, dq_2/dt = p_1 + ip_2 = p \qquad (5\text{-}10)$$

hence

$$m\, d \ln q/dt = p/q = (p_1 + ip_2)/(q_1 + iq_2) =$$

$$(q_1 - iq_2)\,(p_1 + ip_2)/(q_1^2 + q_2^2) =$$

$$= [q_1 p_1 + q_2 p_2 + i\,(q_1 p_2 - q_2 p_1)\,]/qq^* \qquad (5\text{-}11)$$

It exists an invariant, Inv, linking the q's and p's

$$Inv = q_1 p_2 - q_2 p_1 = \text{constant} = C \qquad (5\text{-}12)$$

Demonstration:
Let us take the time derivation of Inv

$$d\, Inv/dt = d[q_1p_2 - q_2p_1]/dt = p_1p_2/m + q_1dp_2/dt$$

$$- p_2p_1/m - q_2dp_1/dt = + q_1dp_2/dt - q_2dp_1/dt \qquad (5\text{-}13)$$

because

$$p_1p_2 = p_2p_1 \qquad (5\text{-}14)$$

Now, from the equation of the harmonic oscillator (eq. (5-5))

$$d^2q/dt^2 + \omega^2 q = 0 \qquad (5\text{-}15)$$

one obtains

$$dp/dt = -m\omega^2 q \qquad (5\text{-}16)$$

and

$$d(p_1 + ip_2)/dt = -m\omega^2(q_1 + iq_2) \qquad (5\text{-}17)$$

so

$$dp_1/dt = -m\omega^2 q_1 \qquad (5\text{-}18a)$$

$$dp_2/dt = -m\omega^2 q_2 \qquad (5\text{-}18b)$$

which are put in eq. (5-12)

$$d\,Inv/dt = +\,q_1dp_2/dt - q_2dp_1/dt = -\,q_1m\omega^2q_2 + q_2m\omega^2q_1$$

$$= m\omega^2(-\,q_1q_2 + q_2q_1) = 0 \qquad (5\text{-}19)$$

which is zero, because

$$q_1q_2 = q_2q_1 \qquad (5\text{-}20)$$

so the invariant has a constant value, C, that will be determined

$$Inv = q_1p_2 - q_2p_1 = C \qquad (5\text{-}21)$$

With the invariant, Inv, eq. (5-11) becomes

$$p/q = (q_1p_1 + q_2p_2 + iC\,)/qq^* =$$

$$(q_1p_1 + q_2p_2)\,/qq^* + i\,C/qq^* \qquad (5\text{-}22)$$

The wave function (5-6) can be re-written as

$$\Phi(x,t) = \pi^{-1/4}\,q^{-1/2}\exp[\,i\,(mx^2/2\hbar)\,d\ln q\,/dt] =$$

$$\pi^{-1/4}\,q^{-1/2}\exp[\,i\,(x^2/2\hbar)\,p/q] \qquad (5\text{-}23)$$

So eq. (5-22) can be introduced in eq. (5-23)

$$\Phi(x,t) = \pi^{-1/4}\,q^{-1/2}\exp[\,-\,(x^2/2\hbar)\,C/qq^* +$$

$$i\,(x^2/2\hbar)(q_1p_1 + q_2p_2)\,/qq^*] \qquad (5\text{-}24)$$

and the probability density of presence is given by

$$|\Phi(x,t)|^2 = (\pi\,qq^*)^{-1/2}\exp[-(x^2/\hbar)C/qq^*] \qquad (5\text{-}25)$$

In defining the value for the constant, C, by the Planck constant

$$Inv = q_1p_2 - q_2p_1 = C = \hbar \qquad (5\text{-}26)$$

eqs. (5-24) and (5-25) become

$$\Phi(x,t) = \pi^{-1/4}\,q^{-1/2}\exp[\,-\,x^2/2qq^* +$$

$$i\,(x^2/2\hbar)\,(q_1p_1 + \quad q_2p_2)\,/qq^*] \qquad (5\text{-}27)$$

$$|\Phi(x,t)|^2 = (\pi\,qq^*)^{-1/2}\exp[-x^2/qq^*] \qquad (5\text{-}28a)$$

or

$$|\Phi(x,t)| = (\pi\,qq^*)^{-1/4}\exp[-x^2/2qq^*] \qquad (5\text{-}28b)$$

From eq. (5-28b), the phase velocity, u, is calculated

$$-mu/\hbar = d[\ln(qq^*)^{-1/4} - x^2/2qq^*]/dx = -x/qq^* \qquad (5\text{-}29)$$

or

$$u = (\hbar/m)x/qq^* = \hbar x/mqq^* \qquad (5\text{-}30)$$

From eq. (5-30), the two Quantum Potentials Q_1 and Q_2 are given by

$$Q_1 = (\hbar/2)\,\partial u/\partial x = \hbar^2/2mqq^* \qquad (5\text{-}31)$$

$$Q_2 = -\,mu^2/2 = -\,\hbar^2x^2/2m\,(qq^*)^2 \qquad (5\text{-}32)$$

and the ratio Q_2/Q_1 is

$$Q_2/Q_1 = -\,\hbar^2x^2\,2mqq^*\,/\,2m\,(qq^*)^2\,\hbar^2 = -x^2/qq^* \qquad (5\text{-}33)$$

So, we can state that the probability density of presence (5-28a) is related to the ratio of the two Quantum Potentials

$$P = |\Phi(x,t)|^2 = (\pi\,qq^*)^{-1/4}\exp[-x^2/qq^*]$$

$$= (2m\,Q_1\,/\,\hbar^2\pi)^4\exp[\,Q_2\,/\,Q_1\,] \qquad (5\text{-}34)$$

with

$$Q_1/\pi = (\hbar/2\pi)\,\partial u/\partial x = \hbar^2/2m\,\pi\,qq^* \qquad (5\text{-}35)$$

$$(\pi\,qq^*)^{-1/4} = (2m\,Q_1/\,\hbar^2\pi)^4 = [m\,(\partial u/\partial x)\,/\hbar\pi]^4 \qquad (5\text{-}36)$$

$$Q_1/Q_2 = -\,(\hbar/m)\,(\partial u/\partial x)/u^2 = (\hbar/m)\,(\partial u^{-1}/\partial x) \qquad (5\text{-}37)$$

or

$$Q_2/Q_1 = m\,/(\,\hbar\,\partial\,u^{-1}\,/\partial x) \qquad (5\text{-}38)$$

hence eq. (5-34) can be re-written as

$$P = [m\,(\partial u/\partial x)\,/\,\hbar\,\pi\,]^4\exp[\,-\,mu^2\,/\,(\hbar\,\partial u/\partial x)] =$$

$$[m\,(\partial u/\partial x)\,/\,\hbar\,\pi\,]^4\exp[m\,/(\,\hbar\,\partial\,u^{-1}\,/\partial x)] \qquad (5\text{-}39)$$

Let us now give the general solutions of eq. (5-5)

$$q_1 = a_1\cos\omega t + b_1\sin\omega t \qquad (5\text{-}40a)$$

$$p_1 = -m\omega a_1\sin\omega t + m\omega b_1\cos\omega t \qquad (5\text{-}40b)$$

$$q_2 = a_2\cos\omega t + b_2\sin\omega t \qquad (5\text{-}40c)$$

$$p_2 = -m\omega a_2\sin\omega t + m\omega b_2\cos\omega t \qquad (5\text{-}40d)$$

For determining the values of the parameters of the eqs. (5-40abcd), the eq. (5-26) of the invariant will be used:

$$Inv = q_1p_2 - q_2p_1 = \hbar \qquad (5\text{-}41)$$

The invariant is always true, also for t=0

$$q_1(0) = a_1 \qquad (5\text{-}42a)$$

$$p_1(0) = m\omega b_1 \qquad (5\text{-}42b)$$

$$q_2(0) = a_2 \qquad (5\text{-}42c)$$

$$p_2(0) = m\omega b_2 \qquad (5\text{-}42d)$$

so

$$q_1(0)\,p_2(0) - q_2(0)\,p_1(0) = a_1m\omega b_2 - a_2m\omega b_1 =$$

$$m\omega(a_1b_2 - a_2b_1) = \hbar \qquad (5\text{-}43)$$

thus

$$a_1b_2 - a_2b_1 = \hbar/m\omega \quad (5\text{-}44)$$

Let us take the following values for b_1 and a_2

$$b_1 = 0 \quad (5\text{-}45a)$$

$$a_2 = 0 \quad (5\text{-}45b)$$

so eq. (5-44) becomes

$$a_1b_2 = \hbar/m\omega \quad (5\text{-}46)$$

and

$$b_2 = \hbar/m\omega a_1 \quad (5\text{-}47)$$

Let us define qq* by $\sigma_q^2(t)$

$$\sigma_q^2(t) = qq^* = q_1^2 + q_2^2 = a_1^2\cos^2\omega t + b_2^2\sin^2\omega t \quad (5\text{-}48)$$

At time t = 0, the width

$$\sigma_q^2(0) = a_1^2 \quad (5\text{-}49)$$

In determining the parameter a_1 as

$$a_1 = \sigma_q(0) \quad (5\text{-}50)$$

eq. (5-47) of the parameter b_2 becomes

$$b_2 = \hbar/m\omega\sigma_q(0) \quad (5\text{-}51)$$

hence eqs. (5-42abcd) become

$$q_1 = \sigma_q(0)\cos\omega t \quad (5\text{-}52a)$$

$$p_1 = -m\omega\sigma_q(0)\sin\omega t \quad (5\text{-}52b)$$

$$q_2 = [\hbar/m\omega\sigma_q(0)]\sin\omega t \quad (5\text{-}52c)$$

$$p_2 = [\hbar/\sigma_q(0)]\cos\omega t \quad (5\text{-}52d)$$

and eq. (5-48) becomes

$$\sigma_q^2(t) = qq^* = q_1^2 + q_2^2 = \sigma_q^2(0)\cos^2\omega t + [\hbar/m\omega\sigma_q(0)]^2 \sin^2\omega t = \sigma_q^2(0) + [[\hbar/m\omega\sigma_q(0)]^2 - \sigma_q^2(0)]\sin^2\omega t \quad (5\text{-}53)$$

Let us define pp* by $\sigma_p^2(t)$

$$\sigma_p^2(t) = pp^* = p_1^2 + p_2^2 = m^2\omega^2\sigma_q^2(0)\sin^2\omega t + [\hbar/\sigma_q(0)]^2\cos^2\omega t \quad (5\text{-}54)$$

At time t = 0, the width

$$\sigma_p^2(0) = [\hbar/\sigma_q(0)]^2 \quad (5\text{-}55)$$

or

$$\sigma_q(0)\sigma_p(0) = \hbar \quad (5\text{-}56)$$

and, in defining

$$\Delta x^2 = \sigma_q^2(0)/2 \quad (5\text{-}57a)$$

and

$$\Delta p^2 = \sigma_p^2(0)/2 \quad (5\text{-}57b)$$

one obtains the Heisenberg Uncertainty Relation, at time t = 0:

$$\Delta x\Delta p = \hbar/2 \quad (5\text{-}58)$$

For the particular case

$$\sigma_q(0) = \hbar/m\omega\sigma_q(0) \quad (5\text{-}59)$$

let us define the constant σ_q^2 as follows

$$\sigma_q^2(0) = \sigma_q^2 = \hbar/m\omega \quad (5\text{-}60)$$

With eq. (5-48), eqs. (5-31) and (5-32) are re-written as

$$Q_1 = \hbar^2/2mqq^* = \hbar^2/2m\sigma_q^2(t) \quad (5\text{-}61)$$

$$Q_2 = -\hbar^2x^2/2m\,(qq^*)^2 = -\hbar^2x^2/2m\sigma_q^4(t) \quad (5\text{-}62)$$

For the particular case given by the eqs. (5-59) and (5-60), the eq. (5-53) of $\sigma_q^2(t)$ becomes

$$\sigma_q^2(t) = qq^* = q_1^2 + q_2^2 = \sigma_q^2(0) = \sigma_q^2 = \hbar/m\omega \quad (5\text{-}63)$$

and the eqs. (5-61) and (5-62) of the two Quantum Potentials become

$$Q_1 = \hbar^2/2m\sigma_q^2 = \hbar\omega/2 = E_0 \quad (5\text{-}64a)$$

$$Q_2 = -\hbar^2x^2/2m\sigma_q^4 = -m\omega^2x^2/2 = -V(x) \quad (5\text{-}64b)$$

where the first Quantum Potential, Q_1, is the ground state energy E_0, and the second Quantum Potential, Q_2, is the anti-potential.

Finally, let us give the action S

$$S = -(\hbar/2)\,\mathrm{atan}(q_2/q_1) + (x^2/2)(q_1p_1 + q_2p_2)/qq^* = -(\hbar/2)\,\mathrm{atan}(q_2/q_1) + (x^2/2\sigma_q^2(t))(q_1p_1 + q_2p_2) \quad (5\text{-}65)$$

where

$$q_1p_1 + q_2p_2 = -\sigma_q(0)\cos\omega t\, m\omega\sigma_q(0)\sin\omega t + [\hbar/m\omega\sigma_q(0)]\sin\omega t[\hbar/\sigma_q(0)]\cos\omega t = [-m\omega\sigma_q^2(0) + \hbar^2/m\omega\sigma_q^2(0)]\sin\omega t\cos\omega t = [-m\omega\sigma_q^2(0) + \hbar^2/m\omega\sigma_q^2(0)]\sin(2\omega t)/2 \quad (5\text{-}66)$$

So

$$S = -(\hbar/2)\,\mathrm{atan}(q_2/q_1) + x^2\,\hbar\,(\sigma_q^2/\sigma_q^2(0) - \sigma_q^2(0)/\sigma_q^2)\sin(2\omega t)/4\sigma_q^2(t) \quad (5\text{-}67)$$

From the action S, the momentum, mv, can be calculated as

$$mv = \partial S/\partial x = x\,\hbar\,(\sigma_q^2/\sigma_q^2(0) - \sigma_q^2(0)/\sigma_q^2)\,\sin(2\omega t)/2\sigma_q^2(t) \quad (5\text{-}68)$$

From the action S, the energy, E, can be calculated as

$$E = -\,\partial S/\partial t \quad (5\text{-}69)$$

So

$$E = d[(\hbar/2)\,\mathrm{atan}\,(q_2/q_1)]/dt - x^2\,\hbar\,(\sigma_q^2/\sigma_q^2(0) - \sigma_q^2(0)/\sigma_q^2)\,d[\sin(2\omega t)/4\sigma_q^2(t)]/dt$$

$$= \hbar^2/2m\sigma_q^2(t) - x^2\,\hbar\,(\sigma_q^2/\sigma_q^2(0) - \sigma_q^2(0)/\sigma_q^2)\,[(2\omega\cos(2\omega t)\sigma_q^2(t) - \sin(2\omega t)d\sigma_q^2(t)/dt)/4\sigma_q^4(t)] \quad (5\text{-}70)$$

where

$$d\sigma_q^2(t)/dt = \omega\,[[\hbar/m\omega\sigma_q(0)]^2 - \sigma_q^2(0)]\,\sin(2\omega t) \quad (5\text{-}71)$$

and, with this eq. (5-71), the energy, eq. (5-70), becomes

$$E = \hbar^2/2m\sigma_q^2(t) - x^2\hbar(\sigma_q^2/\sigma_q^2(0) - \sigma_q^2(0)/\sigma_q^2)[(2\omega\cos(2\omega t)\sigma_q^2(t) - \omega[[\hbar/m\omega\sigma_q(0)]^2 - \sigma_q^2(0)]\sin^2(2\omega t))/4\sigma_q^4(t)] \quad (5\text{-}72)$$

The expression of the energy can also be calculated from the relation

$$E = T + V + Q_1 + Q_2 = (mv)^2/2m + m\omega^2x^2/2 + Q_1 + Q_2 \quad (5\text{-}73)$$

With the momentum, mv, given by eq. (5-68), and the Quantum Potentials, Q_1 and Q_2, given by the eqs. (5-61) and (5-62), the energy eq. (5-73) becomes

$$E = (1/2m)[\,x\,\hbar\,(\sigma_q^2/\sigma_q^2(0) - \sigma_q^2(0)/\sigma_q^2)\sin(2\omega t)/2\sigma_q^2(t)]^2 + m\omega^2x^2/2 + \hbar^2/2m\sigma_q^2(t) - \hbar^2x^2/2m\sigma_q^4(t) \quad (5\text{-}74)$$

In the particular case, given by eqs. (5-59) and (5-60), the kinetic energy becomes null:

$$T = (mv)^2/2m = (1/2m)[\,x\,\hbar\,(\sigma_q^2/\sigma_q^2(0) - \sigma_q^2(0)/\sigma_q^2)\sin(2\omega t)/2\sigma_q^2(t)]^2 = 0 \quad (5\text{-}75)$$

and, with eqs. (5-64ab), the energy E, given by eq. (5-73) becomes the ground state energy of the quantum harmonic oscillator

$$E = T + V + Q_1 + Q_2 = 0 + m\omega^2x^2/2 + \hbar\omega/2 - m\omega^2x^2/2 = \hbar\omega/2 = E_0 \quad (5\text{-}76)$$

The total energy of the quantum harmonic oscillator is given by the first Quantum Potential, Q_1, and the second Quantum Potential, Q_2, is an anti-potential which counterbalances exactly the harmonic oscillator potential, V(x).

In conclusion, it is interesting to see that there are two complex harmonic oscillators in this quantum system. The two oscillators exist because there is an invariant which is given by equation (5-26)

$$\mathrm{Inv} = q_1p_2 - q_2p_1 = C = \hbar \quad (5\text{-}26)$$

in defining the value for the constant, C, by the Planck constant that is linked to the discrete level of energy.

The classical limit is obtained by $\hbar$ tending to zero, so there is only one oscillator.

In the same way, the hyperincursive harmonic oscillator with the discrete interval of time Δt is given by two discrete inversible incursive oscillators. When the interval of time Δt tends to zero, the two incursive oscillators tend to one oscillator.

6. The Solutions of the Equation of the Schrödinger Quantum Harmonic Oscillator

This section is an introduction for the next section.

Let us consider the Schrödinger equation

$$i\hbar\,\partial\Psi(x,t)/\partial t = -\,(\hbar^2/2m)\,\partial^2\Psi(x,t)/\partial x^2 + V(x)\,\Psi(x,t) \quad (6.1)$$

in one spatial dimension x for a particle in a potential $V(x) = (m\omega^2x^2/2)$, where $\Psi(x,t)$ is the wave function depending on space x and time t.

A classical method to obtain the analytical solution consists in separating the space time wave function into time and space functions as

$$\Psi(x,t) = \phi(t)\,\psi(x) \quad (6.2)$$

giving

$$(i\hbar/\phi(t))\partial\phi(t)/\partial t = -\,(\hbar^2/2m\psi(x))\,\partial^2\psi(x)/\partial x^2 + V(x) = E \quad (6.3)$$

where E is the energy. So the equation of the time wave function $\phi(t)$ is then given by

$$i\hbar\,\partial\phi(t)/\partial t = E\,\phi(t) \quad (6.4)$$

while the equation of the space wave function $\psi(x)$ is

$$-(\hbar^2/2m)\,\partial^2\psi(x)/\partial x^2 + (m\omega^2x^2/2)\,\psi(x) = E\,\psi(x) \quad (6.5)$$

the discrete values for the energy,

$$E_n = \hbar\omega\,(n + 1/2),\ n = 0, 1, 2, 3, ... \quad (6.6)$$

In introducing the dimensionless variables:

$$\tau = \omega t \text{ with } \omega = \sqrt{[k/m]},$$

and $\xi = x/x_c$ with $x_c = \sqrt{[\hbar/m\omega]}$,

and $\varepsilon = E/\hbar\omega$ (6.7a-b-c)

The normalized space wave functions are thus given by (e. g. Messiah, 1965):

$$\psi_n(\xi) = x_c^{-1/2}\,\pi^{-1/4}\,(2^n\,n!)^{-1/2}\,H_n(\xi)\exp(-\,\xi^2/2) \quad (6.8)$$

where the functions $H_n(\xi)$ are the Hermite polynomials

$$H_n(\xi) = (-1)^n \exp(\xi^2)\,d^n \exp(-\xi^2)\,/\,d\xi^n \quad (6.9)$$

Now, let us consider the equation (6.4) of the time wave function $\phi(\tau)$ with the discrete values of the energy

$$\partial\phi(\tau)/\partial\tau = -\,i\,(n + 1/2)\,\phi(\tau),\quad n = 0, 1, 2, 3, \ldots \quad (6.10)$$

The solution of this time wave function is given by

$$\phi_n(\tau) = \exp[\,-\,i\,(n + 1/2)\,\tau\,] = R_n(\tau) - i\,I_n(\tau) = \cos[(n + 1/2)\,\tau] - i\sin[(n + 1/2)\,\tau] \quad (6.11)$$

where $R_n(\tau)$ and $I_n(\tau)$ are the real and imaginary parts of the time wave function.

The probability of time presence given by

$$\phi_n(\tau)\phi_n(\tau)^* = P_n(\tau) = R_n(\tau)^2 + I_n(\tau)^2 = 1 \quad (6.12)$$

is a constant equal to unity.

The frequency of oscillations of the time wave function increases with the energy. Let us now consider the hyperincursive discrete time Schrödinger quantum oscillator.

7. Hyperincursive Discrete Time Equation of the Schrödinger Quantum Oscillator

This section is reprinted from my previous paper on the incursive and hyperincursive discrete quantum harmonic oscillators (Dubois, 2016). This is useful to see that the hyperincursive discrete equations contributes to a unification of classical and quantum mechanics.

The discrete equation of the time wave function

$$\partial\phi(\tau)/\partial\tau = -\,i\,(n + 1/2)\,\phi(\tau),\ n = 0, 1, 2, 3, \ldots \quad (7.1)$$

will begin by separating the time wave function into its real and imaginary parts

$$\phi_n(\tau) = R_n(\tau) + i\,I_n(\tau) \quad (7.2)$$

So this complex equation $\partial[R_n(\tau) + i\,I_n(\tau)]/\partial\tau = -\,i\,(n + 1/2)\,[R_n(\tau) + i\,I_n(\tau)]$ is separable to the two real coupled equations

$$\partial R_n(\tau)/\partial\tau = (2n + 1)\,I_n(\tau)$$

$$\partial I_n(\tau)/\partial\tau = -\,(2n + 1)\,R_n(\tau) \quad (7.3a\text{-}b)$$

Let us introduce a new dimensionless time variable scaling on the dimensionless energy:

$$\tau_n = (n + 1/2)\,\tau = (n + 1/2)\,\omega t \quad (7.4)$$

The system of equations becomes

$$\partial R_n(\tau_n)/\partial\tau_n = I_n(\tau_n)$$

$$\partial I_n(\tau_n)/\partial\tau_n = -\,R_n(\tau_n) \quad (7.5a\text{-}b)$$

For the discretization, the interval of time $\Delta\tau_n$ will be given by h_n as

$$\Delta\tau_n = (n + 1/2)\,\omega\,\Delta t = (n + 1/2)\,\omega\,h = h_n \quad (7.6)$$

The dimensionless discrete time is defined as:

$$\tau_k = \tau_0 + k\,\Delta\tau \text{ with } k = 0,1,2,... \quad (7.7)$$

where τ_0 is the initial value of the time and k is the counter of the number of interval of time.

The first incursive harmonic quantum oscillator is given by

$$R_{n1}(k + 1) = R_{n1}(k) + h_n\,I_{n1}(k)$$

$$I_{n1}(k + 1) = I_{n1}(k) - h_n\,R_{n1}(k + 1) \quad (7.8a\text{-}b)$$

the second incursive quantum harmonic oscillator by

$$I_{n2}(k + 1) = I_{n2}(k) - h_n\,R_{n2}(k)$$

$$R_{n2}(k + 1) = R_{n2}(k) + h_n\,I_{n2}(k + 1) \quad (7.9a\text{-}b)$$

and the hyperincursive quantum harmonic oscillator is given by

$$R_n(k + 1) = R_n(k - 1) + 2\,h_n\,I_n(k)$$

$$I_n(k+1) = I_n(k-1) - 2\, h_n\, R_n(k) \quad (7.10a\text{-}b)$$

The hyperincursive oscillator is separable into two incursive oscillators with, for each of which, two boundary conditions:

$$R_n(0) = C_1 \text{ and } I_n(1) = C_2 \quad (7.11a\text{-}b)$$

$$I_n(0) = C_3 \text{ and } R_n(1) = C_4 \quad (7.11c\text{-}d)$$

Indeed, let us give the 4 first iterations k = 1, 2, 3, 4, of the hyperincursive oscillator (7.10a-b), as follows:

$$(k=1)\ \mathbf{R_n(2)} = \mathbf{R_n(0)} + 2\, h_n\, \mathbf{I_n(1)}$$

$$(k=1)\ I_n(2) = I_n(0) - 2\, h_n\, R_n(1)$$

$$(k=2)\ R_n(3) = R_n(1) + 2\, h_n\, I_n(2)$$

$$(k=2)\ \mathbf{I_n(3)} = \mathbf{I_n(1)} - 2\, h_n\, \mathbf{R_n(2)}$$

$$(k=3)\ \mathbf{R_n(4)} = \mathbf{R_n(2)} + 2\, h_n\, \mathbf{I_n(3)}$$

$$(k=3)\ I_n(4) = I_n(2) - 2\, h_n\, R_n(3)$$

$$(k=4)\ R_n(5) = R_n(3) + 2\, h_n\, I_n(4)$$

$$(k=4)\ \mathbf{I_n(5)} = \mathbf{I_n(3)} - 2\, h_n\, \mathbf{R_n(4)} \quad (7.12)$$

The first incursive oscillator is obtained with its two boundary conditions, $\mathbf{R_n(0) = C_1}$ and $\mathbf{I_n(1) = C_2}$, as follows

$$(k=1)\ \mathbf{R_n(2)} = \mathbf{R_n(0)} + 2\, h_n\, \mathbf{I_n(1)}$$

$$(k=2)\ \mathbf{I_n(3)} = \mathbf{I_n(1)} - 2\, h_n\, \mathbf{R_n(2)}$$

$$(k=3)\ \mathbf{R_n(4)} = \mathbf{R_n(2)} + 2\, h_n\, \mathbf{I_n(3)}$$

$$(k=4)\ \mathbf{I_n(5)} = \mathbf{I_n(3)} - 2\, h_n\, \mathbf{R_n(4)} \quad (7.13)$$

The second incursive oscillator is obtained with its two other boundary conditions, $I_n(0) = C_3$ and $R_n(1) = C_4$, as follows

$$(k=1)\ I_n(2) = I_n(0) - 2\, h_n\, R_n(1)$$

$$(k=2)\ R_n(3) = R_n(1) + 2\, h_n\, I_n(2)$$

$$(k=3)\ I_n(4) = I_n(2) - 2\, h_n\, R_n(3)$$

$$(k=4)\ R_n(5) = R_n(3) + 2\, h_n\, I_n(4) \quad (7.14)$$

For the first incursive oscillator, the boundary condition of the real part of the wave function, $R_n(0) = C_1$, is defined at time k = 0, while the boundary condition of the imaginary part of the wave function, $I_n(1) = C_2$, is defined at the different time k = 1.

For the second incursive oscillator, the boundary condition of the imaginary part of the wave function, $I_n(0) = C_3$, is defined at time k = 0, while the boundary condition of the real part of the wave function, $R_n(1) = C_4$, is defined at the different time k = 1.

The oscillator is hyperincursive because it is the superposition of two separable and independent incursive oscillators.

The two incursive oscillators are non-recursive implicit algorithms, and the order in which the iterations are made is important for incursive oscillators: the first iteration of the first incursive oscillator deals with the real part of the wave function while the first iteration of the second incursive oscillators deals with the imaginary part of the wave function.

The two independent incursive oscillators are linked by the probability of presence:

$$\phi_n(\tau)\phi_n(\tau)^* = P_n(k) = R_n(k)^2 + I_n(k)^2 = 1 \quad (7.15)$$

So, for k = 0,

$$R_n(0)^2 + I_n(0)^2 = C_1^2 + C_3^2 = 1 \quad (7.16a)$$

and for k = 1,

$$R_n(1)^2 + I_n(1)^2 = C_4^2 + C_2^2 = 1 \quad (7.16b)$$

This gives two conditions for determining the boundary conditions. So, there are only two boundary conditions to be fixed.

Each incursive oscillator is the time reverse of the other incursive oscillator, defined by time forward and time backward derivatives.

So the two incursive oscillators are not reversible.

But the superposition of the two incursive oscillators given by the hyperincursive oscillator is reversible.

The theoretical values of the real part R(k) and imaginary part I(k) of the time wave function of the quantum harmonic oscillator are given by:

$$R_k = \cos(2k\pi/N)$$

$$I_k = -\sin(2k\pi/N) \quad (7.17a\text{-}b)$$

where N is the number of iterates for a cycle of the oscillator.

The values of the hyperincursive oscillator represent alternatively the values of the two incursive oscillators. The dimensionless space wave function of the harmonic oscillator is explained in the previous paper (Dubois, 2016).

8. Survey of the Klein-Gordon and Dirac Quantum Relativist Equations

This section deals with the survey of the Klein-Gordon and Dirac quantum relativist equations.

The next section will develop the hyperincursive discrete Klein-Gordon equation which bifurcates to the 4 first order Dirac equations.

The quantum relativist Klein-Gordon equation for spin-zero bosons is given by (e. g. Messiah, 1965)

$$-\hbar^2\partial^2\Psi/\partial t^2 = -\hbar^2 c^2\nabla^2\Psi + m^2c^4\Psi \quad (8.1)$$

where $\Psi = \Psi(\mathbf{r},t)$ is the wave function depending on the time t and the space $\mathbf{r}$ = (x, y, z), with the space derivative nabla operator ∇, $\hbar$ is the Planck constant, c the light speed, and m the rest mass. This equation is second order in space and time and was deduced from forward-backward space-time shifts (Dubois, 2000).

Dirac proposed his first order equations in time and space for spin 1/2 fermion (e. g. Messiah, 1965) given by

$$\gamma_0\partial\Psi/\partial t = \gamma_1\partial\Psi/\partial x + \gamma_2\partial\Psi/\partial y + \gamma_3\partial\Psi/\partial z + (mc^2/i\hbar)\Psi \quad (8.2)$$

where m is the mass, c the speed of light, i the imaginary number, $\hbar$ the constant of Planck, and the 4 γ_0, γ_1, γ_2, γ_3 matrices. The complex wave function $\Psi(\mathbf{r}, t) = \Psi$ is represented by

$$\Psi = \begin{pmatrix}\varphi_a\\ \varphi_b\end{pmatrix} \quad (8.3)$$

where the complex wave functions φ_a and φ_b are represented by

$$\varphi_a = \begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} \quad (8.4a)$$

$$\varphi_b = \begin{pmatrix}\Psi_3\\ \Psi_4\end{pmatrix} \quad (8.4b)$$

defining 4 complex wave functions Ψ_1, Ψ_2, Ψ_3, Ψ_4 to which correspond the 4 matrices γ_i, = 0, 1, 2, 3, satisfying the relations

$$\gamma_0^2 = 1, \quad (8.5a)$$

$$\gamma_1^2 = \gamma_2^2 = \gamma_3^2 = -1, \quad (8.5b)$$

$$\gamma_i\gamma_j = -\gamma_j\gamma_i \quad (8.5c)$$

for all i, j = 0, 1, 2, 3 with $i \neq j$.

The Dirac matrices are given by

$$\gamma_0 = \begin{pmatrix}0 & I\\ I & 0\end{pmatrix}$$

$$\gamma_1 = \begin{pmatrix}0 & \sigma_x\\ -\sigma_x & 0\end{pmatrix}$$

$$\gamma_2 = \begin{pmatrix}0 & \sigma_y\\ -\sigma_y & 0\end{pmatrix}$$

$$\gamma_3 = \begin{pmatrix}0 & \sigma_z\\ -\sigma_z & 0\end{pmatrix} \quad (8.6a\text{-}b\text{-}c\text{-}d)$$

where the identity matrix is given by

$$I = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \quad (8.7)$$

and the Pauli matrices are given by

$$\sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad (8.8a)$$

$$\sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \quad (8.8b)$$

$$\sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \quad (8.8c)$$

These are called spin matrices because they are characterized by the relations

$$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = I \quad (8.9)$$

$$\sigma_x\sigma_y = -\sigma_y\sigma_x = i\sigma_z \quad (8.10a)$$

$$\sigma_y\sigma_z = -\sigma_z\sigma_y = i\sigma_x \quad (8.10b)$$

$$\sigma_z\sigma_x = -\sigma_x\sigma_z = i\sigma_y \quad (8.10c)$$

which are analogous to the relations of the components of angular momentum of a classical particle.

Let us demonstrate that the hyperincursive discrete Klein-Gordon equation corresponds to the Dirac first order space time equations.

9. The Hyperincursive Discrete Klein-Gordon Second Order Equation Bifurcates to the 4 Dirac First Order Equations

In this section, we will consider the case of the Klein-Gordon and Dirac equations in one spatial dimension z, for simplifying the demonstration of the applicability of the hyperincursive discrete field to the Klein-Gordon and Dirac quantum relativist equations. Extensions to the three spatial dimensions will be proposed in a next paper.

So, the Klein-Gordon equation (8.1), with the complex wave function $\varphi = (\varphi(z,t)$ in space z and time t is given by the following second order equation

$$-\hbar^2\partial^2\varphi/\partial t^2 = -\hbar^2 c^2\partial^2\varphi/\partial t^2 + m^2c^4\varphi \quad (9.1)$$

With the change of variables

$$q(z,t) = \varphi(z,t) \quad (9.2)$$

$$a = \omega = mc^2/\hbar \quad (9.3)$$

equation (9.1) becomes

$$\partial^2 q(z,t)/\partial t^2 = c^2\partial^2 q(z,t)/\partial t^2 - a^2 q(z,t) \quad (9.4)$$

In using the hyperincursive discrete difference equations for second order time and space derivatives, we obtain the hyperincursive discrete second order Klein-Gordon equation

$$q(z,t+2\Delta t) - 2q(z,t) + q(z,t-2\Delta t) =$$
$$+\left[\frac{4c^2\Delta t^2}{4\Delta z^2}\right][q(z+2\Delta z,t) - 2q(z,t) + q(z-2\Delta z,t)]$$
$$-4a\Delta t^2 q(z,t) \quad (9.5)$$

where Δt and Δz are the discrete interval of time and space respectively.

The discrete time is defined as

$$t_k = t_0 + k\Delta t, k = 0,1,2,\ldots \quad (9.6a)$$

where t_0 is the initial value of the time and k is the counter of the number of interval of time.

The discrete space is defined as:

$$z_k = z_0 + j\Delta z, j = 0,1,2,\ldots \quad (9.6b)$$

where x_0 is the initial value of the space and j is the counter of the number of interval of space.

With the change of variables

$$B = \pm\, 2c\Delta t/2\Delta z \quad (9.7)$$

and

$$A = \pm\, 2a\Delta t \quad (9.8)$$

the hyperincursive discrete second order Klein-Gordon equation becomes

$$q(j,k+2) - 2q(j,k) + q(j,k-2) =$$
$$\mathrm{B}B[q(j+2,k) - 2q(j,k) + q(j-2,k)] - AAq(j,k)$$
$$(9.9)$$

This hyperincursive discrete equation bifurcates into 2 equations for the discrete time $\pm\Delta t$ which also bifurcate into 2 equations for the discrete space $\pm\Delta z$, as shown in Table 4.

TABLE 4

The hyperincursive discrete wavefunction q bifurcates into four discrete wavefunctions q_1, q_2, q_3, and q_4, where j is the index of discrete space z, and k the index of discrete time *t*.

	j	0	1	2	3	4	5	6	7
k									
0									
1					q1	q4			
2					q2	q3			
3			q1	q4	q1	q4	q1	q4	
4			q2	q3	q2	q3	q2	q3	
5					q1	q4			
6					q2	q3			
7									

There are multiple possibilities to bifurcate into 4 equations. In this paper, we will give the most obvious bifurcation, based on the bifurcation of the hyperincursive oscillators shown in the previous sections.

Thus, let us consider the following hyperincursive bifurcation into the 4 first order in space and time complex discrete equations

$$q_1(j,k+1) = q_1(j,k-1)$$
$$+B[q_1(j+1,k) - q_1(j-1,k)] - Aq_2(j,k),$$
$$j = 1,3,5,\ldots, k = 1,3,5,\ldots \quad (9.10)$$
$$q_2(j,k+1) = q_2(j,k-1)$$
$$-B[q_2(j+1,k) - q_2(j-1,k)] + Aq_1(j,k),$$
$$j = 1,3,5,\ldots, k = 0,2,4,\ldots \quad (9.11)$$

In these two equations, q_1 and q_2 depends of each other and in replacing q_2 from the first equation (9.10) to the second equation (9.11), the resulting equation of q_1 is exactly the second order in space and time discrete Klein-Gordon equation.

The following two equations q_3 and q_4 are given by

$$q_3(j,k+1) = q_3(j,k-1)$$
$$+B[q_3(j+1,k) - q_3(j-1,k)] + Aq_4(j,k),$$
$$j = 0,2,4,\ldots, k = 0,2,4,\ldots \quad (9.12)$$
$$q_4(j,k+1) = q_4(j,k-1)$$
$$-B[q_4(j+1,k) - q_4(j-1,k)] - Aq_3(j,k),$$
$$j = 0,2,4,\ldots, k = 1,3,5,\ldots \quad (9.13)$$

In these two equations, q_3 and q_4 depends of each other and in replacing q_4 from the first equation (9.12) to the second equation (9.13), the resulting equation of q_3 is exactly the second order in space and time discrete Klein-Gordon equation.

Let us remark that the first two equations are independent of the second two equations. This is similar to the hyperincursive discrete harmonic oscillators given by two independent separable incursive oscillators.

For example:

Bifurcation of $+\Delta z$ and $-\Delta t$

$$q_1(1,2) = q_1(1,0)$$
$$+B[q_1(2,1) - q_1(0,k)] - Aq_2(1,1),$$

$$j = 1, k = 1 \qquad (9.10a)$$

Bifurcation of +Δz +Δt

$$q_2(1,3) = q_2(1,1)$$
$$- B[q_2(2,2) - q_2(0,2)] + Aq_1(1,2),$$
$$j = 1, k = 2 \qquad (9.11a)$$

Table 5A gives the positions of the discrete variables q_1 and q_2 in the discrete space and time.

TABLE 5A

Space j and time k position of q_1 and q_2, for equations (9.10a) and (9.11a).

	j	0	1	2	3
k					
0			**q1(1,0)**		
1		**q1(0,1)**	**q2(1,1)**	**q1(2,1)**	
2		q2(0,2)	**q1(1,2)**	q2(2,2)	
3			q2(1,3)		
4					

Bifurcation of −Δz, +Δt

$$q_3(2,3) = q_3(2,1)$$
$$+ B[q_3(3,2) - q_3(1,2)] + Aq_4(2,2),$$
$$j = 2, k = 2 \qquad (9.12a)$$

Bifurcation of −Δz, −Δt

$$q_4(2,2) = q_4(2,0)$$
$$- B[q_4(3,1) - q_4(1,1)] - Aq_3(2,1),$$
$$j = 2, k = 1 \qquad (9.13a)$$

Table 5B gives the positions of the discrete variables q_3 and q_4 in the discrete space and time.

TABLE 5B

Space j and time k position of q_3 and q_4, for equations (9.12a) and (9.13a).

	j	0	1	2	3
k					
0				**q4(2,0)**	
1			**q4(1,1)**	**q3(2,1)**	**q4(3,1)**
2			q3(1,2)	**q4(2,2)**	q3(3,2)
3				q3(2,3)	
4					

The hyperincursive discrete equations (9.10), (9.11), (9.12) and (9.13) are computational difference equations that are inversible for time and space as shown in Table 6.

TABLE 6

The hyperincursive discrete equations equations (9.10), (9.11), (9.12) and (9.13) are inversible of each other in time and space.

	Δz	+Δz	−Δz
Δt			
+ Δt		**q2**	**q3**
− Δt		**q1**	**q4**

Let us now transform the hyperincursive discrete equations (9.10), (9.11), (9.12) and (9.13) to differential equations at the limit when Δx and Δt tend to zero, in view of demonstrating that the the hyperincursive discrete equations represent the original Dirac quantum relativist first order equations.

The two hyperincursive discrete equations (9.10) and (9.11), tend to the first order continuous differential equations

$$+ \partial \Psi_1(z,t)/\partial t = + c\, \partial \Psi_1(z,t)/\partial z - (mc^2/\hbar)\, \Psi_2(z,t)$$

$$+ \partial \Psi_2(z,t)/\partial t = - c\, \partial \Psi_2(z,t)/\partial z + (mc^2/\hbar)\, \Psi_1(z,t)$$

(9.14a-b)

with the continuous functions

$$\Psi_1(z,t) = \lim_{\Delta z \to 0\, \Delta t \to 0} q_1(z,t)$$
$$\Psi_2(z,t) = \lim_{\Delta z \to 0\, \Delta t \to 0} q_2(z,t) \qquad (9.15a\text{-}b)$$

With the Pauli matrices

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (9.16a)$$

and

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad (9.16b)$$

In defining

$$\varphi_a = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} \qquad (9.17a)$$

the equations (9.14a-b) transformed to

$$+\ \partial\varphi_a/\partial t = +\ c\ \sigma_z\ \partial\varphi_a/\partial z - i\ \sigma_y\ (mc^2/\hbar)\ \varphi_a \quad (9.18a)$$

The two hyperincursive discrete equations (9.10) and (9.11), tend to the first order continuous differential equations

$$+\ \partial\ \Psi_3(z,t)/\partial t = +\ c\ \partial\ \Psi_3(z,t)/\partial z + (mc^2/\hbar)\ \Psi_4(z,t)$$

$$+\ \partial\ \Psi_4(z,t)/\partial t = -\ c\ \partial\ \Psi_4(z,t)/\partial z - (mc^2/\hbar)\ \Psi_3(z,t)$$

(9.14c-d)

with the continuous functions

$$\Psi_3(z,t) = \lim_{\Delta z\to 0\ \Delta t\to 0} q_3(z,t)$$

$$\Psi_4(z,t) = \lim_{\Delta z\to 0\ \Delta t\to 0} q_4(z,t) \quad (9.15c\text{-}d)$$

It is interesting to note that Ord and Mann (2003) deduced the same equations (9.14a-b-c-d), from a stochastic model but with real density variables, our variables being complex wave functions.

In defining

$$\varphi_b = \begin{pmatrix}\Psi_3\\ \Psi_4\end{pmatrix} \quad (9.17b)$$

the equations (9.14c-d) transformed to

$$+\ \partial\varphi_b/\partial t = +\ c\ \sigma_z\ \partial\varphi_b/\partial z + i\ \sigma_y\ (mc^2/\hbar)\ \varphi_b \quad (9.18b)$$

With the two Dirac matrices

$$\alpha_z = -\begin{pmatrix}\sigma_z & 0\\ 0 & \sigma_z\end{pmatrix} \quad (9.19a)$$

$$\beta = \begin{pmatrix}\sigma_y & 0\\ 0 & -\sigma_y\end{pmatrix} \quad (9.19b)$$

and in defining

$$\Psi = \begin{pmatrix}\varphi_a\\ \varphi_b\end{pmatrix} = \begin{pmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\\ \Psi_4\end{pmatrix} \quad (9.17c)$$

the two equations (9.18a) and (9.18b), can be regrouped to the following equation

$$+\ \partial\ \Psi/\partial t = -\ c\ \alpha_z\ \partial\ \Psi/\partial z - i\ \beta\ (mc^2/\hbar)\ \Psi \quad (9.20a)$$

In multiplying equation (9.20a) by i $\hbar$, one obtains the equation

$$+\ i\ \hbar\partial\ \Psi/\partial t = -\ c\ \alpha_z\ i\ \hbar\partial\ \Psi/\partial z + \beta\ mc^2\ \Psi \quad (9.20b)$$

And finally, let us introduce the quantum moment p_z

$$p_z = -\ i\ \hbar\ \partial/\partial z \quad (9.21)$$

in the equation (9.20b), hence we obtain the equation

$$+\ i\ \hbar\partial\ \Psi/\partial t = +\ c\ \alpha_z\ p_z\ \partial\ \Psi/\partial z + \beta\ mc^2\ \Psi \quad (9.22)$$

which is the original Dirac quantum relativist equation.

In conclusion, the hyperincursive second order discrete Klein-Gordon quantum equation bifurcates into the Dirac 4 first order equations that is represented by the original Dirac relativist equation.

10. Conclusion

A big challenge is to find a unified method to study classical, quantum and relativist field mechanics. A common characteristic of many systems deals with their representation with continuous second order differential equations that are theoretically reversible.

After the recall of the theory of incursive discrete harmonic oscillator, it is shown the hyperincursive discrete harmonic oscillator is separable into two incursive oscillators.

It is shown that any differential continuous derivative bifurcates into two difference discrete derivatives.

For second order differential equations, a generalized discrete derivative is presented, that can become complex, defining so a complex velocity.

The hyperincursive discrete time equation of the Schrödinger quantum oscillator is recalled, to show that the hyperincursive discrete equations contribute to a unification of classical and quantum mechanics.

Finally, we develop the theoretical presentation of the hyperincursive discrete equation of the Klein-Gordon differential second order equation which bifurcates to 4 first order discrete equations that gives the original Dirac quantum relativist equation.

This is a remarkable result because the Dirac equation is rediscovered from this new method based on the hyperincursive discrete second order equation which bifurcates to first order equations.

We think that this paper demonstrates that the method of the bifurcation of hyperincursive discrete equations is rather general to call it a unified discrete mechanics.

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Fundaments of Ontological-Phase Topological Field Theory

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We posit an Ontological-Phase Topological Field Theory (OPTFT) as the descriptive formalism for imminent 3rd regime of Unified Field Mechanics (UFM). If the author knows one thing for sure, it is that gravity is not quantized! The physics community is so invested in quantizing the gravitational force that it could still be years away from this inevitable conclusion. There is still a serious conundrum to be dealt with however; discovery of the complex Manifold of Uncertainty (MOU), the associated 'semi-quantum limit' and the fact of a duality between Newton's and Einstein's gravity, may allow some sort of wave-particle-like duality with a quantal-like virtual graviton in the semi-quantum limit. Why mention the gravitational field? Relativistic information processing (RIP) introduces gravitational effects in the 'parallel transport' aspects of topological switching in branes. There are A and B type topological string theories, and a related Topological M-Theory with mirror symmetry, that are somewhat interesting especially since they allow sufficient dimensionality with Calabi-Yau mirror symmetry perceived as essential elements for developing a UFM. But a distinction between these theories and the ontology of an energyless topological switching of information (Shannon related) through topological charge in brane dynamics, perhaps defined in a manner making correspondence to a higher dimensional (HD) de-Broglie-Bohm super-quantum potential synonymous with a 'Force of coherence' of the unified field is of interest. Thus the term 'OPTFT' has been chosen to address this issue as best as the *Zeitgeist* is able to conceive at the time of writing…

Keywords: Ontological phase, Topological field theory

It is possible to make 'intelligent guesses and conjectures – Atiyah [2].

Not all who wander are lost. - J. R. R. Tolkien

Reality leaves a lot to the imagination. - John Lennon

And those who were seen dancing were thought to be insane by those who could not hear the music
– Friedrich Nietzche.

1. Abductive *a Priori a Posteriori* Tautology

This is a challenging chapter for the author, the conception of which wasn't even in the list of topics when the book was first conceived in 2014; and not knowing sufficient Group Theory limits current enfolding. I didn't suspect there would be much to say about Anyon – quasi-particle – quantum Hall TQC [3] because it was perceived as an LD 'Toy Model' of the HD UFM UQC architecture proposed to take its place. My expertise at the time on TQFT and TQC was sparse such that quite a can of worms was opened into my world view in bringing myself sufficiently up to speed with study and tad of tutoring given graciously by a world-renowned topologist.

The necessity of r-qubits (relativistic qubits) had already been embraced since first hearing of them at Physcomp96 [4]; and again in the course of getting up to speed, discovered that a corner of the QC R&D community finally began a discussion of their utility for modeling relativistic quantum computing (RIP) with a version of r-qubits [5-7].

I felt that attempting to develop a relativistic-TQFT was not a correct nomenclatural framework for both mathematical and physical reasons. Most acutely that the universe is not fundamentally quantum (anymore) and that gravitation, unlike the other three known phenomenological fields, is not quantized. The hearty belief in a quantum gravity persists only because of a herd mentality confounded by the current belief that fundamental reality is indeed quantum.

Most likely, the imminent age of discovery will be described topologically. Field theory has evolved from classical field theory to the current 2nd regime modes of QFT, RQFT and TQFT. It is proposed that the 3rd regime of reality, Unified Field Mechanics (UFM) will be described by an Ontological-Phase Topological Field Theory (OPTFT). In terms of the nature of reality, quantum information processing and the measurement problem, there has been a recent introduction of relativistic parameters including relativistic r-qubits and not just an Amplituhedron but more saliently a dual-Amplituhedron replacing spacetime, all bringing into question the historically fundamental basis of and need to be restricted to 'locality and unitarity'.

We briefly review this dilemma in terms of Bell's inequalities, the no-cloning theorem and discuss correspondence to the epistemic view of the Copenhagen Interpretation versus the ontic consideration of objective realism and as merged by W. Zurek's epi-ontic blend of quantum redundancy in quantum Darwinism [8-10]. Finally, we delve into the UFM ontological-phase topology requiring a new set of topological transformations beyond the Galilean, Lorentz-Poincairé.

A radical paradigm shift is needed to incorporate the new 3rd regime of Unified Field Mechanics (UFM), which appears to be inherently topological, suggesting extensions of current theory are required. If I was M. Atiya's clone, I would write a seminal introduction to an extended topological field theory as he did in 1986 [11]. UFM does not imply a 5th force, is not quantized, but entails an ontological mediation of information by a 'force of coherence' transferring information (by a form of topological charge) in a Shannon sense in the geometric topology of branes. This process, as we continue to mention, is an energyless process called 'topological switching' utilizing 'topological charge' [12-14].

2. The Phasor (Phase Vector) Complex Probability Amplitude

As the first step in trying to figure out how to develop a new concept of Ontological-phase we wish to adapt the phasor or phase vector concept as a precursor for describing ontological topological phase. In general, a phasor is a complex number for a sinusoidal (π rotation) function with Amplitude A, angular frequency ω and initial phase θ, which are all time invariant. The complex constant is the phasor [15].

Euler's formula allows sinusoids to be represented as the sum of two complex-valued functions:

$$A\cdot\cos(\omega t+\theta)=A\cdot\frac{e^{i(\omega t+\theta)}+e^{-i(\omega t+\theta)}}{2}, \tag{1}$$

or as the real part of one of the functions:

$$\begin{aligned}A\cdot\cos(\omega t+\theta)&=\mathrm{Re}\left\{A\cdot e^{i(\omega t+\theta)}\right\}\\&=\mathrm{Re}\left\{Ae^{i\theta}\cdot e^{i\omega t}\right\}.\end{aligned} \tag{2}$$

The function $A\cdot e^{i(\omega t+\theta)}$ is the analytic representation of $A\cdot\cos(\omega t+\theta)$. Multiplication of the phasor $Ae^{i\theta}e^{i\omega t}$ by a complex constant, $Be^{i\phi}$, produces another phasor that changes the amplitude and phase of the underlying sinusoid**:**

$$\begin{aligned}\mathrm{Re}\left\{\left(Ae^{i\theta}\cdot Be^{i\phi}\right)\cdot e^{i\omega t}\right\}&=\mathrm{Re}\left\{\left(ABe^{i(\theta+\phi)}\right)\cdot e^{i\omega t}\right\}\\&=AB\cos\left(\omega t+(\theta+\phi)\right).\end{aligned} \tag{3}$$

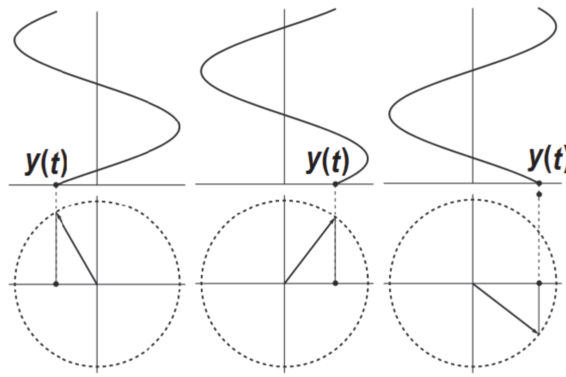

Fig. 1. Top sine waves - Phase transform in the complex plane. Bottom, can also be thought of as 2D rotation of the reference circle, and 1D sliding point on the line segment, helping us ponder the 2D nature of anyon braid topology. Thus elements of the figure can be considered in 1D, 2D and 3D.

When function $A \cdot e^{i(\omega t+\theta)}$ is depicted in the complex plane (Fig. 1), the vector formed by the imaginary and real parts rotates around the origin. *A* is the magnitude, *i* is the imaginary unit $i^2 = -1$, one cycle is completed every $2\pi / \omega$ seconds, and θ is the angle formed with the real axis at $t = n \cdot 2\pi / \omega,$ for integer values of *n* [16].

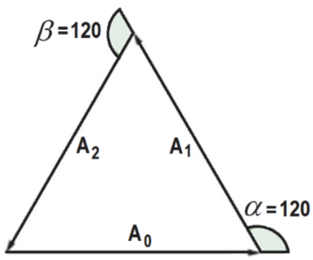

Fig. 2. Phasor diagram of three waves in perfect destructive interference.

This type of addition occurs when sinusoids interfere with each other constructively or destructively. Three identical sinusoids with a specific phase difference between them may perfectly cancel. To illustrate, we take three vectors of equal length placed head to tail so that the last head matches up with the first tail forming an equilateral triangle with the angle between each phasor being 120° (2π/3 radians), or one third of a wavelength $\lambda / 3$. Thus the phase difference between each wave is 120°,

$$\cos(\omega t) + \cos(\omega t + 2\pi / 3) + \cos(\omega t - 2\pi / 3) = 0. \quad (4)$$

In the example of three waves, the phase difference between the first and the last wave is 240º, In the limit of many waves, the phasors must form a circle for destructive interference, so that the first phasor is nearly parallel with the last. This means that for many sources, destructive interference happens when the first and last wave differ by 360º, a full wavelength, λ [16].

2.1. *Complex Phase Factor*

For any complex number written in polar form, such as $re^{i\theta}$, the phase factor is the complex exponential factor, $e^{i\theta}$. As such, the term 'phase factor' is related more generally to the term phasor, which may have any magnitude (i.e., not necessarily part of the circle group). The phase factor is a unit complex number of absolute value 1 as commonly used in quantum mechanics.

The variable θ is usually referred to as the phase. Multiplying the equation for a plane wave $A^{ei(k \cdot r - \omega t)}$ by a phase factor shifts the phase of the wave by θ:

$$e^{i\theta} A e^{i(k \cdot r - \omega t)} = A e^{i(k \cdot r - \omega t + \theta)}. \quad (5)$$

In quantum mechanics, a phase factor is a complex coefficient $e^{i\theta}$ that multiplies a ket $|\psi\rangle$ or bra $\langle\phi|$. It does not, in itself, have any physical meaning in the standard formulation of QM, since the introduction of a phase factor does not change the expectation values of a Hermitian operator. That is, the values of $\langle\phi|A|\phi\rangle$ and $\langle\phi|e^{-i\theta} A e^{i\theta}|\phi\rangle$ are the same [17].

However, *differences* in phase factors between two interacting quantum states can be measurable under certain conditions such as in Berry phase, which has important consequences. The argument for a complex number *z* = *x* + *iy*, denoted arg z, is defined as:

- Geometrically, in the complex plane, as the angle φ from the positive real axis to the vector representing *z*. The numeric value given by the angle in radians is positive if measured counterclockwise.
- Algebraically, the argument is defined as any real quantity φ such that $z = r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$ for some positive real *r* (Euler's formula). The quantity *r* is the *modulus* of *z*, as $|z|$: $r = \sqrt{x^2 + y^2}$.

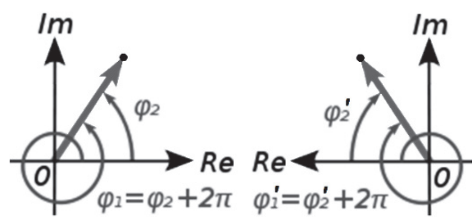

Fig. 3. Left-Right phase argument.

Use of the terms *amplitude* for the modulus and *phase* for the argument are sometimes used equivalently. Under both definitions, it can be seen that the argument of any (non-zero) complex number has many possible values: firstly, as a geometrical angle, whole circle rotations do not change the point, so angles differing by an integer multiple of 2π radians are the same. Similarly, from the periodicity of sin and cos, the second definition also has this property.

An N-particle system can be represented in non-relativistic quantum mechanics by a wavefunction, $\psi(x_1, x_2, \ldots x_n)$, where each x_i is a point in 3D space. A classical phase space contains a real-valued function in 6N dimensions (each particle contributes 3-spatial coordinates and 3-momenta. Quantum phase space involves a complex-valued function on a 3N dimensional space. Position and momenta are represented by operators that do not commute, and ψ lives in the mathematical structure of a Hilbert space. Aside from these differences, the analogy holds.

In physics, this sort of addition occurs when sinusoids interfere with each other, constructively or destructively. The static vector concept provides useful insight into questions like: What phase difference would be required between three identical sinusoids for perfect cancellation? In this case, simply imagine taking three vectors of equal length and placing them head to tail such that the last head matches up with the first tail. Clearly, the shape which satisfies these conditions is an equilateral triangle, so the angle between each phasor to the next is 120° ($2\pi/3$ radians), or one third of a wavelength $\lambda/3$. So the phase difference between each wave must also be 120°. In other words, what this shows is: $\cos(\omega t) + \cos(\omega t + 2\pi/3) + \cos(\omega t - 2\pi/3) = 0.$

2.2. *Geometric Phase - Berry Phase*

A Berry phase difference acquired over the course of a cycle, when a system is subjected to cyclic adiabatic processes resulting from the geometrical properties of the parameter space of the Hamiltonian [18]. This phenomenon was first discovered in 1956, [19] and rediscovered in 1984 [20]. It can be seen in the Aharonov-Bohm effect and in the conical intersection.

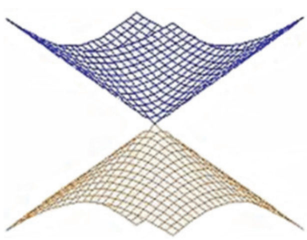

Fig. 4. Conical intersection of two potential energy surfaces.

A conical intersection of two potential energy surfaces is the set of geometrical points where the two potential energy surfaces are degenerate (intersect) and the non-adiabatic couplings between these two states are non-vanishing. For the Aharonov–Bohm effect, the adiabatic parameter is the magnetic field enclosed by two interference paths, and is cyclic because the two paths form a loop. For a conical intersection, the adiabatic parameters are molecular coordinates. In addition to quantum mechanics it can occur whenever there are at least two parameters describing a wave in the vicinity of a singularity or topological hole.

In a quantum system at the n^{th} eigenstate, if adiabatic (adapts to gradually changing external conditions; but for rapidly varying conditions there is insufficient time, so the spatial probability density remains unchanged) evolution of the Hamiltonian evolves the system such that it remains in the n^{th} eigenstate, while also obtaining a phase factor. The phase obtained has a contribution from the state's time evolution and another from the variation of the eigenstate with the changing Hamiltonian.

The second term corresponds to the Berry phase which for non-cyclical variations of the Hamiltonian can be made to vanish by a different choice of the phase associated with the eigenstates of the Hamiltonian at each point in the evolution. However, if the variation is cyclical, the Berry phase cannot be cancelled, it is invariant and becomes an observable property of the system. From the Schrödinger equation the Berry phase γ is:

$$\gamma[C] = i\oint_C \langle n,t | (\nabla_R | n,t \rangle) dR \tag{6}$$

where R parametrizes the cyclic adiabatic process. It follows a closed path C in the appropriate parameter space. Geometric phase along the closed path C can also be calculated by integrating the Berry curvature over surface enclosed by C [21].

One of the simplest examples of geometric phase is the Foucault pendulum [22]. The pendulum precess when it is taken around a general path C. For transport along the equator, the pendulum does not precess. But if C is made up of geodesic segments, precession arises from the angles where the segments of the geodesics meet; the total precession is equal to the net deficit angle, which equals the solid angle enclosed by C modulo 2π. We can approximate any loop by a sequence of geodesic segments, from which the most general result is that the net precession is equal to the enclosed solid angle. Since there are no inertial forces on the pendulum precess, precession, relative to the direction of motion along the path, is entirely due to the turning of the path. Thus the orientation of the pendulum undergoes parallel transport [22].

2.3. *The Toric Code*

The toric code introduced by Alexei Kitaev, is named from its periodic boundary conditions having it the shape of a torus allowing the model to have translational invariance useful in TQC. Putative experimental realization requires open boundary conditions, allowing the system to be embedded on a 2D surface. Toric code and its generalized surface codes provides a basis for anyonic computation by braiding defects. The unique nature of topological codes, like Kitaev's toric code, is that stabilizer violations can be interpreted as quasiparticles [23].

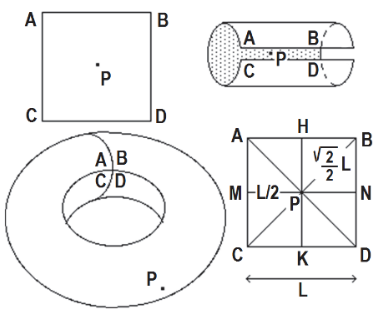

Fig. 5. a) Square Euclidean torus. b) 3-torus.

Kitaev defines the Toric Code on a periodic 2D lattice, usually the square lattice, with a spin-1/2 degree of freedom located on each edge. Stabilizer operators are defined on the spins around each vertex v and plaquette p of the lattice:

$$A_v = \prod_{i \in v} \sigma_i^x,\ B_p = \prod_{i \in p} \sigma_i^z. \tag{7}$$

Where $i \in v$ denotes edges touching the vertex v, and $i \in p$ denotes the edges surrounding the plaquette p. The stabilizer space of the code is where all stabilizers act trivially,

$$A_v|\psi\rangle = |\psi\rangle,\ \forall v,\ B_p|\psi\rangle = |\psi\rangle,\ \forall p, \tag{8}$$

for any state $|\psi\rangle$. For the toric code, this is a 4D space, so it can store two qubits. The occurrence of errors moves the state out of the stabilizer space, resulting in vertices and plaquettes for which the above condition does not hold. The positions of these violations is the 'syndrome of the code', and is used for error correction. The unique nature of topological toric codes, is that stabilizer violations can be interpreted as quasiparticles. Specifically, if the code is in a state $|\phi\rangle$ such that, $A_v|\phi\rangle = -|\phi\rangle$, a quasiparticle called an e anyon exists on the vertex v [23, 24].

Another method introduces a distance truncature at the antipode of each set of points. In Fig. 5, the square is a flat Euclidean torus with null curvature everywhere [25].

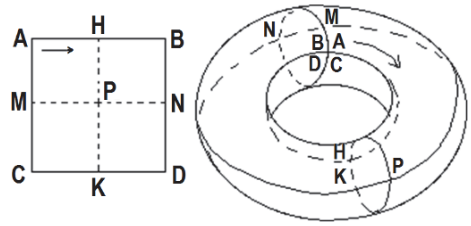

Fig. 6. The P torus point owns three antipodal points (A,B,C,D), (M,N) and (H,K).

From a geometrical point of view, the points A,B,C,D must be identified to an antipode of point P on the torus. For the Euclidean square torus, straight lines are geodesics of the torus. The gravitational action of a mass located at the antipodal point (A,B,C,D) on the point P is zero, which is the same for a mass located in (H,K) or (M,N) [25]. See Fig. 6 (Right). The corresponding geodesic path lengths are basically different (Fig. 5) as shown in (9):

$$\begin{aligned} &\mathrm{PA} = \mathrm{PB} = \mathrm{PC} = \mathrm{PD} = \frac{\sqrt{2}}{2}\ \mathrm{L} \\ &\mathrm{PM} = \mathrm{PN} = \mathrm{PH} = \mathrm{PK} = \frac{L}{2} \end{aligned} \tag{9}$$

Note that on a torus there are an infinite number of geodesics joining two given points, one being the shortest. When computing a corresponding gravitational interaction, both lengths must be considered, $d = \alpha R d' = R(2\alpha\pi - \alpha)$ [25].

3. Transitioning from TQFT to OPTFT

Topological quantum field theories (TQFT) were originally created to avoid the infinities plaguing quantum field theory [11, 26]. Atiyah [11] initially to an axiomatic approach to TQFT, which has been realized in low dimensions and the primary method for modeling anyonic QC. The motivation for topological field theories stems from modern physical theories being defined by invariance under certain group actions like gauge groups in particle physics, diffeomorphism groups in general relativity, or unitary operator groups in quantum mechanics. In topological field theory, the concern is topological invariants, which are objects computed from a topological space (smooth manifold) without any metric [27]. Topological invariance is invariance under the diffeomorphism group of the manifold. Important milestones were Thom's theory of cobordism [28], de Rham cohomology, and knot theory. Through theories such as the Chern-Weil theory linking differential geometry and algebraic topology, abstract formalisms found powerful geometric applications which were applied to physics beginning in the 70's [29] and flourished through the work of Witten and Atiyah [30].

Fundamental strings map out 2D surfaces. The $N = (1,1)$ sigma model quantum field theory is defined on each surface. It consists of maps from the surface to a supermanifold interpreted physically as spacetime and each map is interpreted as the embedding of the string in spacetime. Only certain spacetimes admit topological strings. Classically one must choose a spacetime that allows an additional pair of supersymmetries, so in fact the theory is an $N = (2,2)$ sigma model. This is the case for a Kähler manifold where the H-flux is identically equal to zero [30].

Ordinary strings on special backgrounds are never topological. To make these strings topological, one needs to modify the sigma model by a procedure called a *topological twist* invented by Witten in 1988 [31]. The central observation is that these theories have two U(1) symmetries known as R-symmetries, where the Lorentz symmetry may be modified by mixing rotations and R-symmetries. One may use either of the two R-symmetries, leading to two different theories, called the A model and the B model. After this twist the action of the theory is BRST exact, and as a result the theory has no dynamics, instead all observables depend on the topology of a configuration [26].

Twisting is not possible for anomalies. In the Kähler case where $H = 0$ the twist leading to the A-model is always possible, but that leading to the B-model is only possible when the first Chern class of the spacetime vanishes, implying that the spacetime is Calabi-Yau. More generally $N = (2,2)$ theories have two complex structures and the B model exists when the first Chern classes of associated bundles sum to zero, whereas the A model exists when the difference of the Chern classes is zero. In the Kähler case the two complex structures are the same and so the difference is always zero, which is why the A model always exists [31].

3.1. *The A and B-Models of Topological Field Theory*

The topological A-model comes with a target space which is a real-6D generalized Kähler spacetime describing two objects. There are fundamental strings, which wrap two real-dimensional holomorphic curves. Amplitudes for the scattering of these strings depend only on the Kähler form of the spacetime, and not on the complex structure [30].

The B-model also contains fundamental strings, but their scattering amplitudes depend entirely upon the complex structure and are independent of the Kähler structure. In particular, they are insensitive to worldsheet instanton effects and so can often be calculated exactly. Mirror symmetry then relates them to A-model amplitudes, allowing one to compute Gromov–Witten invariants. The B-model also comes with D(-1), D1, D3 and D5-branes, which wrap holomorphic 0, 2, 4 and 6-submanifolds respectively. The 6-submanifold is a connected component of the spacetime. The theory on a D5-brane is known as holomorphic Chern-Simons theory [29].

3.2. *Dualities Between Topological String Theories (TSTs)*

A number of dualities relate the above theories. The A-model and B-model on two mirror manifolds are related by mirror symmetry, which has been described as a T-duality on a 3-torus. The A-model and B-model on the same manifold are thought to be related by S-duality, implying the existence of several new branes, called NS branes by analogy with the NS5-brane, which wrap the same cycles as the original branes but in the opposite theory. Also a combination of the A-model and a sum of the B-model and its conjugate are related to topological M-theory by a kind of *dimensional reduction*. Here the degrees of freedom of the A-model and the B-models appear to not be simultaneously observable, but have a relation similar to that between position and momentum in quantum mechanics [26, 30].

3.3. *The Holomorphic Anomaly*

The sum of the B-model and its conjugate appears in the above duality because it is the theory whose low energy effective action is expected to be described by Hitchin's formalism. This is because the B-model suffers from a holomorphic anomaly, which states that the dependence on complex quantities, while classically holomorphic, receives non-holomorphic quantum corrections. In Quantum Background Independent String Theory, Witten argued that this structure is analogous to a structure that one finds geometrically quantizing the space of complex structures. Once this space has been quantized, only half of the dimensions simultaneously commute and so the number of degrees of freedom has been halved. This halving depends on an arbitrary choice, called a polarization. The conjugate model contains the missing degrees of freedom, and so by tensoring the B-model and its conjugate one reobtains all of the missing degrees of freedom and also eliminates the dependence on the arbitrary choice of polarization [23, 24, 26, 30].

4. Topological Vacuum Bubbles by Anyon Braiding

According to a basic rule of fermionic and bosonic many-body physics, known as the linked cluster theorem, physical observables are not affected by vacuum bubbles, which represent virtual particles created from vacuum and self-annihilating without interacting with real particles. Here we show that this conventional knowledge must be revised for anyons, quasiparticles that obey fractional exchange statistics intermediate between fermions and bosons. We find that a certain class of vacuum bubbles of Abelian anyons does affect physical observables. They represent virtually excited anyons that wind around real anyonic excitations. These topological bubbles result in a temperature-dependent phase shift of Fabry-Perot interference patterns in the fractional quantum Hall regime accessible in current experiments, thus providing a tool for direct and unambiguous observation of elusive fractional statistics [32].

When two identical particles adiabatically exchange positions $r_{i=1,2,}$ their final state ψ, to dynamical phase, relates to the initial state through an exchange statistics phase θ^*,

$$\psi\left(r_2, r_1\right) = e^{i\theta*}\psi\left(r_1, r_2\right), \tag{10}$$

with $\theta^* = 0(\pi)$ [33].

In many-body quantum theory [33], Feynman diagrams are used to compute the expectation value of observables. This approach invokes vacuum bubble diagrams, which describe virtual particles excited from vacuum and self-annihilating without interacting with real particles. According to the linked cluster theorem [33], each diagram having vacuum bubbles comes with a partner diagram of the same magnitude but of opposite sign that it is exactly cancelled by. Consequently, vacuum bubbles do not contribute to physical observables.

This common wisdom must be revised for anyons because a certain class of vacuum bubbles of Abelian anyons does affect observables. These virtual particles, called topological vacuum bubbles, wind around a real anyonic excitation, gaining the braiding phase $\pm 2\pi v$ [32].

Han's team proposes an experimental procedure for detecting them and $\theta^* = \pi v$, where πv is the anyon phase and θ the interference phase shift [32]. For an interference a_1a_2 between processes a_1 and a_2 for propagation of a real particle, in a_1, a virtual particle-hole pair is excited then self-annihilates after the virtual particle winds around the real particle, forming a vacuum bubble, which is not excited in a_2. The winding results in a braiding phase $2\pi v$ and an Aharonov–Bohm phase $2\pi\left(\Phi / \Phi_0^*\right)$ from the magnetic flux Φ enclosed by the winding path, contributing to the interference signal as $e\exp i\left(2\pi\left(\Phi / \Phi_0^*\right) + 2\pi v\right)$; $\Phi_0^* = h / e^*$ as the anyon flux quantum [32].

The limiting cases of bosons ($v = 0$) and fermions ($v = 1$) imply that this bubble diagram appears together with, and is cancelled by, a partner diagram. The partner diagram has a bubble not encircling the real particle and involves only $2\pi\left(\Phi / \Phi_0^*\right)$. The two diagrams (and complex conjugates) yield

$$\begin{aligned} &\text{Interference signal} \propto \\ &\mathrm{Re}\left[e^{i\left(2\pi\left(\Phi/\Phi_0^*\right)+2\pi v\right)} - e^{2\pi i\left(\Phi/\Phi_0^*\right)}\right] \\ &= -\sin\left(\pi v\right)\sin\left(2\pi\left(\Phi / \Phi_0^*\right) + \pi v\right). \end{aligned} \tag{11}$$

For bosons and fermions, the two diagrams fully cancel each other with $\sin\left(\pi v\right) = 0$ in agreement with the linked cluster theorem; thus, the signal disappears. By contrast, for anyons they cancel only partially, producing non-vanishing interference in an observable, and are topological as the braiding phase is involved [32].

The astute reader will begin to notice, that the anyon braid topology begins to overlap with the UFM OPTFT. The question will be whether the cryogenic TQC will be built as a 'proof of concept' or a 'leap-frog' will occur to the table top room temperature UFM model. If the utility of the Aharonov-Bohm effect remains a key element of 'Topological vacuum bubbles by anyon braiding' interferometry; it is easy to add Aharonov-Bohm effect parameters to the OPTF dynamics.

5. Topological Switching – Key to Ontological-Phase

The 2-state formalism currently forms the basis of QC. Qubits, are 2-state systems. Any QC operation is a unitary operation that rotates the state vector on the Bloch sphere. To move from Hilbert space to ontological-phase space we must begin to define what we mean by topological switching [12-14]. We begin with a number of ways of looking at the ambiguous Necker cube [34].

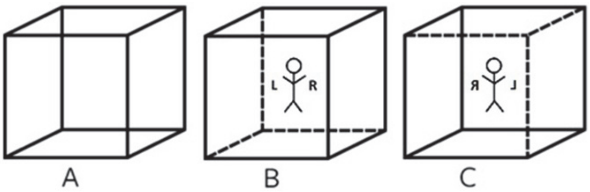

Fig. 7. Ambiguous Necker cube, left, mirror image, center and perceived shift between the two states in 4D.

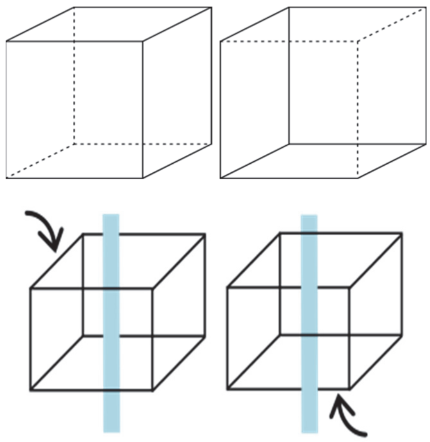

Fig. 8. Two states of the Necker cube. A physically real description is needed.

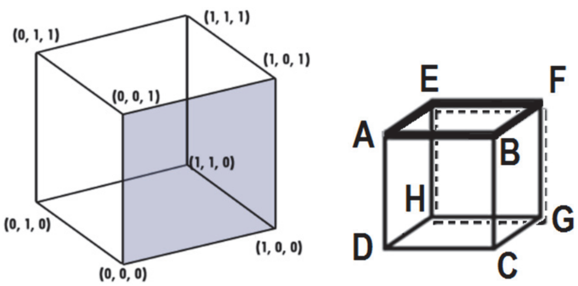

Fig. 9. A first step towards physicality might be distinguishing the vertices.

Quaternions have the ability to represent rotations of 3D space. If we represent 3-space, $\mathbb{R}^3$ as the set of pure quaternions of the form $\mathbb{Z} = ai + bj + ck$ with *a, b, c* real numbers, then g is a unit quaternion mapping $\rho : \mathbb{R}^3 \to \mathbb{R}^3$ defined by the equation $\rho(\mathbb{Z}) = g\mathbb{Z}g^{-1}$ describes a 3-space rotation by angle θ around axis μ when

$$g = \cos(\theta/2) + \sin(\theta/2)\mu. \tag{12}$$

In this manner, μ is a unit length quaternion giving a direction to a vector in 3-space, a rotation is specified by an angle θ about an axis *U*, which in the case below is in the positive direction [35].

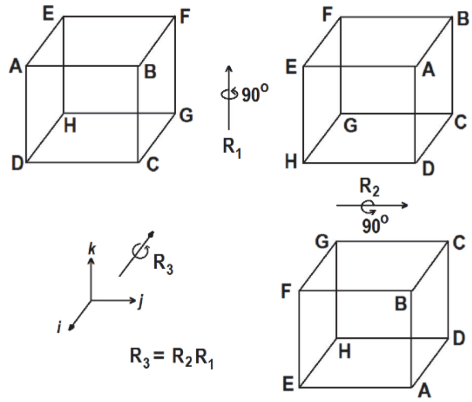

Fig. 10. Denoting two 90° rotations R_1 and R_2, we write $R_3 = R_2R_1$ for the rotation obtained by 1st performing R_1 and then R_2. R_3 fixes the corners B and H; Thus R_3 is a 120° rotation about the diagonal axis.

Thus, following Kauffman [35],

$$e^{j(\pi/4)}e^{k(\pi/4)} = \left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2} + k\frac{\sqrt{2}}{2}\right)$$
$$= \frac{1}{2}(1+j)(1+k) = \frac{1}{2}(1+j+k+jk).$$
$$= \frac{1}{2}(1+i+j+k) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{i+j+k}{\sqrt{3}}\right).$$

$$\therefore e^{j(\pi/4)}e^{k(\pi/4)} = e^{\left[(i+j+k)/\sqrt{3}\right][2\pi/3]/2}$$

$\uparrow$ $\uparrow$

diagonal axis 120°.

These quaternion rotations can be considered phase changes under certain conditions; but they do not correspond to the ontological phase we are looking for because Euclidean geometry has no natural inherent perspective. It appears we need a duo-morphic projection perhaps involving Berry phase because the ambiguous vertices of the Necker cube are not distinguished in Kauffman's quaternion rotation system [35].

To clarify how projective transformations lose orientable information, rotating a triangle in a plane is used as an illustration [36].

The rotation sequences in Fig. 11 are I,II,III for clockwise and I,III,II for counter-clockwise. According to Shaw the direction of rotation reverses if the back and front ranges are interchanged. This is denoted by the connecting lines in the boxes below the rotation triangles. **Bold** letters mark the front range; this system is able to preserve orientation information under projected rotation.

The 3D wire-frame Necker cube can be projected onto a 2D surface, collapsing the cubes six faces into a complex of one to seven coplanar polygons depending on orientation of the cube.

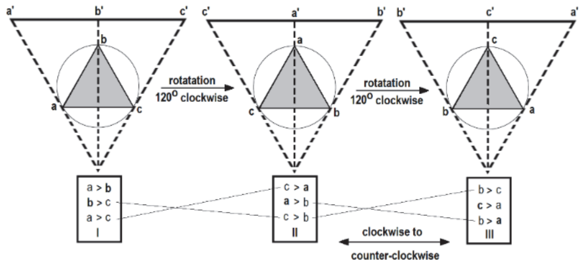

Fig. 11. Removing ambiguity from a projected rotation, with > denoting order of sequence occurrence – to the left on the projective line. **Bold** letters are the front range of projective mapping. Fig. redrawn from [36].

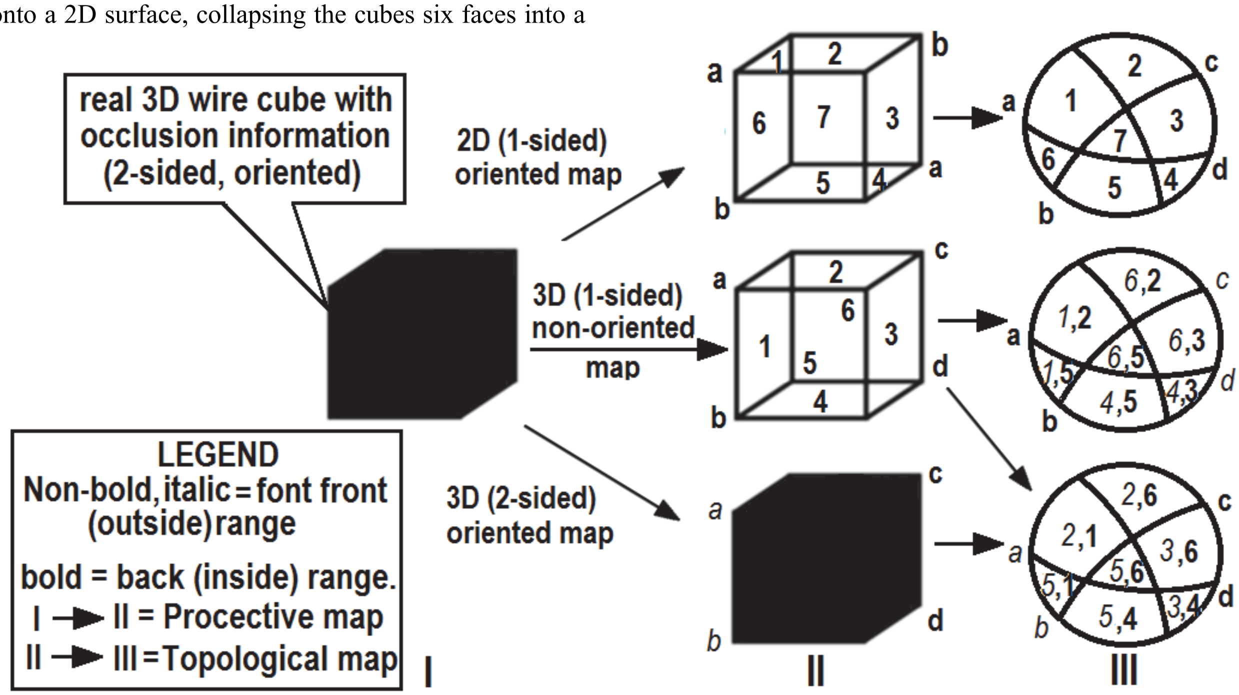

Fig. 12. Contrasting nonoriented - oriented projective geometries. Redrawn from [37].

Figure 12 illustrates three different forms of projection.

- I → II-Top → III-Top: no occlusion information
- I → II-Middle → III-Middle and III-Bottom: occlusion information is specified ambiguously
- I → II-Bottom → III-Bottom: occlusion information is specified unambiguously.

The Necker cube, like the Möbius strip is an ambiguous figure because of the problem of projective mapping. In ordinary projective space, the Möbius strip and Necker cube, are one-sided (Fig. 12). The spherical model of this geometry represents the fact that the projections of a point on the back of the sphere and of a point on its front both have the same image in the Euclidean (projective) plane. All of the projected points, regardless of the hemisphere to which they belong, cover the projective plane in the usual way without any designation of where they originated. The loss of orientation is due to this failure of the projective mapping to preserve the distinction between the front and back range, collapsing both into positive values of the dimension of depth *w*. This loss of orientation is represented by the fact that relationships (e.g., the arrows) invert when the projective angle passes through the points at infinity [36].

To keep the front and back ranges distinguished, traditional computational geometries use the line at infinity as a reference; but this move is not a real solution to the orientation problem in projective geometry because it is tantamount to a return to Euclidean geometry which has no inherent natural perspective.

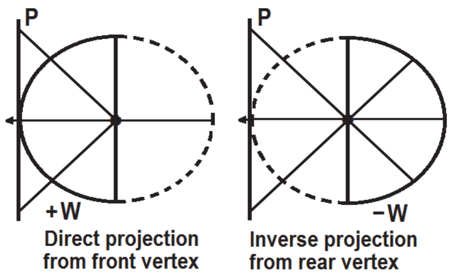

Fig. 13. Duo-morphic oriented projections (+W, -W) yield a double covering of the projective plane, P.

To distinguish front and back ambiguous vertices of the Necker cube is a problem of orientation. Oriented projective geometry introduces a methodology for distinguishing the ambiguous vertices of the Necker cube [36]. Shaw [37] assigns a dual range, +W and -W to represent front and rear ranges of a sphere.

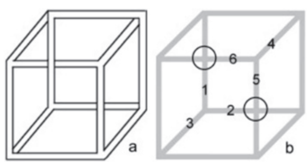

Fig. 14. Ambiguity needs a method of labeling for clarity.

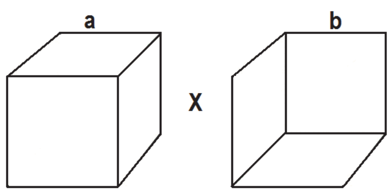

Fig. 15. Visual test of stereoscopic construction of a Necker cube.

Figure 15 separates the ambiguous Necker cube into its component perspectives. Although what we are about to illustrate is usually considered a mental construct, we use it here to illustrate what we mean by ontological phase and an ontological phase transformation. Focus on the 'X' halfway between the 2D L-R Necker perspectives; relax one's eyes and allow them to lose focus and cross. Soon, a 3rd image appears between the two printed L-R images fusing the original perspective into one apparent 3D image, confirmed by noticing the labels 'a' and 'b' are now superposed. This stereoscopic condition is the scenario we want to utilize to define ontological-phase.

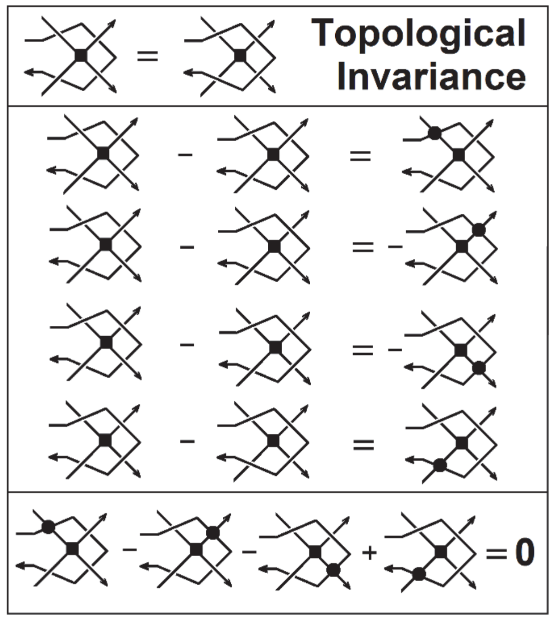

Fig. 16. Topological Invariance must be included in any phase labeling. Figure redrawn from [25].

Masahide & Satoh generalize the class of roll-spun knots for 2-knot theory and show how to calculate the quandle cocycle invariant for any roll-spun knot [38]. For the case $X = S_4 = \mathbb{Z}\left[t, t^{-1}\right] / \left(2, t^2 + t + 1\right)$, the element $w = 1 \cdot t^{-1} \cdot 0 \cdot \left(t+1\right)^{-1}$ satisfies $\varphi_w = \mathrm{id}_{S4}$; such that we have

$$
\begin{array}{ccccc}
0 & \xrightarrow{\varphi_1} t+1 & \xrightarrow{\varphi_t^{-1}} 1 & \xrightarrow{\varphi_0} t & \xrightarrow{\varphi_{t+1}^{-1}} 0 \\
1 & \mapsto 1 & \mapsto 0 & \mapsto 0 & \mapsto 1 \\
t & \mapsto 0 & \mapsto t+1 & \mapsto 1 & \mapsto t \\
t+1 & \mapsto t & \mapsto t & \mapsto t+1 & \mapsto t+1.
\end{array}
\tag{13}
$$

Since $\text{ind}(w) = 0,$ it holds that $w \in G_0(S_4).$ Figure 17 shows that $w^2 = 1$ in $\mathbf{G}_0(S_4)$, and that w is the generator of $G_0(S_4) \cong \mathbb{Z}_2$.

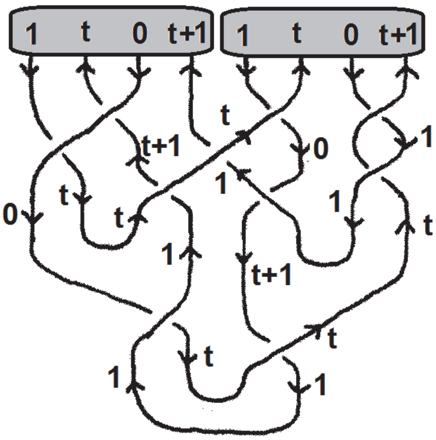

Fig. 17. Deform-spun knot tangle diagram. Redrawn from [38].

The spun knot is explored as a possible component topological move for ontological-phase transitions. When parallel transport creates a deficit angle in brane raising and lowering dynamics, in addition to Reidemeister moves, rotations, reflections and any other topological moves, spun knot components may add another type of phase transition with lattice charge.

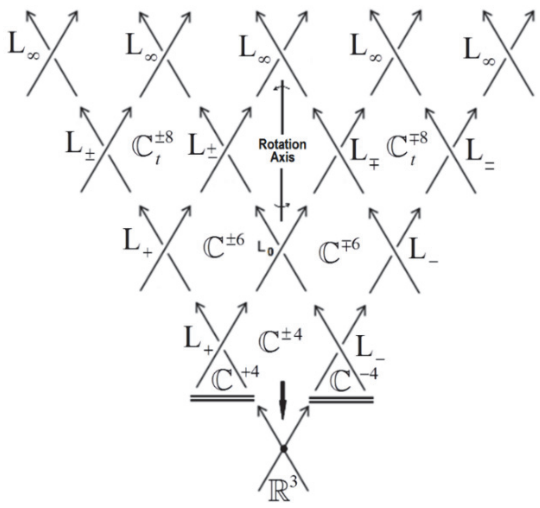

Fig. 18. Rolling spun knots. The infusion of topological charge as a UFM 'force of coherence' driving evolution throughout the multidimensional brane hierarchy can allow multiple types of moves to occur at multiple levels simultaneously.

An important feature of TQFTs is that they do not presume a fixed topology for space or spacetime. In other words, when dealing with an n-dimensional TQFT, one is free to choose any (n - 1)-dimensional manifold to represent space at a given time. Moreover, given two such manifolds, say S and S', one is free to choose any nD manifold M to represent the portion of spacetime between S and S'. Mathematicians call M a 'cobordism' from S to S'. We write $M : S \to S'$, because we may think of M as the process of time passing from the moment S to the moment S'.

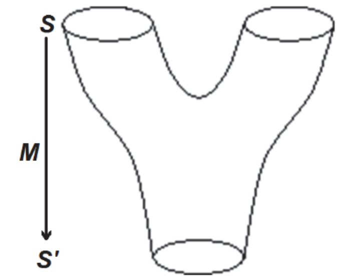

Fig. 19a. A basic cobordism.

For example, in Fig. 19a we depict a 2D manifold M going from a 1D manifold S (a pair of circles) to a 1D manifold S' (single circle). Crudely speaking, M represents a process in which two separate spaces collide to form a single one! This may seem *outré*, but currently physicists are quite willing to speculate about processes in which the topology of space changes with the passage of time [39].

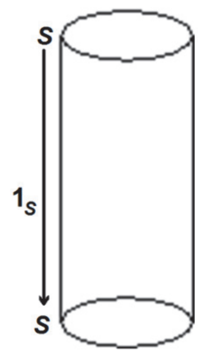

Fig. 19b. Identity cobordism.

There are various important operations one can perform on cobordisms, but we only describe two. First, we may 'compose' two cobordisms $M : S \to S'$ and $M : S' \to S''$, obtaining a cobordism $M'M : S \to S''$, as illustrated in Fig. 20. The idea here is that the passage of time corresponding to M followed by the passage of time corresponding to M' equals the passage of time corresponding to $M'M$. This is analogous to the familiar idea that waiting t seconds followed by waiting t' seconds is the same as waiting $t+t'$ seconds. The big difference is that in topological quantum field theory we cannot measure time in seconds, because there is no background metric available to let us count the passage of time! We can only keep track of topology change. Just as ordinary addition is associative, composition of cobordisms satisfies the associative law:

$$\left(M''M'\right)M = M''\left(M'M\right). \tag{14}$$

However, composition of cobordisms is not commutative. As we shall see, this is related to the famous noncommutativity of observables in quantum theory [39].

Second, for any (n–1)D manifold S representing space, there is a cobordism $1_S : S \to S$ called the 'identity' cobordism, which represents a passage of time without topological change. For example, when S is a circle, the identity cobordism 1_S is a cylinder, as shown in Fig. 19b. In general, the identity cobordism 1_S has the property that for any cobordism $M : S' \to S$ we have $1_S M = M$, while for any cobordism $M : S \to S'$ we have $M1_S = M$ [39].

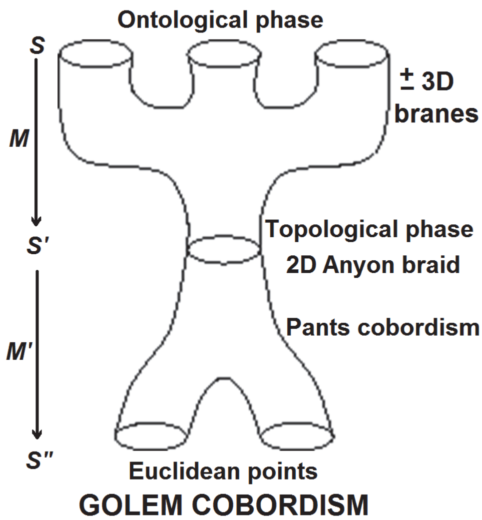

Fig. 20. The Golem, composition of cobordisms designed to handle ontological-phase.

These properties say that an identity cobordism is analogous to waiting 0 seconds: if you wait 0 seconds and then wait t more seconds, or wait t seconds and then wait 0 more seconds, this is the same as waiting t seconds.

These operations just formalize of the notion of 'the passage of time' in a context where the topology of spacetime is arbitrary and there is no background metric. Atiyah's axioms relate this notion to quantum theory as follows. First, a TQFT must assign a Hilbert space $Z(S)$ to each (n – 1)D manifold S. Vectors in this Hilbert space represent possible states of the universe given that space is the manifold S. Second, the TQFT must assign a linear operator $Z(M): Z(S) \to Z(S')$ to each nD cobordism $M : S \to S'$. This operator describes how states change given that the portion of spacetime between S and S' is the manifold M. In other words, if space is initially the manifold S and the state of the universe is ψ, after the passage of time corresponding to M the state of the universe will be $Z(M)\psi$ [39].

In addition, the TQFT must satisfy a list of properties. Let me just mention two. First, the TQFT must preserve composition. That is, given cobordisms $M : S \to S'$ and $M' : S' \to S''$, we must have $Z(M'M) = Z(M')Z(M)$, where the right-hand side denotes the composite of the operators $Z(M)$ and $Z(M')$. Second, it must preserve identities. That is, given any manifold S representing space, we must have $Z(1_S) = 1_{Z(S)}$ where the right-hand side denotes the identity operator on the Hilbert space $Z(S)$ [39].

Both these axioms are eminently reasonable if one ponders them a bit. The first says that the passage of time corresponding to the cobordism M followed by the passage of time corresponding to M' has the same effect on a state as the combined passage of time corresponding to $M'M$. The second says that a passage of time in which no topology change occurs has no effect at all on the state of the universe. This seems paradoxical at first, since it seems we regularly observe things happening even in the absence of topology change. However, this paradox is easily resolved: a TQFT describes a world quite unlike ours, one without local degrees of freedom. In such a world, nothing local happens, so the state of the universe can only change when the topology of space itself changes[3].

The most interesting thing about the TQFT axioms is their common formal character. Loosely speaking, they all say that a TQFT maps structures in differential topology (the study of manifolds) to corresponding structures in quantum theory. In coming up with these axioms, Atiyah took advantage of a powerful analogy between differential topology and quantum theory, summarized in Table 1 [39].

This analogy between differential topology and quantum theory the sort of clue we should pursue for a deeper understanding of quantum gravity. At first glance, general relativity and quantum theory look very different mathematically: one deals with space and spacetime, the other with Hilbert spaces and operators. Combining them has always seemed a bit like mixing oil and water. But topological quantum field theory suggests that perhaps they are not so different after all! Even better, it suggests a concrete program of synthesizing the two, which many mathematical physicists are currently pursuing. Sometimes this goes by the name of 'quantum topology' [2, 11].

Quantum topology is very technical, as anything involving mathematical physicists inevitably becomes. But if we stand back a moment, it should be perfectly obvious that differential topology and quantum theory must merge if we are to understand background-free quantum field theories. In physics that ignores general relativity, we treat space as a background on which states of the world are displayed. Similarly, we treat spacetime as a background on which the process of change occurs. But these are idealizations which we must overcome in a background-free theory. In fact, the concepts of 'space' and 'state' are two aspects of a unified whole, and likewise for the concepts of 'spacetime' and 'process'. It is a challenge, not just for mathematical physicists, but also for philosophers, to understand this more deeply [39].

We begin to explore various types of crossover links and moves to start cataloguing the variety of moves that maybe applicable to ontological-phase transitions.

Fig. 21. Simple crossover links.

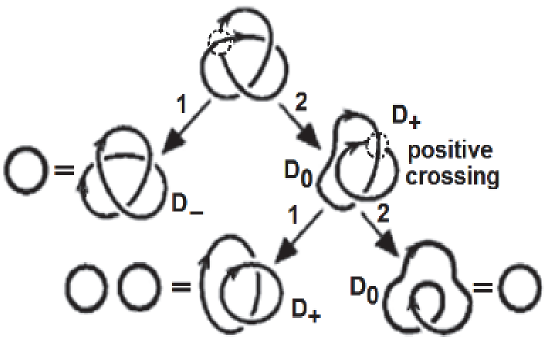

Fig. 22. Crossings for octonion trefoil knots.

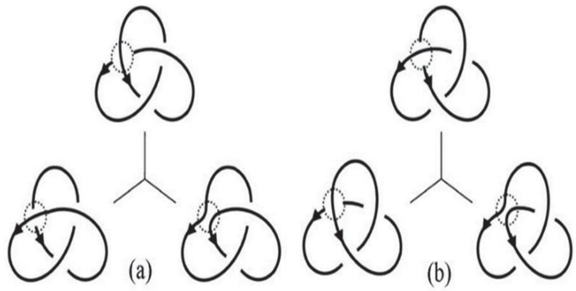

Fig. 23. Reduction schemes for the left- and right-handed trefoil knots. (a) Top: left-handed trefoil knot; bottom: writhe γ_- and a Hopf link H_-, with crossing −1. (b) Top: right-handed trefoil knot; bottom: writhe γ_+ and a Hopf link H_+, with crossing +1. The two knots are mirror images of one another. Figure adapted from [40].

Thus, a true octonion contains three trefoil knots, whereas a split octonion may be specified by mixing a pair of quaternion trefoil lines. To define a tripled Fano plane requires three copies of Furey's particle zoo. It describes a set of 21 = 3 x 7 (left cyclic) modules over a noncommutative ring on eight elements. The ring is given by the upper triangular 2 x 2 matrices over the field with two elements. Similarly, for right cyclic modules [41, 42].

The quaternions, H are a 4D algebra with basis $1, i, j, k$. To describe the product, it is easy to note that:

- 1 is the multiplicative identity,
- i, j, k are square roots of -1,
- we have $ij = k$, $ji = -k$ and all identities obtained from these by cyclic permutations of (i, j, k).

We can summarize the last rule as a diagram

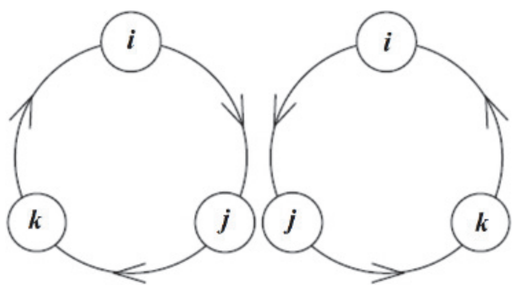

Fig. 24. Clockwise and counterclockwise rule for Quaternion cyclicality.

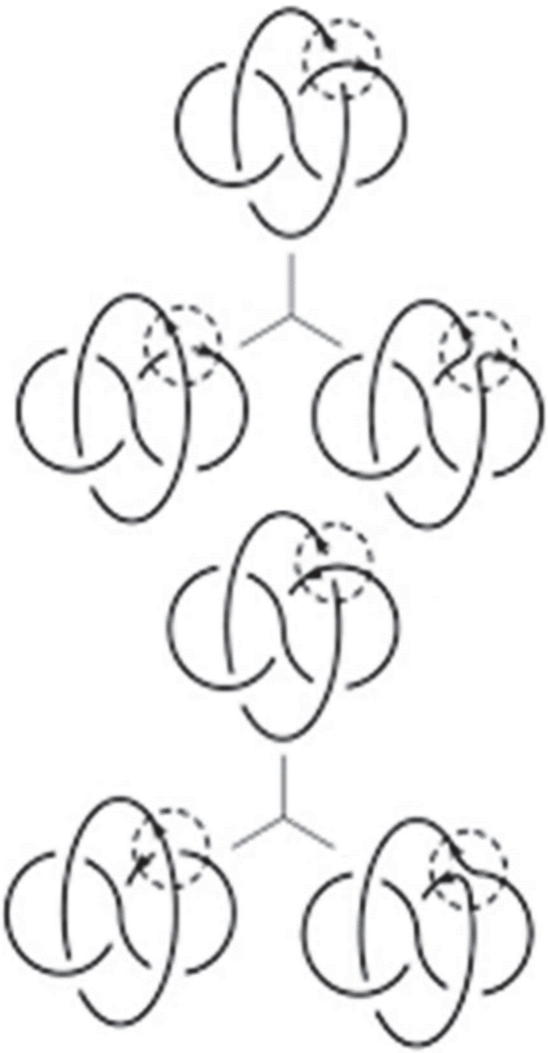

Fig. 25. Reduction schemes for Whitehead links W_+ and W_-. (a) Top: Whitehead link W_+ with crossing +1; bottom: Hopf link H_- and the left-handed trefoil knot T^L. (b) Whitehead link W_- with crossing −1; bottom: Hopf link H_+, and a figure-of-eight knot F^8. Figure adapted from [40].

In multiplying two elements going clockwise around the circle we get the next one: for example, $ij = k$. But when we multiply two going around counterclockwise, we get *minus* the next one: for example, $ji = -k$. We can use the same sort of picture to remember how to multiply octonions:

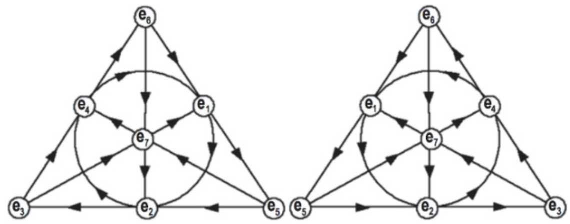

Fig. 26. The Fano plane and its mirror image.

The Fano plane is the finite projective plane of order 2, having the smallest possible number of points and lines, 7 each, with 3 points on every line and 3 lines through every point. The Fano plane has 7 points and 7 lines. The 'lines' are the sides of the triangle, its altitudes, and the circle containing all the midpoints of the sides. Each pair of distinct points lies on a unique line. Each line contains three points, and each of these triples has a cyclic ordering shown by the arrows. If e_i, e_j, e_k are cyclically ordered in this way then $e_ie_j = e_k, \quad e_je_i = -e_k$.

Together with these rules:

- 1 is the multiplicative identity,
- $e_1, \ldots, e_7$ are square roots of -1,

the Fano plane completely describes the algebra structure of the octonions. Index-doubling corresponds to rotating the picture a third of a turn. Interestingly, The Fano plane is the projective plane over the 2-element field $\mathbb{Z}_2$. In other words, it consists of lines through the origin in the vector space $\mathbb{Z}_2^3$. Since every such line contains a single nonzero element, we can also think of the Fano plane as consisting of the seven nonzero elements of $\mathbb{Z}_2^3$. If we think of the origin in $\mathbb{Z}_2^3$ as corresponding to $1 \in \mathrm{O}$, we get the following picture of the octonions:

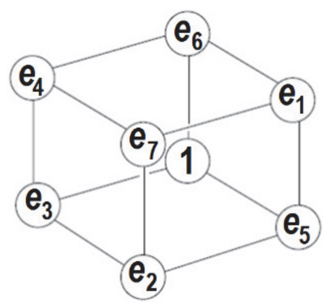

Fig. 27. The octonions for $1 \in \mathrm{O}$.

Note that planes through the origin of this 3D vector space (Fig. 26) give subalgebras of O isomorphic to the quaternions, lines through the origin give subalgebras isomorphic to the complex numbers, and the origin itself gives a subalgebra isomorphic to the real numbers [39].

Now we finally arrive at the fundamental geometric topology for describing ontological-phase topological field theory. When the formalism is next written it will be created by utilizing both topology and complex quaternion/octonions Clifford algebra which is especially suited to handle the manifold embedding [43].

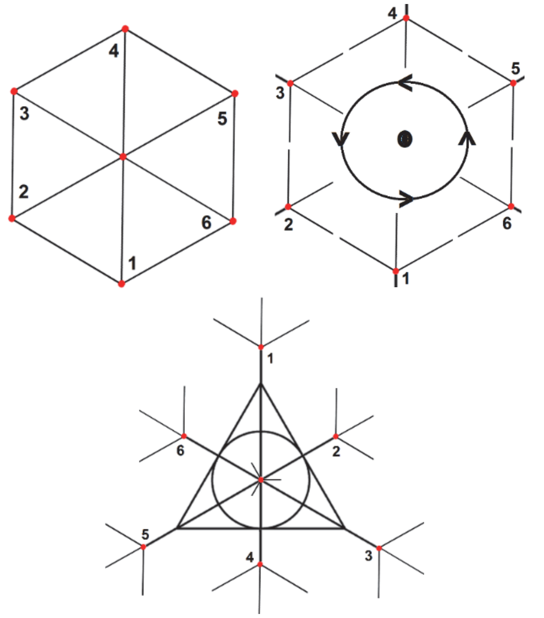

Fig. 28. The 'antennas' (snowflakes) on a Fano plane (top) represent vertices on the circumference of a hexagon or cube (bottom). The center rotates unconnected so position 1 or 2 can create the front/rear vertices of a Necker cube. b) Antennas 1-6 combine to form the outer vertices of a cube/hexagon depending on what dimensional phase the state is in.

The Fano snowflake configuration in Fig. 28 involutes to form a 2D hexagon or vertices of a Euclidean Necker 3-cube. We expect to require a dual set of twin Fano-Snowflakes as would be derived from Fig. 26 to account for all the parameters necessary for 'the mirror image of the mirror image to be causally free of the Euclidean 3-space QED quantum state.

6. Dual Amplituhedron Geometry and 'Epiontic' Realism

The amplituhedron geometric jewel simplifies particle interaction calculations and challenges the notion that space and time are fundamental components of reality, advancing a long effort to reformulate quantum field theory, the body of laws describing elementary particles and their interactions by calculations with formulas thousands of terms long that can now be described by computing the volume of its amplituhedron, yielding an equivalent one-term expression. The new geometric version of quantum field theory could also facilitate the search for a theory of quantum gravity. Attempts thus far to incorporate gravity into the laws of physics at the quantum scale have run up against nonsensical infinities and deep paradoxes. An amplituhedron type geometry could help by removing two deeply rooted principles of physics: locality and unitarity [47].

Locality is the notion that particles can interact only from adjoining positions in space and time. And unitarity holds that the probabilities of all possible outcomes of a quantum mechanical interaction must add up to one. The concepts are the central pillars of quantum field theory in its original form, but in certain situations involving gravity, both break down, suggesting neither is a fundamental aspect of nature. In keeping with this idea, the new geometric approach to particle interactions removes locality and unitarity from its starting assumptions. The amplituhedron is not built out of space-time and probabilities; these properties merely arise as consequences of the jewel's geometry. The usual picture of space and time, and particles moving around in them, is only a useful construct [47].

Because "we know that ultimately, we need to find a theory that doesn't have" unitarity and locality, Bourjaily said, "*it's a starting point to ultimately describing a quantum theory of gravity*." The 1st part of Bourjaily's statement is correct; however, the 2nd part is not. Most physicists still consider the quantum regime the basement of reality and thus automatically think to progress in unification gravity must be quantized. This is not the regime of integration and therefore obviously why there is no quantum gravity. But transition to the 3rd regime of UFM is confounded 'epiontics'. Reality acquires a semi-quantum (epi) limit on the way to the ontological (ontic) regime of UFM [47, 48].

The amplituhedron in HD encodes in its volume "scattering amplitudes," which represent the likelihood that a certain set of particles will turn into certain other particles upon colliding. The twistor theory at the root of it does this kind of simplification. It folds the speed of light into the geometry by mapping point particles to their light cones. The point becomes an intersection of the sphere of light rays that could radiate from it. Then you can do extra stuff like cancelling out the asymmetry of universal expansion by mapping the larger future light cone on to the smaller past light cone [49].

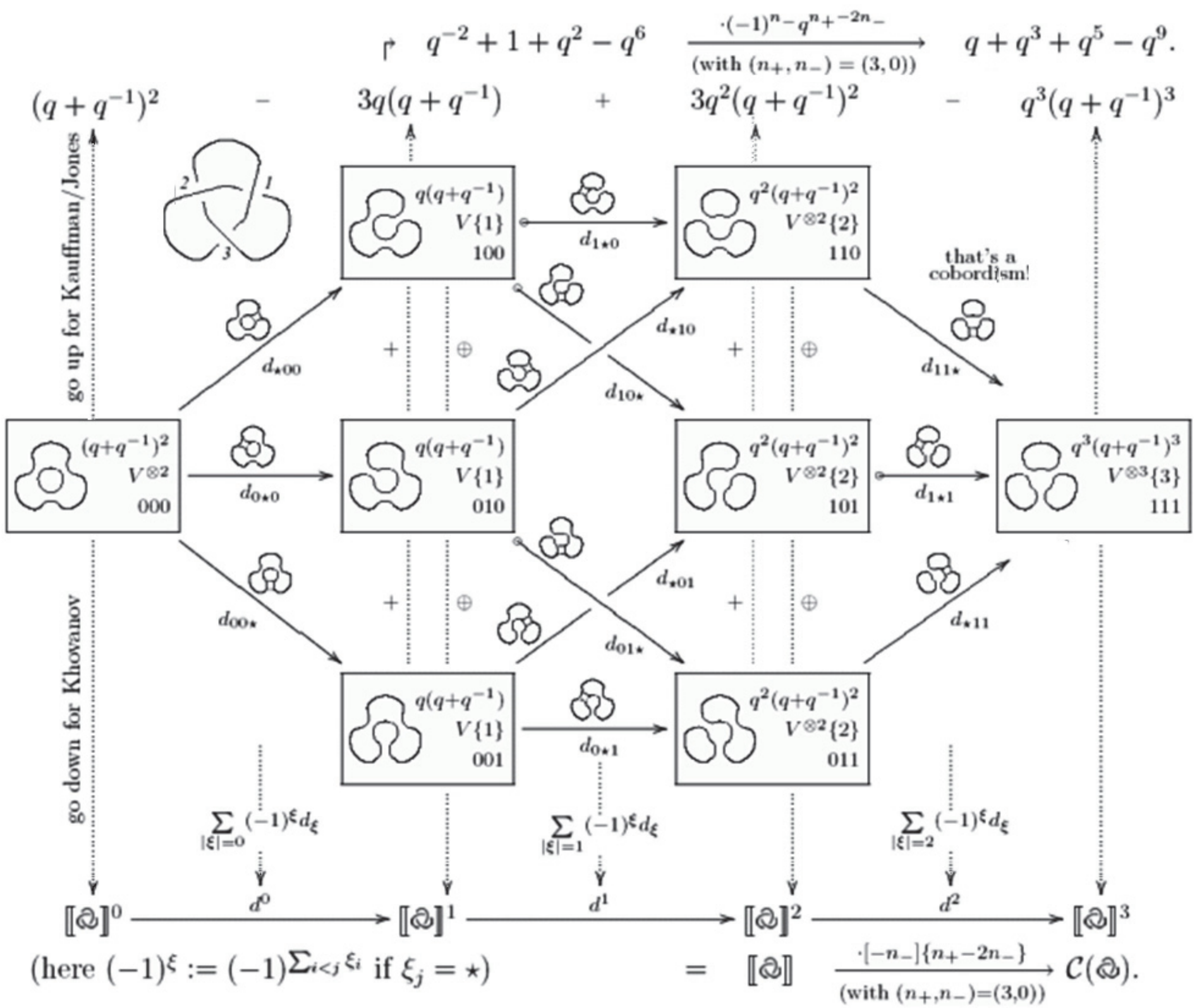

Fig. 29. Construction to improve Khovanov's seminal work on the categorification of the Jones polynomial. Figure adapted form [46].

DIFFERENTIAL TOPOLOGY	QUANTUM THEORY
(n – 1)-dimensional manifold (space)	Hilbert space (states)
cobordism between (n – 1)-dimensional manifolds (spacetime)	operator (process)
composition of cobordisms	composition of operators
identity cobordism	identity operator

Table 1. Analogy between differential topology and quantum theory

Some of the complexity for categorizing the Jones polynomial is shown in Fig. 29 as it might apply to modeling ontological-phase.

Perhaps often, mathematics corresponds perfectly well to physical reality. But maybe now as we move away from a Hilbert space representation of qubit processing to a truly physical basis, we might surmise 'No wonder it has been difficult to implement bulk QC'. For classical digital computing, math itself was sufficient; but as we move to relativistic qubits and topological quantum field theory apparently this is not the case [50].

Jaynes had this to say:

"*... our present formalism is not purely epistemological; it is a ... mixture describing in part realities of Nature, in part incomplete human information about Nature ... if we cannot separate the subjective and objective aspects of the formalism we cannot know what we are talking about ...* ." [50, 51].

The term epistemic is used to represent – not real, mind of observer, in contrast to ontic – real; Zurek coined the term epiontic to merge the two philosophies into what he called Quantum Darwinism. Quantum Darwinism describes the proliferation, in the environment, of multiple records of selected states of a quantum system. It explains how the fragility of a state of a single quantum system can lead to the classical robustness of states of their correlated multitude; shows how effective 'wavepacket collapse' arises as a result of proliferation throughout the environment of imprints of the states of quantum system; and provides a framework for the derivation of Born's rule, which relates probability of detecting states to their amplitude. Taken together, these three advances mark considerable progress towards settling the quantum measurement problem [48].

From copying to quantum jumps Quantum Darwinism leads to appearance, in the environment, of multiple copies of the state of the system. However, the no-cloning theorem [52, 53] prohibits copying of unknown quantum states. If cloning is outlawed, how can redundancy be possible? Quick answer is that cloning refers to (unknown) quantum states. So, copying of observables evades the theorem. Nevertheless, the tension between the prohibition on cloning and the need for copying is revealing: It leads to breaking of unitary symmetry implied by the superposition principle, accounts for quantum jumps, and suggests origin of the "wavepacket collapse", setting stage for the study of quantum origins of probability [50].

Alexander's horned sphere is a convoluted, intertwined surface with a difficult to define inside and outside that is homeomorphic to a ball, meaning that it can be stretched into a ball without being punctured or broken or vice versa. Embedded in Euclidean 3-space, it can be constructed from a torus (Fig. 30) in the following manner:

1. Remove a radial slice of the torus.
2. Connect a standard punctured torus to each side of the cut, interlinked with the torus on the other side.
3. Repeat steps 1 & 2 on the two tori added in step two *ad infinitum*.

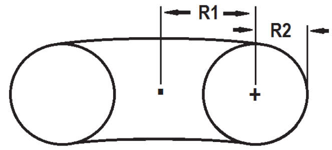

Fig. 30. Torus showing minor and major radii.

States with different topological orders or different patterns of long range entanglements cannot change into each other without a phase transition. In the case of Alexander's horned sphere, we believe this requires an ontological-phase topological transition.

7. Generalizing Topological Phase Transitions in Homological Mirror Symmetric Brane Dynamics

Can Yang-Mills (YM) Kaluza-Klein (KK) correspondence drive the Future of Particle Physics? Although it is generally known that YM-KK theories define equivalence on principle fiber bundles; specific conditions for equating their Lagrangians have not been rigorously specified. Since the origin of KK Theory virtually all corresponding extensions of the Standard Model (SM) rely on a profusion of additional dimensionality (XD); a conundrum that clearly can only be resolved experimentally. Topological phase/phase transition is the most active research arena in contemporary physics and a signpost signifying the entryway to the 3rd regime of Unified Field Mechanics (UFM). Majorana found symmetric solutions to the Dirac equation describing fermionic particles that are their own anti-particle. Derivatives of his formalism now appear in Nuclear, Majorana-Weyl graviton/spinors, particle and solid state/condensed matter physics, suggesting that there is likely to be a 'Majorana quantum number' of adjoint topological order and geometric phase. The Dirac-Majorana duality condition represents a semi-quantum correspondence of Homological mirror symmetry for paired Calabi-Yau manifolds relating the algebraic geometry of X and the symplectic geometry of Y where one maps from a 4D SM Riemann surface into a fixed target 6D Calabi-Yau brane topological manifold.

A protocol utilizing YM-KK equivalence as a path for introducing topological phase transitions beyond the Standard Model (SM) is outlined. For example, Riemannian KK manifolds, M with horizontal and vertical subspaces in the tangent bundle (M = X x G) defined by the YM connection are orthogonal with respect to the KK metric, where X is a 4D spacetime and G an arbitrary gauge Lie group; and for the corresponding YM theory, M is a trivial principle G-bundle. This suggests putative orthogonal extensions of topological phase transitions beyond the limits of the SM. A protocol has been found for empirically testing the model. It is generally known that KK modeling makes correspondence to the SM through YM Gauge Theory.

If one attempts the exchange of two particles in 2 + 1 spacetime by rotation, the rotations are inequivalent, since one cannot be deformed into the other (without the worldlines leaving the plane, an impossibility in 2D space). This is a problem for accessing topologically protected anyon braids, achieving understanding of the key condition for getting experimental access to topological.

Remarkably, the spatial separation between the two MZM's can be arbitrarily large, so that the quantum information is stored in a highly nonlocal manner. This key property endows the qubit with topological protection against local perturbations.

Rowlands' description of dualistic phenomena, splits "*the universe into two halves that are mathematically and physically, if not observationally, equivalent*". Rowlands does not mean equivalent in the sense of a correspondence but rather a dualistic process. He writes, "*Further dualling is possible on the same basis, but it is clear that only three fundamental principles are required to continue the dualling to infinity – opposite signs (or equivalent), the distinction between real and imaginary components, and the introduction of cyclic dimensionality – and to establish every conceivable combination of these, that is to establish every type of dualling, requires a group of 64 elements* [54]."

7.1. *Higher Dimensional Space and the Klein Cycle*

The coordinate system for special relativity with time, t as a 4th dimension is $x^\mu = \left(x^0, x^1, x^2, x^3\right) = \left(t, x, y, z\right)$; later when Einstein introduced general relativity, a 2nd set of indices was required, $g_{\mu\nu}\left(x\right)$ which has the line element,

$$ds^2 = g_{\mu\nu}\left(x\right) dx^\mu dx^\nu \tag{15}$$

Kaluza made his famous attempt to combine electromagnetism with general relativity by postulating a 5th dimension with an additional coordinate, θ collectively denoted, x^M with index M running

$$x^M = \left(x^0, x^1, x^2, x^3, x^4\right) = \left(t, x, y, z, \theta\right) \tag{16}$$

Since curved spacetime was required, Kaluza proposed a Riemannian geometry with the metric tensor, $\hat{g}_{MN}\left(x\right)$, which has the 5D line element

$$d\hat{s}^2 = \hat{g}MN\left(x\right) dx^M dx^N \tag{17}$$

Klein [55-57] then assumed the XD had circular topology so that the coordinate, θ is *periodic*, $0 \le \theta \le 2\pi$ and that at every point on the line there is a little circle that Klein suggested that there is a little circle at each point in 4D spacetime.

This is the current thinking that proposes the XD must be Planck scale because they are unobserved, but the periodicity of θ also means that the fields, $\hat{g}_{MN}\left(x, \theta\right)$ may be expanded as

$$\hat{g}_{MN}\left(x, \theta\right) = \sum_{n=-\infty}^{n=\infty} \hat{g}_{MN(n)}\left(x\right) \exp\left(in\theta\right) \tag{18}$$

which by way of the LCU semi-quantum LSXD duality structure of OPTFT.

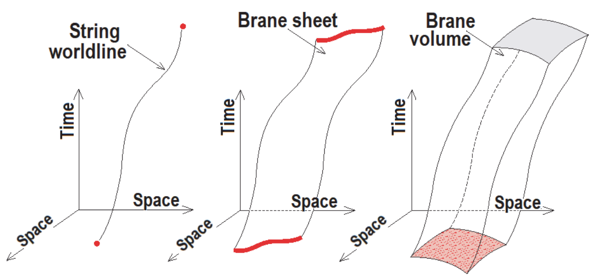

Fig. 31. String worldline to brane sheet and brane volume topology M-Theory.

In the Kaluza-Klein model, em and G were integrated in terms of metrics for their field parameters. We have remained in the dark ages in terms of what particles are; physicists have gotten away with 0D Point particle singularities essentially because we have been able to fudge the math for a century.

String theory now claims a particle is a 1D vibration. It appears that 2D ribbons and brane volumes are considered topological fields of wave-particle duality.

Topological fields? What needs to be realized is that matter must be considered a synergy of wave-particle duality and conformal scale-invariant homological topology. Knowing why will allow it to be acceptable.

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Neo-Cartesian Unified Fluid Theory: From the Classical Wave Equation to De Broglie's Lorentzian Quantized Mechanics and Quantized Gravity

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This paper introduces a fluid aether formed by discrete, 3D-extended energy-like sagions obeying three conservation principles of classical mechanics: total energy, linear momentum, and angular momentum. In contrast to Newtonian mechanics, neither mass, nor force are primitive notions (hence, the Cartesian"), but the theory is atomistic (hence the "neo"). Firstly, the notions of field, continuity, discreteness, extension, and philosophical and empirical reasons leading to reinstating aether are clarified. The collective fluid behaviour is described by the classical wave equation, also known as the homogeneous Klein-Gordon equation (HKGE). Connections of electromagnetism (EM), gravity and quantum mechanics (QM) to the theory of fluids are noted. The goal is to attain a unified field theory that contain as special cases the other "forces". In particular, QM must be relativistic *ab initio* for consistency with Einstein's general theory of relativity (GTR), while GTR must be extended to allow for permanent violation of the principle of equivalence, in the sense that gravity interactions depend upon composition of matter, as effectively observed in the original Eötvös experiment, and in the outstanding, but neglected, experiments of Quirino Majorana. Of particular interest are three families of nonharmonic solutions to the HKGE discovered by this author in the 1990s. The minimum angular momentum in sagion-sagion interactions is identified as Planck constant, thus introducing quantum features in classical mechanics. Coalescence of sagions leads to a kinematic theory of photons and fundamental particles, whose simplest object is a rotating dumbbell, which forms a torus in 3D-space. Acceleration produced by successive pushes of a small projectile (say, a sagion) generates an acceleration curve resembling Einstein's mass increase, thus giving a different interpretation to some claims of special theory of relativity (STR); in particular, Bertozzi experiment is explained as an inefficient transfer of linear momentum, and the fitting of Bertozzi data by neo-Cartesian predictions is superior to STR's predictions.

Keywords: Unified fluid theory, Unified field theory, Kinematic particle model, Particle creation, Force unification, De Broglie quantum mechanics, Relativistic quantum mechanics, Quantized gravity, Relativistic mass increase, Bertozzi experiment

1. On Continuity, Field, Force and Extension

The proceedings of the past IX Vigier Symposium [1, p. vii] begin with an epigraph quoting Einstein [2]: *I consider it quite possible that physics cannot be based on the field concept, i.e., on continuous structures. In that case, nothing remains of my entire castle in the air, gravitation theory included*, [and of] *the rest of modern physics*. The third quotation in the same epigraph belongs to Feynman's Nobel Prize lecture: *Current fashion* [is] *field theory ... the chance is high that the truth lies in the fashionable direction. But, on the off chance that it is in another direction ... who will find it? Only someone who sacrifices himself ... from a peculiar and unusual point of view, one may have to invent for himself* (underlining added here).

1.1. *On Continuity and Discreteness*

Regarding continuity, it may be noted at once that the whole natural world is formed by discrete objects, arranged in non-continuous structures, so that, if the field concept implies continuity as claimed by Einstein, it never had the slightest chance of becoming the final theory of nature. In this context, any continuous mathematical equation is a first order representation of the behavior of a large collection of discrete objects. The apparent continuity in nature is a mere artifact of the distance d from which a detector (say, the naked human eye) observes a group of objects, each of size s. For instance, from a ten to twenty-kilometer distance one sees a forest as a continuous green spot (say, $d/s > 10^3$), while discrete features become manifest as one approaches the forest (say $10 < d/s < 100$), and at close distances ($d/s < 1$) one sees the trees but not the forest (see fig. 1). This artifactual dichotomy tree-forest, may also help understand the long-standing controversy regarding the wave-particle issue: a wave, as in the sea, is the collective motion of myriads of individual water molecules. Needless to say, water appears continuous to the naked eye because d/s is very large.

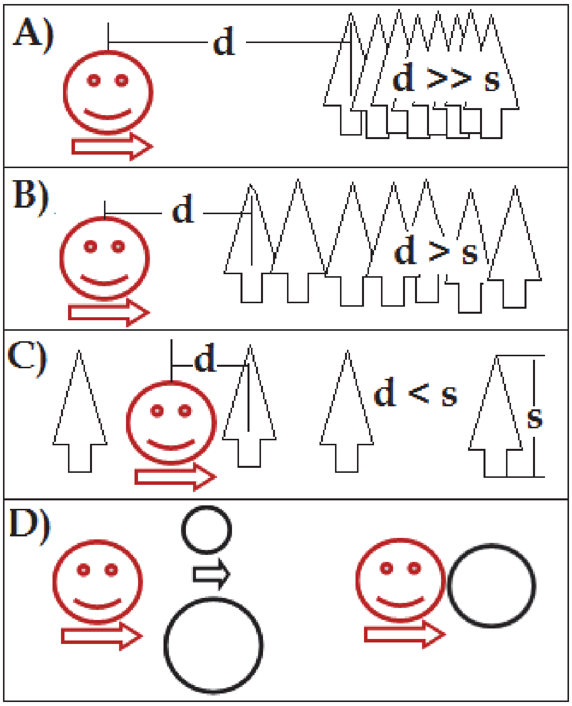

Figure 1. Linear momentum (the arrow under the observer's face) is transferred to the system being observed. Panels A, B, C sketch observation of a pine forest from different distances *d*. Panel D is a cartoon with two instances of detection by collision in the microworld (see text for details).

Another instance of artifactual continuity appears even in commensurate situations, i.e. $d/s \sim 1$, when a fast moving object, say a dumbbell rotating with high frequency around an axis perpendicular to the rod, is observed with a detector of relatively poor temporal resolution, say the human eye. In this example, the dumbbell is seen as a continuous torus, but a high-speed camera brings out the discrete structure.

It is well known that quantum mechanics (QM) was formulated to explain phenomena at the atomic scale that were beyond classical physics at the turn of the 20th century. As a result, quantization is usually considered to be a property of the microscopic world: *discontinuous changes characteristic of atomic processes* [3, p. 3]. However, the atomic-like structure of our solar system was noted by Titius and Bode in the 18th century, and at the laboratory scale quantum-like phenomena in fluids were observed quite recently [4, 5, 6]. So, it seems rather that discrete structures and phenomena may be associated with a small number of objects seen from a near-range, or very-close range as defined by small d/s. Any process of measurement involves an exchange of linear momentum, which in the case of the forest occurs as reflection of sunlight that eventually reaches the observer's eyes; linear momentum (qualitatively represented by arrows in fig. 1) is usually negligible in macroscopic measurements. Contrariwise at the microscopic scale (panel D, fig. 1), the linear momentum associated with detection may be larger than the linear momentum of the particle being observed, leading to a large disruption in the system undergoing observation.

Obviously, the behavior of the observed system is not the same after being observed, but it does not follow that it is impossible to predict the evolution of an *identical unperturbed system*. A limiting case involving two identical particles is illustrated on the right side of panel D: the observed particle, originally at rest, exchanges linear momentum with the detector, and moves after the collision to the right carrying the detector's original linear momentum, while the detector is now at rest. A QM-experimenter might interpret the observation as quantum tunneling, with the detector traversing the target without being affected! However, the classical mechanics interpretation is that the target particle was initially at rest, and that it will continue the same while unperturbed.

1.2. *On Field, Force and Discreteness*

This paper is a continuation of the exposition of a classical unified fluid theory, intended to be applicable to all interactions in nature [7, 8]. Our use of the word "field" is not the same as Einstein's; rather "field" is a short-name or alias for the collective behavior of an aether fluid formed by a *very large ensemble* of discrete individual energy-like objects called sagions. The relevance of the "very large ensemble" condition (many times the Avogadro's number) cannot be under-estimated. Thirty years ago this writer noted that another continuous equation, the Poisson equation, did not accurately represent static discrete arrangements for a small number N < 20 charges on the surface of a sphere; in that case a simple classical discrete calculation exhibited quantum like-effects [9, 10]. Our opinion regarding the approximate status of continuous equations to represent discrete systems runs counter to the received view. For example, in his 1929 lectures at the University of Chicago, Heisenberg acknowledged *the success of the classical mechanics in explaining the Wilson photographs ... Nevertheless, one can regard* *these achievements of classical theories only as a proof of the similarity of the classical and quantum theories, in the sense of the correspondence principle; for the answer to all quantitative questions an appeal must be made to the exact quantum theory* [3, p. 161], underlining added. That is, according to Heisenberg, the continuous Schrödinger equation provides an exact representation for all discrete atomic processes. However, to attain such feat *ad hoc* quantization rules were imposed by Bohr right at the inception of QM, and, when the need arose at a later time, by Hartree [3, p. 164].

Rather than inventing something new as suggested by Feynman in the epigraph quoted above, our theory is based on the completely *unfashionable* return to the roots of classical mechanics. Newton was a convinced atomist that believed in absolute time and three-dimensional space [11]; we concur, but we retrocede further back to the twin notions of physical and mathematical absolute space propounded by Patrizi [12] in Ferrara one hundred years before Newton's *Principia*. Both Patrizi and Newton considered that physical objects occupy a finite portion of absolute space Σ, so that there was a clear distinction between the homogeneous and continuous geometrical space Σ, and the discrete physical objects contained therein. Such distinction is notoriously absent in 20th century physics.

The shortcomings of Newtonian mechanics are well-known. Let us mention two of them: the faulty circular definition of mass in the first line of first page of the *Principia* [13; 14, p. 329], and the absence in the *Principia* of a mechanism for the generation and propagation of gravitational force. From philosophical considerations on causality, the latter was immediately criticized by the disciples of René Descartes. About the same time, Bentley, a dedicated follower of Newton, misinterpreted Newton on two accounts: gravity is inherent to matter, and gravity acts at a distance; both of them were strongly resisted by Newton, as in his third letter to Bentley [15, p. 54]: *That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws, but whether this agent is material or immaterial I have left to the consideration of the readers* (underlining added). Misconceptions never disappeared. On the contrary, around 1860 Boscovich [16], who also was a convinced Newtonian, based his physics on the existence of a unique force emanating from punctual centres of force, and acting at a distance. Both misconceptions are still alive, even in 20th century textbooks. As an anecdote, the present writer was taught that the gravitational field is associated with gravitational masses entering Newton's universal law of gravity. In the limit, when the distance between the two small masses tends to zero, both force and potential energy blow up to infinity. If one recalls Einstein's special theory of relativity, a contradiction arises because the energy associated with mass should be finite; all this resulted in a paper trying to solve the contradiction [17]. My paper would had been different if back in 1971 I had been aware of Newton's letters to Bentley.

Paying attention to Newton's words, and to the ensuing developments by Fatio and Lesage [18], the present writer postulated an energy-like aether as the main pillar for a unified fluid theory, and developed its properties along the lines suggested by Lesage [19, 20].

Returning to Newtonian mass, if it is ill-defined, then Newton's second law becomes also suspect. A simple solution is to derive classical mechanics from the principle of conservation of linear momentum, rather than from Newton's three laws. Such approach, rooted in Cartesian thinking, is superior because it is by far more economical on fundamental principles [21, 22, 23]; a self-consistent scale for mass may be easily constructed by observing the collision of a body against a reference mass, as described in [24, ch. 4]. Such approach implements a kinematic definition of mass proposed by Barré de Saint Venant in 1845 [25, pp. 89-91], [26, pp. 216-217].

In the Cartesian approach force is not a primitive notion; the same is true in Einstein's general theory of relativity. Then, force merely is a convenient name for the average exchange of linear momentum in a contact collision between two discrete objects [27], and may be also viewed as a current or flow of linear momentum [28, pp. 35-52], [29].

1.3. *On Extension, Atomism and Discreteness*

Our theory is also Cartesian in the sense of a close scrutiny of received views, and of a rigorous adherence to logical rules, including the principle of continuity proposed by Leibniz in the 17th century [30, p. 293], and defended by Boscovich in the 18th century [31, p. 390]. However, this writer does not agree with Descartes on two issues. Firstly, helical or swirl structures, similar to hurricanes and tornadoes, may form in the aether, but there may exist many other temporary and permanent structures. Secondly, and more important, Descartes was not an atomist, he considered that his ether was a plenum, that could be indefinitely divided [32, ch. 4].

On the contrary, the present writer is, as Newton, a convinced atomist.

Daily experience indicates that matter occupies a finite volume in Σ, that matter is soft, that matter has inner structure, and that matter is deformable. It is our strong opinion that there are no compelling philosophical reasons suggesting that matter at scales bigger or smaller than the human scale may be characterized by properties differing from daily experience. Hence, the smallest bit of matter must *a fortiori* have a finite 3D-volume, and must have internal structure capable of deformation. A huge bonus on logical consistency ensues: since matter is deformable at all scales, a collision between material objects never violates Leibnitzian continuity. For Descartes, matter was infinitely divisible, for us it is not; this is why our atomistic theory is neo-Cartesian, rather than simply Cartesian.

Quite obviously, if the smallest bit of matter has parts, it may be further divided into its components, which cannot be material by definition (for otherwise the smallest bit of matter would not be the smallest). This explains why we postulated that the simplest object in Nature — the elementary component of aether — is the sagion described as an *energy-like*, 3D-extended structureless spherical rotating object permanently moving in Σ with a high-speed $\mathcal{C}$, of the same order of magnitude as the speed of light c. Then, a sagion is described by only four parameters (script letters): linear momentum $\mathcal{P}$, speed $\mathcal{C}$, diameter $\mathcal{D}$, and inherent spin $\mathcal{S}$, whose positive (negative) sign results from counter-clockwise rotation, CCW (clockwise rotation, CW). Note that neither force, nor charge appear at the fundamental sagion level.

In the 1920s during the early development of quantum theory, particles were assumed to be geometrical points endowed with physical properties as mass and spin, in particular, it was claimed that spin was a unique quantum feature. Even today, the electron is still considered in mainstream physics as a punctual structureless object. Such naive model led to physical inconsistencies (as infinite mass density), disguised under the neutral name of singularity, and "removed" by the questionable procedure of renormalization. On the contrary, in our theory matter is extended and has structure at all scales, so that the spin of earth, the spin of a rotating top, and the spin of an electron all of them have the same classical explanation and origin.

Let us recall that Newton demonstrated in the *Principia* that for calculations involving translation, the mass of an extended 3D-body may be treated *as if* it were concentrated at its center of mass (CM). Likewise, the gravitational attraction exerted by a homogeneous spherical body of mass M and radius R at distance $r > R$ may be calculated *as if* a punctual mass M were placed at the center of the sphere. The latter theorem was demonstrated by Newton in 1685, and it provided the mathematical support for Newton's treatment of extended bodies as punctual particles. This very importance result led Kuhn to conjecture that Newton delayed publication of the *Principia* until he had demonstrated that theorem to his satisfaction [33, p. 258]. Unfortunately, the creators of QM forgot the "as if" part, forgot that the extended object was still there occupying a 3D-volume around the CM, and forgot that a calculation of spin actually requires the details of the geometrical extended configuration. Singularities automatically disappear by re-introducing extension in physical theory.

Let us close this lengthy introduction recalling some relevant steps in my fifty-year personal voyage. While attending a colloquium at the ICTP in Trieste in the summer of 1971 this writer became aware of the internal contradictions built in the conventional concept of point particles. I have been ruminating, ever since, over the philosophical bases of both QM and Einstein's special theory of relativity (STR). A relevant preliminary question was: Is probability a physical object? No. The only species in nature that makes conscious conjectures about future events is humankind. Since the rest of Nature does not make conjectures, probability is a mere construct of the human mind. Then, QM cannot possibly be a theory about the natural world, but merely a model about how some human beings perceive nature. My own view of probability is causal as in Laplace [34], and is close to Feynman's path integrals, but it evolved in the context of nuclear power risk studies as a manner to quantify probability in non-repetitive large scale events [35, 36].

During the second half of past century there was a claim that empirical evidence demonstrated that the QM-view of Nature was right, while the classical view was not. After checking Bell's theorem, I was not convinced by the troubling claims that classical physics was dead [37]. My own conclusion was the the only logically acceptable version of QM is the stochastic interpretation [38], which is just an approximate but very useful methodology to predict experimental outcomes; however, it cannot possibly be a final theory of nature. This writer completely shares Chebotarev's opinion [38, p. 1]: *In the 70 years following the advent of quantum theory ... it has not been possible to achieve a satisfactory understanding of the fundamental physics underlying the mathematical scheme of quantum mechanics ... nor is there a satisfactory answer to the question of the physical nature of the wave function.*

My next question was: what is the evidence against the aether and absolute space? On the experimental side the Michelson and Morley experiment (MMX), and on the theoretical side Einstein's STR. It is well known in some quarters that Einstein had to reintroduce aether and/or absolute space in the context of his general theory of relativity (GTR) [39]. In the current revival of aether — disguised as dark matter, as "physical vacuum", or as zero-point field ZPF — there are some brave efforts to clear Einstein of any guilt in the killing of aether a hundred years ago: *Quite undeservedly, the ether has acquired a bad name. There is a myth, repeated in many popular presentations and textbooks, that Albert Einstein swept it into the dustbin of history* [40]. However, Dirac and Heisenberg had a different perception about Einstein's role; ninety years ago, Heisenberg father of quantum mechanical uncertainty wrote [3, p. 63]: *Faraday and Maxwell explained electromagnetic phenomena as the stresses and strains of an ether, but with the advent of the relativity theory, this ether was dematerialized; the electromagnetic field could still be represented as a set of vectors in space-time* (underlining added).

Since final arbiter in natural science is experiment, rather than theory, present writer checked all MMX from 1881 to 1930, period of inception of both QM and STR; results tended to be positive, rather than negative [41]. So, it was decided to repeat the MMX using laser light, and we calculated beforehand the expected outcomes assuming that light travels with constant speed relative to a preferred frame [42]; our two-year experiment was consistent with predictions [43-47]. Since my own experiment is consistent with motion of earth relative to a preferred frame, the present writer confidently feels that both absolute space and aether may be reinstated [48], thus returning to the Greek, Cartesian and Newtonian roots of logic, atomism, and classical physics.

Unaware of Feynman's challenge to find a unfashionable alternative theory, I slowly followed that program over decades. My only "new" contribution is to explicitly acknowledge that all bodies in nature have 3D-extension, contain a finite amount of energy, and carry finite linear momenta; of course, it also applies to the smallest bit of matter, and to the sagion — modern version of Democritus atom. Our intuitive theory contrasts with the fashionable model of punctual mathematical particles endowed with physical properties, as energy, linear momentum, mass, and spin, model that is logically inconsistent *ab initio*.

2. Reinstatement of Aether

In 1938 Einstein, Infeld, and Hoffmann [49] revisisted the link between GTR and the classical equations of motion, summarized by Jammer [26, pp. 262-264]: *the nonlinear character of the field equations in general relativity ... made it possible to deduce the dynamical law from the field equations ... the approximation method employed by Einstein, Infeld, and Hoffman applies only to the case of slowly varying fields ... a unified field theory that subjects electromagnetic and possibly also nuclear forces to a similar treatment as gravitation, then it would lead us to a final stage in the history of the concept of force ... classical mechanics still admitted, tolerantly ... force as a methodological intermediate, the theory of fields would have to banish it even from this humble position* (emphases added). Note that Einstein and co-authors were talking, of course, about classical Newtonian mechanics, not about Cartesian mechanics (where force does not exist, see section 1). The remainder of the quotation is a good summary of the intentions of the present paper.

2.1. *Sagion Aether as a Classical Fluid Equation*

Let us postulate that the whole absolute space Σ is populated by a fluid aether formed by discrete energy-like sagions. In intergalactic space and other regions free of matter, the collective fluid obeys conservation of total energy, and conservation of linear momentum as given by [7, 8]

$$\square\Psi = 0, \square\boldsymbol{\Psi} = 0, \square \equiv \frac{\partial^2}{\partial w^2} - \nabla^2, w \equiv Ct \qquad (1)$$

In the scalar equation, Ψ is content of energy in the aether per unit three-dimensional volume in Σ, and in the vector equation, $\boldsymbol{\Psi}$ is flow of linear momentum carried by sagions per unit area. The D'Alembertian operator is $\square$, and time *t* is given by *w* in length dimensions, where C is the average local speed of sagions. In the presence of matter the right-hand side is non-zero due to inelastic interaction of sagions with matter, and to sagion-matter interconversion. The classical equations of fluids are often formulated as vector equations [50], rather than wave Eqs. (1), but they are equivalent [7, 8].

Several remarks are in order:

1. Classical fluid equations are non-linear on both time and space, and thus consistent with GTR.
2. The non-objective concept of probability does not appear in the field equations. The pair of field variables (Ψ, $\boldsymbol{\Psi}$) represent physical properties of an objective fluid formed by discrete sagions.
3. Continuous field Eqs. (1) represent the collective behaviour of zillions of sagions, and are not appropriate to represent the local interaction of a small number of sagions, which must be treated using the methods of discrete classical mechanics, as in [9] and section 4 below.

4. Our theory is causal in the strong classical sense — the same underlying Laplacian probability [34] — but it does not follow that the future may be easily predicted. Uncertainty arises because initial conditions are not accurately known, or because the process of measurement disturbs the observed system (section 1 above). Additionally, there is a novel *inherent uncertainty*, resulting from the *logical* impossibility to measure lengths with accuracy better than $\mathcal{D}$. For, if there exist rulers smaller than $\mathcal{D}$, then the sagion is not the smallest object in nature. For a similar reason, time cannot be measured with accuracy better than $\mathcal{T}$.
5. As a consequence, in our theory derivatives are not defined as in the usual mathematical non-discrete approach, but are physical derivatives defined for any physical magnitude W as [7]:

$$\left(\frac{dW}{ds}\right)_{phys} \equiv \lim_{\Delta s \to \mathcal{D}} \left(\frac{\Delta W}{\Delta x}\right), s = x, y, z, r$$
$$\left(\frac{dW}{dt}\right)_{phys} \equiv \lim_{\Delta t \to \mathcal{T}} \left(\frac{\Delta W}{\Delta t}\right), \quad \mathcal{T} \equiv \mathcal{D}/\mathcal{C} \tag{2}$$

6. Poincaré demonstrated the Lorentz invariance of Eq. (1) independently and before Einstein's STR [51-54]. This implies that speed in Poincaré's transformations is relative to absolute space Σ, and is completely compatible with aether and Eqs. (1).

The one-dimensional version of Eq. (1) is the well-known travelling wave equation. In the 19th century propagation of sound was described by the classical wave equation with C being the speed of sound. For time-independent, static and stationary problems Eq. (1) reduces to Laplace's equation, used in 1787 by the Marquis of Laplace to study the rings of Saturn. Hence, all physical problems described by *1/r*-static potentials (both in gravity and electromagnetism) are consistent with the version of aether propounded here. A related case is Yukawa's static potential, derived from the non-homogeneous wave equation [55, pp. 748-750]. Other connections of Eqs. (1) with gravity, electrodynamics, and QM are listed next.

2.2. *Classical EM Theory and Fluid Equations*

Maxwell developed his electromagnetic (EM) theory guided by the transport of fluids [56, 57]. It is rather curious that the electromagnetic field due to electrons may be expressed as two scalar potentials F and G, both obeying Eq. (1) [58].

Maxwell's equations may be reduced to non-homogeneous wave equations in terms of (**E**, **B**) [59, section 7.1], or in terms of a pair of scalar and vector potentials (φ, $\boldsymbol{A}$) [59, sections 6.4-5]; the standard procedure introduces the Coulomb or transversal gauge $\nabla \cdot \boldsymbol{A} = 0$ as an additional constraint. In the vacuum, electric charge density and electric current density are zero, thus obtaining the homogeneous wave Eqs. (1). Present writer reached similar wave equations from his symmetrical Maxwell's equations, without imposing the transversal gauge [60, p. 2095], which suggests that transversality is not an intrinsic trait of Maxwell's equations, as conventionally believed; indeed, the mathematical existence of simple longitudinal solutions compatible with Maxwell's equations may be checked by direct substitution [61]. Derivation of wave equations does not require all four Maxwell's equations; this is easy to see in our symmetrical version [60], thus suggesting that the pair (**E**, **B**) contains redundant information, and pointing toward existence of more fundamental and simpler physical processes. The claim here is that such physical reality is the aether fluid described by Eqs. (1).

The opposite approach is to explicitly derive Maxwell's equations from the wave equations of the aether fluid, Eqs. (1). This was done in France in 1926 by Henri Malet [62], who defined

$$\phi = \rho C^2, \mathbf{A} = \rho C \mathbf{V} \tag{3}$$

The scalar potential φ is the average kinetic energy density carried by the fluid, and the vector potential **A** is the linear momentum density transported by the convection of a fluid moving with velocity **V**. These definitions are similar to the meaning attached to (Ψ, $\boldsymbol{\Psi}$) in Eqs. (1). Unaware of Malet's prior work, the present writer re-discovered a similar derivation of Maxwell's equations [7, 8, 63, 64] from the vector fluid equations for the particular case of incompressible and constant density aether [63, p. 73]. By the same epoch, at least three other authors independently rediscovered Malet's ideas [65-67]. Since Maxwell's equations are a special case of fluid theory, a richer electrodynamics may be based on the pair of potentials (φ, **A**) = (Ψ, $\boldsymbol{\Psi}$).

However, it is not usually realized that Maxwell's equations only apply to absolute space, or to systems at rest therein. This explains the absence in Maxwell's equations of the speed of the laboratory relative to Σ, and justifies (at least in part) Maxwell's use of partial derivatives rather than total derivatives as it should be in a complete theory. Both before and after Maxwell, other electrodynamic theories were proposed in France and Germany: Ampère (1823) [68], Gauss (1835), Grassman (1845), Neumann (1845), Weber (1846), Helmholtz (1873), and Riemann (1875); for references see [69].

Some theories are based on the Newtonian concept of force which some authors [69, 70] consider superior to the field concept (the latter implicit in Maxwellian and Einsteinian approaches). Velocity of laboratory or detector explicitly appears in Weber's force revived by Assis [71], and in the neo-Hertzian field theory revived by Phipps [72, 73, pp. 17-67], which is Galilean invariant, and may be derived from a fluid equation similar to Eq. (1), the only difference is that time is the proper time of the detector [72, p. 78].

In the 20th century Jefimenko formulated a self-consistent causal theory applicable to both EM and gravity [74, 75]. Jefimenko notes that a "sourceless" homogeneous wave equation cannot originate EM fields [75, p. 8 & 17]. There is no contradiction with our Eq. (1) describing transport of linear momentum and energy in fluid aether, rather than "force" which indeed requires the presence of matter on the right-hand side of the wave equation, and leads to a force denstiy already defined in [7, p. 253], [63, p. 73]; recall that "force" is just a name for sagion-matter interactions.

Finally, many EM phenomena cannot be explained by Maxwell's theory. Examples are the experiments by Tesla at the end of 19th century, and by Graneau in the 1980s, see references in [70]. The observed apparent "excess energy" may be related to the unaccounted motion of earth relative to Σ.

2.3. *Gravity and the Fluid Equations*

The similitude of Newton's and Coulomb's laws led Maxwell to explore possible connections of gravity and ether, but he gave up noting that undisturbed aether had *an enormous intrinsic energy ... the presence of dense bodies influences the medium so as to diminish this energy wherever there is a resultant attraction* [57, pp. 492-493], i.e., according to Maxwell, material bodies extracted energy from the aether in the process of gravitational interaction. Such finding is consistent with Le Sage's attenuation of the aether field [18, 19], which manifests as gravity.

More than twenty years before Einstein's GTR, Oliver Heaviside described in 1893 a gravito-magnetic field [76], which explained precession of Mercury's perihelion. Heaviside's equations were analogous to Maxwell's equations, and thus equivalent to Eqs. (1) as noted in previous subsection. Jefimenko's causal EM and gravitational theory [75] is similar to Heaviside's.

In 1903 Whittaker considered [77, p. 355] *gravitation and electrostatic attraction explained as modes of wave-disturbance ... gravitational force in each constituent field will be perpendicular to the wave-front, i.e. the waves will be longitudinal ... this undulatory theory of gravity ... propagated with a finite velocity,... need not be the same as that of light, and may be enormously greater* (emphasis added). Such notions are contrary to the usual view that Maxwell's equations only have transversal solutions; this author [61] agrees with Whittaker.

A connection between Einstein's GTR and the fluid equations is established in [7, pp. 252-254], [8], but GTR is additionally constrained by the equivalence principle — made by Einstein a postulate of his theory. In contrast, for Newton equivalence between gravitational and inertial mass was an empirical question, amenable to experimental testing. For Einstein, gravitation is independent of chemical composition of matter (i.e. of atomic number Z of its constituents), while electromagnetic, atomic and nuclear interactions depend on Z [55]. No wonder that Einstein never succeeded in his dream for unification! Another question arises: Is Einstein's equivalence principle right? Two experiments violating such principle are briefly described next.

1. The Eötvös, Pekár and Fekete (EPF) experiment. Torsion balances were used in Hungary by EPF to compare earth's attraction on materials of different Z; two series were carried out with copper (Cu, Z =29) and platinum (Pt, Z = 78) as reference materials [78; 79, pp. 130-136].

According to mainstream views, the EPF experiment supports Einstein's principle of equivalence. There is some counter-evidence. Thirty years ago Fischbach and co-workers suggested the existence of a fifth-force, while searching for empirical support they revisited the EPF experiment. Fischbach identified a variation of Newtonian gravity with the nuclear structure of the interacting bodies: baryons (panel A of fig. 2), and isospin (in panel B). Many experiments looked for a Yukawa-type fifth-force, similar to the exponential part in Eqs. (6) and (8) below; however, not all experiments in [79] included in the interpretation the Z-dependence, contained take in gravitational mass, and build up of eqs. (6) and (8). Fischbach did not find enough support for his form of fifth-force, and abandoned the search by the end of 20th century [79].

However, Nieto, Hughes and Goldman noted in 1989 that *even though the original analysis of Fischbach et al. had been corrected and their proposed coupling to hyper charge is ruled out ..., the correlation with baryon number is present in the Eötvös data*" (emphasis in the original) [80]. The unavoidable conclusion is that Einstein's equivalence principle did not have empirical support when it was formulated a century ago.

This writer also revisited the EPF by himself, and identified two groups of data, clearly associated with atomic number of the reference substance Cu or Pt (see fig. 2, panels A & B). We attributed this effect to the different <u>atomic</u> structure of Cu and Pt [19, 81]. On his part, Fischbach ignored the orbital electrons, and attributed the observed effects to the structure of the <u>nucleus</u> only [79].

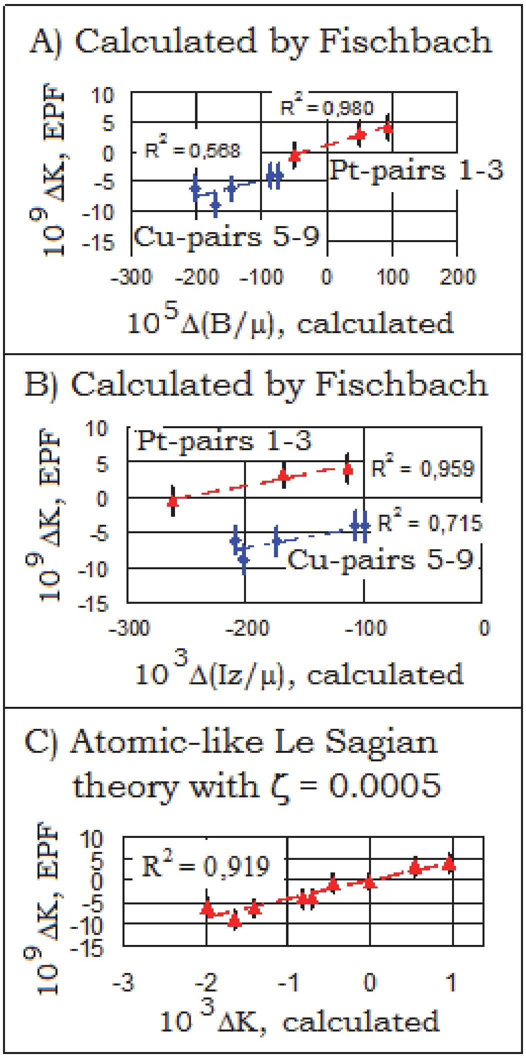

Figure 2. Panels A & B: re-analysis of EPF experiment show dependence of gravitational interaction (y-axis) with baryonic composition of matter (x-axis in panel A), and nuclear isospin (x-axis in panel B) [79]. Panel C: a Le Sagian model leads to high correlation of Z (horizontal axis) with all 9 pairs of data in EPF experiment (y-axis) [19].

In our atomic-like Lesagian model for generation and propagation of gravity [19, 81], the small mass of the Z-orbital electrons, is taken into account by the cross-section of the sagion-electron interaction σ_E, while the baryons in the nucleus interact according to the proton-sagion and neutron-sagion cross sections (σ_P & σ_N). The atomic cross section $\sigma(A,Z)$, for a nuclide formed by Z protons and $(A-Z)$ neutrons is

$$\begin{aligned} &\sigma(A,Z) = Z\sigma_P + (A-Z)\sigma_N + Z\sigma_E, \\ &\frac{\sigma(A,Z)}{\sigma_P} \cong A + \zeta Z, \ \text{ for } \ \sigma_P \approx \sigma_N, \ \zeta = \frac{\sigma_E}{\sigma_P} \end{aligned} \quad (4)$$

The electron-proton cross-section ratio ζ was treated as a free parameter. Its value was varied to optimize the correlation between EPF data (y-axis in panel C) and our model based on Eq. (4), x-axis in panel C. Highest correlations attained for $0.0005 < \zeta < 0.001$. As intuitively expected, the electron/proton mass ratio (1/1836) is in that range. That is, EPF's data supports our Le Sagian model, which implies that gravity also depends on *Z*. So there is no fundamental difference between gravity and the other three "forces" in nature.

2. Majorana's gravity attenuation. Reports on the possible absorption of gravity appeared at least since 1897, which led Quirino Majorana to start in 1918 his own well-designed experiments at the Polytechnic Institute of Turin, continued after 1922 at University of Bologna [82-85]. Contrary to Newton's warnings noted in section 1, Majorana treated gravity as an inherent property of matter such that *the force of gravitation could be explained by a kind of energical flux, continually <u>emanating</u> from ponderable matter* [83, p. 489] (underlining added), and argued that a spherical body of radius r and density δ has apparent mass M_A and true mass M (M_V in his notation) related by

$$M_A = M\exp(-h\delta r) \quad (5)$$

where h is a new constant of nature. According to Majorana, the mass to enter Newton's gravitational law should be M_A rather than M so that *Newton's Law would only be exact in the first approximation* [83, p. 490]. Although Eq. (6) does not appear in Majorana's papers [82-84], he was suggesting that

$$\begin{aligned} F &= G\frac{M_{A,1}M_{A,2}}{r^2}\exp\left(-\int h\delta dr\right) \\ F &= G\frac{M_1 M_2}{r^2}\exp\left(-\int h\delta_1 dr - \int h\delta_2 dr - \int h\delta dr\right) \end{aligned} \quad (6)$$

where δ_1, δ_2 are the densities inside the interacting bodies 1 and 2, and δ is the density of external matter along the line joining the centers of mass of bodies 1 and 2; the integrals are taken over the appropriate distances. In the context of this paper, the apparent mass M_A is gravitational mass, while true mass M is inertial mass. In his critical article, Russell wrote Eq. (6) as in the second line, and (correctly?) associated it *with the "law* *of progressive absorption" which holds good for radiation* [86, p. 334].

Majorana designed the experiment to test for the attenuation of gravity by a high Z-material between M_1 (the earth) and M_2 (a test ball), according to first line in Eq. (6). Gravitational force F manifests as weight W measured by a laboratory balance, so that

$$\frac{W(Z)}{W(air)} = \frac{F(Z)}{F(air)} = \frac{\exp\left(-\int h(Z)\delta(Z)dr\right)}{\exp\left(-\int h(air)\delta(air)dr\right)}, \quad (7)$$

$$\text{where } \int h(air)\delta(air)dr \approx 0$$

Expansion of the exponential on the right-hand side immediately allows calculation of $h(Z)$. Majorana used a modified Ruprecht laboratory balance in vacuum to weigh a small leaden sphere (3 cm radius, 1.274 kg mass). In a given experimental session two alternating measurements were carried out: series 1, with test mass in free air, and series 2, with test mass surrounded by a high *Z* material. First experiment began in Turin in April 1918, and included the day of the general strike (July that year), which provided a particularly quiet environment, free of vibrations. The shielding material was a cylinder (height: 22 cm, diameter: 22 cm), with a capacity of 109.6 kg of mercury (Hg, Z = 80) when full. Majorana reports 104 kg, and that the center of mass (CM) of the test ball coincided with the CM of Hg, but it is not clear whether he referred to a full or to a partially filled cylinder. Each series produced a curve of weight versus time of day, the two curves were parallel, with series 2 consistently below series 1 [82, p. 92; 83, p. 499], from which Majorana calculated h(Hg) = 6.73 x 10^{-12} cm^2/g [83, p. 502].

In a second experiment the same leaden ball was surrounded by a massive lead cube (9,603 kg, side 95 cm) [84; 85, p. 28]. After experiencing operational difficulties with the heavy shielding, Majorana carried out a third experiment with a small lead attenuator (180 kg) [85, p. 30]. Error analysis for Hg experiment is fully described [82, 83]; for Pb experiments it is omitted in [82-85]. The first part of table 1 summarizes findings reported in [82-85]; second part is our recalculation that includes details left out by Majorana. Our values for $h(Z)$ do not change substantially, confirming two fundamental facts: $h(Z) \neq 0$, and $h(\text{Hg}) \neq h(\text{Pb})$.

Table 1. Gravity Attenuation by Hg and Pb

Exp.	*r*, cm	*ΔW*, μg	*h*(*Z*), cm²/g	Reference
1/Hg	8.4*	0.97	6.73 E-12	[83, p. 502]
2/Pb	47.5&	1.72	2.5 E-12	[84, p. 478]
2/Pb	47.5&	1.57	2.28 E-12	[85, p. 28]
3/Pb	12.5#&	0.51	2.8 E-12	[85, p. 30]
Average *h*(Pb) =			2.5 E-12	
1/Hg	6.51§u	0.97	8.63E-12	[83, p. 498]
2/Pb	43.5§	~2	3.18E-12	[84, p. 478]
3/Pb	8.5#§	0.51	4.12E-12	[85, p. 30]
Average *h*(Pb) =			3.65E-12	

*Value is not clear. & Point Pb-ball. # Cubic Pb shield assumed. §Subtracting ball cavity (radius: 3.95 cm). u Unfilled Hg cylinder; Pb-ball at CM of Hg (104 kg).

No much attention should be paid to the relative values of h(Hg) and h(Pb); two reasons: (a) shielding geometries are different (cylindrical versus cubical), (b) possible assymmetry in position of Pb-ball when Hg cylinder is partially filled. Majorana expected $h = h(\text{Pb}) = h(\text{Hg})$, and noted that he could not tell whether there was an error in his first experiment, or whether there was a defect in his theory. Majorana's truly scientific attitude must be praised: *the experimental research represents the real foundation of science, and its results are facts which in any case enrich our patrimony of scientific knowledge. As to my researches, one can leave aside the a priori theories I proposed* [85, p. 28]. Since Majorana was a good experimenter, the failure surely was in his theory as explained next.

Obviously, this writer i looking in retrospect, and has the benefit of current knowledge on the interaction of radiation with matter. Neither Majorana, nor Russell had to know in 1920 that interaction of EM radiation with matter is a complex phenomenon. Two separate questions merit attention. Firstly, no reason for Majorana's coefficient h to be a constant of nature. Secondly, besides gravitational absorption there is also scattering, leading to the little known phenomenon of build-up, which manifests in thick attenuators. Interaction of photons with matter is described by the attenuation coefficient $h(Z, E_0)$ which has contributions of four main microscopic processes: photoelectric absorption, Rayleigh scattering, Compton scattering and absorption, and absorption by pair-production (i.e. matter-antimatter pairs of particles). All mechanisms depend on both Z and the energy of the photon E_0 [55, 87]. So, in the case of gravity, there is no contradiction in having h(Hg) different of h(Pb), observation that puzzled Majorana [84, p. 478].

Consider a flux I_0 of monoenergetic photons E_0 per unit area and unit time traversing matter of thickness r, density δ, and linear attenuation coefficient μ. A flux $I(E)$ of photons appears at other side of the material:

$$I(E) = B(E_0, Z, h\delta r) I_0 \exp(-h\delta r), \text{ where } h = \frac{\mu}{\delta} \quad (8)$$

$$\text{For } r \to 0: B(E_0, Z, h\delta r) \to 1, I(E) \to I(E_0)$$

There is a complex spatial and energy E distribution of photons at the other side of attenuating material. Eq. (8) is used for quick estimates in radiation protection, where B is called the build-up factor and represents the mixture of undisturbed photons carrying energy E_0, and scattered and re-emitted photons with energy $E < E_0$ travelling in directions non-parallel to the incoming photons. Factor B strongly depends on the thickness of material δr, geometry of attenuating material, and directional distribution of incoming photons [87]; for precise calculations MonteCarlo simulations are used.

Table 1 and Eqs. (8) yield a linear attenuation of gravity μ(Pb) = 4.1E-11 cm^{-1}, and the inverse mean free-path λ(Pb) = 2.4E+10 cm, which is similar to earth-moon distance (3.84E+10 cm). At laboratory scale, B factor should be unity, and only very small effects should be noticeable at the earth-moon scale, as in eclipses. Additionally, the average Z of moon and earth are much smaller than Z = 82 for lead, so that any expected effect is even smaller (see table 2).

In Majorana's papers [82-84], he only gave a couple of passing mentions to the Le Sagian approach, which underlies our theory, where gravity depends on the solutions (Ψ, $\mathbf{\Psi}$) to Eqs. (1). Qualitatively, gravity near surface of earth is due to the difference between the incoming flow of sagions $(\Psi, \mathbf{\Psi})_{IN}$ from above, and the ougoing flow of sagions from below $(\Psi, \mathbf{\Psi})_{OUT}$ (the latter smaller due to attenuation by earth). In the case of Majorana experiment there is attenuation with a high-Z absorber of incoming flow and/or outgoing flow. Let attenuator thickness be r_A and r_B (A = above, B = below), and let $f(h\delta r)$ be attenuation factor, then attenuated local terrestrial gravity acceleration g^* is proportional to

$$\begin{aligned} g^* &\propto (\Psi,\mathbf{\Psi})_{IN}\, f(h\delta r_A) - (\Psi,\mathbf{\Psi})_{OUT}\, f(h\delta r_B) \\ g^* &\propto \left[(\Psi,\mathbf{\Psi})_{IN} - (\Psi,\mathbf{\Psi})_{OUT}\right] f(h\delta r) = g f(h\delta r) \end{aligned} \tag{9}$$

In most Majorana experiments the test mass was symmetrically placed at the center of shielding, and the observed decrease in acceleration of gravity agrees with second line of Eq. (9). Majorana also mentions a few measurements with the test ball above the Pb cube, or below it [84, pp. 478-479]. Those results seem compatible with the first line in Eq. (9).

Table 2 illustrates relative attenuation of lead (Z_0 = 82) to several possible functional dependences of $h(Z)$ according to Z, Z^2 and Z^3. Majorana used $h = h$(Hg) as a constant to calculate gravitational self-absorption in the sun. Russell [86] used same h to evaluate self-absorption in other solar planets and moons, and concluded that the observed stability of our solar system requires h to be smaller by a multiplicative factor from 1/500 to 1/10,000 depending on the planetary bodies involved [86, p. 339], factors that are compatible with fourth column in table 2. Then, Majorana's observations are not incompatible with the stability of our solar system. Furthermore, existence of gravity attenuation implies gravitational self-attenuation; hence, the validity of Eq. (5) contradicting once again Einstein's principle of equivalence.

Table 2. Relative Attenuation h(Pb)/h(Z)

Substance	Z	Z0/Z	$(Z0/Z)^2$	$(Z0/Z)^3$
H	1	82	6,724	551,368
He	2	41	1,681	68,921
Li	3	27.3	747	20,421
Water	7.2*	11.4	129	1,464
Earth's crust	12.1*	6.8	46	311

*Average Z according to percentage of mass composition.

Although Russell criticized Majorana, he fairly wondered *what then becomes of Professor Majorana's long and careful series of experiments?* [86, p. 342], and ended his paper stating that *further evidence regarding the reality of the experimental effect appears to be urgently called for* [86, p. 346]. Sadly, in his April 1957 farewell papers Majorana was still pleading for an independent repetition of his experiments [88, pp. 397 & 402]; Majorana passed away on July 31/1957.

Geologists became interested in Majorana's absorption at least since the solar eclipse of 30 June 1954 [89]. Afterwards various groups have used sensitive gravimeters with the hope of observing a decrease of gravity coincident with maximum eclipse. During the solar eclipse of March 9/1997 a nice experiment was carried out at an isolated geophysical station in northern China; two significant anomalous valleys (about 6 and 7 μGal deep) were automatically recorded thirty minutes before first contact C1, and just after last contact C4. The authors suggested a *possible shielding effect of the Moon on the gravitational force of the Sun* [90, p. 041101-3], without explaining that it did not coincide with Majorana's expected absorption of gravity. They were immediately criticized in the same journal because *the expected shape of the signal in any reasonable model of shielding would be a bell-shaped curve* (underlining in the original) [91, p. 062002-2]. The critics are only correct regarding absorption of parallel radiation by a thin piece of matter; but obviously are unaware of a huge technical literature on the scattering of radiation, of which ref. [87] is just the elementary tip of the iceberg. Unfortunately, the Chinese group reconsidered their initial interpretation [92], and later invoked a *rapid air mass movement for the bulk of the atmosphere ... as a sufficient explanation ... of the anomaly* [93, p. 022002-1]. However, the validity of the latter explanation is quite controversial since it *presumes that air streams in from the surrounding area with speeds on the order of several hundred meter per second (imposing a hazard to airplanes flying at cruising altitudes during solar eclipses which has never been reported)* [94, p. 271]. Nonetheless, the related changes of pressure and temperature at totality are often invoked by other writers as sufficient explanation for eclipse gravity anomalies [95]. To avoid steep changes of temperature and pressure during totality, two possible locations to carry out gravity experiments during solar eclipses are: (1) Outside the totality band, at several hundred, even into the one to two thousand kilometers, and (2) The antieclipse band, antipodal to the optical shadow.

Within a unified theory of nature, gravity should exhibit interactions similar to photons, so that by analogy to gamma ray interaction with matter, the present writer interprets gravimeter results as due to scattering, reflection, and absorption on the surface and the in layers of the moon close to surface. Preliminary qualitative arguments indicate that if attenuation is dominated by scattering, the residual gravity curve may exhibit two lateral valleys, as effectively observed in at least six solar eclipses from 1954 to 1999, listed in [96, 97]. On the contrary, the mythical bell-shaped curve associated with pure gravitational absorption has never been observed.

However, the best way to (dis)confirm Majorana's findings a hundred years ago is to repeat his laboratory experiments using modern technology.

2.4. *Quantum Theory and Fluid Equations*

Old quantum theory began in 1913 from Bohr's ad hoc postulate *that an atomic system can exist in particular stationary or quantized states, each of which corresponds to a definite energy of the system. Transitions from one stationary state to another are accompanied by the gain or loss, as the case may be, of an amount of energy equal to the energy difference between the two states* [98, p. 4].

Schiff notes that the first candidate for a quantum equation was *the most familiar one-dimensional wave equation, that which describes the motion of transverse waves on a string or plane sound waves in a gas* [98, p. 21], which Schrödinger discarded for some properties of its harmonic solutions; he opted for a differential equation with a first-order derivative with respect to time. Such equation does not guarantee conservation, and requires that electrostatic, gravitational, and nuclear forces be included by hand as derivable from a real potential energy.

In our view, Schrödinger's choice constituted the crucial fork that led physics into the blind alley where it is now: gravity is not inherently-quantized, and quantum theory is not inherently-Lorentz-invariant, thus posing almost unsurmountable difficulties to the old dream of unification.

The connection between Schrödinger's equation and Madelung's fluid has been known since 1926 [38, pp. 4-8]. Due to some classical mechanical content, De Broglie and Vigier gave consideration to Madelung's equation, as attested in several articles collected on the occasion of Vigier's 80th birthday [38].

Present paper proposes to follow the branch that Schrödinger did not pursue in the early 1920s. That is, it is postulated that quantum phenomena are mere manifestations of an aether fluid described by a pair of scalar and vector homogeneous classical wave equations (HCWE), Eqs. (1). The scalar HCWE is a particular case of the Klein-Gordon equation (KGE) for a zero spin particle; properties of KGE are well known and appear in any textbook on relativistic quantum mechanics [99]. In the description of Dirac's relativistic quantum mechanics appear both the KGE for particles, and the HCWE to represent the aether (eqs. (21) in [100, p. 42]).

The new thing in this subject is our discovery of novel non-harmonic and non-dispersive solutions for the HCWE [101-103], inherently exhibit quantum-like structure, without invoking Bohr's *ad hoc* quantization postulate. Also, since solutions to the HCWE are Lorentz invariant, gravity and quantum theory are unified *ab initio*.

3. New Solutions of Classical Wave Equation

3.1. *Traditional Harmonic Solutions*

Generic solutions for Eq. (1) were obtained by Poisson around 1820, Kirchhoff by 1883, and Whittaker in 1903 [77]; note that solutions to Laplace's equation constitute particular solutions of Eq. (1). Consider Whittaker's general solution for the differential equation of wave motions, obtained as an extension of the general solution of the time-independent potential equation as Φ_0 given by the integral of an arbitrary function *f*. According to Whittaker *it is clear from the proof that no generality is lost by supposing that f is a periodic function of θ* [77, p. 333]:

$$\Phi_0 = \int_0^{2\pi} f\left(i\rho + z, \theta\right) d\theta, \rho = x\cos\theta + y\sin\theta \quad (10)$$

Whittaker's claim of generality is unacceptable, for the assumption of periodicity ignores all nonperiodic solutions of Φ_0. By the mid-twentieth century other authors softened Whittaker's claim thus: *in most practical cases the solution is expected to vary harmonically in time* [104]. Note that Eq. (10) implies a rotation of coordinates to the complex plane (z, $i\rho$), perpendicular to the X-Y plane, with ρ directed along angle θ on the X-Y plane.

In the same way, to obtain a general time-dependent solution for Eqs. (1) Whittaker projected z and ρ onto ray r directed at angle φ relative to the Z-axis, thus implicitly shifting to spherical coordinates (r, θ, φ) [77, p. 345]:

$$\Phi = \int_0^{2\pi} \int_0^{\pi} f\left(w + \rho \sin\phi + z\cos\phi, \phi, \theta\right) d\phi d\theta \quad (11)$$

The standard solution for the three-dimensional wave Eq. (1) in spherical coordinates (r, θ, φ) is of the form [104; pp. 372-381]:

$$\Psi(w,r,\theta,\phi) = T(w)R(r)Y(\theta,\phi;\ell,m),$$
$$\lambda = -\ell(\ell+1),\ \lambda_1 = -m^2, \quad (12)$$
$$\ell \geq m \text{ where } \ell, m = 0,1,2,...$$

In Eq. (12), the $Y(\theta,\varphi;\ell,m)$ are spherical harmonics, separation constants λ and λ_1 implicitly define quantum numbers ℓ and m with limitations set forth therein. Our new solutions were described already in [7, pp. 256-260], [101-103]; a quick summary follows, together with information not previously reported.

3.2. *Helicoidal Solutions*

A straightforward extension of eq. (12) is to allow for positive values of separation constant λ_1, without imposing the constraint that m must be an integer:

$$\Psi(w,r,\theta,\phi) = T(w)R(r)H(\theta,\phi),$$
$$\lambda = -\ell(\ell+1), \text{ where } \ell = 0,1,2,.., \quad (13)$$
$$\lambda_1 = +m^2 = -\mu^2,\ \mu = \pm im = \pm i\sqrt{\lambda_1}$$

The (new?) helicoidal solutions $H(\theta,\varphi)$ are given by

$$H(\theta,\phi) = F(\phi)P(\theta;\ell,\mu),$$
$$F(\phi) = A_1 + A_2\phi \text{ for } m = 0, \text{ and} \quad (14)$$
$$F(\phi) = A_1 e^{+m\phi} + A_2 e^{-m\phi} \text{ for } m \neq 0$$

Function $P(\theta;\ell,\mu)$ represents Legendre functions [104, 105], and A_1, A_2 are arbitrary constants. Elevation angle θ varies in the closed interval [0, π], and azimuth angle φ is unbound. The relevant fact is that $P(\theta;\ell,\mu)$ in the real domain only for some values of angle θ_1, θ_2,...; hence, $P(\theta;\ell,\mu)$ is inherently quantized, and the solutions in eq. (14) are helices on the surfaces of quantized cones with half-angle θ_1, θ_2,...; some explicit values are listed in [103].

Electron microscopy found helicoidal structures at the scale of 125 nm in the growth of $YBaCu_3O_6$ films prepared by sputtering at T = 880 K [106]. At a higher 100 micrometer scale helices appear in the growth of silicon carbide crystals [107, vol. 3, p. 636]; helical growth is evident in various living beings as sea shells (see figure 3). A testable prediction is that the cone angle in all helicoidal objects and events is quantized. Helicoidal solutions are reminiscent of Cartesian vortices, but, of course, they are neither unique, nor the most stable fluid structures observed in Nature.

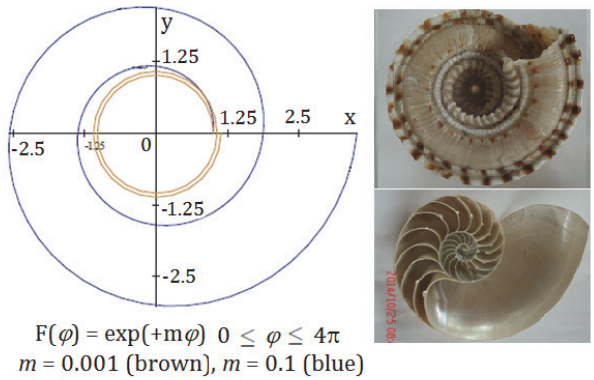

Figure 3. Spiral function $F(\varphi)$ yields helicoidal patterns $H(\theta,\varphi)$ on the surface of quantized cones. Left: Positive exponents produce curves spiraling outward. Right: Sea shells exhibit helicoidal growth similar to $H(\theta,\varphi)$ in Eq. (14).

3.3. *Quingal Solutions*

In spherical coordinates, motion along a ray r directed at any arbitrary direction (θ, φ) is analogous to one-dimensional motion. It is known that solutions to one-dimensional travelling wave equations are proportional to first term in eq. (15), see for instance [58, p. 370]). Redefining in terms of the dimensionless variable q, with time and distance entangled in a non-usual way that resembles an inverse average speed, one gets

$$\frac{1}{r} f\left(t - \frac{r}{c}\right) = f_1\left(\frac{ct}{r} - 1\right) = f_1(q-1) = Q(q)$$
$$\text{where } q \equiv \frac{ct}{r} = \frac{w}{r} \quad (15)$$

The entangled time-distance function $M(w,r)$ and a directional function $D(\theta,\varphi)$ are as in Eq. (16), where

$D(\theta,\varphi)$ may be spherical harmonics $Y(\theta,\varphi;\ell,m)$ or the helicoidal functions $H(\theta,\varphi)$ of previous sections:

$$\begin{aligned}&\Psi(w,r,\theta,\phi) = M(w,r)D(\theta,\phi),\\&\Psi(w,r,\theta,\phi) = \Psi_0(r,\theta,\phi) + Q(q)D(\theta,\phi)\\&\text{where } M(w,r) = I(r) + Q(q),\ \lambda = -\ell(\ell+1),\\&\Psi_0(r,\theta,\phi) = I(r)D(\theta,\phi),\ \ell = 0,1,2,...\end{aligned} \qquad (16)$$

Details appear elsewhere [7, 101-103], λ is the usual separation constant; a new constant η, which plays the role of principal quantum number in Schrödinger's equation, appears in the process leading to $Q(q)$ in terms of three new functions $Q_1(q;\ \ell)$, $Q_2(q;\ \ell)$ and $Q_3(q;\ \ell)$, now called quingal functions of the first, second and third kinds:

$$Q(q;\ell,\eta) = A_1Q_1(q;\ell) + A_2Q_2(q;\ell) + \eta Q_3(q;\ell) \qquad (17)$$

Quingal stands for Quantized Universal Isomorphic under Neo-GAlilean and Lorentzian transformations. The connection of $Q(q)$ to Boscovich force was noted already [7]. The isomorphism exhibited by $Q(q)$ is sketched next.

Lorentz invariance of the homogeneous KGE is well known; it applies to systems in Cartesian coordinates moving with speed V relative to the inertial frame [51-54]. In spherical coordinates consider motion along ray r arbitrarily oriented at (θ,φ) and assuming isotropy of the three-dimensional Euclidean space, the direction of any ray r may be chosen as X-axis, thus reducing to two the number of dimensions relevant for linear motion: x (renamed r) and time w. New independent variable q transforms under Poincaré rules as:

$$\begin{aligned}&r' = \gamma l(\beta)(r-\beta w), \quad w' = \gamma l(\beta)(w-\beta r),\\&q'\big|_L = \frac{w'}{r'} = \frac{w-\beta r}{r-\beta w} = \frac{q-\beta}{1-\beta q},\\&Q(\ q) \to Q\left(q'\big|_L\right) \neq Q(q)\\&\text{where } \beta \equiv V/c, \quad \gamma = \left(1-\beta^2\right)^{-1/2}\end{aligned} \qquad (18)$$

Since the Poincaré transformation was not imposed to the directional (θ,φ)-pair, it is not surprising that the quingal function $Q(q)$, which only contains the (r,w)-pair, is not Lorentz invariant.

Let us consider now a neo-Galilean transformation defined by

$$\begin{aligned}&r' = r-\beta w, \quad w' = w-\beta r,\\&q'\big|_{NG} = \frac{w'}{r'} = \frac{w-\beta r}{r-\beta w} = \frac{q-\beta}{1-\beta q},\\&\text{then } \ q'\big|_{NG} = q'\big|_L,\ Q\left(q'\big|_{NG}\right) = Q\left(q'\big|_L\right) = Q(q)\end{aligned} \qquad (19)$$

In Eq. (19), distance r obeys the usual Galilean transformation, while time w transforms symmetrically in a Galilean-like way — tangentially mentioned by Poincaré in 1900 [108, p. 273]. Then, both Lorentzian and neo-Galilean transformations lead to the same $Q(q')$, which is the strong case of isomorphism.

3.4. *Four Families of Relativistic Aether*

Our new solutions to the homogeneous KGE add three new families (F2 to F4) of inherently quantized solutions to the standard harmonic solution F1. In contrast, in the conventional F1, quantization is introduced from outside; just remember Bohr's old quantum theory where quantization was imposed as circular stationary state orbits, whose *angular momentum must be an integral multiple of $h/2\pi$* [98, p. 4]. Families S1 and S2 are solutions to Laplace's equation, which always form part of the general solution to the KGE and correspond to: (1) time-independent conditions, (2) stationary and steady-state conditions defined by a constant time derivative.

Table 3. Solutions of the Homogeneous KGE

F	Solution	Inherent quantization	Strong isomorphism
1	$T(w)R(r)Y(\theta,\varphi)$	no	no
2	$T(w)R(r)H(\theta,\varphi)$	yes	no
3	$Q(q)Y(\theta,\varphi)$	yes	yes
4	$Q(q)H(\theta,\varphi)$	yes	yes
S1	$\Psi_0 = I(r)Y(\theta,\varphi)$	no	no
S2	$\Psi_0 = I(r)H(\theta,\varphi)$	yes	no

4. Sagion-Sagion Interactions and Coalescence

Let us switch now to interactions involving a small number of sagions, which are outside the range of applicability of fluid equations (1). Both the speed and the direction of motion of unperturbed sagions remain constant until they collide with another free sagion, or with a group of sagions. The large majority of sagion-sagion collisions are elastic, in the sense that both total linear momentum and kinetic energy of the pair of objects before the collision is equal to total linear momentum and kinetic energy of the pair after the collision. As already stressed in section 2.1, spatial or temporal scales below $\mathcal{D}$ and $\mathcal{T}$ cannot be known with certainty because there are no "rulers" smaller than a sagion, i.e., it is intrinsically impossible to know the exact time and precise location at which colliding sagions change direction of motion. This leads to an unavoidable classical indeterminacy in the sagion-sagion collision; this writer also identified another instance of classical uncertainty [109]. In sagion-sagion interactions the only observables are linear momentum and kinetic energy "before" (say at time t_1) and "after" the collision (say at time t_2); details of the physical processes in the interval of time $\Delta t = t_2 - t_2$, from "before" to "after", will always remain hidden, as if the processes occurred inside a "black" box, shown as a transparent box in figure 4.

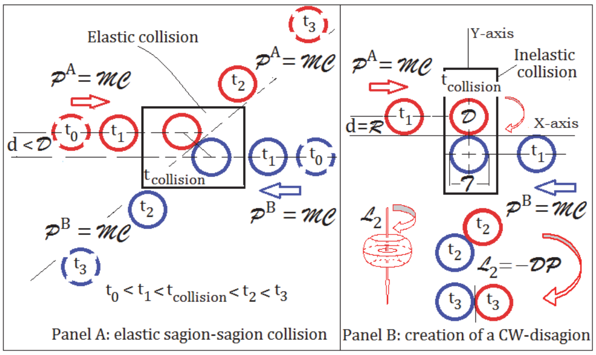

Figure 4. Stroboscopic view at t_0, t_1, t_2, ... of two identical colliding sagions moving with speed $\mathcal{C}$ in opposite directions along X-axis. Panel A: elastic collision. Panel B: inelastic slanting collision leading to a CW-disagion (see text).

Sagions may exist in two different states: (1) Free sagions in straight line motion with linear momentum $\mathcal{P}$, and (2) Coalesced sagions in orbital motion, conserving the original linear momentum $\mathcal{P}$ as orbital tangential speed. The two states of sagions are simpler than the traditional states of macroscopic matter (gas, liquid, and solid).

4.1. *Coalescence of Sagions*

In a slanting collision, the two sagions may coalesce to form a disagion, conserving the initial angular momentum as orbital momentum of magnitude $\mathcal{L}_2 = \mathcal{PD}$ = L; in panel B of fig. 4 orbital motion is clockwise (CW), so that L= -1 (in $\mathcal{PD}$ units). Counter-clockwise (CCW) disagions with L= +1 may form when sagion A touches sagion B from below in said figure. The disagion is created at rest in absolute space Σ; the "photographs" at times t_2 and t_3 in fig. 4 show disagion at the place where it formed, but the individual sagions are at different absolute positions due to their orbital motion with tangential speed $\mathcal{C}$.

Since the individual sagions move with tangential speed $\mathcal{C}$, in 3D-space the disagion occupies a toroidal region of radius $\mathcal{D}$ + ε, pierced by a central hole of a vanishingly small radius ε, generated by the tiny gap between the surfaces of the two mutually orbiting sagions. A disagion observed with a low speed camera is seen as a continuous toroidal object. This constitutes a classical view for the topological objects described by Thoules, Haldane and Kosterlitz (THK) — winners of the Nobel prize in physics for 2016.

The disagion is the simplest structure of coalesced sagions. Figure 5 shows the eight classes of disagions, depending upon the individual CCW-spin or CW-spin of each sagion, and the sense of orbital rotation. In this illustration sagion spin is taken tentatively as equal to orbital angular momentum L. As usual, J = S + L.

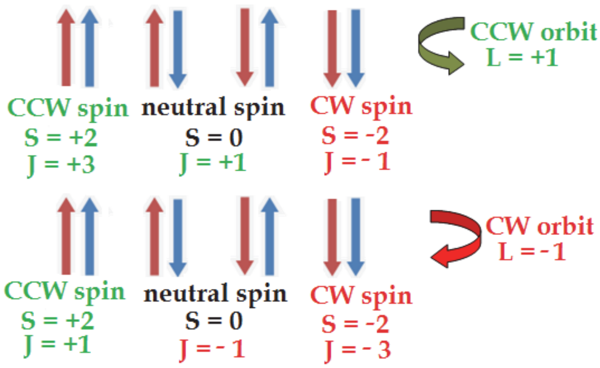

Figure 5. Eight different classes of disagions, leading to six triplets (S, L, J). Neutral spin disagions are basic components for the composite photon [110, 111].

Some slanting collisions between a free sagion and a disagion at rest may lead to capture of the free sagion to form a trisagion, with the CM of the individual sagions forming an isosceles triangle. De-excitation leads to an equilateral triangle, which is the minimum potential energy configuration [9].

The disagion and the trisagion are the elementary components of the photon and of the fundamental material particles; groups SU(2) and SU(3) are associated with them. Two stacked disagions, one of them twisted 90°, lead to a tetrahedron; likewise, two stacked trisagions, one of them twisted 60°, produce a twisted hexasagion. The tetrahedron and the twisted hexasagion are 4-sagion and 6-sagion arrays with lowest potential energy [9]. Those geometrical arrays are in permanent rotation with tangential speed $\mathcal{C}$ around the z-axis (whose direction is locally defined by the stacking). The 3D-space occupied by the rotation of such symmetrical sagion arrays have beautiful shapes, similar to those of THK. Our kinematic theory of particles will be described elsewhere.

4.2. *Principle of Intrinsic Discreteness*

In the formation of a disagion each sagion contributes angular momentum $\mathcal{L} = \mathcal{PR}$ to the total orbital angular momentum of the disagion $\mathcal{L}_2 = 2\mathcal{PR} = \mathcal{PD}$. In Newtonian language a sagion is an object moving with speed $\mathcal{C}$ and mass $\mathcal{M}$; orbital motion within a disagion has period T given by

$$\mathcal{M} \equiv \frac{\mathcal{P}}{\mathcal{C}}, T = \frac{2\pi R}{\mathcal{C}} = \frac{2\pi(\mathcal{R}+\varepsilon)}{\mathcal{C}} \cong \frac{2\pi\mathcal{R}}{\mathcal{C}} \quad (20)$$

Our principle of intrinsic discreteness introduces quantumness in classical mechanics, it states that $\mathcal{L}$ is the minimal angular momentum in nature, and equals Planck's reduced constant:

$$\mathcal{L} = \mathcal{PR} \equiv \hbar \quad (21)$$

Substitution of Eqs. (20) into Eq. (21) leads to

$$\mathcal{MC}^2 = \frac{h}{T} = hf, \text{where} f = \frac{1}{T} \quad (22)$$

Table 4. Physical Properties of the sagion

Symbol	Value & units	Description (comments)
$\mathcal{D}$	1.6162E-35 m	Diameter, Planck length
$\mathcal{R}$	8.0810E-36 m	Radius ($\mathcal{R} = \mathcal{D}/2$)
$\mathcal{A}$	2.0515E-70 m^2	$\mathcal{A}=\pi\mathcal{R}^2$ sagion cross section (smallest area)
$\mathcal{V}$	2.2104E-105 m^3	Sagion volume (smallest volume in Nature)
$\mathcal{P}$	1.3050E+01 kg.m/s	Sagion linear momentum eq. (21) (human scale!)
$\mathcal{C}$	2.9979E+08 m/s	Local average speed of sagions = light speed
$\mathcal{S}$	2.1092E-34 kg.m^2/s	Sagion spin (= $\mathcal{PD}$ tentatively)
$\mathcal{PR}$	1.0546E-34 kg.m^2/s	Min. angular momentum (reduced Planck const.)
$\mathcal{M}$	4.3530E-08 kg	Sagion mass equivalent, eq. (20) (43 micrograms!)
$\mathcal{MC}^2$	2.4419E+28 eV/c^2	Sagion mass energy (twice Planck mass)
$\mathcal{PC}$	3.9123E+09 kg.(m/s)^2	Kinetic energy carried by sagion (3.9 GJoule)
$\mathcal{T}$	5.3910E-44 s	Shortest measurable time $\mathcal{T} = \mathcal{D}/\mathcal{C}$
$\mathcal{F}$	5.9044E+42 turns/s	Disagion frequency (highest frequency)
$\mathcal{P/V}$	5.9038E+105 kg/(m^2 s)	Highest linear momentum density
$\mathcal{E/V}$	1.1047E+133 eV/m^3	Energy density in sagion (highest energy density)
$2\mathcal{P}$	2.6100E+01 kg.m/s	Maximum momentum exchange of two sagions
$2\mathcal{P}/\mathcal{T}$	4.8414E+44 kg m/s^2	Maximum momentum exchange rate (max. force)
$2\mathcal{P}/\mathcal{TA}$	2.3599E+114 kg/(m.s^2)	Maximum "force" through $\mathcal{A}$ (maximum pressure)

Equation (22) is a remarkable result providing a simple kinematical interpretation to Einstein's mass-energy equation, to Einstein's photon, and to De Broglie's matter-wave duality. Moreover, since the sagion is an energy-like object, eq. (22) provides a mechanical model for frequency f of the photon and similar massless objects, i.e. photon frequency merely is the frequency of rotation of a disagion or dipole, object similar to a macroscopic dumbbell obeying conservation of elementary angular momentum.

4.3. *Physical Properties of the Sagion*

Since our neo-Cartesian theory intends to be a general covering theory, it must reduce to Newton's, Einstein's and quantum theories in the appropriate limiting cases. Consider the disagion as a Newtonian rotating dipole with two masses $\mathcal{M}$, concentrated at their center of mass and located at the ends of a massless bar of length $\mathcal{D}$; each mass moves with tangential speed $\mathcal{C}$ on a circumference of radius $\mathcal{R}$.

The Newtonian centrifugal force F_C is balanced by Newtonian gravity F_N exerted by the other member of dipole, Eqs. (20) and (21) are used in the last step of the calculation for $\mathcal{D}$, which turns out to be Planck's length:

$$\begin{aligned} F_C &= \frac{\mathcal{MC}^2}{\mathcal{R}} = \frac{2\mathcal{MC}^2}{\mathcal{D}} = F_N = \frac{G_N \mathcal{MM}}{\mathcal{D}^2} \\ \Rightarrow \mathcal{D} &= \frac{G_N \mathcal{M}}{2\mathcal{C}^2} = \sqrt{\frac{G_N \hbar}{\mathcal{C}^3}} = L_P = 1.6162x10^{-35} m \end{aligned} \quad (23)$$

Physical properties of the sagion collated in table 4 (above) are obtained with the additional assumption that local average velocity of sagions equals speed of light; Eqs. (20) thru (23) are used in the calculation.

5. Neo-Cartesian Universal Acceleration Curve

To explain acceleration Newton postulated force, but for Descartes acceleration originates in exchange of linear momentum in collisions.

5.1. *Acceleration by a Succession of Pushes*

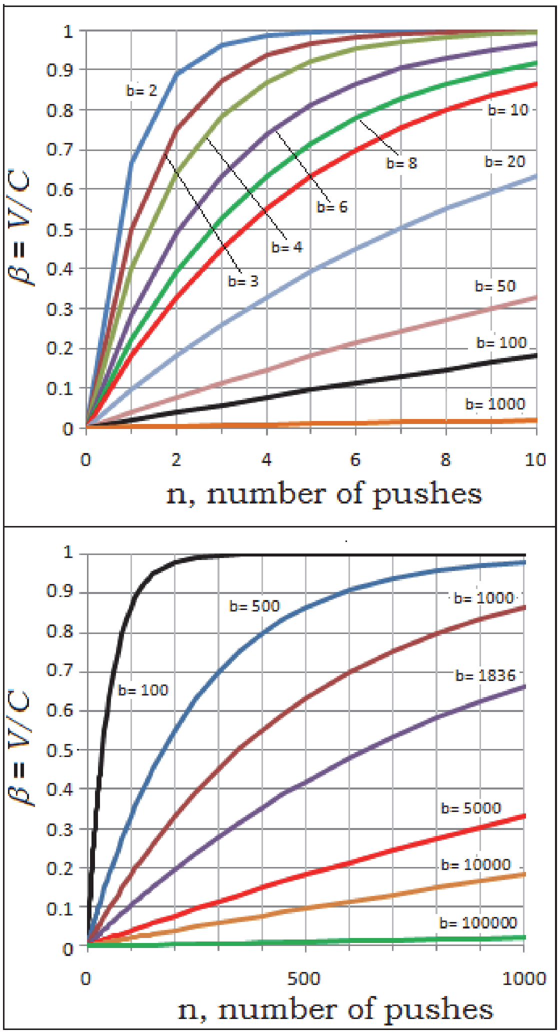

Figure 6. Number of pushes required to accelerate target of mass b to speed C. Target $b = 5$ requires about ten pushes (top panel), for $b = 100$ about 400 pushes are required (bottom panel). A neutron pushed by an electron, $b = 1836$, requires 5,000 pushes to attain C.

Consider a collinear elastic collision of an extended three-dimensional projectile or bullet B of mass m, speed C and linear momentum $P_B = mC$ moving in the positive direction of X-axis and a target of mass M, initially at rest, $b = M/m \geq 1$. At end of first collision target moves forward along X-axis with speed V_1; a second identical bullet hitting target from behind transfers an additional amount of linear momentum, smaller than the first one, and so on. Each collision pushes the target forward, and in the n^{th} push transfers impulse I_n:

$$I_n = \Delta P_T = \frac{2\left(bP_B - P_T\right)}{b+1}, b \geq 1$$
$$\Rightarrow \Delta V_n = V_n - V_{n-1} = \frac{2\left(C - V_{n-1}\right)}{b+1} \qquad (24)$$

After a little elementary algebra, eq. (24) becomes

$$\beta \equiv \frac{V_n}{C} = 1 - \left(\frac{b-1}{b+1}\right)^n, b \geq 1, n = 1,2,3,\ldots \qquad (25)$$

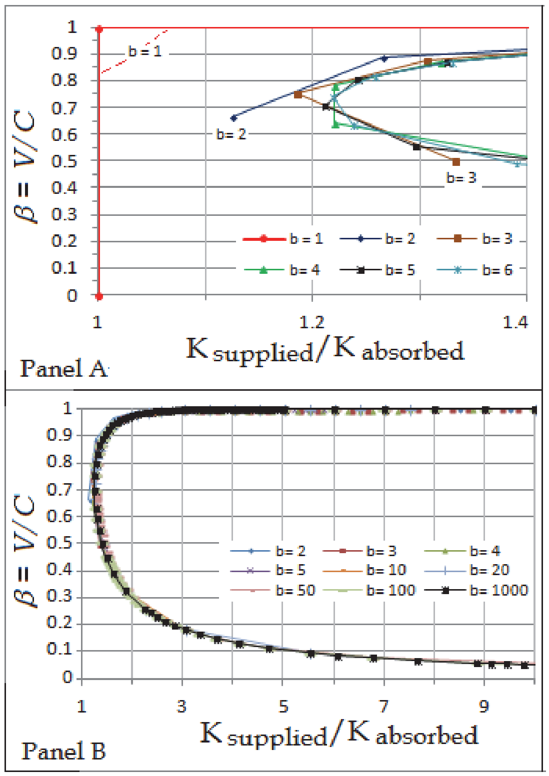

Figure 7. Increase of target speed as function of kinetic energy carried by n bullets (KSUPPLIED) Panel A: Discrete structure for a small number of pushes 1, 2, 3, and $b = 1$ thru 6. Panel B: Target from $b = 2$ thru 1,000. The discrete structure is lost at scale of panel B; at intermediate target speed $0.3 < \beta < 0.95$, energy ratio stays in the range 1.2 to 2.

For $b > 1$, fig. 6 shows that speed V_n of target approaches C after a finite number of pushes n; note that as V_n increases, the impulse I_n transferred in a push decreases. If the pushing bullet is identified with the sagion, then the sagion local speed C is a limiting speed, without invoking Einstein's STR. This simple physical process is not related to relativistic increases of target's mass, but rather with the obvious fact that if target moves with speed C, a bullet with the same speed C will never catch up from behind! In Newtonian language, it means that force is a function of velocity.

Since I_n decreases with n, kinetic energy transferred to the target, or absorbed by it (K_{ABSORBED}), decreases relative to the kinetic energy carried by the projectile. From the view point of the total kinetic energy of n bullets (K_{SUPPLIED}), the efficiency of energy transferred to target decresases as n increases:

$$\frac{K_{SUPPLIED}}{K_{ABSORBED}} = \frac{n}{b\beta^2} \tag{26}$$

Figure 7 shows the increase in the speed of target given by Eq. (26) as function of the kinetic energy supplied (K_{SUPPLIED}) in n successive impacts of the pushing projectile. This process is seen by an external observer as a much larger total energy entering the spatial region where acceleration is taking place; in STR the implicit (incorrect) assumption is that all energy entering the region of collision is absorbed by the target, and becomes mass increase. For $b = 1$, all the energy of the projectile is transferred to the target in the first collision, so that additional projectiles entering the spatial region can not transfer more energy to the target, which moves with speed C at the end of the first collission; the energy of the second and succesive projectiles is completely wasted (see panel A). Panel B shows all curves from $b = 2$ up to 1000. Note that individual curves are superimposed, and appear to the eye as a continuous curve with a finite width; details due to discreteness disappear as the horizontal scale is compressed.

Panel B shows that, from the view point of the total energy entering the collision region, acceleration of bodies is inefficient both at low and at high speed relative to Σ. The low end appears in macroscopic observations in a variety of guises: "static friction", "hysteresis", "inertial resistance", and so on. The high end of figure 7 provides an alternative fresh view to the old controversy that started in the last decades of the 19th century as to whether mass is an electromagnetic or mechanical notion, and as to whether mass is variable or not; it also leads to a novel interpretation of the 1902 Kaufmann experiments, discussed in [25, pp. 136-153], that eventually led to Einstein's special theory of relativity. Our neo-Cartesian approach provides a coherent and unified approach to interactions at all speeds under a single theory and a unique universal acceleration curve (panel B, fig. 7).

5.2. *Bertozzi Experiment*

Consider now the neo-Cartesian interpretation of Bertozzi's experiment, conventionally exhibited as proof of the mass increase predicted by Einstein's STR [112]. In STR, the kinetic energy K_{STR} of a body moving with speed β is the difference between total energy $E(\beta)$ and rest mass energy $E(0)$:

$$\frac{K_{STR}}{E(0)} = \left(1-\beta^2\right)^{-1/2} - 1 \tag{27}$$

Table 5. STR Analysis of Bertozzi Experiment

Run	$E(\beta)$, MeV	$E(\beta)/mC^2$ Bertozzi	$\beta = V/C$ observed	$E(\beta)/E(0)$ this paper
a	0.5	1	0.867	0.978
b	1.0	2	0.910	1.957
c	1.5	3	0.960	2.935
d	4.5	9	0.987	8.806
e	15	30	1.0	29.354

Panel A in fig. 8 shows eq. (27) versus the five observations reported by Bertozzi (table 5). It is evident that runs "a", "b" and "c" (the square dots) are to the left of STR predictions. This is quite a surprise because fig. 3 in Bertozzi's paper shows those three points almost coinciding with the curve for β^2 versus $K_{\text{STR}}/E(0)$. Bertozzi used an approximate value $E(0) = 0.5$ MeV (column 3 in table 5), while we used the more exact $E(0) = 0.511$ MeV (column 5 in table 5), but this difference does not account for the observed inconsistencies. Figure 8 also shows β^2 versus $K_{\text{STR}}/E(0)$ and $E(\beta)/E(0)$ with $E(0) = 0.5$ MeV as used by Bertozzi; the three data points continue at left side of STR predictions. So, it seems that Bertozzi's figure 3 is not accurate, and the claimed consistency between STR and Bertozzi experiment becomes even weaker.

In our Cartesian theory, $E(\beta)$ is the same as total kinetic energy supplied, K_{SUPPLIED}, a fraction of which becomes kinetic energy (K_{ABSORBED}) transferred to a nonrelativistic particle of mass $M = bm$, where m is the (rest) mass of the electron. The kinetic energy is multiplied by 0.511 MeV and divided by the rest mass of the electron to convert kinetic energy to MeV:

$$\frac{K_{SUPPLIED}}{K_{ABSORBED}} = \frac{2E(\beta)}{0.511 b\beta^2} \tag{28}$$

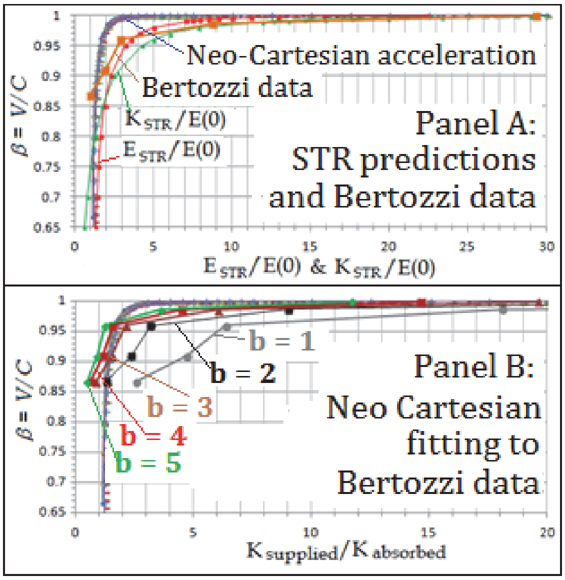

Figure 8. Bertozzi's data and STR predictions (panel A), and neo-Cartesian predictions (panel B). Our neo-Cartesian fit is better than Einstein's STR fit (see text for details).

Panel B in figure 8 shows eqs. (28) versus the five observations reported by Bertozzi, for five values of the parameter $b = 1,2,3,4,5$ in table 6. The experimental data for $b = 3$ & 4 closely fit the Cartesian acceleration curve. It is clear that our fitting is better than STR fitting to same data. The scanty number of points does not merit any statistical analysis. The good fitting for b = 4 suggests that an electron is formed by four sagions, empirical information that is consistent with our kinematic model of the electron as a rotating tetrahedron (section 4.1).

Table 6. Neo-Cartesian Analysis

Run	$\beta = V/C$ observed	Ksupplied/Kabsorbed		
		b =2	b =3	b =4
a	0.867	1.302	0.868	0.651
b	0.910	2.363	1.575	1.182
c	0.960	3.185	2.123	1.593
d	0.987	9.040	6.027	4.520
e	1.0	29.354	19.569	14.677

6. Closing Remarks: is Curved Motion Natural?

A neo-Cartesian unified field theory based on a fluid aether obeying the homogeneous Klein-Gordon equation (HKGE) was proposed, and three novel families of nonharmonic solutions to the HKGE briefly reported. Laboratory evidence indicating that gravity interacts with matter in a way similar to EM was discussed. Neo-Cartesian theory is kinematical, rather than dynamical. Sagion-sagion interactions and sagion-matter interactions are contact collisions obeying three conservation principles: energy, linear momentum, and angular momentum, all relative to absolute space Σ. By postulating discreteness of angular momentum at the sagion level, Einstein's mass equivalence and De Broglie's matter-wave duality smoothly follow from sagion-sagion interactions. As in other field theories (say, Einstein's GTR), force is not a primitive concept in Cartesian theory. Acceleration produced by succesive pushes of a small projectile (say, the sagion) leads to a universal acceleration curve resembling Einsteinian mass increase. This provides a completely different interpretation to several claims of STR. In particular, Bertozzi experiment is simply explained as an inefficient transfer of linear momentum; fitting of neo-Cartesian predictions to Bertozzi data is superior to STR's fitting.

The physical principle underlying the coalescence of sagions in section 4 of this paper is the open question in our theory. Since force does not exist in the Cartesian approach, the ensuing question is: what is the mechanism that keeps the two sagions orbiting each other? A detailed calculation of local pressure gradients in the sagion aether may suffice, but, if it does not, let consider a bolder alternative.

Twenty-five centuries ago Aristotle's believed that the natural state of matter was rest, and that motion required an explanation; hence, a body would only move when a force was acting. The paradigm changed in the 17^{th} century with Galileo's principle of inertia, which postulated that linear motion was the natural state of bodies. So, no agent was required to keep a moving object in motion, rather, an agent was required to stop it, or, in general, to change initial velocity. Galileo's principle is a particular case of the more general principle of conservation of linear momentum, foreshadowed by Descartes and formulated in modern terms by Huygens; Newton's three laws become mere theorems in that context [21].

A hundred years ago Einstein formulated his general theory of relativity (GTR), that he interpreted as representing a deformable spacetime. In GTR the straight trajectories of light become curved geodesics. It is our opinion that the equations of GTR are just a particular case of the more general classical equations of fluids [7, 8], as Eqs. (1) inhere. In fluid theory neither space, nor time are deformed, but the aether fluid filling the 3D-absolute may change shape over time, as all fluids do.

But, what happens if Einstein was right regarding the geodesics? What if light actually moves on a curved path in the Euclidean absolute space Σ? If that is the case, then the principle of conservation of linear momentum only has local validity, as a first order approximation to a general principle of conservation of curved motion, that manifests as large scale geodesics. At very short distances the new principle might account for the highly-curved orbital motion of a sagion moving over a circle in a rotating disagion. Then, no additional explanation would be required for the existence of the disagion.

From a geometrical viewpoint, an equivalent approach might be to accept that our universe is a huge closed spherical space, that locally manifests as a flat Euclidean space. Perhaps, the curvature of that space is related to the cosmological constant — recently reintroduced, once again! In such scenario Eqs. (1) used here to represent the aether only have validity over a local domain.

In the same conjectural mood, if absolute space has the shape of a huge sphere, then the natural system of coordinates is spherical, rather than Cartesian. This might explain why our new solutions to the wave equation were found in spherical coordinates.

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Quantized Space-Time Structure: The 0-D Point/Twist Void Co-Creator of the Continuum and Single Field

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A nagging problem has existed in the way we regard the local physical world around us and the non-local universe at large since the very beginning development of our philosophical and scientific attitudes toward the external world. That problem deals with the dualistic way in which we parse the physical world itself through geometry. Geometry can be based upon two different elements: the extension- or metric-element of Riemann and the point-element. Riemannian geometry can be fixed by expanding it to include the point-element, but even that is not enough. A further physical advance can be made by adopting the idea of the 0-D point Void, first developed intuitively by Sperry Andrews, but understanding the physical role of the 0-D point Void can only be realized by expanding that notion by adopting the physical concept of a discrete geometrical point/twist. It is only when a discrete 0-D point/twist Void replaces the simple point-element missing in the Riemannian system of differential geometry of surfaces that post-modern physics fulfills its promise. Understanding the concept of a point-element, of course, is necessary to understand how the Riemannian geometry has been used in general relativity as well as how it can be expanded to unify all of modern physics, including quantum theory, under a single geometrical paradigm. Whether a scientist is considering the discrete point-particles of the Standard Model or the existence of point singularities in relativity theory, the concerns are exactly the same, which forces the concept of an individual 0-D discrete point void to the center of the unification process. In either case, the human Mind and Consciousness are perceiving and interpreting the physical/material world that science is attempting to theoretically describe so the ultimate question of Consciousness and how it interacts with the Mind/brain as well as our commonly experienced physical reality also needs to be answered within the context of the 0-D point/twist. In other words, this is the point (no pun intended) where scientific logic and non-scientific intuition come together to give a complete theoretical structure of our commonly shared physical reality. Toward that end, the only logical scientific precedent to understand anything like the 0-D point/twist in all of the history of science is only found in the notion of a tesseract, which dates from the late nineteenth century attempts to 'realize' the concept of a hyperspace in the absence of being able to detect them through astronomical observations so that a hyperspace geometry could be used to explain nature. The end product of understanding these concepts is a greater insight into how the single field theory explains a much wider range of physical phenomena than any single previous paradigm of physics.

Keywords: Riemannian geometry, point-element, 0-D point/twist, Single Field Theory, Unified Field Theory, unification, five-dimensional space-time, Standard Model, quantized curvature, anti-symmetric tensor, Big Bang, cosmic inflation, particle creation

1. Introduction

Single Field Theory (SoFT) is a unified field theory based on Einstein and other's attempts to represent all of physics upon the basis of a unique single space-time continuum. [1, 2] It utilizes a five-dimensional space-time framework consisting of a three-dimensional curved in a four-dimensional space connected to time as space-time. This framework is filled with the single field which lends to the space-time framework its substantiality from which all matter (animate and inanimate) and physical fields emerge as patterned variations in the single field density with our three-dimensional material world being the densest three-dimensional surface in the continuum. A modified and enhanced Riemannian geometry, used which posits a three-dimensional surface (space) curved in a four-dimensional manifold (space) that varies over time like a rippling sheet, is used to geometrize our experienced reality of the world.

Time and space are literally bound together point-to-moment and distance-to-duration (reflecting the dual nature of geometry and physical space) by a specific binding constant commonly called Planck's constant ($h/2\pi$ or $\hbar$). [3] But when space and time are unbound forcing an unnatural experimental condition on any given material event and considered separately, science is faced with either the Euclidean flat world of Newtonian physics and its simple relativity in the everyday world of experience or the non-geometric picture in the world of the extremely small where Planck's constant is invoked and science is faced with the uncertainty inherent in Heisenberg's quantum mechanics.

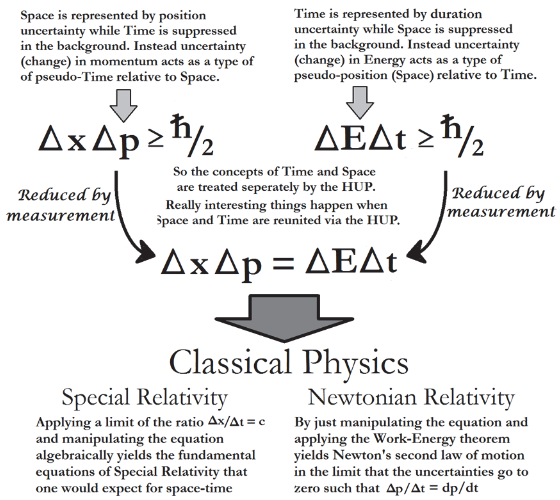

Figure 1. Reduction of the HUP to classical physics.

Otherwise, the world follows the Riemannian geometry described by a more complete picture of Einstein's original curved space-time continuum The central point, quite literally, to this unnatural unbinding of space and time is the simple fact that Riemannian geometry and the classical world picture both rely on a metric (measurable or extension) and thus relative definition of surfaces and spaces, while the quantum world relies on a non-geometry of discrete points not unlike Newton's absolute (in a non-relative sense) space and time. [4]

Riemann based his differential geometry of surfaces completely and wholly on metric-elements, reflecting a strictly limited geometric view of physical reality. He purposely ignored the idea that geometry could also be based upon points, or point-elements as he called them before dismissing them, because doing so was extremely difficult and highly problematical. He either failed to realize the fact, or at least did not act upon it, that real physical space is point/extension dualistic and this dualism must be ultimately accounted for in any scientific theory dealing with space and time in any manner whatsoever. In other words, explaining the Riemannian differential geometry of surfaces by adding in the effects of point-elements would complete Riemannian geometry with respect to the real world as well as later generalize relativity theory. By generalizing Riemannian geometry in this manner, it can be rendered completely compatible with both electromagnetic theory by using a combination of the Einstein-Schrödinger anti-symmetric [5, 6] and the Kaluza five-dimensional [7] models and the quantum (quantum mechanics as well as the Standard Model) when past philosophical interpretations and some subsequent physical misinterpretations are taken into account, if not thrown out altogether.

Doing so results in a concept of a discrete point/twist that replaces both the geometrical concept of an infinitesimal point and the quantum concept of a discrete point with one single concept which can be equated to the discrete 0-D point/twist Void within the Riemannian context. The 'twist' property of points in physical space guarantees and maintains the physical integrity of Andrews' discrete 0-D point Void [8, 9] in its differentiation from the absolute Void from which it emerged as well as its discreteness from other such points later when a fully developed 'space' (Riemannian surface) has emerged. From this concept and its modification of Einstein and other's attempts to develop a unified field theory, the single field theory, which has specific applications to many of the present problems facing modern physics, has been developed. Or, as the ancient Chinese philosopher Lao Tzu said,

In the beginning there was one; One begat two;
Two begat three; And three begat all things.

One could be physically interpreted as the nothingness or 'no-thing-ness' that was the pre-creation absolute Void, two is the differentiation and primal awareness of the localized 0-D discrete point/twist Void and the not-localized absolute Void, three is space, time and the single field. From these simple beginning our whole modern universe evolved.

The single field potential is the physical manifestation of reality that emerged from the initial singularity of the Big Bang, which amounted to the rapidly expanding 'wave front' of the three-dimensional material/physical universe expanding three-dimensionally within itself as well as the physical fourth direction of space. The discrete geometrical point/twists that constitute the simple internal geometry of the expanding three-dimensional surface which is our normally perceived universe manifest gravitationally in the space-time continuum as the gravnetic vector potential field which accounts for what are mistakenly called Dark Matter and Dark Energy. In other words, Dark Matter is just an additional (non-local curvature) effect of normal baryonic matter that causes normal (local) gravity effects.

This non-local effect can be expressed by the

Heaviside equation (gravitational equivalent of the Lorentz equation) [10] in classical Newtonian physics or the anti-symmetric tensor (Einstein-Cartan-Schrödinger) [11, 12] in relativity theory. Keeping this in mind, the fourth spatial dimension of the embedding space in Riemannian geometry (the fifth dimension in four-dimensional space-time) can be geometrically modified (to account for point-elements or twists) and well defined to allow the unification of gravity and electromagnetism (Kaluza-Einstein-Bergmann). [13] The resulting macro-extended embedding spatial dimension can then be quantized into fixed (quantized) groups of parallel three-dimensional surfaces which constitute sheets with an 'effective width' along the fourth spatial direction, [14] literally quantizing the space-time curvature of the continuum.

The problem of unification is not with gravity theory itself, but rather with the mentally-derived mathematical expression of physical space in which gravity acts. The space through which gravity acts is thought of as a single thing and has thus been traditionally expressed by only a simple (Euclidean) extension geometry, but it can be expressed in two different but equivalent ways as either a three-dimensional collection of extensions or metric elements (relative space) or as a three-dimensional collection of individual discrete points (an absolute space) or point-elements. Even Newton noted this difference although he expressed it differently. So if how we mathematically express space has always been the problem and its solution resulted in an extension only based geometry to only partially explain gravity, then the inherent dualism of physical space should affect both electromagnetism and gravity in a similar manner.

Since gravity is by far the weakest of the natural forces, the effects of the dual nature of space have not been so easily observed or detected, therefore our present theories of gravity (Newton and Einstein) do not take into account the effects of the point-like geometry of space. However, common Maxwellian electromagnetic theory clearly takes the dualism into account to explain the differences between the electric and magnetic fields because electromagnetic effects are so much stronger and thus the effects of the dualism are more easily observable. In other words, gravity should have two fundamental components instead of one (whether the one is Newtonian m**g** or Einsteinian $\mathbf{R}_{ik}$) just as electromagnetism (Maxwellian dualism of q**E** and m**v** cross **B**) does. And just as the electric field **E** acts as an extension space (center to center) like gravity, the secondary effects of the gravity field should act point-space-like (around a point center) just like magnetism. So the second fundamental term of gravity would yield gravnetism (the source of DE and DM) just like electricity yields magnetism.

The discrete versus continuity debate that has raged throughout physics for several decades is actually as misstatement or misrepresentation of the point-space versus extension-space geometrical problem and understanding the concept of the dualistic nature of physical space (point quantum versus metric curvature) resolves that problem. When this simplified view of nature is realized, the determinism versus indeterminism debate reduces to no more than 'much ado about nothing' since neither viewpoint alone represents physical reality, just human vanity with regard to physical reality.

This means that the quantum and relativity are not incompatible as has long been thought, but are in fact totally and completely compatible. So once the Einstein unified field theory has been completed by combining the anti- or non-symmetric approach of Schrödinger and Einstein (to account for DM and DE) with the higher embedding dimension approach of Kaluza (to account for a unified EM and GR), the natural dualism of space – point versus extension – leads to a full unification of quantum and relativity in the form of a quantized space-time curvature that emerges by utilizing Oscar Klein's suggestion that quantizing the embedding dimension, even though it is now macroscopically extended, quantizes the four-dimensional space-time continuum.

The whole space-time/single field structure can be summarized in a single graphic which indicates how a single six-dimensional tensor located at the single pole in five-dimensional space-time splits according to the extension/point duality (stronger force/weaker force) into two anti-symmetric tensors representing the electromagnetic and gravito-gravnetic fields in five-dimensional space-time. These split once more when reduced to their four-dimensional space-time counterparts, the electric field (extension), magnetic field (point), gravity field (extension or metric curvature) and the gravnetic field (point or what some call the torsion field). [15].

In this theoretical structure, Kaluza mathematized the split by using a cut-transformation that yielded electromagnetism (literally a fourth-dimensional cut across three-dimensional space filled by electric field stress) and the four-transformation to yield the effects of gravitational curvature (literally a three-dimensional slice across the top of a three-dimensional surface or 'sheet').

If further reduced by splitting space and time according to their classical Newtonian interpretation in terms of physical forces, the extension/point dualism is represented by the Lorentz equation for electromagnetism and the Heaviside equation for gravito-gravnetism. In the world from the sub-microscopic to infinitesimally small, the split yields the

Heisenberg uncertainties whereby location in space and location in time are split into two different relationships with respect to changes in momentum and energy, respectively.

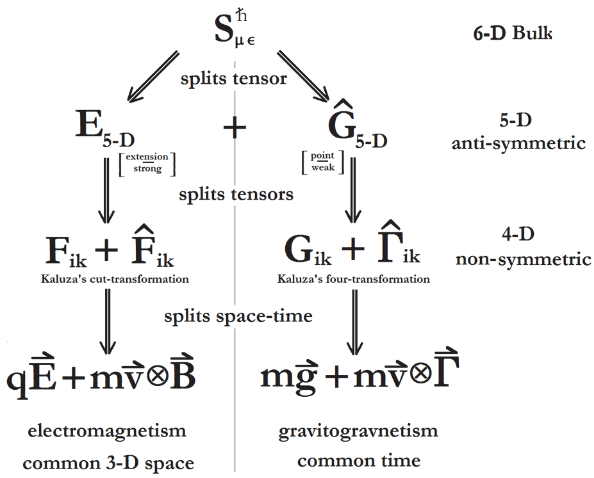

Figure 2. Summary of Single Field Theory.

At this final fundamental level of physical reality, the point extension duality reduces to a problematical interpretation of measurement and disappears altogether at the extreme limits of measurement, thus invoking the final split between space and time and simultaneously invoking the application of Plank's constant as space and time are unbound. Only then is the possibility of a probabilistic interpretation of nature raised (or unnaturally forced on nature by experimental and mathematical intervention) in the mathematical expression of the Heisenberg uncertainty principle. It is within this perspective that the physical role that the discrete 0-D point/twist Void plays in the inner workings (explained by physics) of our perceived natural world must be interpreted.

2. The Tesseract

The tesseract is the four-dimensional equivalent of a three-dimensional cube. It is considered more of a mathematical curiosity today, which belies the fact that is original intent was completely physical, so its history is more enlightening with regard to new developments in unifying physics than previously (and presently) thought. After the mid-nineteenth century, W.K. Clifford popularized the physical concept of hyperspaces in Britain through his own theoretical work and his translation (1873) of Riemann's *Habilitationsschrift* (1854, 1868) [16] which dealt with the concept of multi-dimensional surfaces curved in higher-dimensional manifolds. Clifford was attempting to develop the mathematics to understand his friend J.C. Maxwell's electromagnetic theory more fully as a four-dimensional electromagnetic effect in our physically experienced three-dimensional space.

However, Clifford died at the age of thirty-four in 1879 without completing his attempted theoretical work in either magnetism or his planned work on a new theory of matter and gravity, [17] which unfortunately ended in 1876 due to his failing health. Yet Clifford did have a lasting influence on further developments in science and mathematics before the scientific revolution in 1900, after which his theoretical work was subverted and ultimately all but forgotten except for anecdotes (some derogatory) about his anticipating Einstein's later successful use of Riemannian geometry to explain gravity. [18] At the very least, introduction and popularization of the concept of hyperspace geometry, as it was commonly called in the late 1800s, to the English-speaking world made it that much easier for the world in general to accept Einstein's radical explanation of gravity as a result of space-time curvature in 1915. In particular, Clifford's work directly influenced the later work, both mathematical and physical, of Charles Howard Hinton, an Oxford geometrician. [19, 20]

Hinton initially took a different view of the hyperspace concept in the 1880's. He first noted the fact that humans could not think in terms of a hyperspace but were limited to thinking in terms of a three-dimensional space with time as explained by Newtonian physics. So Hinton tried to develop a geometrical system whereby we could envision or think in terms of the hyper-dimensional space that he and others were sure represented our real physical world.

Within just a few years after Clifford's death, the concept of a real hyperspace (the Riemannian spherical curvature of space) had become so popular that astronomers were attempting to verify the positive curvature of physical space by looking for discrepancies from flat Euclidean space in the measurements of the largest possible triangles they could determine in their search for distant stars that exhibited parallax. No such discrepancies were ever detected so all that the astronomers and scientists could determine was that the positive curvature was so small due to the vast extent of space that their best parallax measurements were very close to flat Euclidean and fell within the experimental error of their observational and measuring devices.

The inability to detect curvature on the large scale through astronomy complicated the scientific issue of the existence of hyperspaces since we cannot directly observe or directly detect any other physical influences of the suspected higher dimension, but Hinton still tried to develop a logical geometrical method to teach people how to ‘realize’, visualize or think in terms of the suspected higher-dimensional space. The tesseract was a part of his geometrical system of meditation, the important thing about the tesseract being that it is an imaginary geometrical object that is supposed to help people concentrating on it to create a sense or higher-state of consciousness.

The tesseract is an implied geometrical object rather than a physical/materially real object, so it is only considered a mathematical curiosity today. Scientists and people in general take little heed of either it or the higher dimension it implies since it has no ‘effect’ in our physical world, or at least that ‘seems’ to be the case. Even Einstein seriously questioned the real existence of a higher-dimensional embedding space even though he adopted and worked on unifying physics based for nearly the whole decade of the 1930s on Theodor Kaluza’s theoretical model of a five-dimensional space-time continuum.

Einstein wondered how Kaluza’s five-dimensional model could work so well even though it was so strictly limited when we are unable to either detect of observe any other effects (beyond the mere fact of successfully unifying gravity and electromagnetism) of the higher embedding dimension of space. Einstein (1956) even made explaining this lack of perceiving the higher dimension a condition for future science to accept a five-dimensional physical theory.

Hinton thought that meditation and intense concentration on the tesseract and other features of his geometrical system would ultimately teach a person how to ‘realize’ the higher-dimensionality of our shared physical reality. His methods did not catch on as far as history reports and no one has ever ‘realized’ the higher dimension of space using Hinton’s methods, but the tesseract itself caught on and is widely known today, more than a century and a quarter after it was initially developed by Hinton.

From the very beginning, by design, the concept of the tesseract was associated with mentally reaching a higher state of mind or consciousness that was associated with a higher embedding dimension (hyperspace) than that represented by our normally sensed three-dimensional physical/material space. Hinton merely assumed that hyperspaces were physically real (a hyperspace could refer to either a non-Euclidean spaced or a higher-dimensional embedding manifold, or both) as did a large number of scientists, other academics and commoners, so we should be able to perceive or ‘realize’ them with proper training. By analogy, we cannot directly perceive gravity although we feel weight when we lift an object even though we do not feel our own weight, so we know and accept the fact that gravity exists without directly perceiving it.

3. Enfolding/Unfolding a 4-D Object

The tesseract is the best known four-dimensional object, but it cannot exist in a real three-dimensional physical/material world. It is just a mathematical (geometrical) figure that can be ‘realized’ (imagined since it is not real) through analogy with other dimensioned geometrical structures, at least as a static object.

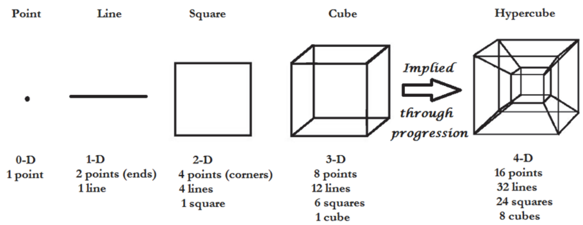

Figure 3. Dimensional progression from point to tesseract.

We cannot picture the four-dimensional cube as it really exists in four-dimensional space in our mind, at least statically, because our mind is three-dimensional fixated due to a lifetime of observation and directly experiencing only three- or lesser-dimensional objects and events in our material world. Yet the idea of a higher-dimensional embedding manifold or space has proven quite useful and extremely convenient in modern theoretical physics, enough so that modern physics actually implies its physical although not material reality.

The only way that this could be possible would be if some type of a tesseract-like object were to exist in our three-dimensional world of reality and if such an object did exist then it would need to be constantly enfolding into itself from moment-to-moment from our three-dimensional perspective because our perception of a physically real higher dimension cannot be attained through observing or measuring extensions in the higher-dimensional space. They can only come through experiencing (directly sensing) the higher space on a geometrical point-by-point basis, which is materially impossible although not necessarily physically impossible. Were a person to actually become aware of objects and interactions within the fourth dimension of space, the person would be unable to use common language to describe the witnessed events simply because our language and conceptual background are all three-dimensionally biased and limited.

On the other hand, we can logically simulate a ‘sense’ of the four-dimensional world by manipulating the tesseract through an unfolding/enfolding process. We can imagine unfolding the four-dimensional tesseract into its three-dimensional component cubes, and then enfolding those same three-dimensional cubes back into the four-dimensional tesseract.

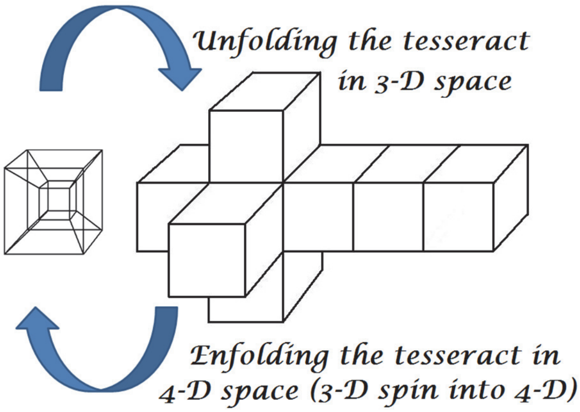

Figure 4. Enfolding/unfolding the tesseract.

The unfolded tesseract yields equal three-dimensional cubes, so the enfolded tesseract would have six equal three-dimensional cubes folded into a seventh cube to create an eighth cube in the higher fourth dimension.

Each section of the tesseract, no matter its shape in a two- or three-dimensional rendering, is still an equally-sized cube. The central cube only appears smaller because it is just further away in the higher dimension (by visual perspective from three-dimensional space) into which the cube is being enfolded (and thus pushed further back into the higher dimension). A better mental image of this can be formed if the enfolding is continued constantly through time as in a continuing video of the enfolding/unfolding process. [21] The time dimension merely simulates the higher fourth dimension of space making 'realization' easier. So from the common three-dimensional perspective, a static tesseract, one that is not changing in any matter over time, would merely exhibit a 'propensity' or 'desire' and thus a 'tendency' to enfold its three-dimensional aspect into its four-dimensional reality that we cannot directly sense by our three-dimensional sensations, brain or mind.

4. Sphere, Discrete and Singularity

Keeping the analogy of the tesseract in mind as well as the mental difficulty in imagining or visualizing the tesseract generates, the next step would be to imagine four-dimensional spheres (spherical surfaces) as three-dimensional spheres enfolding/unfolding in themselves since this situation would mimic the case of 0-D discrete point Voids, more commonly called singularities in mathematics, in three-dimensional space. We are here talking about a real physical space rather than just a logical mathematical system with a new degree of freedom.

Within this context, the case of a common sphere of radius 'R' is more useful for scientific understanding since a 0-D point in space is just a three-dimensional sphere shrunken to zero dimensions, much as a distance 'S' is shrunk to zero (a point) in the case of analyzing the curvature of a surface at a point in Riemannian geometry.

Even trying to visualize a four-dimensional sphere as three-dimensional spheres enfolding/unfolding into itself, like the tesseract, has a real physical purpose or goal in mind, in this case understanding the Big Bang and ultimately other physical singularities. Our three-dimensional universe is thought to have expanded equally in all three of its dimensions from an initial (0-D point) singularity (the Big Bang) in the Void approximately 13.5 billion years ago. This model of the origin of our universe, although implied by both theory (general relativity 1915-23) and observation (Edwin Hubble's work in 1928/29), raises numerous questions, not the least of which is how something could emerge from the nothing of the Void.

So science needs to pass from the mathematical concept of a discrete singular point or singularity to the concept of a 0-D point/twist Void to logically model how the physical universe (something) emerged or evolved from nothing. In this matter, the enfolding/unfolding of a tesseract only goes so far since our universe is constructed of geometrical points and extensions, not cubes, which alone can model the simple physical attribute that space is isotropic (equal in every direction).

Our universe is thought to be a three-dimensional expanding spherical surface (Riemannian double-polar) embedded in a higher-dimensional manifold. The observed expansion alone should automatically imply the reality of a higher embedding dimension of space, since no one really understands how such an expansion could take place unless the three-dimensional expanding spherical surface of our universe were expanding in a higher embedding dimension. Quite simply, our universe must be expanding into something from the nothing of the singularity (or no-thing of the absolute Void), so it is more convenient and easier to apply the concept of enfolding/unfolding to 0-D spheres than tesseracts, even though the tesseract was originally invented to visualize such higher spaces.

A distinction is being made here between the unthinkable singularity of a no-thing (the absolute Void) divided by nothing (0/0) and the thinkable but infinitesimally small 0-D point Void which can be thought of as a three-dimensional sphere whereby the radius is thinkable but approaching or all but zero in extension and dimension. Unfortunately, the basic notion of spheres enfolding into spheres to create a higher-dimensional spherical surface is much harder to mentally picture than the cube to tesseract analogy above, so it is better to think of spheres enfolding into a sphere as the three-dimensional inward 'spin' of a sphere toward its point center from all points on its surface.

No real extended three-dimensional 'object' could act in this manner except for an individual discrete 0-D (zero-dimensional and thus having no extension by which to determine dimensions) geometrical point. The only 'thing' in our universe that is capable of 'spinning' simultaneously in all three-dimensions of space and thus into itself (imploding or collapsing) is the geometrical point since any and all geometrical points in the universe are capable of becoming or acting as point/centers of rotation or circular motion for extended objects, a property of points termed 'twist' by W.K. Clifford. [21] A 'twist' of this nature would appear as a virtual 'stress' at each discrete point in three-dimensional space that would cause a virtual-torsional 'strain' in the surrounding continuous space.

In other words, the higher embedding dimension would literally be within (or accessible through) every discrete geometrical point (a 0-D point/twist Void in a physical sense) in our three-dimensional space of experience while the only way that the concept of 'twist' could be understood is if each and every discrete geometrical point in the whole of three-dimensional space was enfolding back into itself (back into the Void from which it emerged physically) and consequently emerging into the fourth dimension of space. Yet our stable physical reality (and logic) would dictate that such a 0-D point/twist could only be and therefore must represent an individual state of stable equilibrium (since our space, which is made of such points, does not 'collapse' but remains constant) to exist.

Therefore the 0-D discrete point/twist Void must be a dynamical object, *i.e.*, a stable object whose stability depends upon a dynamic equilibrium. This means that each geometrical point would be constantly and continuously trying to enfold into itself–more-or-less like an object spinning three-dimensionally toward its center point in three-dimensional space–while an equal and opposite attempt to unfold outward (centrifugally) occurs to stabilize it.

A three-dimensional 'spin' of this type is not real in itself, but the 'twist' or tendency to 'spin' inward in this manner is real. Each point exhibits a real static tendency or potential to 'twist', which has important consequences for the physical nature of our reality. Without such a 'twist', our experienced three-dimensional space could neither support nor sustain real circular/rotational motions of any type. So such a 'twist' is necessary, even though it is missing from nearly all previous science dealing with the properties of space and time. Moreover, the existence of 'twist' results in other special unsuspected physical features of our universe. In order to understand these features, the 'twist' must be likened to the normal spin of an ideal object.

Normal three-dimensional bodies or objects can only spin two-dimensionally about a one-dimensional axis in three-dimensional space, but three-dimensional objects in a three-dimensional surface curved in an embedding fourth dimension of space can spin three-dimensionally around a two-dimensional axial plane in four-dimensional space, which in three-dimensional space would appear as a static 'twist'. This concept easily explains what is normally called the half-spin of fundamental particles such as protons and electrons.

The enfolding exhibited by a 'twist' in three-dimensional space could be abstractly described, by analogy, with a circular wheel spinning in two-dimensional space that produces a torque through or out of the wheel's one-dimensional axle in three-dimensional space. We could say that the two-dimensional space of the so-called 'spin' (a dynamical motion rather than a static situation) is embedded in the three-dimensional space corresponding to the complete mechanical system being described.

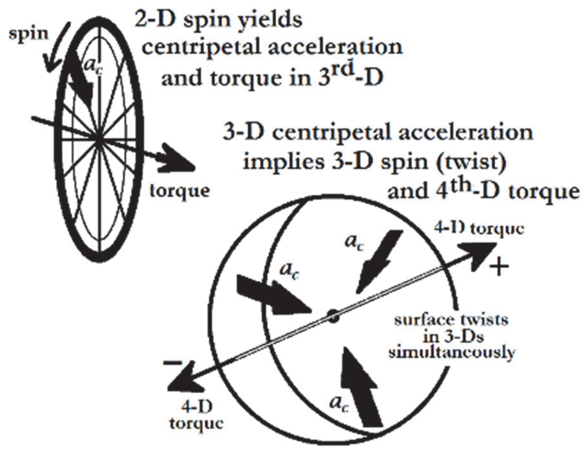

Figure 5. Centripetal acceleration of 2-D spin implies 3-D spin or 'twist' for imploding sphere.

The wheel itself is approximately a two-dimensional surface (flat disk) and the spin is two-dimensional, yet the torque (a resultant force of the spinning motion) is in the third dimension (the embedding dimension of the two-dimensional wheel) of space.

By analogy, we could say that the 'tendency' or 'propensity' of a sphere enfold or 'spin' into itself would produce a sphere spinning simultaneously in all three-dimensions toward a point at its center whose axle is a virtual torque perpendicular to the three-dimensional spin (enfolding) into a four-dimensional space. This enfolding could also be characterized as a virtual centripetal acceleration along the spherical surface of the point acting inward toward the center of the point.

A discrete 0-D point/twist can thus be approximated or pictured in the same way as a three-dimensional object in three-dimensional space spinning or 'twisting' inward toward its center, in so far as it can be imagined as a dimensionless point-sphere in three-dimensional space of (or approaching) zero radius (analogous to the concept of metric extensions $\Delta S \rightarrow 0$ in Riemannian metric geometry). In other words, we can logically determine the physical properties of a discrete geometrical point by assuming it is an infinitesimal sphere and decreasing the radius (measure of its extension ΔR in the three-dimensions of space) to zero (a dimensionless point). Doing so gets rid of the extension in space but not the spin, leaving the dimensionless point with the physical property of 'twist'.

So a 0-D point/twist Void can be characterized as a sphere whose radius has been reduced to or approaches its infinitesimal limits of zero simultaneously in each of the sphere's three dimensions, yet the three-dimensional spin/twist would still result into the 0-D point's enfolding into itself along the lines of virtual centripetal acceleration creating a virtual torque projected into both directions of the fourth dimension of space. This virtual torque in the fourth embedding direction of our real physical space is thus a product of the 'twist' portion of every 0-D point/twist Void, but it also creates a virtual 'torsion' in the three-dimensional space surrounding each and every 0-D point/twist Void.

All the discrete geometrical points that constitute our real perceived space-time continuum are actually physically discrete 0-D point/twists of Void attempting to collapse back into the original absolute Void, but they are prevented from doing so since they are maintained or stabilized in a dynamic equilibrium by their 'twist'.

5. Cosmic Inflation and Space

The virtual torque corresponds to an equivalent amount of expansion into the higher (perpendicular) embedding dimension that would be proportional to the amount of time passed. The 0-D point/twist is relatively but not absolutely dimensionless. Thus it is discrete relative to the three-dimensional space, but still not collapsed back into the absolute Void (where it would become absolutely dimensionless) because the new element of time has emerged during which the completely spatial process of expansion take place. Technically it is a single moment of time that separates the 0-D point/twist Void's relative dimensionless-ness from the absolute Void's dimensionless-ness. The 'twist' maintains the integrity of successive moments of time while maintaining the spatial integrity of contiguity between neighboring geometrical 0-D points to create the continuity across extensions in all four dimensions of space and the continuous duration of time.

The passage of time forward continues in exact proportion to expansion/torque into the higher embedding dimension, but it is never zero because time still passes within each and every 0-D point/twist independent of other such 0-D point/twists in the three-dimensional space from which individual discrete geometrical points emerged. This is analogous to the calculus concept of the limit of speed = $(\Delta x/\Delta t)$ as $\Delta t \rightarrow 0$. In other words, this is the point (no pun intended) where the calculus of motion and the differential geometry of surfaces (where the point is defined and approximated as the limit as $\Delta s \rightarrow 0$) meet and combine. This means that as along as time continues the 0-D discrete point/twist Void cannot completely collapse (it all but collapses in spatial, but not temporal dimension) back into the total absolute pre-existing absolute Void out of which it emerged.

However, the same amount of time passes for normal three-dimensional space outward from the center of the discrete point/twist which is geometrical (and arithmetical/mathematical along a number line of moments) as opposed to the 0-D point/twist Void which is physical. The discrete 0-D point could not increase in size to compensate, as it expands equally in each of the three dimensions in the three-dimensional space, so it creates new discrete 0-D point/twists in each of the three directions of three-dimensional space. Each newly created discrete 0-D point/twist Void does likewise each and every moment of time that passes. Expansion would thus be equivalent to time and effectively become time itself in a manner of speaking.

In so far as this process of expansion occurs in three-dimensional space by the creation of new 0-D point/twists in our three-dimensional experiential space, it rapidly reaches an explosive state of expansion after the initial singularity (0-D discrete point/twist Void) is formed that could be called 'cosmic inflation'. Each individual discrete 0-D point/twist Void in three-dimensional space would thus form a dynamic equilibrium state creating equivalent new discrete point/twist Voids in three-dimensional space as both three-dimensional and the fourth dimension of space expand from an initial singularity (the original unique 0-D point/twist Void) into our commonly experienced and fully extended continuous space with time or equivalently the three-dimensional space-time continuum described as a three-dimensional surface embedded in a fourth dimension by the Riemannian metric geometry.

Just as time and space are directly related and connected to one another through expansion, physically rather than just mathematically bound together, the binding constant for our particular physical/geometric space could only be Planck's constant, which could only be determined by physical rather than mathematical means. Under these circumstances, our physical space-time continuum could be perceived as a constant co-creation of something-ness out of the Void of no-thing-ness from an initial singularity, which is itself a 0-D point/twist Void although not discrete from other such points because they have not yet emerged or been created.

Since the expansion is continuing unabated over time, space must be re-establishing itself on a moment-to-moment or point/twist-to-point/twist basis just to remain physically viable and stable. In each 0-D discrete point/twist in three-dimensional space (moment of space) is recreating physical space over time (moment-by-moment) from the absolute Void. This co-creation from moment-to-moment establishes a dynamic equilibrium between expansions of the fourth dimension of space and three-dimensional space, as well as within each 0-D point/twist Void itself. In yet another sense, this dynamic equilibrium of the discrete 0-D point/twist Voids creates the necessary continuity exhibited by the space-time continuum that we perceive, interpret and model as logical systems of geometry as used in physics.

In other words, there is a moment-to-moment (temporal equivalent of spatial point-to-point) 'desire', propensity, potential, tendency or necessity for the discrete 0-D point/twist Voids in three-dimensional space to collapse (enfold) into its point/twist 'self' and return to the formless and thus pointless absolute Void from which they emerged. However, this 'tendency' causes the physical moment of each three-dimensional 0-D point/twist Void to create the expansion vector along its fourth direction of space, thus establishing the dynamic equilibrium of each and every discrete 0-D point/twist Void so all of space cannot collapse back into the absolute Void. In turn, according to action/reaction or Newton's third law, this expansion vector (and the associated virtual torque in the fourth direction of space) co-creates (with the enfolding) an equivalent expansion (unfolding) of three-dimensional space.

In this model of a physical reality that emerges from nothing, all things being considered equal (such that each action balances with another equal and opposite reaction which balances the whole to nothing), the discrete 0-D point/twist Voids are created in equal amounts and configuration in both directions in the fourth dimension of the developing space-time continuum although this diagram only depicts creation of the 'virtual torque' half. A second 'negative virtual torque' is also created in the opposite direction of the fourth dimension. Both are simultaneously created from the 'twist' of the original discrete 0-D point/twist Void.

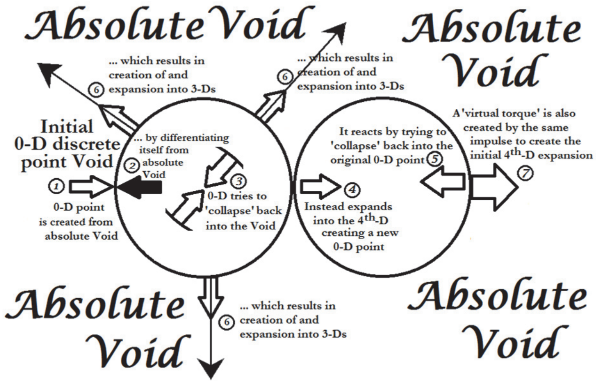

Figure 6. The co-creative dynamic equilibrium of the 0-D point/twist Void and its expansive physical results.

Both then react to their creation by rebounding the creation impulse back into the original 0-D discrete point/twist Void which in turn reacts by expanding and thus creating new such entities in the three-dimensional space-time continuum that is emerging from the creative process. The various 'virtual torques' correspond to pure 'potential' and thus form the beginning of the single field that corresponds to the geometrically structured space-time continuum. So they are expansions of potential (tendencies), not energy or matter themselves, but the potential to later form matter and energy given the quantum and geometric restrictions of the space-time continuum and single field by which matter/energy and other physical fields are defined. By comparison, a real physical torque is a force, not an energy or matter itself, although that force can be alternately interpreted as the cause of a change in energy carried by a material body.

In essence this physical process not only co-creates new three-dimensional exact duplicates of the three-dimensional discrete 0-D point/twist Voids, but they co-create the single field itself which fills (inhabits) all of four-dimensional space. These points are thus continuous (extended) with each other even while they remain discrete in metric or measurably extended space.

The expansion vector potentials (equivalent to the virtual torques) in the fourth dimension of space are physically interpreted in the real physical world as Dark Energy points in free-space and as inertial mass points (analogous and equivalent to the Standard Model's Higgs particles) under the metric curvature that defines material particles and objects in general relativity. Since space and time, or rather space-time, obviously does not naturally collapse back into the absolute Void from which it emerged during the Big Bang (or some other primal pre-physical process), each and every 0-D point/twist Void must be constantly re-co-creating itself each and every moment that passes in time to form the stable equilibrium of geometrical 0-D discrete point/twists that we experience as the spatial extension of our three-dimensional double-polar spherical universe, but also that we experience as different density configurations in the single field that gives substantial (material) reality to our world.

The physical situation for 0-D point/twists within three-dimensional matter is obviously different from 0-D point/twist Voids in empty (devoid of matter or external to existing material particles) three-dimensional space. Matter in the form of material particles equates to a maximum three-dimensional surface curvature that has 'congealed' (or 'coalesced') into a different physical state than space due to the restrictions of the quantum.

This is another way of saying that some event must have occurred whereby some portions of extended space with a greater density of 0-D point/twist Voids collapsed into particles rather than Void. In this case two possibilities exist: (1) the space inside material particles is expanding at the same rate as empty three-dimensional space. This expansion goes completely unnoticed because particles are so small relative to the vastness of empty space that their internal expansion goes unnoticed over the short periods of time covered by scientific observation and experimentation; or (2) enfolding and unfolding within material point/twists is potential instead of actual since material 0-D point/twists, although still discrete Void, result in measurably distinguishable extended four-dimensional single field densities (interpreted as three-dimensional curvature in the embedding dimension).

6. The Driving 'Force' Behind Inflation

The moment-to-moment temporal 'tendency' of the discrete 0-D point Void to 'collapse' into itself creates a 'twist' or virtual three-dimensional spin as described above. That 0-D point/twist that results creates a virtual torque as well as an expansion in the fourth dimension of space. The 'twist' associated with each and every geometrical point also guarantees its discreteness from other neighboring discrete 0-D point/twists even while the individual geometrical points or point-elements taken together form the continuous extension or metric-element that is the basis of a Riemannian n-dimensional surface embedded in the $n+1$-dimensional manifold that is used in general relativity to describe our universe.

In a sense we can say that the 0-D point/twist Void in three-dimensional space is a localization against (or in comparison to) the lack of localization or not-localization of the absolute Void. However, this property of not-localization of the absolute Void is not to be confounded and confused with the physical concept of non-localization, which refers to positional location (or place) of particles, events and material objects within three-dimensional space because there is no position or place within the absolute Void of 'no-thing-ness'.

While the 0-D discrete point/twist Void has no finite or measurable spatial extension in itself, the fact is that it does have an infinitesimal extension that is non-zero, all but zero but not exactly zero. So the mere fact that it exists physically gives it a virtual 'moment' relative to its center that creates the 'twist'. In an extended rotating object, the 'moment' or the average positon of the rotating matter (the moment of inertia) relative to the center of rotation (or axle) coincides with the extended position of the momentum of the rotation of the object only as it rotates. The expansion vector and the virtual torque in the fourth direction of space are both products of that virtual 'moment' that are realized (made real) from the 0-D point/twist's 'tendency' to collapse into itself to return to the absolute Void.

The expansion vector and the virtual torque are both directed into the fourth direction (what becomes the dimension) of space and are equivalent (overlap) to each other. The virtual torque becomes potential and with the expansion vector co-creates an equivalent expansion (thrust) in three dimensions through the 0-D point center that stabilizes the discrete 0-D point/twist Void so it cannot collapse, thus giving the 0-D point/twist Void a sense of dynamic equilibrium as well as stability. The 'twist' is the essential feature of the geometric dimensionless point that guarantees it cannot collapse back into the absolute Void. The absolute Void is itself a not-localizable 'no-thing-ness', so the 'twist' is what keeps the 0-D point/twist Void separate from the absolute Void, rendering it discrete within itself and distinguishable from the absolute Void as well as discrete from other such points as the expansion continues in all four dimensions of space with the passage of time.

The simple fact that the original 0-D discrete point/twist Void is distinguishable from the absolute Void follows a process of differentiation between the two which implies that there exists some form of primal awareness, something like a mutual reciprocation beyond a normal physical action/reaction process, between the two for each other. This mutual awareness, whether primal or not, is important because it is the precursor to the emergence of a virtual and thus semi-physical pre-consciousness field that guarantees the development of life, mind and consciousness in the later expanded universe. It could also be interpreted as either a potential for the later development (emergence) of consciousness or some undifferentiated type of virtual consciousness itself. Consciousness in living organisms is thus the evolutionary product of an internal awareness of the universe as a whole for its 'self' as well as an external awareness in everything sensed or otherwise interacted within the material world that has evolved from its elementary 0-D point/twist Void parts.

However, just as the outward expansion of the 0-D point/twist creates the dynamic equilibrium that prevents the 0-D point/twist from collapsing, the expansion also extends further in all four directions that become the dimensions of space (up/down, right/left, in/out, inward toward the center/outward from the center) to duplicate itself. In this manner, there is a dualistic spherical expansion that creates the universe itself. This process is commonly and popularly known as the Big Bang. Even though the individual 0-D discrete point/twists are themselves dimensionless, they also form relative dimensional directions that are necessary as parts of the greater whole since the greater whole is itself spherical relative to the original center of the original discrete 0-D point void. This process is difficult to picture, but it can be imagined with the help of a simple diagram.

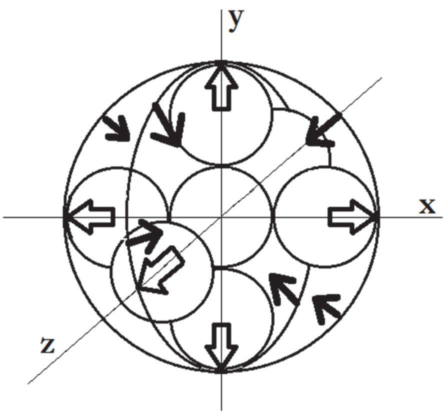

Figure 7. First round or moment of expansion of 0-D point/twist establishes an equilibrium state in 3-D space.

This diagram depicts smaller spheres representing the original 0-D point and the first expansion 0-D points in three-dimensional space. They are encased within a larger sphere which corresponds to the advancing 'front' of expansion as a whole. Each virtual (imaginary) point on the hypothetical 0-D point/twist Void acts like a keystone that keeps the sphere from collapsing inward at an actual zero point (nothing) as $\Delta r \rightarrow 0$ in a Riemannian fashion. The hypothetical 0-D point/twist sphere is literally a nothing trying to collapse into a 'no-thing' of its 'self' in an absolute Void.

The individual 0-D point/twist Voids within the sphere and the spherical front of the whole represent the combined point/extension duality of space and time. So just as the individual 0-D point/twist Voids exist in a state of dynamic equilibrium, the spherical front that is expanding outward from moment-to-moment also has a 'desire' or 'tendency' to collapse back into the individual 0-D point/twist Voids and the absolute Void as a whole as shown by arrows in the diagram. This arrangement yields a space that is isotropic (the same in all directions) with no specific measurable or discoverable center along the expanding three-dimensional surface of the whole.

However accurate this diagram is, it is also misleading. It depicts three-dimensional objects inside a three-dimensional sphere with a two-dimensional surface curved in the higher three dimensions of space, but it actually represents three-dimensional objects that are a part of a three-dimensional surface that is curved and expanding in the higher embedding fourth dimension of space. So it is at the same time as misleading as it is informative and another diagram that depicts this particular aspect of the 0-D point-twist Void contribution to the overall expansion of the surface is in order.

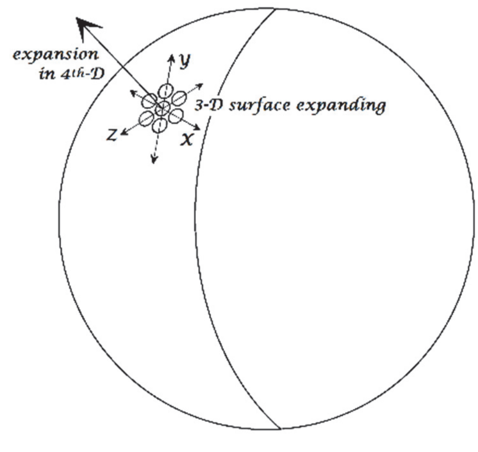

Figure 8. 3-D surface of the 4-D Riemannian sphere is expanding intrinsically and extrinsically.

In this case, the original 0-D point is surrounded by its derivative 0-D point/twist Voids in three-dimensional space, which are expanding the three-dimensional surface itself, represented by the surface of the larger four-dimensional sphere, while also indicating the corresponding expansion of the sphere surface in the higher embedding fourth dimension.

This picture is less misleading, but still incomplete as are all attempts to depict things and events in four dimensions within our lower three-dimensional space, let alone on a two-dimensional sheet of paper. This diagram better shows that (1) the original 0-D point/twist Void cannot be distinguished from other such 0-D point/twist Voids within the overall surface, *i.e.*, the universe technically has no center in three-dimensional space, (2) the surface is isotropic, *i.e.*, the same in each direction although the same is not necessarily true of the matter content of the universe, *i.e.*, when material bodies, represented by local variations in the overall curvature in the surface, are placed in various positions on the surface, and (3) the universe is closed with respect to the three dimensions of our normally perceived space which renders our universe unbounded (because it could possibly expand forever growing ever larger) yet finite. These three physical characteristics fit the observational data that no matter which direction we look in, the vast majority of stars and galaxies are receding (moving away) from us at equal speeds at equal distances as observed by Edwin Hubble in 1929.

Each 0-D discrete point/twist Void that is created during this expansion process has its own center relative to all other such 0-D geometrical points and thus sees itself as equivalent to the original discrete 0-D geometrical point. This relativity is necessary to guarantee that the discrete geometrical points that constitute the universe are continuous with one another while remaining discrete relative to other such points. The process thus depends on the necessity that fourth spatial dimension is closed and finite in extent rather than open, unending and thus infinite in extent. This last dualistic characteristic of expansion–discrete versus continuous (the quantum perspective) or more fundamentally point versus extension (the geometric perspective)–guarantees the emergence of a physically oriented three-dimensional space that must be embedded in a higher four-dimensional manifold that emerged from the absolute Void as a three-dimensional reaction to the quantized action of virtual torque directed in the fourth dimension of space.

7. Physicality of Space and Matter

Perhaps the strangest thing about our universe is how matter and energy came from the suspected Void of 'no-thing-ness'. The dominance of matter and energy in the universe seems to break all kinds of symmetry laws (there is missing anti-matter). This fact implies that something in nature is out of balance with itself since it seems that equal amounts of anti-matter and/or negative energy are necessary to balance out the equations of nature, but this view is not necessarily correct. While time and expansion have been equally accounted for in the discrete 0-D point/twist Void model as the sources for our space-time continuum, the virtual torque In the fourth dimension of space should be thought of as the source of a single potential field from which all matter, energy, and everything of a physical nature that we have ever observed or even will observe in our commonly perceived universe.

The twist portion of every discrete 0-D point/twist Void is a static property of the geometrical points or Riemannian point-elements of space-time, but since the universe is both dynamical and expanding the static twist is rendered dynamic by the production of the virtual torque that it 'creates' in the embedding fourth dimension of space. That torque creates a single field of potential in its collective form as the fourth dimension of space expands outward in conjunction with the three-dimensional spherical surface expansion.

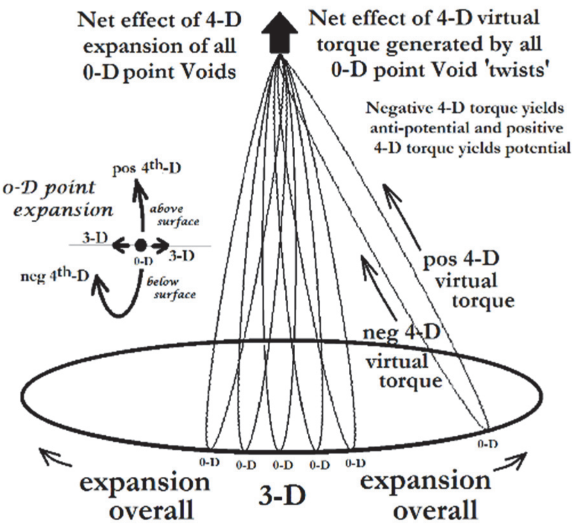

Figure 9. 0-D point/twists in 3-D surface (common space) are all connected via the single-pole in the 4^{th}-D.

While the net virtual torque increases exponentially for any given surface each and every moment as time advances forward, the whole amount of virtual torque would be internalized point-by-point as potential which manifests physically as the single field in the fourth dimension of space. Since the fourth dimension expands in two directions (above and below) the original primary three-dimensional spherical surface, but the two directions are connected by closure (they form a four-dimensional single-polar spherical surface of their own), the expansion would seem or appear from our three-dimensional perspective to be a push from inside the three-dimensional surface that we perceive as our whole universe on the single 0-D point/twist (the single-polar position in four-dimensional space) where the outermost 0-D point/twist Voids come together for closure.

All geometrical discrete points in the three-dimensional surface of our universe are directly (causally) connected to one another since their four-dimensional projections come together with each other at or through this single-pole 0-D point/twist without the intervention of other points (through some measurable distance) extended across the three-dimensional surface, *i.e*. there is no three-dimensional type of distance between different 0-D point/twists in three-dimensional space via the fourth dimension because physical distance is a term defined as relative to the various material objects that occupy only the three-dimensional surface of our material universe.

Distance is itself a purely three-dimensional concept or perceived construct and thus does not apply to the fourth direction of physical space. So the seemingly instantaneous signal or communication between distant points or places in three-dimensional space that is called 'quantum entanglement' is merely communication passing between three-dimensionally separated distant points through the single-pole in the embedding dimension of space where all three-dimensional 0-D point/twist Voids meet.

Having said that, the role of the single-pole and the collective virtual torque of individual 0-D point/twists acting on or through it (as expansion continues) needs further clarification, especially in its role in the emergence of material particles. A single-polar surface extended in the fourth direction of space from a single discrete geometrical point in the three-dimensional surface of our universe would look and act more-or-less like a Möbius strip, or better yet the collection of all such geometrical points in our three-dimensional material surface curved in the fourth dimension would yield a four-dimensional Klein bottle-like structure.

In other words, if you could travel completely around the fourth dimension from the three-dimensional surface upwards through the single-pole and return to the surface from below, you could not tell the difference except for the fact that very part of your body would be exchanged right for left. In the case of individual 0-D point/twist Voids, this reversal appears as a half-spin, so when real material particles emerged at the end of the eras of cosmic inflation they had a virtual half-spin. Elementary particles are not really spinning in any sense of the word, they just have an intrinsic property of virtual half-spin due to the geometry of the space-time continuum.

All real material particles must have a half-spin, either positive or negative, but it must and can only be a half-spin to be in full accord with the geometry of the universe. All of the other temporary energy resonances, single field patterns that do not quite meet the full complement of geometric and quantum requirements of the universe to be a real particle, that are presently deemed point-particles with spins of 0 or 1 by the vast majority of theoretical physicists are just pseudo- or temporary-particles. These point-like pseudo-particles, which are quite common in the Standard Model of the quantum where they are mistakenly dubbed as real particles, very rapidly decay to either real particles (with half-spin) with/or without energy in the form of kinetic energy carried by the real particles, gamma photons, or both. In fact, these single field resonance patterns or pseudo-particles decay so rapidly simply because they do not conform to the full complement of geometrical and quantum requirements to even 'exist' as real 'things' in our universe, let alone as real material particles.

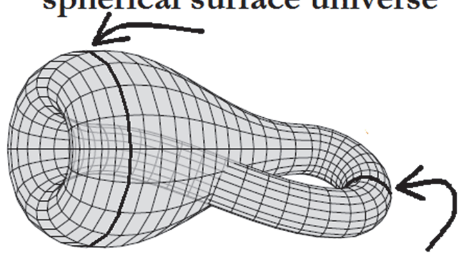

Figure 10. A 2-D embedded Klein bottle analogy to our 3-D embedded space.

Each subsequent 0-D point/twist Void that is created during the expansion process must be somehow connected to the next discrete 0-D point/twist Void while remaining discrete itself, therefore each geometric point is connected to its neighbor by a physical binding constant. Electric permittivity can be interpreted as the binding constant between points in three-dimensional space is the, between each 0-D point/twists in three-dimensional space while the magnetic permeability can be interpreted as the corresponding binding constant between neighboring 0-D point/twists in the fourth direction of space.

This leaves Planck's constant for the binding constant between the moments of time and all of the 0-D point/twists that constitute all four dimensions of physical space. In other words, time only began to progress forward when it became bound to space by Planck's constant since and the original 0-D point/twist Void differentiated itself from the overall absolute Void. Therefore, Planck's constant is subdued or hidden (invisibly from observation) within each and every 0-D point/twist Void or geometrical discrete point in physical space and time independent of any overall geometrical features of the universe as long as space and time are considered to act in tandem.

This suppression of time within each and every discrete geometrical point of space (or their physical equivalents in the discrete 0-D point/twist Voids) is re-expressed and comes out of hiding in the various equations of the Heisenberg uncertainty principle since these equations attempt to separate space from time while 'change' is occurring in some semi-related manner, *i.e.*, through the uncertainties in momentum and/or energy normally associated with 'change' in the physical status or material particles. These binding constants, in conjunction with the overall geometry of the universe determine which single field resonance patterns will become real particles or which will become pseudo-particles meant to decay into real particles as expressed by the Standard Model, within the limits established by how particles are philosophically and mathematically defined as opposed to how they actually represent themselves materially in the universe.

From the beginning of time and expansion, there has been a physical differentiation between the two directions in the fourth dimension since the expansion process moves in two directions in every dimension, including the fourth dimension. One direction (above the three-dimensional surface) yields a positive virtual torque from each 0-D discrete point/twist center as duplication and expansion occurs, while the other direction (corresponding to below the three-dimensional surface) creates a negative virtual torque. However, collectively these become a potential (from positive virtual torques) and an anti-potential (from the negative virtual torques) whose mathematical absolute value, 0-D point/twist by 0-D point/twist, becomes the pure potential of the single field.

These potentials together form the single field which give the universe its substantiality beyond the normal space-time continuum, which is described by the geometrical discrete point and their extension/metric equivalents. Since the anti-potential and potential are oppositely directed, they occur in equal amounts and were the universe ever to collapse back into the absolute Void, all substantiality (matter/energy and fields in three-dimensional space) would not necessarily cancel each other back into the nothing or nothingness of the absolute Void, although it might. Other than that there is no other reason for anti-matter and/or negative energy to exist, although anti-matter does exist, at least naturally in the form of anti-particles.

Anti-particles are merely equivalent particles that emerge from the anti-potential below or on the bottom side of the positively curved three-dimensional surface that we call our universe in equal numbers to their counterpart particles by pair production only after the initial rapid inflationary rate has decreased significantly to its slower expansion counterpart and through the occasional decay process of pseudo-particles. Since they exist as local metric curvatures in surface below or on the underside of the three-dimensional surface, they can mutually decay into photons by mutual annihilation only when they meet their counterpart positive particles that appear as small regions of localized metric curvature above the surface. In this case, the two localized regions of equal curvature above and below the three-dimensional surface just cancel each other out to a nearly flat local curvature around a single point in space, which amounts to a massless (non-curved point localization in space) photon or photons, containing an amount of energy equal to the mass of the original two particles according to the equation $E = mc^2$.

No energy exists or is created until matter itself is created by the cosmic inflation ending event since energy is a measure of the motion of matter relative to other bits of matter, except for the case of the photon which is another matter altogether. In this context, there is no such thing as an absolutely negative energy (nor is there even any need for the concept) and energy is always positive according to the relative scale of measurement. Any reference to negative energy could only indicate that any energy associated with a material body is moving in a negative direction in three-dimensional space relative to another material body's positive direction. In systems of material bodies, energy can be either added to (gained by) or removed from (lost to) the overall energy of the material systems which would also indicate positive (added) or negative (lost) energy relative to the overall energy of the material systems at any given time, but still no such thing as absolute negative or anti-energy exists.

Both the anti-potential and potential of the single field in the fourth dimension can only create energy, but that energy can be directed in the opposite directions in three-dimensional space. Furthermore, this means that it is useless to talk about energy in the universe or the universe having energy and thus a temperature before material particles are created at the end of the cosmic inflationary period of the universe's history. Before that moment in time, there is only the four-dimensional single field potential (anti-potential and potential), and that is a potential to create matter, energy and physical fields in three-dimensional space.

As such, space, whether three-dimensional or four-dimensional is being considered, is dualistic and all that is in space, *i.e.* physically real, must conform to that dualism, including common physical fields. In physics before the twentieth century that dualism was expressed in Newton's concepts of absolute space (point-based geometry) and relative space (extension based geometry) and did not play any role in either Newton's geometrical interpretations of the world or in his physics other than implying the real unverifiable existence of his absolute space. Newton's classical gravity theory uses a simple form of relativity and is thus distance/measurement-based (or metric). Therefore Newtonian gravity theory is not complete in the same sense as Maxwell's electromagnetic theory, which accounts for the dualism of space in the form of the electric field (extensive) and the magnetic field (point based).

The geometric dualism of electromagnetism is thus expressed by the Lorentz electromagnetic force equation $\mathbf{F} = q\mathbf{E} + q\mathbf{v}\ \theta\ \mathbf{B}$, while it is not expressed in the Newtonian gravity equation $\mathbf{F} = m\mathbf{g}$. However, in 1893 Oliver Heaviside did express the dualism of space without calling it that, he was just working by analogy to electromagnetism, when he wrote and explained the gravitational equation $\mathbf{F} = m\mathbf{g} + m\mathbf{v}\ \theta\ \mathbf{S}$. He explained the second term in his gravity force equation as a true centrifugal force of gravity, which is true, based on the attraction of the rest of matter in the universe.

In twentieth century modern physics that very same duality vexes and confounds science as the quantum theory (discrete point-based non-geometry) and general relativity (extension-based or metric geometry). However, the fundamental notion of that dualism and what it really means has been lost in the modern fallacy (a phallacy of fysics) that the quantum and relativity are mutually incompatible. They are actually perfectly if not absolutely compatible, but this cannot and will not be understood until the discrete versus continuous debate as well as the determinism versus indeterminism historical propaganda biases are completely discarded as purely philosophical (and thus distinguished from physical) gibberish that they are in favor of the more fundamental, realistic and physical dualism of point-versus extension-based geometry (what Riemann termed point-elements and metric-elements).

The dualism will only loose its mystery and negative retarding influence over physics and science, which has prevented true unification, when it is recognized as just that by its proper identification and analysis as a real difference between point and extension. Only then can the quantum and relativity be demonstrated and accepted as compatible within the Riemannian context of a three-dimensional spatial geometry embedded in a fourth-dimensional single-polar spherical geometry.

8. Big Bang Begat the Big Blowout

The term 'Big Bang' was originally meant as a derogatory yet descriptive appellation (Fred Hoyle, 1949) for a specific characteristic of the early expansion – the extremely rapid and thus explosive-like expansion of the early universe from nothing to a very large something. The term cosmic inflation or inflationary period came much later (Alan Guth, 1979) to denote an even more rapidly expansive but very short-lived period of time in the early universe that generated a large part of the volume of the space of our present physical universe. The cosmic inflationary period is thought to have started at 10^{-36} seconds after the Big Bang and ended abruptly at about $10^{\mathbf{-33}}$ or $10^{\mathbf{-32}}$ seconds.

These might seem like very precisely measured and very short time periods compared to our present perception of time, but given the time scale of the early universe as measured in infinitesimal moments of time even this short a period of time was quite long. After the inflationary period ended, the rate of expansion slowed to about what it is today with some small occasional variations. The big theoretical question then became 'what ended the inflationary period', yet very few scientists have ventured to speculate on either an answer to the cause of inflation or how it ended, let alone offer a valid and logical theoretical model of these events and occurrences. On the other hand, the single field theory presents a clear model that answers both of these cosmological questions.

Rapid rate of Inflation

# of moments	1	2	3	4	5	6	7	8	9
0-D points in each D	1	3	9	27	81	243	729	2187	6561
0-D points in all 3-Ds	1	7	42	252	1512	9072	54132	326592	...

Rate increases even more rapidly when doubling of parallel surfaces (above and below) in the 4th dimension is added to these numbers

Figure 11. Rapid rate of inflation in first few moments after the Big Bang.

The key to this answer is again understanding the concept of a discrete 0-D point-twist Void. How the simple expansion of the universe occurred as a byproduct of the dynamic equilibrium of the 0-D point/twist has already been explained, but how that became an explosive rate of expansion and how that explosive rate of inflation ended is still subject to further elaboration. A simple calculation of the first few moments after the singularity became a 0-D point/twist Void easily demonstrates how rapidly the initial cosmic inflation evolved/emerged from nothing.

With respect to our three-dimensional space, every infinitesimal moment saw the individual three dimensions expand at an exponential rate of three to the n-power, where n = 0, I, 2, … for succeeding moments of time, but the overall rate of collective expansion in the combined three dimensions of space was much greater as shown in the table above. After only eight infinitesimal moments, physical space had expanded to a total of 326,592 discrete 0-D point/twists.

Looking at the inflation problem in this way easily demonstrates the explosive nature of early expansion, but it must be remembered that this was all occurring before either the duration of time or extensions in space even became measurable quantities, although still infinitesimally small. Given the fact that an infinite number of infinitesimal geometrical point/twists make up any extended line, area or in this case three-dimensional volume, the time of 10^{-36} seconds after the initial Big Bang must have been enough time for number of geometrical point/twists per moment to reach an infinite number and only then did our relative measurable universe even begin.

To further complicate the situation and render the inflation period even more 'explosive', with each new moment a new three-dimensional surface of 0-D point-twists is added above and below the primary three-dimensional surface that is our perceived universe (the other surfaces being imperceptible). Each moment after that the primary surface not only expands three-dimensionally and adds new parallel surfaces above and below it in the fourth dimension of space, but each of the surfaces above and below it do the same and thus triple the rate of expansion of three-dimensional space across the whole of the fourth dimension.

During this whole process of inflation the universe was expanding not only three-dimensionally but also four-dimensionally. This expansion was accompanied by the creation of virtual torque in each direction (above and below) the three-dimensional surface which is our common space. The collective nature of the virtual torque above and below rendered an amount of substantiality to the space-time continuum that later manifested as the single field. In other words, after each infinitesimally small moment of time passed the total volume of the three-dimensional surface that is our universe not only tripled, but that tripling also doubled.

While there is a collective virtual torque element (torque is a vector and thus additive in both magnitude and direction) that corresponds to the overall expansion in the fourth direction of space, the individual virtual torques corresponding to the individual 0-D point/twists that constitute the successive parallel surfaces that occupy the fourth direction of space. Collectively, the 0-D point/twist associated virtual torques in the fourth direction of space constituted a real physical potential above the primary surface and an anti-potential below the primary surface.

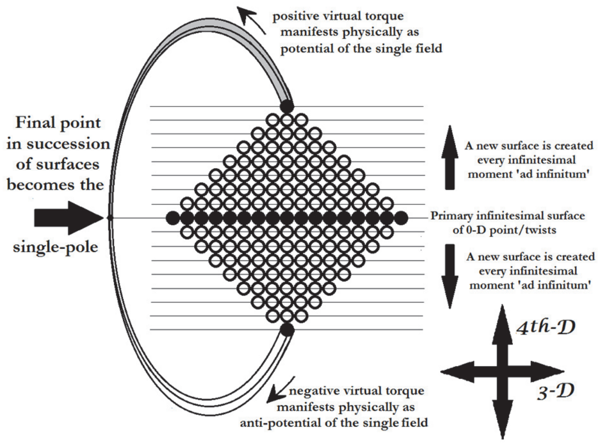

Figure 12. Each successive 3-D surface in 4-D space is smaller until the single-pole point is reached.

Together, the potential and anti-potential form the physically real single potential field that is the precursor to all matter, energy and three-dimensional potential fields in the universe.

Moreover, the tripling effect of each three-dimensional parallel surface reproducing itself both above and below every infinitesimal moment more-or-less concentrates the virtual torques within the three-dimensional surfaces and rendering the primary three-dimensional surface denser with respect to the single field than other surfaces. Each successive surface above and below the primary surface is less dense with respect to internal virtual torque and thus potential than the surface that preceded it in creation. The single field that is derived from or caused by the collective nature of the virtual torques associated with each and every 0-D point/twist Void thus gets less dense exponentially as the distance in the fourth direction of space from the primary surface increases.

This structure yields not only our common space-time continuum, but also the single field which gives us matter and energy, all of which cancels out to nothing if the individual 0-D discrete point/twist Voids were to collapse back into the absolute Void that existed before the first 0-D point Void and the subsequent Big Bang. This process thus explains how everything in our universe of 'somethings' can be created from the 'no-thingness' of the absolute Void.

9. The Big Blowout

At some point or moment in time determined by the geometrical restrictions and quantum limitations of the space-time continuum and universe as it then stood, quantum fluctuations constituting weaknesses in the surface emerged. These weaknesses caused a 'blowout' in the direction of the expansion (above the surface) in the fourth dimension of space. This blowout occurred at random points in the three-dimensional surface to create the very first material particles.

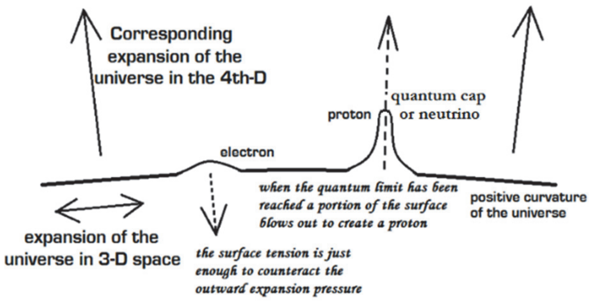

Figure 13. No anti-particles are created when inflation ends.

In other words, when the single field density of the surface reached a specific minimum value determined by the quantum and speed of light, individual 0-D point/twists in three-dimensional space would have erupted four-dimensionally outward (blown out not in) along the favored direction of expansion, creating three-dimensional protons in the surface that appeared four-dimensionally as exponential curves in the surface leaking potential into the fourth dimension.

The leakage stopped when the three-dimensional inner diameter of the particles reached a crucial quantum limit, determining the four-dimensional 'thickness' of a quantum measurement of infinitesimally thin three-dimensional surfaces that formed a sheaf or 'sheet'. [22]

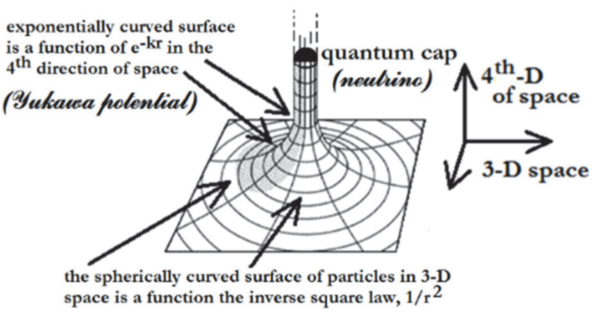

Figure 14. The exponential curvature of a particle in 4^{th}-D.

At this point in time, a quantum cap formed on top of the exponential curves in the surface that ended the blowout and created the very first protons. The quantum cap stabilized the blowout in three-dimensional space and the fourth dimension as well as smoothed out the singularity at the center of the curvature predicted by general relativity at the particle's mass center, thus guaranteeing that mathematical singularities (infinities) could not form or even exist theoretically in real physical space-time.

During this initial inflation-ending event, no anti-particles could have been created because anti-particles would have curved inward toward the non-existent spherical center of the expanding bubble (or below the primary three-dimensional surface) and thus against the 'momentum' of the outward direction of the expanding universe. Anti-particles display the same curvature, footprint (in the three-dimensional surface) and characteristics in three-dimensional space except for the single fact that their curvature is away from the expansion of the universe on the opposite side of the three-dimensional 'sheet', which gives them their characteristic opposite electrical charge.

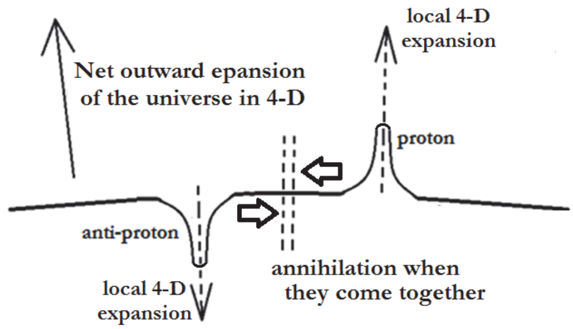

Figure 15. Mutual annihilation of a proton and anti-proton.

Anti-particles are inherently unstable in our universe because of their charge and oppositely directed internal stress (due to expansion in the fourth direction pulling on their quantum cap), but they cannot decay until they come into contact with their oppositely curved particles because something cannot collapse into nothing even though they do not decay into 'nothing' but equivalently with respect to matter/energy into other somethings. Furthermore, the mass of the anti-particles would be ever so slightly less (an asymmetry formed) than that of the counterpart particles because of their curvature on the underside of the curved surface and 'sheet'.

When any particle and its corresponding anti-particle make physical contact (their metric curves in the surface touch and overlap) and come together, occupying the same position in three-dimensional space even though their locations in four-dimensional are not the same, they annihilate their three-dimensional geometrical aspects by cancelling each other's geometrical curves. The interaction of merely touching or coming together at the same 'place' in three-dimensional space (which is actually a different 'place' in four dimensions) results in quantum cancellation due to the geometrical conditions which defined them as individual three-dimensional particles and thus completely annihilates the particle/anti-particle pair or curvatures that defined them.

However, their four-dimensional aspects or physical characteristics still remain, *i.e.* their single field potential, which manifests in three-dimensional space as gamma photons equivalent in energy to the previous proton's material content by the prevailing physical conditions of $E = mc^2$ (a three-dimensional restriction) and $E = hf$ (a four-dimensional restriction). The potential that was packaged within their curvature (in the form of point mass-inertia that becomes point Dark Energy when the metric curvature is annihilated) cannot be annihilated, so it is lost as gamma rays that have no curvature but still have energy content.

The extremely rapid rate of inflationary expansion in the early universe slowed drastically with the initial blowout during the creation of protons–like wind going out of a balloon–beginning to end the inflationary period. The newly formed protons act like excessive drag to a moving vehicle or a boat's anchor to quickly slow the overall inflation of the surface. However, the production of protons alone would not be enough to completely stop or sufficiently slow the rate of expansion. The 'virtual momentum' of the expanding bubble was still too great and a second almost-blowout occurred.

This second event would not have had enough local potential at any given geometrical 0-D point/twist Void in the three-dimensional surface to create more protons because the surface tension of the three-dimensional 'sheet' of three-dimensional surfaces would enough to counteract the blowouts, point-by-point, leaving smaller bumps in the surface – electrons. So electrons are curves or little 'hills' in the overall curvature of the surface/universe equivalent to the maximum amount of curvature up to the moment blowout or rupture of the overall surface to form protons.

This second slowing process of expansion all but ended the period of cosmic inflation by very nearly equalizing the overall expansion in all four dimensions of space, above and below the primary three-dimensional surface except for the amount of expansion favored (above) due to the positive curvature of the three-dimensional surface. The actual rupture or blowout at the three-dimensional location of the electrons did not become a complete rupture (and thus form a proton) because a 'surface tension' of the three-dimensional surface resulting from the opposing directed virtual torque (below the curved surface) held the three-dimensional surface intact.

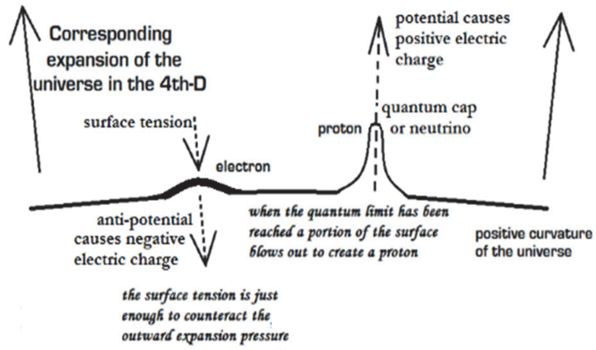

Figure 16. Creation of electrons in the inflation ending event.

In this case, the net effect was a downward tug (in the direction below the three-dimensional surface) or downward push by the 'surface tension' that gave the electron a negative electric charge as opposed to the proton which has a positive electric charge because the prevailing potential or virtual torque pulls upward (above) on the quantum cap that closed the three-dimensional curved surface that is the proton.

Any further quantum fluctuations in the single field that rose to the levels set by the quantum and geometrical conditions would have only caused excess 'burble' (point-centered turbulence) or small puckers of curvature in the three-dimensional surface in the form of neutrinos. So neutrinos represent the minimum amount of local curvature in the three-dimensional surface that can be distinguished as an individual particle from the overall positive (non-local) curvature of the three-dimensional surface, and thus their quantum 'width' would be equal to the 'effective thickness' of a 'sheet' of three-dimensional surfaces. They would not have any electrical charge because the potential and anti-potential of the four-dimensional single field would be equal and opposite along the minimal three-dimensional curvature.

The period of cosmic inflation would certainly have slowed sufficiently by the time any neutrinos were created and no anti-particles would or could have been created since the motion of the universe expanding outward (the favored direction of expansion due to the positive curvature of the sphere) created only the outward protrusions of curvature that are protons, electrons and neutrinos. Thus ended the inflationary period with the production of protons, electrons and some neutrinos, as observed in nature today, but without any production of anti-particles as required and predicted by the Standard Model of the quantum.

10. Conclusion

Given this process of particle creation, protons are not made of quarks as is claimed in the Standard Model of quantum theory. In fact, quarks are not particles at all and they only exist as a misleading description of the internal three-dimensional structure of protons, neutrons and some temporary pseudo-particles. Quarks have only been detected and misinterpreted as individual but not independent particles during high energy collisions because specific geometric and quantum conditions have been reached for their detection, but they are not real extended material particles in any sense of the word 'particle'.

Nor is there any need for intermediary particles such as gluons which do no more than fill a logical and/or mathematically driven need for completion of a theoretical model that is more false than true, rather than a physical necessity. Such particles were invented merely to make mathematical sense of a seriously flawed physical paradigm and maintain the scientific status quo. In fact, many of the particles of the Standard Model zoo are nothing more than machinations to prove and thus justify the mathematics rather than real measurable and thus verifiable physical entities, as for example gravitons, gravitinos, axions, super-symmetric particles, wimps, and many others.

Many other particles within the quantum zoo are no more than pseudo-particles which are just temporary intermediate energy resonance patterns of and within the single field that fulfill at least one and as many as several of the conditions for real extended material particle creation, but not quite all of the conditions necessary for true stable particle creation. Therefore they destabilize rapidly and decay into real particles with or without kinetic energy and/or gamma rays. In other words, pseudo-particles are just intermediate potential states of single field density resonances that occur during real particle (proton, electron and/or neutrino) creation.

Yet many portions of the Standard Model and its predictions as well as the quantum theory's overall physical features are still perfectly valid and correct, so all that is really needed for unification is a change in the interpretation of the fundamental concepts of the quantum theory that would allow its better portions to be incorporated into the single field theoretical model and its subsequent validation as a major contributor to a unified field theory such as the single field theory. The coming together and merging of the two paradigms–relativity and quantum–in this manner is necessary for physics and the scientific understanding of both the external world of matter and fields as well as the internal mind/consciousness that perceives and interprets our world.

Several facts, or rather physical truths, have become apparent with the adoption of the single field model of reality that originates with 0-D discrete point/twists of Void which are the physical correlates of purely geometrical discrete points. The quantum theory as it stands within the present paradigm is, was and will always be incomplete which has led to the discrepancies between portions of the Standard Model which are correct and those which are merely speculative conjecture, but the general theory of relativity, as presently interpreted as a four-dimensional space-time continuum with intrinsic curvature, whatever 'intrinsic curvature' means, is also incomplete.

Relativity theory has previously defied unification in any form because it has either ignored or misinterpreted the fundamental problem of the space-time continuum as defined by the point/extension duality. When this duality and its physical consequences are taken into consideration, general relativity, quantum theory in both its modern (Standard Model) and classical (quantum and wave mechanics) configurations, Maxwell's electromagnetic theory and classical Newtonian mechanics can be unified into a single generalized geometrical model of the space-time continuum.

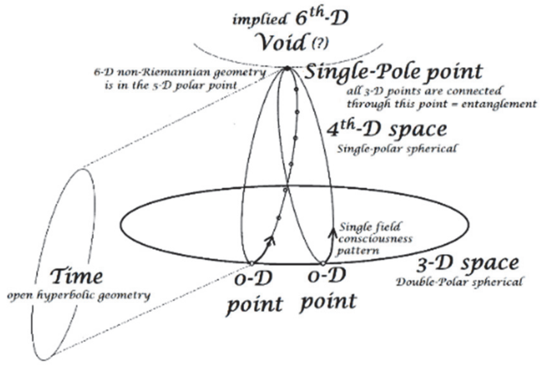

Figure 17. A 6^{th}-D is implied by the physics of the 4^{th}-D space.

The Riemannian geometry that expresses the unification starts with the discrete 0-D point/twist Void which creates our commonly experienced three-dimensional physical space as well as the embedding fourth dimension of space upon expansion in all four dimensions. While this geometry accounts for and describes the creation of the four-dimensional space-time continuum it also accounts for the dynamical substantiality of our world from the creation of world.

The twist portion of the three-dimensional 0-D point/twist void maintains the discreteness and integrity of this basic unit of co-creation while itself creating the 'virtual torques' (pre-force) in both directions of the fourth dimension which are collectively the precursors for the potential and anti-potential of the single field.

The discrete nature of the individual 0-D point/twists of Void allows the quantum field to be rendered in terms of Riemannian geometry while quantum and wave mechanics can be adequately explained as physical characteristics of the geometrical point/twists (discrete quantum field centers) within the context of the single field (which is equivalent to Bohm's quantum potential field as well as the superposition of all possible Schrödinger wave functions for all possible quantum events).

The single field is the precursor to classical three-dimensional fields such as gravity, electricity and magnetism, while matter/energy, life and consciousness are accounted for as the various single field density patterns in five-dimensional space which appear as extrinsic four-dimensional space-time curvature in the overall five-dimensional continuum. Scientists and non-scientists alike mentally perceive local curvature and the time variation of local curvature of the three-dimensional surface curved extrinsically in the higher embedding dimension as the solid material bodies that constitute relative three-dimensional space and one-dimensional time with our three-dimensional brains and the sensations of our five normal senses (which are three-dimensionally biased). We collectively interpret these sensations against the context of our minds and within our universal sense of consciousness.

What we perceive or detect as Dark Matter is merely the interaction of local curvature with the overall positive curvature (non-local) of the universe, or rather the interactions of local matter with the rest of the matter (non-local) of the universe as local material bodies orbit or move relative to a more massive central material body. This interpretation of mater and motion strictly embodies Mach's principle and offers a solution to the three- or more-body gravitational problem that has vexed physics for centuries.

These gravitational concepts are accompanied by the electric and magnetic fields as perceived by us as well as the emergence of life, mind and consciousness through the evolutionary process as special complexities of matter/energy, electric and magnetic fields. The evolution of life and consciousness itself has been influenced by and proceeded from a primordial or primal awareness based on the reciprocal relationship between the absolute Void of nothingness that preceded the Big Bang and the 0-D discrete point/twist Void that emerged from that absolute Void as the original singularity. The 0-D discrete point/twist Void thus introduces a way to explain how the 'some-thingness' of our perceived physical/material universe emerged and evolved from the 'no-thingness' of an assumed absolute Void (with no physical properties) that existed before the Big Bang.

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Modern Applications of Boscovich's Unified Field Theory

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Independent
Incorporating work of Dragoslav Stoiljkovich and Augustus Prince
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Boscovich provided a unified theory of point-particles, and this served as basis for Modern quantum mechanics. Applications of Boscovich's theory to quantum problems, such as to Modern chemistry will be considered based on the work of Dragoslav Stoiljkovich. The works of Stoiljkovich having not been published very much in English, and thus has not been more widely known among English-speaking scientists. This author is now engaged in translating these works in corroboration with Stoiljkovich; works that highlight the modern work that is still being pursued based on Boscovich's unified field theory. Also, briefly the work of Augustus Prince will be dealt with.

Keywords: Boscovich, unified field theory, polymerization

1. Introduction

This paper is associated with issues raised in my talk for Vigier Conference 2014.

2. The First Unified Field Theory

As far as I know what I am talking about is the first unified field theory. John D Barrow [1] says: "Boscovich was a passionate Newtonian who was the first to have a scientific vision of a theory of everything." Of course, the influence of Einstein on physics has diverted much of the attention from looking at Newtonian unified theories. However according to Einstein in 1954, the year before he died he admitted to a friend that he might have been on the wrong track [2]: "I consider it quite possible that physics cannot be based on the field concept, that is, on continuous structures. Then nothing remains of my entire castle in the sky, including the theory of gravitation, but also nothing of the rest of modern physics." And indeed, it is my contention that much of the quest for unified field theory has been on the wrong track. In my Nexus article 2001 [3] I point out the similarities of Boscovich's theory with superstring theory in its use of higher dimensions etc. It's merely that mistakes have been made since the 18th century and physics could be put back on the correct path.

3. Translation Project

Due to the lack of interest in Boscovich because of the diversion set up by Einstein, I have taken it upon myself to start translating from Latin some of the numerous works of Boscovich 18th Century physics into English; in the hope that this might attract interest in getting his work more widely known.

My latest translated book is "On Living Forces, Roger J Boscovich 1745". [4] In it Boscovich says: "Et quidem ita putamus, luminis repulsum in reflexione non fieri ab impactione in eam superficiem, a qua id reflectitur, ut eam propositionem censeamus a Newtono demonstrari, quantum in physicis licet."

I want to call your attention to the last bit which translates as: "…proposition concluded from Newton demonstrated, quantum physics allowed." This is from 18th Century and is talking about Quantum Physics, where "quantum" is a Latin word. However, it roughly means "as much as." So, might it be read as: "…proposition concluded from Newton demonstrated, as much as physics allowed."

We have this problem in translation that Latin words have been adopted by the English language. So, what Boscovich meant "quantum" in the context of the 18th Century does not now necessarily mean what we have adopted the word to mean since. But still Boscovich is talking about Quantum Physics!

So, let's get onto history of physics: Theories of matter. From early times, there was question of what was matter, was it continuous or discontinuous. The atomic theory of matter came from ancient Greeks such as Democritus.

This is the sort of typical thing said by science education online [5]: “460 BC Democritus, from Greece developed the concept of dividing matter into smaller and smaller pieces until you could divide it no more. He called these smallest pieces atoms (atomos = indivisible in Greek). This was a philosophical idea rather than a scientifically based theory. After the Greek era, scientific investigations greatly diminished. The Romans, although great engineers, were less interested in the nature of matter. And with the onset of the Middle Ages dogma became more important than science. As a result - for over two thousand years - the atomic theory lay dormant.

“1803 - Atomic theory John Dalton, English chemist, revived the term of the atom when he suggested that each (chemical – my comment) element was made up of unique atoms and the atoms of an element are all the same. He formulated his theory that chemical reactions result from the union and separation of these atoms and that atoms have characteristic properties. Combinations of atoms bound to each other he designated as molecules. For a while it seemed that atoms were fundamental. As their name suggests, they could not be split into anything simpler. However, towards the end of the 19th century, it became clear that atoms are not fundamental - they themselves are made of smaller particles. One of these atomic particles is the electron, which we now think is fundamental.”

And that is wrong. First of all, it misses out a lot between Democritus and Dalton, so that is a very condensed history. Second – it is mixing up two different concepts by the word “atom”. What Dalton meant by “atom” is not what the ancient Greek Democritus meant by “atom.”

That might be one of the reasons for the confusion. The ancient Greek “atom” could not be split, but the Dalton “atom” could be. The ancient Greek idea of “atom” that cannot be split, is closer to the idea in Boscovich’s version of atomic theory where he has such an “atom” as point-particle.

It should also be noted that modern physics is based on the point-particle. Hence Boscovich is sometimes known as father of modern atomic theory. [6]

It’s strange that this is usually omitted from science education. But science education is a brief summary of some important parts of science, and they miss out a lot; so, the educators must have unfairly decided to omit Boscovich. What they miss is that: modern physics comes from Boscovich’s theory. Quantum physics originated with Boscovich, and he dealt with relativity of Galileo so he gave a unified theory of quantum and relativity physics, published in his theory in 1758. [7]

Yale University Library says [7]: “Dubrovnik-born Ruđer Bošković (Italian Ruggero Boscovich) was a mathematician, physicist, astronomer, philosopher, diplomat and poet. Educated at Collegium Ragusinum and the Jesuit *Collegium Romanum* in Rome, he became a professor of mathematics at Collegium Romanum in 1740. After his studies Bošković joined the Jesuit order. In 1736 he began publishing mathematical, physical and astronomical treatises and continued to do so until his death in Milan in 1787. His main theories proposed in “*Theoria Philosophiae Naturalis*,” first published in Vienna in 1758, are in harmony with conclusions arrived at by the methods of modern scientific research two centuries later. He wrote about the constitution of matter, the law of gravitational forces, atoms, space and time, relativity, and the theory of light. His work anticipated the modern theories of physics.”

So, what we have is an 18th century version of a unified theory of quantum and relativity physics. Of course, with revolutions caused by Einstein most physicists are thinking in terms of a relativity theory and quantum theory that has so far refused to be combined into a unified theory.

My position is that modern physics in 20th-21st century with its quest for unified theory has gone wrong, and the unified theory was in the 18th Century.

What does not help with the modern quest for unified theory, is the fact that educators are teaching a bad version of history. For instance, they credit quantum theory starting with Planck 1900. [8] ESA (European Space Agency) website says [8]: “At first his [i.e. Planck’s] theory met resistance but, due to the successful work by Niels Bohr in 1913 calculating positions of spectral lines using the theory, it became generally accepted. The quantum theory was born. Planck himself said that, despite having invented quantum theory, he did not understand it himself at first. Nevertheless, he received the Nobel Prize for Physics in 1918 for his achievement.”

So, in general such people are ignoring those dealing with quantum-type ideas before Planck. This is a distortion of the actual facts, and is what is called “Whig history.” The term “Whig history” arising from historian Butterfield in his book “The Whig Interpretation of History” (1931), where he criticized Victorian historians [9]: “who wrote history as if the liberal values and institutions of their day were the end-point of historical progress. His devastating critique quickly became commonplace within the profession, and the “Whig interpretation” came to be stigmatized as the hallmark of an unprofessional style of history practiced only by politicians and popular historians.”

There is a similar biased retelling of history by modern physicists. They don’t say what actually happened, instead they give a biased version, a "Whig history." Quantum theory actually starts much earlier than Planck, with Boscovich. Child in his translation of Boscovich's book says [10]: "[Boscovich's] theory also suggests curious - almost uncanny - intimations of general relativity and quantum mechanics."

So, what is this point-particle theory of Boscovich?

Well we all get taught about point-particles in physics, and treated like the idea is obvious. So, that maybe another reason why they miss out teaching who came up with the idea for a comprehensive theory of it; the educators might think it trivial. But it is not trivial, Boscovich had a unified theory of point-particles. And that is not an obvious a theory, because it deals with a unified force law.

The basis of theory comes from Newton query 31, which reads as follows [11]: "Have not the small Particles of Bodies certain Powers, Virtues, or Forces by which they act at a distance, not only upon the Rays of Light for reflecting, refracting and inflecting them, but also upon one another for producing a great part of the Phaenomena of Nature? For it's well know that Bodies act one upon another by the Attractions of Gravity, Magnetism and Electricity; and these Instances shew the Tenor and Course of Nature, and make it not improbable but that there may be more attractive Powers than these. For Nature is very consonant and conformable to herself." Which the Yale site goes onto say: "Here Newton suggests that many forces stronger than those already known to man (Gravity, Magnetism, etc.) may exist. These strong attractive forces might explain how "small particles" interact with each other and light."

Paolo Casini says [12]: "In fact, Boscovich borrowed from Query 31 the idea of reducing to "a single law of nature" the elementary interactions between his puncta." Puncta meaning "point-particle".

Boscovich explained that matter "consists of points that are perfectly simple, indivisible, of no extent, and separated from one another; that each of these points has a property of inertia, and in addition a mutual active force depending on the distance in such a way that, if the distance is given, both the magnitude and the direction of this force are given; but if the distance is altered, so also the force is altered; and if the distance is diminished indefinitely, the force is repulsive, and in fact also increases indefinitely; whilst if the distance is increased, the force will be diminished, vanish, be changed to an attractive force that first of all increases, than decreases, vanishes, is again turned into a repulsive force, & so on many times over; until at greater distances it finally becomes an attractive force that decreases approximately in the inverse ratio of the squares of the distances" (form Boscovich's Theoria, Synopsis of the whole work). (See Fig. (a)-(c).)

By Boscovich, these elementary points combine producing more complex particles of first order; two first order particles combine producing second order particles, etc. By Boscovich, the same law of forces explaining their interaction is valid whatever is the order of particles.

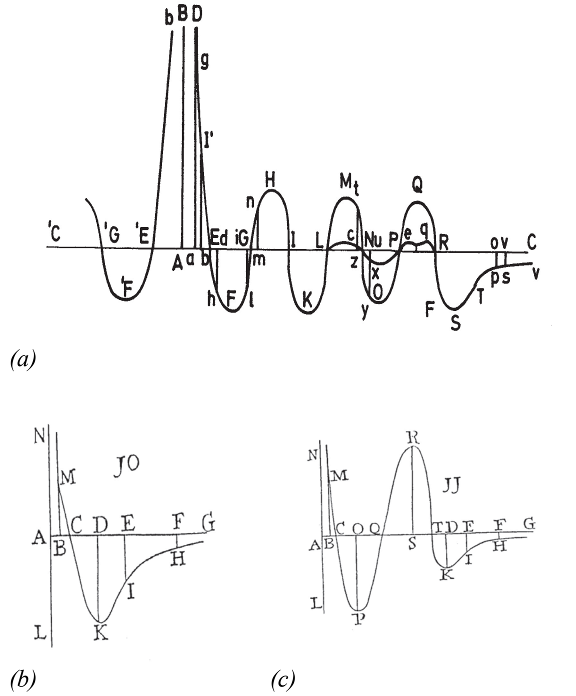

Figure 1. General (a) and special shapes of Boscovich curve show the change of attractive and repulsive forces (lower and upper ordinate, respectively) with the change of the distance (abscissa) between the elementary points of matter (Figure 1 in Boscovich's Theoria).

The difficulty in modern physics is unifying Einstein's relativity with quantum mechanics, which results in various attempts such as quantum gravity and superstring theory. However, Boscovich's theory has no such difficulty, and unifies the relativity of his day which was Galileo's relativity with his quantum ideas. It is my contention that relativity as developed from Galileo's time to Einstein via Boscovich has incorporated many mistakes along the way [13]. So Boscovich's unified theory is a purer form of physics.

So now onto modern applications:

4. Dragoslav Stoiljkovich's Work

Dragoslav Stoiljkovich since 1981 up to retirement in 2012 was employed at the University of Novi Sad, Serbia, at the Faculty of Technology, Department of Materials engineering, as regular professor of the production and processing of polymeric materials, as well as for the case "The methodology of scientific research work" [14].

In a Science Interview of Dragoslav Stoiljkovich [15] says: "The booklet of eighty pages ("Roger Boscovich – The founder of modern science", published by the Research Center Petnica in Serbia) [by Dragoslav] [16], will cause a storm among the local physicists. And only [cause a storm] among foreign [physicists], if they read [it]?" So, what we have is a chemist pointing out that physicists have overlooked Boscovich's theory.

Ideally there should have been big publicity about Boscovich's theory after Rutherford experiment decided matter was made of point-particles in mostly empty space. Purdue University says [17]: "When he published the results of these experiments in 1911, Rutherford proposed a model for the structure of the atom that is still accepted today. He concluded that all of the positive charge and essentially all of the mass of the atom is concentrated in an infinitesimally small fraction of the total volume of the atom, which he called the nucleus (from the Latin for little nut)." In other words, matter is concentrated in small regions of space as if were points, i.e. point-particle theory of Boscovich. There were various theories of matter such as it was continuous. Boscovich's point-particle was the one being confirmed.

Picking up now from Stoiljkovich [18]: "In 1912, after seven months spent with Thomson in Cambridge and four months spent with Rutherford in Manchester, Niels Bohr in 1913 calculated the possible paths of electrons, taking into account that electrons can move from one orbital to another only if they receive or lose a certain amount of (quantum) energy – as Boscovich said a century and a half earlier… Today, this model of the atom is called "Bohr model.""

The Boscovich part is explained as [19]: "Boscovich indicates that it stems from his theory that as a particle approaches another particle, and when it passes from one to the other limits of cohesion, it will lose or gain exactly a certain amount of energy. That "quantum energy", as it is now called, between the two limits of cohesion is equal to the difference between areas delimited by repulsive and attractive arches."

William P D Wightman says [20]: "It is doubtful whether the reputation of any other natural philosopher of the first rank has suffered such an undeserved eclipse as that of Roger Joseph Boscovich SJ. Nor is the word 'eclipse' an idle metaphor; for less than fifty years ago this luminary shone with a light undiminished by the passage of a century." That was 1962 so forgotten in period 50 years before that is about 1912, and still mostly forgotten today 2015, so forgotten so far as about 100 years.

Dragoslav Stoiljkovich in his paper "Importance of Boscovich's theory of natural philosophy for polymer science" [21] admits that Hermann Staudinger is generally presented as having given the macromolecular hypothesis in 1920. However, states that really Roger Boscovich in his book "Theoria philosophiae naturalis" (1758) was the first to give the macromolecular hypothesis.

According to Stoiljkovich, Boscovich's first, second and other higher order particles can be recognized in modern physics as protons, neutrons, atomic nucleus, atoms etc.

The term "atom" can be a problem in the context of Boscovich, and Stoiljkovich says [22]: "The term 'atom', Boscovich implies for a particle that is composed of parts, and these parts remain together in an atom owing to the force described by his curve." Hence, in Part 6 of this article we have several types of "atom" to consider.

Boscovich in his book (paragraph 440) says that the atoms could be connected: "In such a way atoms might be formed like spirals; and, if these spirals were compressed by a force, a very great elastic force or propensity for expansion would be experienced." And then he goes on to further details.

According to Stoiljkovich [21] Boscovich gave statements that the series of points and spirals of atoms could be long and the large number of slight bends meant that he was proposing a high degree of polymerization. Thus: "Boscovich suggested all the basic characteristics of macromolecules: the chain structure, the high degree of polymerization, the possibility of spiral chain conformation, the change of the conformation as a result of the slight bendings of chemical bonds and also the elastic properties of macromolecular materials."

Boscovich stated that the same law of forces explaining their interactions is valid at whatever is the order of particles. Stoiljkovich collected empirical data from modern physics concerning the interactions of various particles at nine levels of the hierarchy of matter (form nucleons, atoms, molecules, nano-particles, macromolecules up to colloidal particles) and confirmed this Boscovich's statement.

Stoiljkovich then goes onto explain that he has applied Boscovich's theory several times to solve problems of polymer science as follows:

1. Physical meaning of cohesion and non-cohesion limits on the molecular scale. By applying the theory of Savic-Kasanin for the calculation of specific volume of matter whose molecules are at the cohesion or at the non-cohesion limits of Boscovich's curve. Proving for 143 substances that the cohesion and non-cohesion limits correspond to the characteristic states of matter

such as critical point, triple point, absolute zero temperature etc. These states are the inherent properties of matter, which do not depend on the pressure and temperature.

2. Free radical polymerization of compressed ethylene gas. The peculiarity of the ICI (1933) polymerization process is that it can only be performed if the ethylene gas is compressed to a very high pressure. Hunter suggested that ethylene molecules were regularly packed, properly oriented and highly distorted at polymerization conditions, and that a supra-molecular organization of ethylene was a prerequisite for polymerization. Usually the ethylene molecules interaction is as presented by Lennard-Jones potential curve (1924), and which is similar to one type of the Boscovich's curve (1745). Instead of that, another type of Boscovich's curve was proposed by Stoiljkovich as more appropriate. Based on that the supra-molecular particles of compressed ethylene were suggested, which have been confirmed by thermodynamic, physical and spectroscopic methods.

3. Effect of pressure on melting temperature of low density polyethylene (PE-LD) was predicted by Boscovich's law of continuity, and confirmed by the empirical data.

4. Polymerization of liquid methyl methacrylate has accelerating rate, so called the gel effect or Norrish-Trommsdorff effect. To explain that phenomena, Stoiljkovich used a curve proposed by Boscovich's for the interactions of particles in a liquid. The supra-molecular organization and the fractions of order and disorder domain in MMA were calculated, and confirmed with experimental measurements.

Stoiljkovich concludes that Boscovich's theory is very important for polymer science. However, he says: "His theory is of greatest significance for other scientific fields, such as the particle theory, the electric and magnetic field theory and the quantum mechanics. Great contributions have been made to mathematics, astronomy, theory of relativity, optics and physics of elementary particles on the basis of Boscovich's theory…"

Stoiljkovich also points out that: "Werner Heisenberg in 1958 placed even greater emphasis on the importance of Boscovich's ideas for the 20th century science: The remarkable concept that forces are repulsive at small distances, and have to be attractive at greater ones, has played a decisive role in modern atomic physics."

5. Part of Paper by Augustus Prince (Given Permission to Include) Dealing with Unified Force of Boscovich

A very interesting comment concerning Boscovich and the possibility of a unified field interpretation was made by J.C. Graves [23] where he speaks of Boscovich's field of force: "It is this 'substantialization of force' which is one essential requirement for the notion of a field. In field theory, a particle interacts directly (i.e. by spatio-temporal contiguity) with the field at the point where it is located, and only indirectly with any possible sources of that field. For Boscovich seems in fact to be creating a trichotomy of space, matter (identified with the point inertial masses), and force. While it is true that mass and force appear to be proportional, they are different sorts of entities, and Boscovich would certainly want to keep inertial and gravitational masses as separate concepts only accidentally related. Inertial mass is localized at the center of force, but gravitational mass really extends throughout space. But most important of all, insofar as Boscovich may be said to have a field of theory, it is a unified field theory. There is no multiplicity of forces surrounding the central mass and exerting independent influences on any test particles elsewhere in space, but only one.*

[*By this is meant one unified force. But Stoiljkovich would like to point out: The title of Boscovich theory is: "Philosophiae naturalis theoria redacta ad unicam legem virium in natura existentium", i.e. "Theory of natural philosophy reduced to a unique law of forces that exist in the nature." That means, there is multiplicity of forces but only one unique law presented by Boscovich's force-distance curve. By this unique law, Boscovich discussed different forces, i.e. gravitation, electrical and magnetic forces.]

Although the total force-function may include many terms, they are all functions of r which may be simply added together, i.e.,

$$F = \sum_i f_i(r) r$$

This force F will then affect all bodies in the same way, depending (presumably only) on their respective inertial masses. Boscovich's vision is certainly admirable. Its main weakness is that he never gave analytic (algebraic) expression for the total force; the most he achieved was a graphical representation of it. A reasonable expression might be a sum of increasing negative powers of r, so that

$$f_i(r) = \alpha_i r^{-i}$$

where the first term would be I = 2,

$$\alpha_2 = -Gm$$

(the gravitational term). The terms would alternate in sign, with the last term being opposite to that of the gravitational. (We will see that general relativity introduces correction terms of just this sort into the law for the gravitational field of a single mass-particle.) But there is no indication of what the magnitude of these other terms might be, what physical interpretation could be given to each of them, or whether the α's would require introducing any new parameters which might have to refer to other essential properties of matter."

In a footnote, Graves [24] continues: "Boscovich might in fact have been able to resolve Olbers' paradox that the night sky would be infinitely bright in an infinite universe with uniform average density of matter under Newton's law; he could simply have introduced an additional term (proportional to 1/r, say) effective at great distances."

It is this footnote that draws one's interest. Throughout the scientific literature regarding the quantum science of galactic structure, many theorists have commented on the non-Newtonian aspects of the universe on the large scale.

Following the hint of the 1/r term, a search of the literature finds several investigations. One of the many findings is how many cosmologists modify Newton's gravitational law using a Yukawa functional form for the potential given by:

$$V_{Yukawa}(r) = -g^2 \frac{e^{-kmr}}{r}$$

where g is a magnitude scaling constant, m is the mass of the affected particle, r is the radial distance to the particle, and k is another scaling constant. The potential is monotone increasing, implying that the force is always attractive.

There have been many efforts to modify Newton's law. Von Seeliger [25] felt that Newton's inverse square law was not exact and stated that it was only an empirical formula. In his modification, he made the assumption that an attenuation factor be used to express the gravitational force F between bodies m and m′ be given as:

$$F = -Gmm' \frac{e^{-\lambda r}}{r^2}$$

Seeliger's effort was followed by another modification of Newton's law given by C. Neumann [26] who felt that the problem could be resolved by using a potential of the form

$$\phi(r) = \frac{Aa^{-\alpha r}}{r}$$

Which led to a generalized force F given by:

$$F = -Gm_1m_2 \frac{1+\alpha r}{r^2} e^{-\alpha r}$$

These ideas of Seeliger and Neumann in which a slight adjustment of modification of Newton's inverse square dependence has recently been address by others. One of these is E. Fischbach et al. [27] whose modification of the Newtonian effects is described using a modified expression for the potential energy V(r) is given by:

$$V(r) = \frac{-G_\infty m_1 m_2}{r}\left(1+\alpha e^{-r/\lambda}\right) = V_N(r) + V'(r)$$

Here G_∞ is the Newtonian constant of gravity, and the parameters λ and α, respectively, give the range of the new force and its strength relative to gravity. Also, V'(r) describes the correction to the effective gravitational potential arising from the particular non-Newtonian interaction we are considering (which in this case is a Yukawa).

This in turn leads to a force F(r) given by:

$$F(r) = \nabla V(r) = \frac{-G_\infty m_1 m_2 r}{r^2}\left[1+\alpha(1+r/\lambda)e^{-r/\lambda}\right]$$

Fischbach et al. continue their modification by assuming a model which contains two canceling Yukawa potentials which result in an approximate exponential. It is suggested that the reader consult Fischbach's assumption, since it is too detailed to be presented here.

The fact to be considered is that in all of the efforts regarding the modification of Newton's law with a Yukawa potential, substantiates Grave's [28] comment regarding Boscovich's 1/r dependence for a field of force, in this case gravitation with this in mind, a modified Yukawa force based on the Boscovich curve in the S, V and S′ V′ regions was developed in an ad hoc manner is given by:

$$F(x) = \frac{ne^{(px)}}{x}$$

The curve using this equation is shown in the Figure 2 below

n = 0.0041215

p = 0.0549

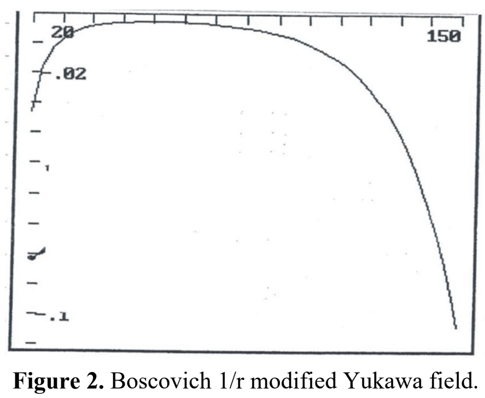

Figure 2. Boscovich 1/r modified Yukawa field.

One will note that the exponent is positive instead of the negative exponent that exemplifies the conventional Yukawa force. The reason for this is that in Boscovich's Fig. 14 the curve drops sharply negative in the S′ V′ and S V region.

As an aside, G. Bertin and C.C. Lin in their book, "Spiral Structure in Galaxies, A Density Wave Theory" [29] produced a curve based on a positive exponent in what appears to be a Yukawa type force which is depicted in Figure 3 below:

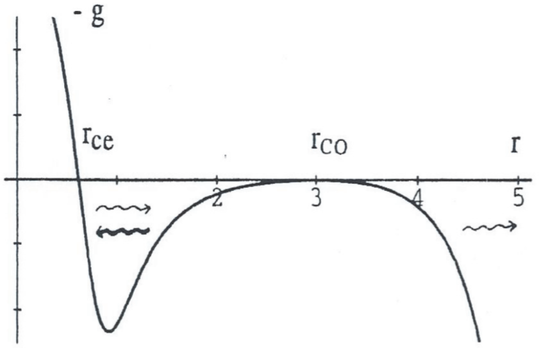

Figure 3. Positive exponent Yukawa force.

They mention that there are two turning points (i.e. two zeros of the function g). One is at $r = r_{ce}$, a simple turning point, and the other at r_{co} a double turning point.

The relation between the Newton inverse square law and the 1/r dependence seems obvious and compares to first figure.

This might indicate that Boscovich was ahead of his time again, since an extremely large negative force might imply that this might be due to large masses at extreme distances beyond our observation. Such a situation might explain the so called "dark matter" that is spoken about in today's cosmology.

The concept of dark matter was initiated by J. Oort [30] who was studying stellar motions in the galactic region.

This was closely followed by F. Zwicky [31] in his study of clusters and galaxies. Then in the 1960 to 1970 interval, V. Ruben and W.K. Ford Jr. [32] established a method using more sensitive instruments to analyze velocity curves of distant galaxies with much more precision.

Based on the findings of Oort, Zwicky, Rubin and Ford Jr. it can be said that here might be some substantiation of the Boscovich curve. With this information, it might be said that this concludes the empirical description of Boscovich's famous curve.

Its various regions for the microscopic and possibly solid state range is denoted as Region A. The Newtonian range as Region B and the "dark matter" distance range as Region C in Figure 4 below.

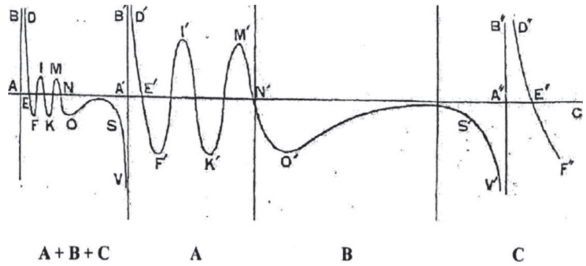

Figure 4. Newtonian range as Region B and the "dark matter" distance as Region C.

From *Theoria* Fig. (4)

A: $F(x) = \frac{Be^{(-kx)} sin\left\{\frac{1}{5}\pi\left(e^{(kx)} - e^{(-kx)}\cos(\pi x)\right)\sqrt{5}\right\}}{x^3}$

$$k = 0.481211825$$

B: $F(2) = -\frac{gk}{x^2}$

$$g = 6.668$$

C: $F(3) = \frac{-ne^{(px)}}{x}$

$$n = 0.0041215$$
$$p = 0.0549$$

The numerical values for the constants shown in the above figure have been chosen so as to make the Boscovich curve continuous throughout.

One must remember that the curve is a qualitative one and cannot be shown in scale due to the fact that in Region A we measure distances in 10^{-13} cm or less, while in Regions B and C we measure distances in light years, e.g., in Region C we speak of 46-47 million light years.

It goes without saying that much needs to be done especially in defining the various constants used in the curve description.

Referring to Seeliger, Kragh [33] states: "The modified force law was essentially ad hoc and also arbitrary, since many other modifications might resolve the gravitation paradox in a similar way. The idea of modifying Newton's inverse-square law was not, by itself, very original, as many modifications were proposed in the nineteenth century. The exponential correction factor can be found in 1825 in Laplace's Mecanique celeste, which can hardly have avoided Seeliger's attention. However, what was original in Seeliger's approach was that he used it in a cosmological context and not, as in most other proposals, to solve problems of planetary astronomy (such as Mercury's anomalous revolution around the Sun)."

Kragh [34] also mentions Boscovich and his cosmological ideas: "Apart from those already mentioned, several other Enlightenment natural philosophers took up cosmological questions. One of them was the Croatian-Italian astronomer and physicist Roger Boscovich, a Jesuit scholar, who in 1758 published his main work Theoria philosophiae naturalis. Although best known for its contribution to dynamic atomism and matter theory, the book also included considerations of a cosmological nature. For example, Boscovich imagined that, apart from our space, there might exist other spaces with which we are not causally connected. His conception of the universe was relativistic, such as illustrated by a passage from the end of Theoria, which may bring to mind much later cosmological ideas."

He continues to quote Boscovich's ideas about space and time: "If the whole Universe within our sight were moved by a parallel motion in any direction, & at the same time rotated through any angle, we could never be aware of the motion or of the rotation…Moreover, it might be the case that the whole Universe within our sight should daily contract or expand, while the scale of forces contracted or expanded in the same ratio; if such a thing did happen, there would be no change of ideas in our mind, & so we should have no feeling that such a change was taking place. (Theoria, p. 203)"

"Boscovich imagined all matter to consist of point-atoms bound together by Newtonian-like attractive and repulsive forces. If no forces were present, a body might pass freely through another without any collision (after all, points have no extension in space). The possibility led him to a daring cosmological speculation: "There might be a large number of material universes existing in the same space, separated one from the other in such a way that one was perfectly independent of the other, & the one could never acquire any indication of the existence of the other." (Theoria, note 518)

"Boscovich did not elaborate. Here we have, in 1758, a new version of the many universe scenario: not different universes distributed in space and time, but co-existing here and now. It was surely a scenario that harmonized in spirit with ideas that some cosmologists would propose more than two hundred years later."

Perhaps we should end this paper here with the hope that some of the ideas herein presented will come to fruition in the future.

6. Point-Atom Versus Point-Particle

In the last section the term "point-atom" was quoted from a historian. Stoiljkovich points out that Boscovich never used the phrase "point-atom". The Latin term that Boscovich used was "puncta" which in English gets translated to "point". In Boscovich's Theoria paragraph 8, 391 and 392 to see that Boscovich insists on distinctions between his points and Greek atoms. Hence, it is not correct to state that "Boscovich imagined all matter to consist of point-atoms…" by many scholars; it is better to say "point-particles."

It is true that Boscovich's point was named by Thomson as "Boscovichian atom", but it is not correct. Namely, Thomson used only Boscovich force-distance curve and orbitals at cohesion and non-cohesion limits, and suggested its application to a "planetary model of atom" which is complex particle, having nucleus and electrons. This planetary atom referred to by Thomson was neither Boscovich's point nor Greek atom.

The differences between Boscovich's points, Greek atoms and modern atoms are discussed in our monograph "Roger Boscovich – The founder of modern science" (table 4.1.) [16].

Boscovich used the term "point particle", but never used the term "point-atom", which was appeared later introduced by some other authors (e.g. J. J. Thomson, I. Supek etc.).

Boscovich had to be very cautious concerning the atomic concept. Aristotle did not believe that void (emptiness) really exists. Hence, Aristotle discarded atomic concept of Leucippus and Democritus. Church accepted Aristotle's philosophy. According to the Bible, God created Earth, water and lands, all living beings etc. It took six days, but all beings were created at once. Hence, atomic concept was forbidden by church. In 16th century, Pierre Gassendi made some compromise. He stated that God created atoms and definite rules for atoms to form Earth, water, beings… But, still atomic concept was not popular for Jesuits. Hence, Boscovich as Jesuit was very cautious concerning atomic concept and materialism, too.

Writers that talk of "point-atom" instead of "point-particle" are ignoring Boscovich's comprehension concerning atoms, because Boscovich wrote in his Theory about atoms, too. But, when Boscovich wrote about atoms, he referred to Greek atoms or Gassendi's atoms. Boscovich stated that Greek atoms are indivisible but have different sizes and shapes, i.e. occupied some space, and they are different from points which are all identical, indivisible and non-extended. Concerning Gassendi's atoms, Boscovich stated that they are composed of points, i.e. have parts and explained how they are connected to more complex particles such as molecules and arrays of atoms (macromolecules, according to modern terminology).

7. Conclusion

Boscovich's theory should not be ignored. The basis of modern physics is the point-particle idea so the unified theory of that should be taught and more generally known. Extended particles are built up from this idealization of point particles. A person may or may not agree with such a theory but it should at least be talked about what the basis of 20th Century physics was; so, we know what we are agreeing or disagreeing with.

There was a transition from classical physics to modern physics, but if you are thinking in terms of a sudden jump then you are not taking into account the transition that really happened. Whether Boscovich's theory is classical physics is hard to say. In transition from classical particle theory to modern quantum physics it is via Boscovich's theory.

And I reiterate 18th Century physics had a unified theory. If we are thinking in terms of modern relativity theory and modern quantum theory not combining, then we need to account why in the 18th century their versions of relativity and quantum were unified. In my view mistakes were made after that which caused the disunification in the mainstream.

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Centres of Force and Point-particles in Boscovich's Unified Theory

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Modern physics has the problem of singularities which are zero dimensional points in space which when assigned mass then have infinite density, and this leads to the difficulty of not knowing how to handle infinity. However, in the conception of Boscovich's theory an object can be treated as if all its mass were at its centre of gravity, effectively then treating mass as a dimension. This centre of gravity is then treated as the point-particle upon which everything else is built. Thus it could be deemed that the conception of modern physics is on the wrong track and the conception should be as per Boscovich's unified theory. This study is based on recent translation of Boscovich's book "The centre of gravity dissertation", highlighting what was missed in modern physics development from Newton to early form of quantum physics under Boscovich.

Keywords: Boscovich, Point-particles, Center-of-gravity, Centres-of-force

1. Introduction

As is known there are problems with unifying Einstein's relativity with quantum physics. One of the issues is that point-particles in the two theoretical constructions are being dealt with in different ways. The proposal here is that the correct way of dealing with point-particles is by Boscovich's theory.

2. Historical Perspective

A The Copernican Revolution became the transition from Aristotelian physics to what became called Newtonian physics. Highly important in this transition was Roger Boscovich. [1] Issues that had been understood in the context of Aristotelian physics had to be re-examined. Boscovich gave us the unified theory based on Newtonian physics [2] that replaced the old unified physics. Unfortunately, the old Aristotelian physics that had got things wrong from a perspective based on Newtonian physics then got unfairly denigrated, thus discouraging sufficient attention by modern physicists as to what were (1) the reasons for changes, and (2) what was still good within Aristotelian physics. Thus from a 'wisdom of learning from experience' perspective they lost sight of the progress being made in physics as to why the changes happened.

Often Aristotle and his followers were accused of not paying attention to observations. However, this was not the case, and the real problem might originate from when Aristotle said [3]: "Of things that exist, some exist by nature, some by other causes." Which seems to lead to the bias of only wanting to observe natural phenomena, and not bother observing unnatural phenomena; where unnatural phenomena would be experimentation, i.e. discouraged experimentation in favour of nature being observed by artificially changed. As Weinberg says [4]: "It is not that Aristotle neglected the observation of natural phenomena. From the delay between seeing lightning and hearing thunder, or seeing oars on a distant trireme striking the water and hearing the sound they make, he concluded that sound travels at a finite speed. … he also made good use of observation in reaching conclusions about the shape of the Earth and about the cause of rainbows. But this was all casual observation of natural phenomena, not the creation of artificial circumstances for the purpose of experimentation."

So Aristotelian physics had success in certain observations. In other areas it failed, such as in the motion of falling bodies, because it failed to observe in natural surroundings all objects falling at the same rate of acceleration in vacuum, because to create a vacuum was an unnatural act of experimentation in having to create a vacuum. Of course when Galileo did experiments with falling objects, he was not able to do it in vacuum, there not being the technology available in his day. He had to instead idealise what would happen in vacuum based on experiments that he could perform.

Thus to be fair to Aristotelian physics, the changes that came about due to the Copernican revolution was that it had to react to new observations from Galileo, Kepler et al., that were invalidating its perspective based on older observations. Thus although we have with the scientific method of making observation and doing experiment, it still remains the issue of how to connect those observations into a coherent theoretical description; both before and after the Copernican revolution. And as we said based on the new observations and Newtonian physics, Boscovich then provided that unified description in 1758. This then became the basis for the modern physics of the 20th century. However, changes were made between 1758 and the modern day, and it does not seem to make sense why these changes were made. Hence the lack of 'wisdom from experience' that modern physicists seem to have; they should have learnt why certain changes happened, but they seem to totally ignore that.

This leads us to one of the big problems in modern physics that of "singularities". And the question from a historical perspective of trying to have 'wisdom from experience' their introduction seems an anomaly. There was an existing way of dealing with 'points' so why suddenly replace that by 'singularities.' There seems no good reason for this change; it seems just a change that came about by accident for no reason from a person or persons totally not wanting to adhere to 'wisdom from experience' [5]. And of course the introduction of 'singularities' leads to all kinds of problems in physics, namely of what is called its breakdown.

3. Singularity Versus Classical Point-Particle

The Penrose–Hawking singularity theorems are a set of results in general relativity which attempt to answer the question of when gravitation produces singularities. [6]

A singularity in solutions of the Einstein field equations is one of two things:

1. a situation where matter is forced to be compressed to a point (a space-like singularity);

2. a situation where certain light rays come from a region with infinite curvature (time-like singularity).

Wikipedia explains as: Hawking's singularity theorem is for the whole universe, and works backwards in time: in Hawking's original formulation, it guaranteed that the Big Bang has infinite density. Hawking later revised his position in A Brief History of Time (1988) where he stated that "there was in fact no singularity at the beginning of the universe" (p50). This revision followed from quantum mechanics, in which general relativity must break down at times less than the Planck time. Hence general relativity cannot be used to show a singularity. [7]

It should be noted that - Hawking was referring to "infinite density" as at a point. Although Boscovich allows to a certain extent infinite density as possible (See §89, §90 and §381 of Appendix) it is not how the theory was built up as from singularities (i.e. point-particles with infinite density).

Boscovich's point-particles do not have infinite density; however, there is a problem with meaning to "infinite density"; we will treat it for the moment that Boscovich's point-particles do not have infinite density. (The issue of how infinite density dealt with in relation to Boscovich's point-particles will be picked up in section 5.)

This building on a singularity is contrary to how point-particle was built from Newtonian physics, as will now explain:

Isaac Newton [8] proved the shell theorem and stated that:

- A spherically symmetric body affects external objects gravitationally as though all of its mass were concentrated at a point at its centre.
- If the body is a spherically symmetric shell (i.e., a hollow ball), no net gravitational force is exerted by the shell on any object inside, regardless of the object's location within the shell.

A solid, spherically symmetric body can be modelled as an infinite number of concentric, infinitesimally thin spherical shells. If one of these shells can be treated as a point mass, then a system of shells (i.e. the sphere) can also be treated as a point mass.

Thus from Newtonian physics we have this idea of centres of gravity and centres of mass that act as point-particles variously called mass-points and other names. It is not that one of these point-particles really has infinite density, rather that it acts as a centre of force. Theory based on this was taken up by Boscovich to give a unified theory, and this led to modern quantum physics [1].

The transition might be clearer to physics students if textbooks still dealt with atomic physics in the way of such book by Tolansky and Bragg in 1942 [9] in their introduction to atomic physics start from classical physics before talking about quantum physics, and when they do get to quantum physics say: "In Bohr's simple theory for the Balmer series, the nucleus and the electron both rotate about their common center of gravity."

The relevant word is "center of gravity" that is a classical concept, so quantum physics is being built up from classical concepts; an insight we think missed out of most modern quantum physics textbooks. And even in the context of Tolansky and Bragg's book, insights that could be gleaned by first starting from consideration of "centers of gravity" from Boscovich's theory is not presented.

A more unified approach starting from classical physics: Newton, Boscovich et al. as it transitions to modern quantum physics is not offered to modern physics students, leaving those students puzzled as to the steps involved.

Boscovich for instance has an interesting way of dealing with light in terms of his point-particles that does not seem fully realised in its presentation to modern physics. According to Martinovic [10]: "In treatises from 1748 Bošković applied his theory of forces in order to explain numerous physical phenomena. He promoted this approach later as well. In the treatise De Centro Gravitatis (1751) he pointed out that his theory of forces depending on distances (theoria virium a distantiis pendentium) quite simply explains density of light."

Unfortunately, the passage referred to resists easy translation and comes out very roughly as [11]:

"But if there is rarity (thinness) of light; at first sight it seems cannot be completely explained, such an effect as gathering the burning rays (of the sun) by a mirror, causes an enormous perturbation and movement of particles, that can dissolve metals, burn wood into ashes, and reduce lime stones in the shortest possible time (i.e. burning lime to make quicklime).

"But that indeed is quite easy to explain, in the same dissertation it is granted, by means of force, that particles of matter mutually exercise on each when placed in the same state, which had changed from another state that might have been completely calm, can be accomplished by much expeditious (i.e. speed and efficiency) depending on the theory of forces and distances, where very little change happens on the shortest distance, and we proposed this three years ago in our dissertation on light.

"From this fourth proposition, and its obvious corollary is deduced, every movement of the mass-particles, such that the centre of gravity does not promote toward any particular direction, as being produced by the particles of the mutual interactions among themselves on one another, then no external action on the body from any certain quarter relayed, that impels the particles.

"With therefore, immense agitation the centres of gravity are not advanced (i.e. moved); the agitation cannot be attributed to impulse of particles of light, but instead as the agency of mutual action of the particles on one another, when things if they do not already, a new kind of arrangement must be obtained by certain what is very small, very sensitive movement, induced by radiation of particles, in which a new disposition now forces exert, which previously not exercised, and experience a huge movement, that indeed will befall, and as I said depends on the theory of the distances of the forces, so, even those distances little changed, these forces can vary a lot.

"But this, and many other things such as explaining the reason for the increase in the weight of substances such as when reduce lime (i.e. by heating), will not deal with here, that demonstrate the usefulness and necessity of geometry, and Mechanics in Physics where there is never enough accumulation of proposed experiments with sun's rays can illustrate, thus might only be proposed as the noblest property of the centre of gravity accurately referred to as rarity (thinness) of light, where mutual forces of particles changes the disposition of the particles, thrusting to what is achievable."

So there is something like a photoelectric effect of the particles of light interacting with particles of matter using Boscovich's force curve. Unfortunately, translation of the Latin does not come out very clearly to be able to give precise details.

Hooper [12] is a bit clearer and points out the Michelson-Morley experiment could be understood from Boscovich's point-particle treatment of light based ether.

There have been proposed various types of ether; the idea essentially being that light as a wave must have a medium which was called ether. Boscovich has light being made of his point-particles, and so light's medium is in a sense those point-particles.

Modern physics at the end of the 19th century dealt was favouring the wave theory of light in a medium/ether as opposed to a particle theory of light, until Einstein (and others such as deBroglie) gave us wave-particle duality of light. It seems that when there was a demarcation between wave theory and particle theory of light; treating them both as completely different theories, there was not proper consideration of Boscovich's theory of light which treated light as a mix of wave and particle, and was more in line with wave-particle duality.

A light particle based ether also ties to work of Osmaston [13] based on his secret research 1957-1961 for MOD concluded there was a 'particle-tied ether' able to explain the Michelson- Morley experiment.

However, in more modern times, the treatment of point-particles has not been so good, we have for example 1996-2007 Eric W. Weisstein [14] saying: "Point Mass A geometric (0-dimensional) point that may be assigned a finite mass. Since a point has zero volume, the density of a point mass having a finite mass is infinite, so point masses do not exist in reality. However, it is often a useful simplification in real problems to consider bodies point masses, especially when the dimensions of the bodies are much less than the distances among them."

(For example as to use: Ideal gas is a theoretical gas composed of many randomly moving point particles that do not interact except when they collide elastically. [15]) (See also Boscovich, §171 and §405 in Appendix.)

This definition is incorrect. A subtlety in mass point particle is that it is as if mass acts through a centre of force point, not that mass is at that point; if it were at that point then would be infinite density which is absurdity.

Difference: mass point where mass acts "as if through" a point, rather than "as at" a point.

One way of overcoming the difficulty of a mass point particle naively being thought of as having infinite density, is to treat mass as another dimension. [16] This seems to tie in with Rowlands treatment in the maths he has developed [17].

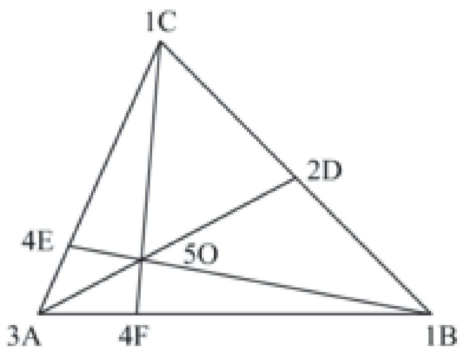

Figure 1. Center of force: The approximation is actually rigorous in the case of the gravity of a solid body having mass distributed with spherical symmetry, since an analytic derivation shows that the body acts exactly as a point mass at the body's center containing the body's entire mass.

Example of type of uses the mass point concept can be put to; and type of things dealt with in Boscovich's book on centre of gravity; though easier from wiki example [18]:

Problem. In triangle ABC, E is on AC so that CE = 3AE and F is on AB so that BF = 3AF. If BE and CF intersect at O and line AO intersects BC at D, compute OB/OE and OD/OA.

Solution. We may arbitrarily assign the mass of point A to be 3. By ratios of lengths, the masses at B and C must both be 1. By summing masses, the masses at E and F are both 4. Furthermore, the mass at O is 4 + 1 = 5, making the mass at D have to be 5 - 3 = 2 Therefore OB/OE = 4 and OD/OA = 3/2.

It has been pointed out that Einstein's relativity does not combine with quantum physics, therefore one of the areas that is the split between them is that Einstein's relativity is treating for a point-particle that it is possible for collapse to a singularity of infinite density, whereas quantum physics based on idea of treating point-particle as centre of force.

In the context of Boscovich's theory when a massive object starts collapsing due to gravity towards becoming a singularity, it never actually becomes a singularity, because the force will reverse from being attractive to becoming repulsive. (See Boscovich's curve in Fig. 2). Thus on this issue unification of physics comes from abandoning the idea of Einsteinian relativity's singularity existing. So in Boscovich's theory we have what is called an "almost black hole". [19]

4. Failure of Newton's Theory of Light

As mentioned Boscovich's particle theory of light is successful, but it success might have been obscured in many people's minds by the failure of Newton's particle theory of light.

Newton explained refraction with his light particle theory, [20] but got some things wrong. He thought that when light particles entered a denser medium, such as passing from air to water, that the light particles were attracted to the particles of that medium due to an attractive force (gravity) between the particles. The flaw being that the speed of light in a dense medium is greater than the speed of light in a thin medium, which is contrary to what happens. So although it is the same form for the equation of refractive index, just the index for light slower in denser medium instead of faster; for this and related reasons Newton's light particle theory was deemed to have failed.

However, whereas Newton only considered the force as attractive, so expected speed of light to increase when entered a denser medium due to attractive force, Boscovich dealt with unified force that can be repulsive as well as attractive. Thus in Boscovich's theory it is possible for the force to be repulsive in denser medium so that light goes slower.

There are other issues such as in Newton's theory light has mass, and Einstein treats it as massless, but I think that is other mistakes and will not go into here; because in our view the process of physical theory development should have been building upon Newtonian physics and transition to Boscovich's theory, instead of many of the things Einstein did.

Stanford Encyclopedia of Philosophy [21] illustrates the type of physical theory development that happened after Newton leading to Boscovich as follows: "Force was to prove a productive addition to experimental science in no uncertain manner in the eighteenth century. Force laws in addition to the law of gravitation, involving elasticity, surface tension, electric and magnetic attractions and so on were experimentally identified and put to productive use. In the domain of science, scruples about the ontological status of forces were forgotten and this attitude spread to philosophy. Eighteenth-century updates of mechanical atomism typically included gravity and other forces amongst the primary properties of atoms. Acceptance of force as an ontological primitive is evident in an extreme form in the 1763 reformulation of Newtonian atomism by R. Boscovich. In his philosophy of matter atoms became mere points (albeit possessing mass) acting as centres of force, the forces varying with the distance from the centre and oscillating between repulsive and attractive several times before becoming the inverse square law of gravitation at sensible distances. The various short-range attractive and repulsive forces were appealed to as explanations of the cohesion of atoms in bulk materials, chemical combination and also elasticity. Short-range repulsive forces varying with distance enabled Boscovich to remove the instantaneous rebounds of atoms that had been identified as an incoherency in Newton's own atomism stemming from their absolute hardness and inelasticity."

Our view is that further development should have been made along these lines of developing Boscovich's light particle theory with its connections to modern quantum mechanics. Then it might have given better explanation for issues like em (electromagnetic) drive, where M.E. McCulloch is trying to think of it in terms of quantised light inertia [22].

5. Infinite Density and Boscovich's Point-Particles

Picking up now the issue - of infinite density in relation to Boscovich's point-particles: as noted Boscovich allows infinite density. However, Branislav Petronievic points out: "Now, infinite density, if not to all of us, to Boscovich at least is unimaginable." [23] And we have from Boscovich that he did not believe in absolute infinity. (See Appendix §404) There is thus the issue of different meanings that can be assigned "infinite density"; infinity is a problematic concept really involving more than one concept hidden in the term "infinity".

There is a subtle issue in relation to point-particles, as mentioned we are treating these entities as mass operating as if it were at a certain point, not that the mass was necessarily at that point. Now in regard to density we have mass divided by volume; thus if mass is finite and divided by zero volume this would give infinity; however, having mass acting "as if" it was at a point is not saying the mass was at that point, and so avoids this type of infinity by not assigning mass a definitive position. The other type of infinity is that for a given non-zero volume can have mass as any value, in this type of infinity; mass can tend to infinity in a finite volume and with that density tend to infinity, but because of Boscovich's curve flips from attractive to repulsive force (and vice versa) this type of infinity would never be reached. This is because as an object might shrink in volume as its mass increases due to an attractive force, eventually the force would reverse preventing further compression. So infinite density is effectively avoided by both effects.

Alternatively, we can consider the issue of the two ways that "infinity" can be used which we need to take into account when we consider that Boscovich allows infinite density. There is a subtlety in what "infinite" means; it can mean an actual "infinity" or as a "infinite" process that is never finished. So in the case of the point-particles with mass acting "as if" it was at a point, this is avoiding the problem of actual infinity. While consideration of an object compressing with its density increasing to what might seem infinite process, Boscovich's curve is preventing that actual infinity being reached.

Thus in a sense there is infinite density allowed if think in terms of infinity as a process of counting that never ends; for then density can have any finite positive value. However, if thinking in terms of "actual infinity" then there isn't; with Boscovich's point-particles avoiding having that type of infinite density. And Petronevic notes various people have had problems with this aspect of Boscovich in his peculiar way of dealing with mass and density, thus having tendency to revert to thinking of point with mass as having infinite density; when really it is mass acting through a point, as if mass were at a point (centre of gravity, centre of mass… of an object) despite mass being spread over a finite volume.

There is then a hierarchy of matter built up from these point-particles. (See §239, §398 and [1].) Where the foundation of Boscovich's theory is his curve (Fig. 2.) that describes the change in force between the particles of matter depending on the distance between them; leading to several levels in the hierarchy of matter - of nucleons in atomic nucleus to the colloidal particles.

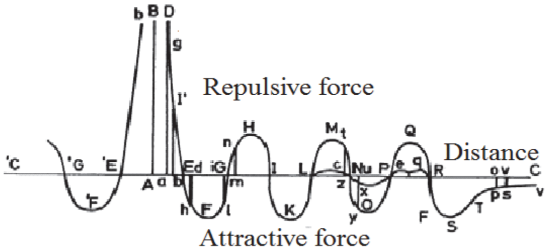

Figure 2. Boscovich's curve: Force vs. distance of two points (or particles) [24].

UFM becomes developed it will become clear that LSXD Calabi-Yau dual mirror symmetric C-QED brane dynamics will allow in contrast 'low energy' particle creation by inherent background symmetry coupling

As UFM becomes developed it will become clear that LSXD Calabi-Yau dual mirror symmetric C-QED brane dynamics will allow in contrast 'low energy' particle creation by inherent background symmetry coupling interactions of the continuous-state spin-exchange dimensional reduction compactification process - a mirror symmetric component coupling.

6. Conclusion

Physical insights that could have been offered by introducing Boscovich's theory to modern physics students have been missed. And this extends up the hierarchy of physics from lowly student to esteemed physics professors, resulting in failures of comprehension such as those pointed out in this paper. Namely: Quantum physics was built (or developed) on the classical concept of point-particle that started with Newton and was further developed into a comprehensive theory by Boscovich. Whereas Einstein's relativity under the stewardship of Penrose, Hawking and others seems to have arbitrarily abandoned that concept in favour of the idea of singularity. Then physicists act mystified to the question of why they cannot combine the two theoretical constructions, when really it is just their mistakes made in their works that have mounted up as a result of them improperly understanding what has already been accomplished in physics by their forebears, that have caused the split.

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Appendix: Boscovich's Citations

(Taken from: A Theory of Natural Philosophy, [24])
§89. …But if the elements are points that are perfectly indivisible & non-extended, then, just as their distances can be increased indefinitely, so also can they just as well be diminished in any ratio whatever. For it is certainly possible that a short line can be divided into parts in any ratio whatever; & thus, just as there is no limit to increase of rarity, so also there is none to increase of density.
§90. ...For if the primary elements of matter are perfectly non-extended & indivisible points separated from one another by some definite interval, then the number of points in any given mass must be finite; because all the distances are finite. I proved clearly enough, I think, in the dissertation De Natura, & Usu infinitorum ac infinite parvorum, & in the dissertation De Lege Continuitatis, & in other places, that there are no infinitesimal quantities determinate in themselves. Any interval whatever will be finite, & at least divisible indefinitely by the interpolation of other points, & still others; each such set however, when they have been interpolated, will be also finite in number, & leave room for still more; & these too, when they existed, will also be finite in number. So that there is only an infinity of possible points, but not of existing points; & with regard to these possible points, I usually term the whole series of possibles a series that ends at finite limits at infinity. This for the reason that any of them that exist must be finite in number; but there is no finite number of things that exist so great that other numbers, greater & greater still, but yet all finite, cannot be obtained; & that too without any limit, which cannot be surpassed. Further, in this way, by doing away with all idea of an actual infinity in existing things, truly countless difficulties are got rid of.
§171. …all the universes of smaller dimensions taken together would act merely as a single point compared with the next greater universe, which would consist of little point-masses, so to speak, of the same kind compared with itself, that is to say, every dimension of each of them, compared with that universe & with respect to the distances to which each can attain within it, would be practically nothing.
§239. …From four particles of this kind [*note] arranged to form a larger pyramid, we can obtain a particle of the second order, somewhat less tenacious of form on account of the greater distance between the particles of the first order that compose it; for from this fact it comes about that the forces impressed upon these from external points are much more unequal to one another than they would be for the points constituting particles of the first order. In the same manner, from these particles of the second order we might obtain particles of the third order, still less tenacious of form, & so on; until at last we reach those which are much greater, still more mobile, & variable particles, which are concerned in chemical operations; & to those from which are formed the denser bodies, with regard to which we get the very thing set forth by Newton, in his last question in Optics, with respect to his primary elemental particles, that form other particles of different orders…*note: as Stoiljkovich would say of first order.
§398. The primary elements of matter are considered by most people to be immutable, & of such a kind that it is quite impossible for them to be subject to attrition or fracture, unless indeed the order of phenomena & the whole face of Nature were changed. Now, my elements (i.e. points - DS) are really such that neither themselves, nor the law of forces can be changed; & the mode of action when they are grouped together cannot be changed in any way; for they are simple, indivisible & non-extended. From these, by what I have said in Art. 239, when collected together at very small distances apart, in sufficiently strong limit-points on the curve of forces, there can be produced primary particles, less tenacious of form than the simple elements, but yet, on account of the extreme closeness of its parts, very tenacious in consequence of the fact that any other particle of the same order will act simultaneously on all the points forming it with almost the same strength, & because the mutual forces are greater than the difference

between the forces with which the different points forming it are affected by the other particle. From such particles of the first order there can be formed particles of a second order, still less tenacious of form; & so on. For the greater the composition, & the larger the distances, the more readily can it come about that the inequality of forces, which alone will disturb the mutual position, begins to be greater than the mutual forces which endeavour to maintain that mutual position, i.e. the form of the particles. Then indeed we shall have changes & transformations, such as we see in these bodies of ours, & which are also obtained in most of the particles of the last orders, which compose these new bodies. But the primary elements of matter will be quite immutable, & particles of the first orders will preserve their forms in opposition to even very strong forces from without.

§381. …But as regards the properties of rarity & density, here I indeed differ from the usual opinion. For, as I showed in Art. 89, I have no limiting value for either density or rarity, no maximum, no minimum; whereas others must admit a minimum rarity, or a maximum density, as being possible; &, since this must be something finite, it must of necessity involve a sudden break in continuity; although they may not admit any maximum rarity or minimum density. For with me the points of matter can both increase & diminish their distances from one another in any ratio whatever…

§404. But in things that actually exist, I consider that it is totally impossible that there can be any absolute infinity.

§405. In this way, it may be that the whole aggregate of the fixed stars, together with the Sun, is a single particle of an order higher than those of which the system is composed; & that it belongs to a system immensely greater still. It may even be the case that there are very many such orders of particles, of such a kind that particles of the same class are completely separated from one another without any possible means of getting from one to the other, owing to several asymptotic arcs to my curve, as I explained in Art. 171.

A Not So Impossible Machine Based on the GHZ Paradox

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We investigate the GHZ paradox as embodied in Mermin's machine. We begin by showing that this machine is impossible to implement within the context of classical physics. Then we go on to show how it can be implemented within quantum mechanics.

We push this investigation to its limits using some of the most recent advances in quantum computation and quantum information science. The detailed wiring diagram constructed herein provides an explicit and revealing definition of the machine. In particular, it makes explicit how paradoxical indeterminism and nonlocality can be quantified and mathematically captured by the second elementary Boolean function. It also gives an illustration of the many subtleties involved in the quantum control of distributed quantum systems.

Within this paper, we introduce two new mathematical constructs, i.e., Boolean unitaries and Boolean observables, that provide a useful mathematical formalism for analyzing problems within quantum information science.

Keywords: GHZ Paradox, Quantum Paradoxes, Quantum Algorithms, Quantum Computation, Quantum Information, Quantum Control, Distributed Quantum Algorithms, Quantum Entanglement, Boolean unitary, Boolean onservable

1. Introduction

In this paper, we investigate the Greenberger-Horne-Zeilenger (GHZ) paradox as embodied in Mermin's machine. We begin by showing that this machine is impossible to implement within the context of classical physics. Then we go on to show how it can be implemented within quantum mechanics.

We push this investigation to its limits using some of the most recent advances in quantum computation and quantum information science. The detailed wiring diagram constructed herein provides a revealing and transparent definition of the machine. In particular, it makes explicit how paradoxical indeterminism and nonlocality can be quantified and mathematically captured by the second elementary Boolean function σ_2. [See Corollary 1 in Section 6.] It also gives an illustration of the many subtleties involved in the quantum control of distributed quantum systems.[9]

It should also be mentioned that we introduce [in Section 4] two new mathematical constructs, i.e., Boolean unitaries and Boolean Observables, that provide a useful mathematical formalism for analyzing problems within quantum information science.

2. The device

A blueprint describing Mermin's machine[5–7] is shown below in Figure 1:

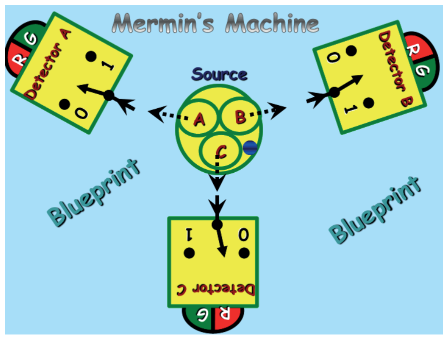


Fig. 1. A blueprint of the Device.

As illustrated, the device consists of two different types of components, i.e., a **source** $\boldsymbol{S}$, and three identical **detectors**, labeled $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$.

The source, as illustrated below in Figure 2, is a device that contains three objects, called **particles**, labeled $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$, and a **blue button**, which when pressed, ejects the three particles $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ in the directions toward the detectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$, respectively.

Each detector, upon encountering an incoming particle, flashes either red $\boldsymbol{R}$ or green $\boldsymbol{G}$. Moreover, each detector has a switch with two settings **0** and **1**, which is randomly set at anytime before the arrival of the particle.

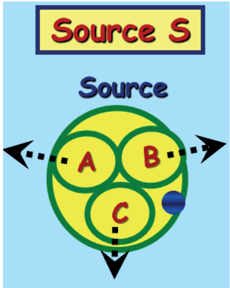


Fig. 2. Source $\boldsymbol{S}$.

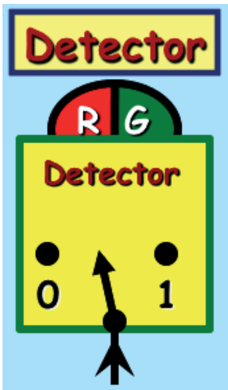


Fig. 3. Detector $\boldsymbol{A}$, $\boldsymbol{B}$, or $\boldsymbol{C}$.

As stated below in the design specifications, the only switch settings of interest are those for which an odd number of the three switches is set to **1**, i.e.,

A	B	C
0	0	1

A	B	C
0	1	0

A	B	C
1	0	0

A	B	C
1	1	1

.

No other switch settings are important, i.e., of interest.

The design specifications are as follows:

Spec 1. After all particles are detected, for switch settings **001**, **010**, and **100**, **only an odd number** of the detectors flash red $\boldsymbol{R}$.

Spec 2. After all particles are detected, for switch setting **111**, **only an even number** of detectors flash red $\boldsymbol{R}$.

These design specifications are subject to the following three constraints:

Constraint 1. The detectors cannot communicate with one another. [They are separated by a spacelike distance.]

Constraint 2. After being ejected from the source S, the particles can no longer communicate with one another.

Constraint 3. The particles only communicate with the detector upon impact.

3. It can't be built!

Because of the above constraints, each particle must locally carry instructions telling its respective detector whether to flash red $\boldsymbol{R}$ or green $\boldsymbol{G}$.

For example, particle $\boldsymbol{A}$ must carry a **local instruction** $f_A(s_A)$ of the form

$$f_A(s_A) = \begin{cases} c_{A0} \text{ if the } \textbf{switch setting } s_A = 0 \\ \\ c_{A1} \text{ if the } \textbf{switch setting } s_A = 1 \end{cases},$$

where $c_{A0} = \boldsymbol{R}$ or $\boldsymbol{G}$ and $c_{A1} = \boldsymbol{R}$ or $\boldsymbol{G}$ for switch settings $s_A = 0$ or 1, respectively. In like manner, the remaining two particles $\boldsymbol{B}$ and $\boldsymbol{C}$ must carry local instructions $f_B(s_B)$ and $f_C(s_C)$, respectively.

Let us rename the colors $\boldsymbol{R}$ and $\mathbf{G}$ as $\mathbf{R} = 1$ and $\mathbf{G} = 0$, respectively. Thus, for each $j = A, B, C$, the local instruction $f_j(s_j)$ is simply a **Boolean function**

$$f_j : \{0,1\} \longrightarrow \{0,1\} .$$

It is now immediate that Specs 1 and 2 are equivalent to the following linear system of equations:

$$\begin{cases} f_A(0) + f_B(0) + f_C(1) = 1 \,(\operatorname{mod} 2) \\ \\ f_A(0) + f_B(1) + f_C(0) = 1 \,(\operatorname{mod} 2) \\ \\ f_A(1) + f_B(0) + f_C(0) = 1 \,(\operatorname{mod} 2) \\ \\ f_A(1) + f_B(1) + f_C(1) = 0 \,(\operatorname{mod} 2) \end{cases}$$

which is obviously inconsistent.

In other words, the device cannot be built! It's simply impossible. Q.E.D.

4. Oh, but it can be built!

However, within the context of quantum physics, it can actually be built, i.e., can be physically implemented.

But before we can show how this device can actually be built, we need a few definitions.

Definition 1. *We define a* ***Boolean unitary transformation*** *as a map from* $\{0,1\}^k$ *into a* *group of unitary transformations. In like manner, a* ***Boolean Hermitian operator*** *is defined as a map from* $\{0,1\}^k$ *into an algebra of observables. If* b $(= 0$ *or* $1)$*, and if* U *is a unitary transformation, then* U^b *will denote the Boolean unitary transformation*

$$U^b = \begin{cases} I \text{ if } b = 0 \\ \\ U \text{ if } b = 1 \end{cases} ,$$

where I *denotes the identity operator. In like manner, if* Ω *is an observable, then* $b\Omega$ *will denote the Boolean observable*

$$b\Omega = \begin{cases} O \text{ if } b = 0 \\ \\ \Omega \text{ if } b = 1 \end{cases} ,$$

where O *denotes the zero operator.*

Remark 1. *In other words, Boolean unitary and Boolean Hermitian operators are unitary and Hermitian transformations controlled by classical bits.*

Remark 2. *There is much more that could be said in regard to Boolean unitary and Hermitian operators. But that would take us too far afield of the intended objectives of this paper. So the following will have to suffice: Let* $\mathbb{B}$ *be a Boolean algebra or Boolean ring. Let* $\mathbb{U}$ *be a unitary group, and let* $\mathbf{u}$ *denote its Lie algebra. The set* $\mathbb{U}^{\mathbb{B}} = map(\mathbb{B}, \mathbb{U})$ *of Boolean unitary operators forms a Lie group containing the group* $\mathbb{U}$ *as a sub-Lie Group. Moreover, the set* $\mathbf{u}^{\mathbb{B}} = map(\mathbb{B}, \mathbf{u})$ *of Boolean Skew Hermitian operators is the Lie algebra of* $\mathbb{U}^{\mathbb{B}}$*, and contains* $\mathbf{u}$ *as a sub-Lie algebra.*

Let X, Y, Z respectively denote the Pauli spin operators

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

Moreover, let H denote the Hadamard gate

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} ,$$

and let U be the single qubit gate

$$U = e^{\left[\frac{i\pi}{3}\left(\frac{X+Y+Z}{\sqrt{3}}\right)\right]} = \frac{1+i}{2} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} .$$

A wiring diagram summarizing a physical implementation of Mermin's machine is shown in Figure 4.

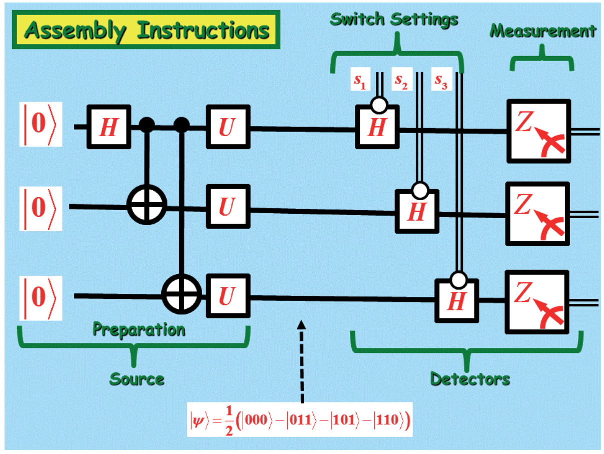
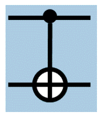
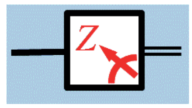
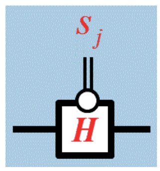


Fig. 4. Wiring diagram of device.

In this diagram, a single line indicates a wire carrying a qubit, a double line indicates a wire carrying a classical bit. The graphics

 and 

denote respectively a **Controlled-Not** and a **measurement in the standard basis**. Finally the graphic



denotes the Boolean gate $H^{s_j^*}$, controlled by the classical bit s_j^*, where s_j^* denotes the **complement** of the j-th switch setting s_j. In other words,

$$H^{s_j^*} = \begin{cases} I \text{ if } \ s_j^* = 0 \Longleftrightarrow s_j = 1 \\ \\ H \text{ if } \ s_j^* = 1 \Longleftrightarrow s_j = 0 \end{cases}$$

Remark 3. *Please note that* $HZH = X$*. Hence, if* $|\varphi\rangle$ *is a single qubit state, then measurement of* $H^{s_j^*}|\varphi\rangle$ *with respect to the observable* Z *is equivalent to measurement of* $|\varphi\rangle$ *with respect to the Boolean observable* $H^{s_j^*} Z H^{s_j^*} = s_j^* X + s_j Z$*. So, each detector portion of the wiring diagram can be simplified to a local measurement with respect to the Boolean observable* $s_j^* X + s_j Z$*, for* $j = 1, 2, 3$*. (Please refer to Figure 5.) In fact, each detector portion of the diagram can be even further simplified to local measurement* *of the GHZ state with respect to Boolean observable* $U^{\dagger}\left(s_1^* X + s_1 Z\right) U = s_j^* Y + s_j X$, *for* $j = 1, 2, 3$.

The three leftmost gates provide a preparation of the GHZ state

$$\frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right) \ .$$

The local unitary[a] transformation $U^{\otimes 3} = U \otimes U \otimes U$ transforms the GHZ state into the entangled state

$$|\psi\rangle = \frac{1}{2}\left(|000\rangle - |011\rangle - |101\rangle - |110\rangle\right) \ ,$$

which will be used to control the flashing light patterns of the three detectors[b].

Let $\mathcal{H}_{even}$ and $\mathcal{H}_{odd}$ denote the Hilbert subspaces of the underlying three qubit Hilbert space $\mathcal{H}$ spanned respectively by the standard basis elements labeled by bit strings of even and odd Hamming weight. It now follows from the following table:

Switch Settings $s = (s_1, s_2, s_3)$	**State** $\left(H^{s_1^*} \otimes H^{s_2^*} \otimes H^{s_3^*}\right)\|\psi\rangle$
111	$\|\psi\rangle = \frac{1}{2}\left(\ \|000\rangle - \|011\rangle - \|101\rangle - \|110\rangle\right)$
001	$\left(H \otimes H \otimes 1\right)\|\psi\rangle = \frac{1}{2}\left(-\|001\rangle + \|010\rangle + \|100\rangle + \|111\rangle\right)$
010	$\left(H \otimes 1 \otimes H\right)\|\psi\rangle = \frac{1}{2}\left(\ \|001\rangle - \|010\rangle + \|100\rangle + \|111\rangle\right)$
100	$\left(1 \otimes H \otimes H\right)\|\psi\rangle = \frac{1}{2}\left(\ \|001\rangle + \|010\rangle - \|100\rangle + \|111\rangle\right)$

that

$$\left(H^{s_1^*} \otimes H^{s_2^*} \otimes H^{s_3^*}\right)|\psi\rangle \in \begin{cases} \mathcal{H}_{even} & \text{if } s = 111 \\ \\ \mathcal{H}_{odd} & \text{if } s = 001, 010, 100 \end{cases}$$

Thus, if the switch setting is $s = 111$, application of each and all local detector measurements with respect to the standard basis (no matter in which temporal order) will project the state $\left(H^{s_1^*} \otimes H^{s_2^*} \otimes H^{s_3^*}\right)|\psi\rangle$ into $\mathcal{H}_{even}$, necessarily resulting in a standard basis state $|c_1 c_2 c_3\rangle$ of even Hamming weight, and corresponding eigenvalues $(-1)^{c_1}$, $(-1)^{c_2}$, $(-1)^{c_3}$ with $c_1 + c_2 + c_3 = 0 (\operatorname{mod} 2)$. Using the same argument for the switch settings $s = 001, 010, 001$, the three local detector measurements of $\left(H^{s_1^*} \otimes H^{s_2^*} \otimes H^{s_3^*}\right)|\psi\rangle$ will result in a standard basis element $|c_1 c_2 c_3\rangle$ of odd Hamming weight with corresponding eigenvalues $(-1)^{c_1}$, $(-1)^{c_2}$, $(-1)^{c_3}$ with $c_1 + c_2 + c_3 = 1 (\operatorname{mod} 2)$.

Thus, using $c_j = 0$ as the control bit instruction to flash Green G, and $c_j = 1$ as the control bit instruction to flash Red, we have shown that the device defined by the wiring diagram satisfies all the required specs and constraints.

So the device can be built after all!

5. Why?

So where has the impossibility argument given in section 3 of this paper gone awry?

Certainly the proof in section 3 of this paper of the following proposition, on which the proof of impossibility is based, is beyond reproach:

Proposition 1. *There exist no Boolean functions*

$$f_A : \{0,1\} \longrightarrow \{0,1\}\,, \quad f_B : \{0,1\} \longrightarrow \{0,1\}\,,$$

$$f_C : \{0,1\} \longrightarrow \{0,1\}$$

[a]It is important to note that $U^{\otimes 3}$ is a local unitary transformation that does not change entanglement type. For a better understanding of the significance of this fact, please refer to.[4]

[b]For a more in depth explanation of the use of entanglement as a distributed control mechanism, please refer to.[9]

such that

$$f_A(s_1) + f_B(s_2) + f_C(s_3) \equiv \begin{cases} 1 \pmod 2 \ \textit{if}\ s = (s_1, s_2, s_3) = 001,\ 010,\ \textit{or}\ 100. \\ 0 \pmod 2 \ \textit{if}\ s = (s_1, s_2, s_3) = 111 \end{cases}$$

The logic is flawless[c]. But the crux of the matter is that the argument of impossibility found in section 3 is only as sound as the assumptions upon which it is based.

More specifically, the argument of impossibility fails because at least one of the following two tacitly assumed premises is false:

PREMISE 1. **Reality Principle:** *What is measured is completely determined before it is measured.* [For a more refined definition of this principle and the concept of an element of reality, please refer to[1] and.[8]]

PREMISE 2. **Principle of Locality:** *Spacelike separated regions of spacetime are physically independent.*

Remark 4. *It is not clear that these are fully independent principles. For how can that which is not fully determined already be localized? Moreover, can that which is not localized already be fully determined?*

The above two premises lead to the following unfounded conclusions:

UNFOUNDED CONCLUSION 1. *Based on Premise 1 (The Reality Principle), the detector lamp instructions f_A, f_B, f_C must already be predetermined well defined total functions*[d] *at the time of particle ejection.*

UNFOUNDED CONCLUSION 2. *Based on Premise 2 (The Principle of Locality), the detector lamp instructions f_A, f_B, f_C must be local. Hence f_j is a function only of the j-th switch setting s_j, and independent of the two other switch settings.*

We will show in the next section that the detector lamp instructions f_A, f_B, f_C are neither predetermined well-defined functions before ejection, nor local independent functions.

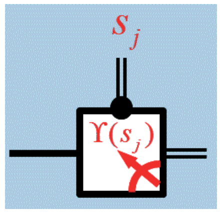
6. Under the mathematical microscope

It is instructive to take a closer look at Mermin's machine.

We will now explicitly compute the random functions f_A, f_B, f_C. In so doing, we will find, contrary to the unfounded conclusions given in the previous section, that these functions are:

1) Random partial functions,
2) Global interdependent functions of the switch settings, and
3) Not fully defined until measured by the detectors.

For reasons of transparency, it will prove more convenient to work with the equivalent wiring diagram shown in Figure 5, where



denotes the s_j-controlled gate for the Boolean observable

$$\Upsilon(s_j) = \Upsilon(s_j) = s_j^* X + s_j Z = \begin{cases} X \text{ if } s_j = 0 \\ Z \text{ if } s_j = 1 \end{cases}.$$

That this wiring diagram is equivalent to the one found in Figure 4 follows from the fact that $HZH = X$. Hence measurement of $H^{s_j^*}|\psi\rangle$ with respect to Z is equivalent to measurement of $|\psi\rangle$ with respect to $H^{s_j^*} Z H^{s_j^*} = s_j^* X + s_j Z$.

We will also need to use the **quantum measurement function** Q, which takes as input a pair consisting of an existing quantum state and a quantum observable, and then upon evaluation produces as output a pair consisting of a resulting eigenstate and the corresponding eigenvalue. For example, if ρ is a density operator representing the state of a quantum system and if Ω an observable with spectral

[c]Actually, as we will see, the above proposition is a proof of the counterfactuality of the lamp instructions being total functions, and not a proof that the device cannot be built.
[d]A **total function** is a function that is defined for all possible values of its arguments. A **partial function** is a function defined for some of its argument values, but not necessarily all. For more information, please refer to any text on recursive function theory.

decomposition

$$\Omega = \sum_{j}^{n} \lambda_j P_j ,$$

then on evaluation $Q(\rho, \Omega)$ produces

$$Q(\rho, \Omega) = \left(\frac{P_j \rho P_j}{Tr(P_j \rho)}, \lambda_j \right) ,$$

where P_j is the projection operator for the eigenspace corresponding to the eigenvalue λ_j.

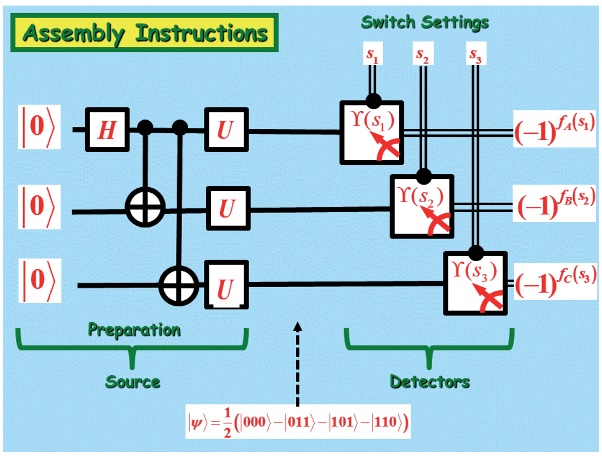


Fig. 5. An equivalent wiring diagram for Mermin's machine, where $\Upsilon(s_j)$ is the Boolean observable $\Upsilon(s_j) = s_j^* X + s_j Z$.

Please note that the function Q is a random output function, very much like the random number generator found on most classical computers, except that it's output is not pseudorandom, but actually truly random. A pseudorandom number generator is a **predeterministic** function, i.e. a function fully predefined before evaluation, which upon evaluation deterministically produces an output. On the other hand, the function Q is **indeterministic**[e], i.e., it is a function that is not fully defined (and not fully determined) as a function until it is evaluated.

We finally are ready to take a closer look at the implementation of Mermin's machine, as described by the wiring diagram found in Figure 5.

After the state preparation of the entangled state $|\psi\rangle$, and before ejection of the particles, the detector lamp instructions f_A, f_B, f_C are indeterministic, i.e., only partially defined (and only partially localized) by the entangled state $|\psi\rangle$. This is a result of the state of each individual qubit of $|\psi\rangle$ being indeterministic, i.e., not yet fully defined, and not yet fully localized.

In section 4, it was pointed out that the property, that the final resulting light pattern always satisfies the machine specifications and constraints, is independent of the temporal order of the detector measurements. For this reason, we focus only on the case for which the detector measurements occur in the temporal order $t_A < t_B < t_C$, where t_A, t_B, t_C denote the measurement times for detectors A, B, C, respectively.

Remark 5. *The topic of the temporal order of measurements is remarkably subtle. To say that the detector light pattern is independent of the order of the measurements is counterfactual, and hence physically meaningless. However, it is meaningful (not counterfactual) to say that the state specifications and constraints are met, independent of the order of measurements. On the other hand, because of relativity, there can be, for each possible temporal order, a different observer that observes the measurements in that order. The fact that each of three different observers sees the measurements in a different temporal order is not counterfactual because all observers are viewing the same measurements.*

We recall that the spectral decompositions of the Pauli spin operators X and Z are respectively

$$X = (-1)^0 P_+ + (-1)^1 P_-$$

and

$$Z = (-1)^0 P_0 + (-1)^1 P_1 ,$$

where

$$\begin{cases} P_+ = |+\rangle\langle+| \\ P_- = |-\rangle\langle-| \end{cases} \text{and} \quad \begin{cases} P_0 = |0\rangle\langle 0| \\ P_1 = |1\rangle\langle 1| \end{cases} ,$$

and where

$$\begin{cases} |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \\ |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \end{cases} .$$

[e]Please note that we have avoided use of the term "nondeterministic" because this term has an entirely different meaning in the theory of computation.

Notation 1. *In the calculations to follow, we use the following notational convention:*

$$j^0 = \begin{cases} 0 \text{ if } j=0 \\ 1 \text{ if } j=1 \end{cases} \Bigg| \quad \textit{and} \quad j^1 = \begin{cases} + \text{ if } j=0 \\ - \text{ if } j=1 \end{cases} \Bigg|$$

Measurement at time t_A:

At the time t_A, the function $f_A(s_1)$ is evaluated as follows:

$$Q\left(Tr_{23}\left(|\psi\rangle\langle\psi|\right), s_1^* X + s_1 Z\right) = \left(\frac{P_{j_1^{s_1^*}} Tr_{23}\left(|\psi\rangle\langle\psi|\right) P_{j_1^{s_1^*}}}{Tr\left(P_{j_1^{s_1^*}} Tr_{23}\left(|\psi\rangle\langle\psi|\right)\right)}, (-1)^{j_1}\right)$$

$$\Longrightarrow f_A(s_1) = j_1 \ ,$$

where $j_1 = 0$ or 1, and where $Tr_{23}\left(|\psi\rangle\langle\psi|\right)$ is the partial trace of $|\psi\rangle\langle\psi|$ over qubits 2 and 3. The resulting state of the three qubits is

$$|\psi'\rangle = \frac{\left(P_{j_1^{s_1^*}} \otimes 1 \otimes 1\right)|\psi\rangle}{\sqrt{\left\langle \psi \left| P_{j_1^{s_1^*}} \otimes 1 \otimes 1 \right| \psi \right\rangle}} \ .$$

Measurement at time t_B:

At the time t_B, the function $f_B(s_2)$ is evaluated as follows:

$$Q\left(Tr_{13}\left(|\psi'\rangle\langle\psi'|\right), s_2^* X + s_2 Z\right) = \left(\frac{P_{j_2^{s_2^*}} Tr_{13}\left(|\psi'\rangle\langle\psi'|\right) P_{j_2^{s_2^*}}}{Tr\left(P_{j_2^{s_2^*}} Tr_{13}\left(|\psi'\rangle\langle\psi'|\right)\right)}, (-1)^{j_2}\right)$$

$$\Longrightarrow f_A(s_2) = j_2 \ ,$$

where $j_2 = 0$ or 1, and where $Tr_{13}\left(|\psi'\rangle\langle\psi'|\right)$ is the partial trace of $|\psi'\rangle\langle\psi'|$ over qubits 1 and 3. The resulting state of the three qubits is

$$|\psi''\rangle = \frac{\left(1 \otimes P_{j_2^{s_2^*}} \otimes 1\right)|\psi'\rangle}{\sqrt{\langle\psi| \left(1 \otimes P_{j_2^{s_2^*}} \otimes 1\right)|\psi'\rangle}} \ .$$

Measurement at time t_C:

At the time t_C, the function $f_C(s_3)$ is evaluated as follows:

$$Q\left(Tr_{12}\left(|\psi''\rangle\langle\psi''|\right), s_3^* X + s_3 Z\right) = \left(\frac{P_{j_3^{s_3^*}} Tr_{12}\left(|\psi''\rangle\langle\psi''|\right) P_{j_3^{s_3^*}}}{Tr\left(P_{j_3^{s_3^*}} Tr_{12}\left(|\psi''\rangle\langle\psi''|\right)\right)}, (-1)^{j_3}\right)$$

$$\Longrightarrow f_A(s_3) = j_3 \ ,$$

where $j_3 = 0$ or 1, and where $Tr_{12}\left(|\psi''\rangle\langle\psi''|\right)$ is the partial trace of $|\psi''\rangle\langle\psi''|$ over qubits 1 and 2 The resulting state of the three qubits is

$$|\psi'''\rangle = \frac{\left(1 \otimes 1 \otimes P_{j_3^{s_3^*}}\right)|\psi''\rangle}{\sqrt{\langle\psi''| \left(1 \otimes 1 \otimes P_{j_3^{s_3^*}}\right)|\psi''\rangle}} \ .$$

Remark 6. *Please note that each of the instructions $f_A(s)$, $f_B(s)$, $f_C(s)$ can only be a non-local function of $s = (s_1, s_2, s_3)$. For from relativity, there can be three different observers Alice, Bob, Charlie each observing the same measurements, but each observing the same measurements in the three different temporal orders $t_A < t_B < t_C$, $t_B < t_C < t_A$, $t_C < t_A < t_B$, respectively. If Alice observes f_A as only a function of s_1, so would Bob and Charlie.*

We are now in a position to explicitly quantify the interdependence of the random Boolean partial functions f_A, f_B, f_C. To do so, we will make use of the following well known combinatorial formula:[3]

Theorem 1. *Let $b = (b_1, b_2, b_3, \ldots, b_n)$ be a binary string of length $n > 0$. The binary expansion of the Hamming weight $Wt(b)$ of b is given by the following formula:*

$$Wt(b) = \sum_{k=0}^{O(\log n)} \sigma_{2^k}(b) \cdot 2^k \ ,$$

where $\sigma_{2^k}(b)$ denotes the 2^k-th elementary symmetric function modulo 2, i.e.,

$$\sigma_{2^k}(b) = \sum_{1 \le \ell_1 < \ell_2 < \ldots < \ell_n \le 2^k} b_{\ell_1} b_{\ell_2} b_{\ell_3} \cdots b_{\ell_n} \pmod 2 \ .$$

In light of the above theorem, an immediate consequence of the above measurement calculations are the following lemma and corollary:

Lemma 1. *If the switch setting* $s = (s_1, s_2, s_3)$ *is of odd Hamming weight, then*

$$\left(P_{j_1^{s_1^*}} \otimes P_{j_2^{s_2^*}} \otimes 1\right)|\psi\rangle \text{ lies in } \left(1 \otimes 1 \otimes P_{j_3^{s_3^*}}\right)\mathcal{H} \ ,$$

where

$$j_3 = j_1 + j_2 + \sigma_2(s) + 1 \pmod 2 \ ,$$

and where $\sigma_2(s)$ *denotes the second elementary symmetric function*

$$\sigma_2(s) = s_1 s_2 + s_2 s_3 + s_3 s_1 \ .$$

Thus,

$$|\psi'''\rangle = |\psi''\rangle \ .$$

Corollary 1. *For a switch setting* $s = (s_1, s_2, s_3)$ *of odd Hamming weight, the detector lamp instructions* f_A, f_B, f_C *are the random partial functions given by:*

$$\begin{cases} f_A(s) = j_1 \\ f_B(s) = j_2 \\ f_C(s) = j_3 \end{cases} ,$$

with the Boolean algebraic dependence

$$f_A(s) + f_B(s) + f_C(s) = \sigma_2(s_1, s_2, s_3) + 1 \pmod 2 \ ,$$

where σ_2 *denotes the second elementary symmetric function*

$$\sigma_2(s_1, s_2, s_3) = s_1 s_2 + s_2 s_3 + s_3 s_1 \ .$$

Hence, the random Boolean instruction functions f_A, f_B, f_C are global and interdependent partial functions, thereby refuting UNFOUNDED CONCLUSIONS 1 and 2, found in section 5 of this paper.

Remark 7. *It is interesting to note that the Boolean function* $\sigma_2(s_1, s_2, s_3)$, *involved in the above algebraic interdependence, in some way fully encapsulates the entire paradox. In other words, this second elementary symmetric Boolean function somehow quantifies the nonlocality and the indeterminism involved in the GHZ paradox.*

7. Conclusion?

We conclude with no conclusion, but with a question:

Question: *Is quantum mechanics trying to tell us that the very fabric of reality is indeterminate, i.e., not fully defined until it is observed?*

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Covariant Formulation of Aharonov-Bohm Electrodynamics and Its Application to Coherent Tunneling

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The extended electrodynamic theory introduced by Aharonov and Bohm (after an earlier attempt by Ohmura) and recently developed by Van Vlaenderen and Waser, Hively and Giakos, can be re-written and solved in a simple and effective way in the standard covariant 4D formalism. This displays more clearly some of its features. The theory allows a very interesting consistent generalization of the Maxwell equations. In particular, the generalized field equations are compatible with sources (classical, or more likely of quantum nature) for which the continuity/conservation equation $\partial_\mu j^\mu = 0$ is not valid everywhere, or is valid only as an average above a certain scale. And yet, remarkably, in the end the observable $F_{\mu\nu}$ field is still generated by a conserved effective source which we denote as $(j^\nu+i^\nu)$, being i^ν a suitable non-local function of j^ν. This implies that any microscopic violation of the charge continuity condition is "censored" at the macroscopic level, although it has real consequences, because it generates a non-Maxwellian component of the field. We consider possible applications of this formalism to condensed-matter systems with macroscopic quantum tunneling. The extended electrodynamics can also be coupled to fractional quantum systems.

Keywords: Maxwell equations; Local charge Conservation; Josephson junctions

1. Introduction

The Maxwell equations play a crucial role in physics and engineering. Their mathematical properties are a beautiful lesson of vector analysis and their Lorentz invariance is a strong precursor of Special Relativity. The quantization of the Maxwell field leads to Quantum Electrodynamics, one of the most successful and accurate physical theories, and the extension of the gauge principle is at the basis of modern theories of elementary particles. So there appears to be little room for improvement of the Maxwell theory. From the experimental point of view, the literature reports a few claims of detection of electromagnetic waves with longitudinal or scalar components [1, 2], which are not compatible with the Maxwell equations.

An extension of Maxwell theory which is compatible with additional degrees of freedom in electromagnetic waves in vacuum is the Aharonov-Bohm electrodynamics [3-7]. The Aharonov-Bohm Lagrangian comprises an additional term proportional to $(\partial_\mu A^\mu)^2$ and has only a reduced gauge invariance; the Lorentz gauge is not applicable to this theory and no quantized versions have been proposed. So it cannot be regarded as a candidate replacement of Maxwell theory at a fundamental particle level, but possibly as adequate to the description of peculiar situations involving electromagnetic waves which are not purely transversal, but comprise a scalar component *S*.

The Aharonov-Bohm equations, however, set a clear limit to their own application when one considers the wave equation with sources. As already remarked in [4], this equation implies that a scalar electromagnetic field *S* can only be generated by a source which does not respect the local charge conservation condition $\partial_\mu J^\mu = 0$. Hively and Giakos [7] discuss whether this could happen due to charge fluctuations at a microscopic level. Van Vlaenderen [5] hypothesized that macroscopic quantum tunnelling could imply that charge is not locally conserved (although in fact this can only happen if the conserved current of the Schrödinger equation or its multi-particle generalization is not applicable - compare below, Sect. 3).

Other possible applications concern situations in which the occurrence of a quantum anomaly or a space-time singularity causes a tunneling process between two vacuum states [8, 9].

The new covariant formulation of Aharonov-Bohm electrodynamics will be discussed in Sect. 2, and possible applications in Sect. 3. Sect. 4 contains a brief conclusion and outlook.

2. Lagrangian Formalism and Field Equations

Let us follow the standard covariant formalism as for instance in [10], with Heaviside units and $c = 1$. In these units, the Maxwell equations are simply written

$$\begin{aligned} &\mathrm{div}\mathbf{E} = \rho \\ &\mathrm{curl}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ &\mathrm{div}\mathbf{B} = 0 \\ &\mathrm{curl}\mathbf{B} = \mathbf{j} + \frac{\partial \mathbf{B}}{\partial t} \end{aligned} \quad (1)$$

The electromagnetic field tensor $F^{\mu\nu}$, whose components are the electric field **E** and magnetic field **B**, is defined as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The four-potential is $A^\mu = (V, \mathbf{A})$. The four-current is $j^\mu = (\rho, \mathbf{j})$ and the "continuity" equation, or local charge conservation equation is

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\mathbf{j} = 0 \Leftrightarrow \partial_\mu j^\mu = 0 \quad (2)$$

The Maxwell equations (1) are written, in terms of $F^{\mu\nu}$ and j^μ, as

$$\begin{aligned} &\partial_\mu F^{\mu\nu} = j^\nu \\ &\frac{1}{2}\partial^\rho \varepsilon_{\rho\sigma\mu\nu} F^{\mu\nu} = 0 \end{aligned} \quad (3)$$

Note that since $F^{\mu\nu}$ is antisymmetric, eq. (3.a) can only be solved if $\partial_\nu j^\nu = 0$. The homogeneous equation (3.b) is a consequence of the definition of $F^{\mu\nu}$.

The field equation (3.a) can be derived from the Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu + \frac{1}{2}\kappa\left[\left(\partial_\mu A^\mu\right)^2 - \partial_\mu A^\nu \partial_\nu A^\mu\right] \quad (4)$$

Here L is seen as a function of the fundamental dynamical variable $A^\mu(x)$ and is the most general possible relativistic invariant Lagrangian constructed with a four-vector field. The term proportional to κ can be written as a four-divergence and gives in fact no contribution to the action $S = \int d^4x L(x)$. When $\kappa = 0$ one speaks of the "minimal" Lagrangian. In a gauge transformation $A^\mu \to A^\mu + \partial^\mu \phi$ the variation of L is

$$\Delta L = j_\mu \partial^\mu \phi \quad (5)$$

However, if the current is conserved, this is equivalent to

$$\Delta L = \partial^\mu(\phi j_\mu) \quad (\text{if } \partial_\mu j^\mu = 0) \quad (6)$$

and therefore, L is gauge-invariant up to a four-divergence.

Aharonov and Bohm [4] have proposed to generalize the electromagnetic Lagrangian by adding a term $\frac{1}{2}\gamma\left(\partial_\mu A^\mu\right)^2$ to the minimal L. (In their paper a parameter $\gamma = \lambda^{-1}$ is introduced.) This modified theory has also been studied by others, with various techniques [3, 5, 6, 7]. Here we would like to further analyze it in the four-dimensional Lagrangian formalism. The new addition to the Lagrangian, not being a four-divergence, changes the field equations as follows:

$$\begin{aligned} &L_{A.B.} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu + \frac{1}{2}\gamma\left(\partial_\mu A^\mu\right)^2 \to \\ &\to \partial_\mu F^{\mu\nu} = j^\nu + \gamma\partial^\nu\left(\partial_\alpha A^\alpha\right) \end{aligned} \quad (7)$$

Under a gauge transformation, the Aharonov-Bohm Lagrangian changes as follows:

$$\Delta L_{A.-B.} = j_\mu(\partial^\mu \phi) + \frac{1}{2}\gamma(\partial^\alpha \partial_\alpha \phi)^2 + 2(\partial^\alpha A_\alpha)(\partial^\alpha \partial_\alpha \phi) \quad (8)$$

This means that the theory is not gauge-invariant anymore (even if $\partial_\mu j^\mu = 0$). It is only invariant under reduced gauge transformations, such that $\partial^\alpha \partial_\alpha \phi = 0$.

Note that since the general invariance is lost, we cannot impose the familiar Lorenz gauge, choosing a new four-potential A'^μ such that $\partial_\mu A'^\mu = 0$. Therefore we must regard the quantity $S = \partial_\alpha A^\alpha$ as a non-trivial dynamical variable, and we are going now to find out more about it. Take the derivative ∂_ν of the field eq. (7) and remember that $F^{\mu\nu}$ is antisymmetric. We obtain

$$\partial_\nu j^\nu = -\gamma\partial_\nu \partial^\nu(\partial^\alpha A_\alpha) = -\gamma\partial^2 S \quad (9)$$

where ∂^2 denotes the D'Alembert operator $\partial^2 = \partial_\alpha \partial^\alpha$. Note that, as expected, the current is not generally conserved. Of course, everything boils down to Maxwell equations with $S = 0$ and conserved current if $\gamma = 0$.

From (9) we obtain an expression for S:

$$S = \partial_\alpha A^\alpha = -\frac{1}{\gamma}\partial^{-2}\left(\partial_\nu j^\nu\right) \tag{10}$$

The well-known operator ∂^{-2} is linear and non-local, as can be seen passing to four-momentum space, where it is represented by k^{-2}. Eq. (10) allows us to write an expression for the variable *S*, essentially integrating over the "source" $\partial_\nu j^\nu$:

$$S(x) = -\frac{1}{\gamma}\int d^4k e^{-ikx}\frac{k_\nu \tilde{j}^\nu(k)}{k^2} \tag{11}$$

Going back to (10), let us now rename the summation index (ν→β), take again the derivative ∂^ν and multiply by λ. We obtain

$$\gamma\partial^\nu\left(\partial_\alpha A^\alpha\right) = -\partial^\nu\partial^{-2}\left(\partial_\beta j^\beta\right) \tag{12}$$

Therefore, the generalized Maxwell equations (7) can be rewritten as follows:

$$\begin{cases} \partial_\mu F^{\mu\nu} = j^\nu + i^\nu \\ i^\nu = -\partial^\nu\partial^{-2}\left(\partial_\beta j^\beta\right) \end{cases} \tag{13}$$

Note that although these are derived from the Aharonov-Bohm Lagrangian $L_{A.B.}$, the parameter γ has disappeared. If the current j^μ is conserved, then the usual Maxwell equations are recovered. The new current component i^ν which now contributes, together with j^ν, to generate the field $F^{\mu\nu}$, is such that the total current $\left(j^\nu + i^\nu\right)$ is always conserved, as can be checked in two ways: (1) by taking the derivative ∂_ν in (13.a); (2) by taking the derivative ∂_ν in (13.b), which yields consistently

$$\partial_\nu i^\nu = -\partial_\nu\partial^\nu\partial^{-2}\left(\partial_\beta j^\beta\right) = -\partial_\beta j^\beta \Rightarrow \partial_\nu\left(j^\nu + i^\nu\right) = 0 \tag{14}$$

Summarizing, we can say that the input of the generalized electrodynamic equations (13) is a four-current j^ν which is not necessarily conserved (computed, for instance, from an "anomalous" microscopic model, as discussed in the following); but the output is an electromagnetic field tensor $F^{\mu\nu}$ which has the usual properties, including that of being generated by a conserved current, namely $\left(j^\nu + i^\nu\right)$. It follows the important property that at the macroscopic level the current is always conserved, as far as it is possible to measure it through the field it generates. In other words, even though in this model the microscopic current j^ν can be not locally conserved, the observable current $\left(j^\nu + i^\nu\right)$ is always conserved. Since eq. (13.a) is linear, the field $F^{\mu\nu}$ is the sum of the fields generated by the currents j^ν and i^ν. In general, the difference between the two currents is that even if the "primary" current j^ν is confined in a certain region of spacetime, the "secondary" current i^ν is not, because of the non-local expression which relates it to j^ν.

Another surprising aspect of this generalized electrodynamics is the following. The new degree of freedom is the scalar quantity $S = \partial_\alpha A^\alpha$; in the traditional view this is a pure gauge mode and cannot contribute to $F^{\mu\nu}$; but here the dynamics is such that *S* affects the observable fields $F^{\mu\nu}$ through the secondary current i^ν which compensates for the local non-conservation of the primary microscopic current j^ν.

For some static solutions with planar and dipolar sources see [11].

3. A First Possible Application: Locally Non-Conserved Current in Phenomena of Coherent Tunnelling

We have seen that the new general equations admit solutions also when the microscopic current is not locally conserved, and yield then an "observable current" which is conserved. This reminds other situations typical of quantum mechanics, where one defines the theory in terms of microscopic quantities, like the wave function, which are not directly observable (while the usual macroscopically observable quantities may be not well-defined at the microscopic level).

We would like to explore the idea, originally proposed in [6] without any formal justification, that in phenomena of quantum tunneling the local conservation of the current might not be ensured. Also in quantum mechanics, however, it is possible to define a microscopic current which is locally conserved. For any solution Ψ of the Schrödinger equation one has

$$\rho = |\Psi|^2; \quad \mathbf{j} = \frac{-i\hbar}{2m}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right); \quad \frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{j} = 0 \tag{15}$$

In some tunneling devices which are operated with high precision, like for instance the tunneling effect microscope, this microscopic expression for the current has been accurately verified [12]. Analogous properties hold for the Ginzburg-Landau equation, which is a non-linear extension of the Schrödinger equation for the description of the macroscopic wave function of superconductors.

There are good reasons to believe, however, that in other less ideal situations it is too restrictive to assume the validity of an equation like (15). In condensed matter systems, macroscopic wave functions obey constrained equations and have therefore in general a non-locally conserved current [13].

In the second-quantization formalism, the current operator is conserved for free fields or in the presence of local interactions. This leaves the possibility of anomalous local non-conservation for certain state averages, or in the presence of non-local interactions.

Let us consider, for instance, a 1D tunneling process, in a stationary situation where a current flows across several barriers in series. Assume that $\frac{\partial \rho}{\partial t} = 0$ everywhere and therefore according to the continuity equation we should have $\frac{\partial(\rho v)}{\partial x} = 0$, i.e. $\rho v = const$. This means that the charged "fluid" must ideally adjust its velocity everywhere in inverse proportion to its density, so that the flux ρv is constant in each section of the material. At those places, deep inside the barriers, where ρ is very small, v must be very large. The Schrödinger or Ginzburg-Landau equations do not enforce any upper limit on v, but in reality we can expect some complications.

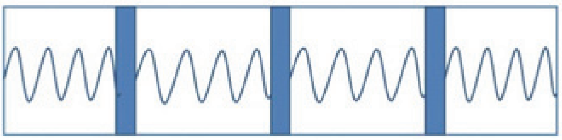

Figure 1. Macroscopic wave function of a supercurrent flowing through a 1D superconductor made of grains (like sintered YBCO). The inter-grain junctions conduct by Josephson tunneling or proximity effect. The amplitude is preserved in the tunneling.

Before discussing further, the implications of the relation $\rho v = const$, we observe that a stationary tunneling flux is realized when a supercurrent flows across Josephson junctions in series (Fig. 1). This is a phenomenological model employed for the description of conduction in superconducting materials like YBCO or BSSCO, which have a granular structure and exhibit Josephson tunneling both of the intrinsic kind (between crystal layers with spacing of the order of 1 nm) and across the inter-grain junctions [14].

Now focus on the junctions (Fig. 2). Note that the amplitude of Ψ on the two sides of the junction must be the same, in order to allow for a complete transmission of the supercurrent. In an ideal case "a la Schrödinger", in order to keep ρv constant, the velocity v of the pairs should increase very much in the center of the junction. However, it is well known from experiments on the proximity effect that the critical current of a SNS junction decreases exponentially as $\exp(-d/\xi_N)$, where ξ_N is a correlation length typical of the normal material. This is also confirmed by the non-local microscopic theory of Gorkov and De Gennes [15]. The physical reason for this exponential decrease is the fact that the upper electron velocity in a solid is limited and of the order of the Fermi velocity, while, on the other hand, the pairs density ρ also decreases exponentially.

If we want to approximate this complex situation with an effective current density $j=\rho v$, the local conservation of j in a stationary flow is only possible if v increases exponentially exactly in parallel with the decrease of ρ. This would require a strong local electric field which is hardly compatible, in our opinion, with all the other interactions present. Two possible situations are depicted in Fig. 2. The law $I_c \propto \exp(-d/\xi_N)$ actually seems to favor the first interpretation. Note that the density ρ only refers to the superconducting pairs. The total charge and current density comprises the density of the normal electrons. Local current conservation requires therefore that the normal electrons compensate locally for any unbalance in the superconducting densities. This may be difficult, especially in the case of high frequency currents.

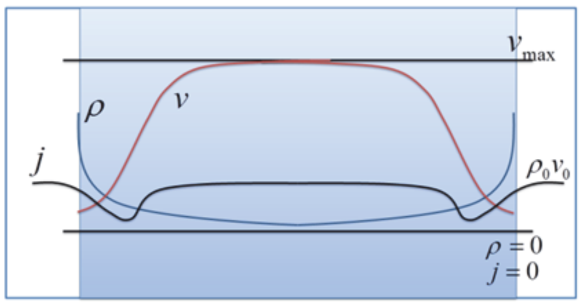

Figure 2. Hypothetical behavior inside a thick inter-grain junction of the pairs density $\rho=|\Psi|^2$, the supercurrent density j, and the pairs velocity v. Top: supposing that the increase in v is slower than the decrease of ρ, but can catch up in the middle of the junction. Bottom: supposing that the increase in v is insufficient to keep j constant.

4. Conclusion

We are led to conclude that when the amplitude $\rho = |\Psi|^2$ decreases very sharply in the junction, one cannot suppose to write a locally-conserved microscopic expression for the current. Fig. 2 shows the qualitative behavior of the supercurrent density, under two different assumptions; the divergence $\frac{\partial j}{\partial x}$ is not zero in two regions of thickness ξ_N inside the junction. It is not clear whether the normal current is able to compensate for this.

At the very least, this shows that the representation $j = \rho v$ is not adequate and therefore, if it is confirmed that $\partial_\mu j^\mu$ is not zero everywhere in certain states, because of a quantum anomaly, this should not be regarded as physically absurd in view of a classical local balance of charge ingoing/outgoing from a region.

A suitable formalism for the description of situations of this kind could be that of fractional quantum mechanics [16, 17]. As recently shown by Wei [18], the probability current of the fractional Schrödinger equation is not in general locally conserved. Our covariant formulation of Aharonov-Bohm electrodynamics allows to couple electromagnetism to charged fractional quantum systems; this would be inconsistent in standard electrodynamics.

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The Second Law and Entropy Increase

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The link between the Second Law of Thermodynamics and the claim that entropy can never decrease is examined afresh in the light of the fact that the entropy of classical thermodynamics does, in fact, decrease on many occasions. Traditional attempts to deduce the idea of entropy decrease will be looked at anew and conclusions will be drawn over the existence, or otherwise, of a general rule.

Keywords: Entropy, Thermodynamics

1. Introduction

Although based on concrete everyday examples, the subject of thermodynamics has caused students, and others, many problems of real understanding over the years. Much of the confusion has been associated with the quantity termed entropy; a quantity which first enters the subject on the introduction of the Second Law into the discussion. This entry takes different forms depending on the approach and language used but using mathematical terminology indicates that the change in this quantity is represented by a total differential, dS, which arises when an integrating factor is found for the inexact differential, $d'Q$, representing an amount of heat added to, or taken from, a system. Hence, in classical thermodynamics, any entropy change is linked irrevocably with a flow of heat via the relation

$$dS = d'Q/T, \tag{1}$$

where T is the absolute temperature and its inverse is, mathematically, the said integrating factor.

However, the discussion to this point in the development of the subject takes no account of the so-called irreversible processes which are prevalent in nature. One of the big stumbling blocks faced by many on their introduction to thermodynamics is the extension to cover these processes since, frequently, people tend to say that, when such processes are involved, the entropy cannot decrease and from this it is often concluded that the entropy can never decrease. Since in the above equation the heat change may be either positive or negative, it follows immediately that the entropy can, in fact, decrease under some circumstances. This raises the question as to when the entropy may be claimed to be non-decreasing?

Several answers have been advanced to this question and here these will be reviewed before an attempt is made to bring all the thoughts together to try and formulate a more general answer.

2. Traditional Attempts to Deduce the Idea of Entropy Decrease

(i) The Approach of Landsberg [1].

Consider a system possessing three independent variables T, V_1 and V_2 and let this system be taken around a cycle. Suppose the initial state of this system is i and suppose it undergoes a non-static adiabatic process to a state f, where i and f are both assumed to be equilibrium states of the system. Then, the entropy change is

$$\Delta S = S_f - S_i \,. \tag{2}$$

During this process, a temperature change may, or may not, have occurred. Whether it has or not, now suppose the system undergoes a quasistatic adiabatic process $f \rightarrow k$ to bring its temperature to that of some arbitrary heat reservoir at temperature T'. Since S_f and S_k are equal,

$$\Delta S = S_k - S_i \,. \tag{3}$$

The system may be brought into contact with the reservoir and caused to undergo an isothermal process $k \rightarrow j$ until its entropy is the same as it was initially. A quasistatic adiabatic process $j \rightarrow i$ returns the system to its initial state and, since S_j and S_i are equal.

$$\Delta S = S_k - S_j \,. \tag{4}$$

The only heat transfer, Q, that has taken place in the cycle is during the isothermal process where

$$Q = T'(S_j - S_k). \tag{5}$$

Also, a net amount of work, W, has been done in the cycle where

$$W = Q. \tag{6}$$

From the Second Law, it is clear that the heat Q cannot have entered the system - that is, Q cannot be positive - for then, the performance of an equivalent amount of work.

Hence,

$$Q \leq 0, \tag{7}$$

from which it follows that

$$T'(S_j - S_k) \leq 0 \tag{8}$$

or

$$\Delta S = S_k - S_j \geq 0 \,. \tag{9}$$

Here it has been assumed that an entropy change is associated with the original non-static adiabatic process. If this were not so, it would be possible to return the system to state i by one quasistatic adiabatic process. Since the nett heat transferred in this cycle is zero, the nett work would be zero also. Under these circumstances, the system and its surroundings would have been restored to their initial states without producing changes elsewhere - implying that the original process was quasistatic. This is contrary to the original assertion, and so the entropy of the system cannot remain unchanged.

Again, the system considered was assumed homogeneous and of uniform temperature and pressure. If this were not so, it would be necessary to subdivide the system into parts - each one infinitesimal in an extreme case - and to ascribe a definite temperature and pressure to each part, so that each part would have a definite entropy depending on its coordinates. The entropy of the system as a whole would be defined to be the sum of the entropies of the various parts. If it is possible to return each part to its initial state in the manner described earlier, using the same reservoir for each part, it follows that ΔS is positive for the whole system.

The final result is that the entropy of a system in a given state cannot be decreased adiabatically for a thermodynamics in which the absolute temperature is positive and heat tends to flow from high to low absolute temperatures. This is a statement of the principle of the increase of entropy of systems in *adiabatic enclosures* but it is definitely restricted to behaviour within *adiabatic enclosures* and doesn't obviously extend beyond this restriction.

(ii) The approach of Münster [2].

Having considered a homogeneous system and subsequently derived the equation

$$dS = d'Q/T \tag{10}$$

as the mathematical formulation of the Second Law for reversible processes, Münster then proceeds to consider a heterogeneous system, firstly with no heat exchange between phases and then with the different parts of the system possessing different temperatures. For simplicity, in the second case an isolated adiabatic system consisting of two parts was considered. The two parts were assumed to have temperatures T^α and T^β with

$$T^\alpha > T^\beta. \tag{11}$$

It was supposed that heat exchange between the phases was slower than heat equilibrium within a phase. It follows that each phase will be in internal equilibrium during the obviously irreversible process which will enable the two phases to come into thermal equilibrium with each other. In the approach to such equilibrium, one phase will lose an amount of heat while the other will gain the same amount, say dQ_i, where the suffix i indicates the fact that the process occurs internal to the system. If no external work is done, it is shown that the equation

$$dS = d'Q/T \tag{12}$$

holds for both parts of the system separately and, since the amount of heat lost by one part equals the amount gained by the other part, it follows that the total entropy change is given by the sum of the two separate entropy changes. After some trivial manipulation, this leads to

$$dS_i = dQ_i\left(\frac{1}{T^\beta} - \frac{1}{T^\alpha}\right). \tag{13}$$

Due to the above inequality, it follows that

$$dS_i \geq 0. \tag{14}$$

In other words, the change in entropy brought about by the irreversible process of heat conduction must be either zero or positive.

Münster then proceeds to generalise this result by noting that, in order to find the total entropy change, any exchange of heat with the surroundings has to be considered. He also noted that any such heat exchange with surroundings had to be shared between the two phases:

$$Q = Q_\alpha + Q_\beta. \tag{15}$$

He then commented that, if the increase of entropy due to absorption of heat from the surroundings is dS_a, the total entropy change will be given by

$$dS = dS_a + dS_i. \tag{16}$$

or, using earlier relations,

$$dS = \frac{dQ_\alpha}{T^\alpha} + \frac{dQ_\beta}{T^\beta} + dQ_i\left(\frac{1}{T^\beta} - \frac{1}{T^\alpha}\right). \tag{17}$$

By defining an 'effective temperature' and dQ' appropriately, Münster eventually writes dS in the form

$$dS = \frac{dQ}{T} + \frac{dQ'}{T} \tag{18}$$

and then shows that $dQ' \geq 0$. After further discussion, he shows that

$$dS \geq 0 \tag{19}$$

for an isolated adiabatic system.

However, as with the particular case discussed here, questions may be raised as to the generality of the result as well as to whether entropy, as a function associated with classical thermodynamics, is even defined in the cases under discussion. Of course, this question relating to the definition of entropy is one of the biggest problems associated with irreversible thermodynamics. Again, though, it should be noted that Münster does restrict all his considerations to situations involving heat flow. Hence, the entropy function he discusses is always associated with changes of heat and, as such, remains fully within the orbit of classical thermodynamics; there is no mention here of statistical or information issues to cloud the issue.

3. Some Further Comments Relating to Irreversible Thermodynamics

Bearing in mind the above results due to Münster, it is interesting to note that, in discussions of irreversible thermodynamics [3,4], the effective starting point is to assume that the entropy change dS of a system is composed of two, and only two, terms – firstly a term dS_e which arises due to a transfer of heat from external sources across the boundary of the system, and secondly a term dS_i due to changes within the system. It is often then claimed that the Second Law demands that dS_i must be greater than, or equal to, zero. It may be noted immediately that it is by no means obvious how this conclusion is reached since it seems, as illustrated above, that the inequality is derived purely for adiabatic processes. However, possibly the most interesting outcome is to read what regularly follows in discussions of irreversible thermodynamics:

In de Groot's classic text [3], the starting point of the discussion is to write the change in entropy in a certain interval as

$$dS = d_eS + d_iS = \frac{dQ}{T} + d_iS, \tag{20}$$

where d_iS is the entropy produced inside the system by irreversible processes and dQ is the heat supplied to the system by its surroundings. As de Groot points out, the system under consideration is a closed one so there is only heat exchange with the surroundings; if there was matter exchange as well, there would have to be an additional term in d_eS. He then notes quite clearly that the term d_eS may be positive, zero or negative, but then simply states that the term d_iS is positive definite

However, when calculating entropy production, dS_i, he points out immediately that two assumptions have to be made, one of which is that the entropy production is positive definite. In the text due to Yourgrau, van der Merwe and Raw [4], it is pointed out that 'the assertion that the entropy production in any process is zero or positive constitutes one of the basic postulates of irreversible thermodynamics' or, in other words, it forms a basic assumption. De Groot continues by explaining that the second fundamental assumption is that the relation

$$TdS = dU + pdV - \sum_i \mu_i dM_i \qquad (21)$$

where U is the energy, V the volume, p the pressure, and M_i the mass and μ_i the chemical potential of the ith component of the system, is assumed valid even outside equilibrium. It follows immediately that this means that the entropy, S, is an explicit function of only energy, volume and concentrations. De Groot then proceeds to consider the validity of these assumptions. He points out that, from the statistical standpoint, the first assumption is just a result of the H-theorem. Immediately, therefore, it is seen that this theory goes outside the realm of classical thermodynamics in that a statistical element is introduced from the very beginning. He then notes that validation of the second assumption is model dependent and he draws on the Chapman-Enskog theory of non-uniform gases to justify it. As a consequence, he shows that, with this model, limitations on the validity of the assumption are exposed.

This whole approach to unravelling the problems associated with the thermodynamics of irreversible situations highlights a number of points of confusion over the whole idea of entropy. Possibly most importantly it raises again the question of whether the function referred to as entropy in several branches of physics is, in fact, one and the same function. Here, in this very brief review of the situation obtaining at the outset in irreversible thermodynamics, it is seen that statistical ideas are introduced immediately and it follows that this moves considerations away from the subject of macroscopic classical thermodynamics. However, in some ways, the biggest question raised relates to what is really meant by the term 'entropy'?

4. Further Thoughts on 'Entropy'

Entropy has been the source of much confusion and uncertainty in various areas of science for many years; indeed, probably from the point where it was first introduced. However, possibly the first and most important point to note is that it first arose in classical thermodynamics. There, it arose from considerations of the Second Law which led to the idea that, mathematically, the inexact differential representing an amount of heat given to, or taken from, a system at a particular temperature possessed an integrating factor. Hence, this quantity of heat multiplied by its integrating factor was an exact differential which was denoted by dS, and this was termed a change in entropy. Hence, a change in this classical thermodynamic entropy was associated quite specifically with a *flow* of heat into, or out of, a system; it was definitely *not* associated with the specific system as a property of that system. Hence, the entropy of classical thermodynamics must be seen to be a different quantity from those other 'entropies' which are properties of the systems themselves. It might be noted at this point that, in the above considerations relating to so-called irreversible thermodynamics, the change in entropy is assumed to consist of two parts – one due to a flow of heat, the other due to changes within the system itself. Of course, these internal changes could be brought about by heat flow within the system but, if not, the question of what they are must arise. Further, if the internal changes are not due to heat flow, the question of whether or not they bring about entropy changes, in the sense of changes in the classical thermodynamic entropy, arises also.

This is a huge question whose answer must have far reaching consequences for science, especially as far as true understanding is concerned. One extra problem must be that the theory in place seems to have worked in practice extremely well for many years but, if truth be told, this has always been against a background of a fuzzy understanding of the basic ideas involved. There is no doubt that, within the established boundaries of statistical theory and information theory, the notion of an entropy function is securely established and is undoubtedly a valid notion. However, this function is usually taken to be identical with the entropy function of classical thermodynamics. This almost automatic assumption has been found to work, seemingly, on many occasions so that it has not been questioned and, nowadays it is not really seen as an assumption, if it ever was, but as an established scientific fact. The above considerations show this to be, in fact, an incorrect assumption since, as shown, the two entropies refer to totally different entities – the classical thermodynamic entropy being linked irrevocably with a heat flow and *nothing else*; the other entropies being linked to systems or distributions.

There are at least two further important points which need to be taken into consideration. The first refers to the use of all that has been said of the notions of ‘reversible’ and ‘irreversible’ processes. This point has been discussed at length previously [5] and there seems little point in reiterating what has been said before. Suffice it to say that here the two words have simply been used to conform with the terminology of earlier work but it should be remembered that use of these two terms generally can lead to complications. The second point to contemplate concerns the implication in much of what has been written that the ‘entropy’ is a state function. It is this assumption which allows a result for purely adiabatic processes to be generalised to include all processes. The idea is that entropy is seen to increase for an irreversible adiabatic process but such a process links two states of a system and, therefore, if entropy is a function of state, any other process linking the two states must be accompanied by the same change in entropy; that is, by an entropy increase. As has been pointed out previously [6, 7], although entropy is undoubtedly a function of state in some circumstances, doubts have been raised over the claim that this is always so. Hence, care must be exercised when making use of this assumption.

5. Conclusions

This article does not set out to give a complete and definite answer to the basic question raised but rather to make people aware of some of the very real, but unstated, problems associated with the quantity, or more probably the quantities, referred to as ‘entropy’ in modern science. In particular, we have highlighted some of the assumptions that have been made within the development of the subject and which have since been overlooked as assumptions. Very real questions have been raised which, if we are ever to understand unequivocally what physical property is associated with entropy, and by extension how different processes change this property, are worthy of further consideration. In this respect, problems associated with the notions of ‘reversibility’ and’ irreversibility’ of physical processes have again been raised, as has the query concerning whether or not entropy is always a state function. Definite answers have not been advanced, but nevertheless some conclusions can be drawn. It does seem that, whatever the success of present day notions in some practical situations, the idea that all the entropies discussed so freely are the same is simply not true. The crucial point highlighted in this context is that, in classical thermodynamics, a change in the entropy function is linked irrevocably with a flow of heat but this is not the case in other areas. In addition, it has been shown that the seemingly blanket claim that the “Second Law of Thermodynamics states that the entropy can never decrease” is, at the very least, misleading and, at worst, totally incorrect.

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Some Thoughts on Magnetic Reconnection

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The idea of 'magnetic reconnection' has been around since the work of Ronald Giovanelli who proposed it while searching for an explanation of solar flares. The idea seems to have received a boost from the work of James Dungey but he made it very clear in his writing that magnetic lines of force were not physical entities and shouldn't be treated as such. Bearing this point in mind, attention is concerned here with noting the non-physicality of the whole notion of 'magnetic reconnection' and suggesting an alternative approach to seeking explanations for such phenomena as solar flares via the study of plasmas.

Keywords: Magnetic reconnection, Plasma, Solar flares

1. Introduction

The whole idea of so-called magnetic reconnection seems to have been brought to the fore again recently with the appearance of the obituary of James Dungey in the October issue of A & G [1], the house journal of the Royal Astronomical Society (RAS) and the right-up, in the December issue [2], of the meeting on the precise issue at an RAS discussion meeting held on December 12th.

In very simple terms, from what is written to explain it, the entire notion of magnetic reconnection seems, on the face of it, to be based on the belief that enormous quantities of energy are produced by magnetic lines of force breaking and then reconnecting. Undoubtedly there is some physical process occurring which produces the enormous quantities of energy observed in the situations concerned in such physical phenomena as solar flares but this explanation, at least in its simplest forms, seems unacceptable. This follows because, crucially, lines of force are simply not real; they're not physical entities. Since so much is attributed in the origins of this topic to James Dungey, it seems worth noting his own words at this point:

"*The familiar method of representing a magnetic field by lines of force provides a picture that is particularly useful in cosmic electrodynamics. As so much use is made of this picture, it is perhaps necessary to remind the reader that the use of lines of force is a mathematical device and that they are not physical objects: the motion of lines of force is a further device, which is to be explained in this chapter, and it will be found that their motion is to some extent arbitrary, so that part of the motion can have no physical meaning*."

This very clear statement provided the introduction to chapter 3 of Dungey's book *Cosmic Electro-dynamics* [3] and makes the non-physical nature of lines of force crystal clear.

Magnetic lines of force, typically, are represented by those lines of iron filings observed mapping out the magnetic field of some magnet or magnets. The direction in which an individual iron filing lies simply indicates the direction of the magnetic field at that point. Note only the direction is given, not the magnitude. In other words, at any point in the magnetic field, the tangent to the line of force passing through that point gives the direction of the magnetic field at the point. However, especially if you search for information on the internet, you find it claimed that the strength of the magnetic field at a point is said to be indicated by the number of lines of force passing through an element of area at that point. This seems an extremely dubious notion since a magnetic field is a continuum filling all space in a region and a line of force is just that – a line – and any line is infinitely thin. However, as is clearly mentioned by Thomson [4] and explained in some detail by Dungey [3], when it comes to discussing the strength of a magnetic field, it is so-called tubes of force which prove of use. A tube of force might be defined as the surface generated by the lines of force passing through a closed curve with a small closed curve being chosen normally so that the resulting tube of force is narrow. Then, from the definition, the component of the field of force normal to the surface of the tube vanishes. Use of the Maxwell equation

$$\nabla.\boldsymbol{H} = 0, \quad (1)$$

then indicates that the magnetic flux

$$\int \boldsymbol{H}.dS \quad (2)$$

through any cross-section of a given tube of force is constant. As Dungey makes absolutely clear, a narrow enough tube of force gives complete information about the strength of the field, while an individual line of force only gives the direction of the field as noted above.

It must be acknowledged that the technique involved here has been extremely successful in enabling people to explain and, in a sense, understand much that is going on but it must be remembered always that this discussion here centres on a model, *not* on the actual physical system under investigation. In truth, the whole notion of lines of force, although very useful in limited circumstances, is a tricky one to contemplate outside its use for determining the direction of a magnetic field at a point.

This whole argument may be summed up point by point as follows:

Lines of force in pictures representing magnetic fields are not real; they are simply a way of picturing what is happening in a magnetic field.

The tangents to these lines of force merely represent the direction – and only the direction - of the field at any point; that is, if the lines of force are drawn, they are curves and the tangent to such a curve at a particular point represents the direction of the field at that point.

The lines of force do not represent the magnitude of the field.

Hence, when two fields come together, the field is either strengthened, or weakened, at any particular point. This in itself will not affect the lines of force but any change in the resulting direction of that field will.

It follows that the pattern of lines of force will alter but to describe such alteration as a result of lines of force breaking and reconnecting is misleading. It is especially dangerous since it attributes a genuine physicality to these lines of force as well as to their behaviour and that is something they definitely do not possess; - lines of force, as introduced by Faraday are merely an aid to help in understanding.

It is crucial to remember that the magnetic field is a vector quantity and so, at any point, possesses both magnitude and direction. The lines of force only picture the direction at any point. Possibly the magnitude is, in a sense, more important.

It does seem that the popular concept of magnetic 'reconnection' is at least misleading.

It seems worth contemplating the situation of two bar magnets approaching one another in order to further clarify some points raised here. As the two magnets approach one another, the strength of the magnetic field at points will either be enhanced or diminished, depending on the actual physical situation occurring. There are four possibilities to be considered here:

(i) The magnets parallel to one another with unlike poles opposite each other,

(ii) The magnets parallel to one another with like poles opposite each other,

(iii) The magnets lying along the same line with unlike poles opposite each other,

(iv) The magnets lying along the same line with like poles opposite each other.

In each case, as the fields combine, at different points the directions of the two fields may differ and the resulting direction may well be different from both the originals. Such a change in direction of the magnetic field at a particular point would be reflected in the relevant line of force. Of course, where the two fields cancel each other, neutral points will arise and this will be reflected by an absence of lines of force in a small region.

All this is very elementary and well-known, with pictorial representations of the merged fields in each of the four above-mentioned cases readily available to view. However, it is easy to see how a naïve interpretation of events might be in terms of lines of force breaking and then reconnecting but such an interpretation is misleading and effectively masks what is really happening physically.

2. Astrophysical Applications

As has been mentioned already, in astrophysics, the idea of magnetic 'reconnection' is used to explain some huge surges of energy in, for example, solar flares. This point needs to be considered in the light of what has been said above but it might be noted that, when separate magnetic fields come together and combine into a single field, the strength of that resulting field will be enhanced at some points and weakened at others. The overall result might be expected to be a completely new pattern of lines of force due to the totally new magnetic field brought about by the natural merging of the original two fields. Again, there is no breaking of lines of force; merely the natural emergence of a new pattern due to the said merging.

As for the enormous quantities of energy produced in some physical occurrences such as solar flares, it seems far more likely that the true explanation is to be found within the realms of plasma cosmology. Many of the experiments associated with plasma are performed in laboratories on earth and there is no real reason to suppose the results of such experiments extrapolate to the cosmic scale. Nevertheless, the general method of performing experiments in laboratories here on earth and extrapolating the results obtained to help explain cosmic features has worked extremely well in the past in a variety of areas since the days of Newton at least. However, although much of this work started with experimental work by Birkeland carried out in a laboratory, there are real problems in attempting to scale down cosmic phenomena to laboratory size. Also, with the advent of satellite and probe investigations, latterly laboratory experiments have aimed at clarifying basic phenomena of importance in cosmic physics rather than attempting to produce a scaled-down version of the cosmic phenomenon involved. This more recent laboratory work has led to the demonstration of the existence of several basic phenomena associated with plasmas that had been neglected previously. As well as providing more useful knowledge about plasmas themselves, this work has led also to the identification of cosmic phenomena with fundamental properties of plasmas.

As far as, for example, solar flares are good clues to help in the search for genuine physical explanations, as is seen by examining the contents of Anthony Peratt's book *Physics of the Plasma Universe*[5]. In this volume, one reads of double layers and their properties and in section 5.5, which deals with the basic properties of these double layers, one reads of the phenomenon of exploding double layers – a topic discussed also in Alfven and Arrhenius's book *Evolution of the Solar System* [6]. In the latter book, on page 250, it is pointed out that

"if a current flows through an electrostatic double layer (which is often produced by the current itself), the layer may cut off the current. This means that the voltage over the double layer may reach any value necessary to break the circuit (in the laboratory, say 10^5 or 10^6 V; in the magnetosphere 10^4 - 10^5V; in solar flares, even 10^{10}V). The plasma 'explodes' and a high-vacuum region is produced"

It was while conducting experiments with low pressure discharges in the 1920's that Irving Langmuir first encountered what are now called 'double layers', although originally he called them double sheathes. These phenomena have been observed regularly since that time. Essentially, a double layer is a structure in a plasma consisting of two thin parallel regions of opposite charge excess which give rise to a potential drop and, therefore, an electric field, across the said layer. Ions and electrons entering such a double layer are accelerated, retarded or deflected by the electric field. It was in 1958 that Alfvén originally suggested the possible existence of double layers in a cosmic setting but roughly twenty years had to elapse before instruments on earth satellites proved their existence in the earth's magnetosphere. Nowadays, double layers, as well as being found in plasmas in discharge tubes, have been noticed in, for example, space plasmas and in so-called Birkeland currents. These currents are named after the Norwegian experimentalist, Kristian Birkeland, who provided much of the inspiration for most of the recent work on plasmas. It was his work that originally led to the notion that charged particles which originated in the Sun and guided by the Earth's magnetic field produced the rings of the auroras but it was many decades before this suggestion, originally scoffed at by theoreticians, was found correct following satellite measurements.

As far as Birkeland currents are concerned, Birkeland showed that electric currents flow along filaments shaped by current induced magnetic fields. As plasma filaments come together, they rotate about one another and the end effect is that the filaments combine into a shape which is reminiscent of a twisted rope. The term 'Birkeland current' seems to have appeared in the scientific literature first in 1969 [6]. However, it is well before that that the likes of Langmuir and Alfvén began to expand on his pioneering work.

All the information relating to double layers gathered since those early days would seem to indicate at least the possibility of a more physically realistic explanation of some cosmological phenomena such as solar flares. Indeed, these and other phenomena are discussed specifically in Anthony Peratt's book [5] in a section devoted specifically to examples of cosmic double layers. Undoubtedly, there will be astrophysical phenomena not covered by Peratt which are regarded as admitting an explanation based on the notion of 'magnetic reconnection' but, given the dubious nature of that so-called effect and bearing in mind the wealth of information gathered by plasma scientists in a variety of ways over many years of study, it seems all such should be re-examined on the basis of all this knowledge relating to plasmas.

3. An Important Aside

As is pointed out by Thornhill and Talbott [7], Hannes Alfvén received the Nobel Prize in 1970 for his fundamental discoveries in magnetohydrodynamics but, in his acceptance speech, he pleaded with scientists to ignore some of his earlier work, particularly his concept of magnetic fields being frozen in to superconducting plasmas. Unfortunately, his plea was ignored and it is this notion which underpins the conventional interpretation of magnetism in space; it has enabled astrophysicists to ignore the electrical currents necessary to generate and maintain these magnetic fields. Returning to Dungey's book, it is seen that in his proof of the magnetic field being frozen into the material, he introduces the condition of perfect conductivity and this, in turn, is shown to mean that

$$\boldsymbol{E} = -\boldsymbol{u}\mathrm{x}\boldsymbol{H}/c. \quad (3)$$

cosmic systems'. It is claimed that this is 'nearly always a good approximation for cosmic systems'. However, it is just that – an assumption! Alfvén himself came to believe that magnetic fields are but one component of plasma science. He felt it dangerous to overlook the electric currents that generate magnetic fields and inadvisable to attempt to model plasma in the absence of electric currents and circuits. It seems it might be sensible to heed his warning here and note the restrictions placed on work by imposing the above-mentioned condition of perfect conductivity. Perfect conductivity might be a convenient restriction in a mathematical context but whether or not it is a reliable one physically is, possibly, another question.

4. Conclusion

Bearing in mind the seeming physical illogicality of the notion of magnetic reconnection, together with the realisation that a physically acceptable alternative is there for all to see, it is difficult to see how the presently accepted position can remain tenable and accepted as yet another piece of scientific 'conventional wisdom'. At the very least, as suggested above, all these phenomena said to admit explanation via the mechanism of so-called 'magnetic connection' should be re-examined using knowledge associated with the study of plasmas – both plasmas in laboratories here on Earth and those observed in space.

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Are Adiabatic Work Processes in the Classical Ideal Gas Intrinsically Irreversible?

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The notion that a reversible thermodynamic process is necessarily quasi-static is one of the cornerstones of thermodynamics, yet to the author's knowledge the notion has never been subjected to any sort of critical analysis. The idea of reversibility rests on the assumption that a small change of constraint, say the external pressure, automatically leads to the restoration of the initial state upon being reversed. Yet, the equation of state for an ideal gas contains three variables, P, V and T. In an adiabatic process in which only one of the variables is altered, the system can only return to the initial state if an additional variable of state is also reversed. If the work done on the gas during compression is the same as the work done by the gas during expansion, the initial state must be restored, but as the total external work done in the different directions is not the same, with the work of compression being greater than the work of expansion owing to the different external pressures acting in the different directions, true reversibility requires an additional constraint. However, that constraint is never specified in the thermodynamic literature. In this paper, the thermodynamics of this idea are examined critically. It is argued that the idea of quasi-static adiabatic reversibility is incompatible with known thermodynamics. The consequences for our understanding of entropy are explored.

Keywords: Quasi-static process, reversibility, adiabatic, entropy, Clausius

1. Introduction

The idea of a quasi-static process as reversible dates back to Clausius' Fourth Memoir of 1854, in which he includes among a number of examples of irreversible processes, "… all cases where a force, in doing mechanical work, has not to overcome an equal resistance, and therefore produces a perceptible external motion, with more or less velocity, the *vis viva* of which afterwards passes into heat". [1]. Later, in his Sixth Memoir of 1862 [2], he was more explicit: "When a change of arrangement takes place so that the force and counterforce are equal, the change can likewise take place in the reverse direction under the influence of the same forces". The idea of no "perceptible external motion" coupled with equal force lead directly to the notion of a quasi-static process.

There is a difficulty, however. Clausius acknowledged that if one force is greater than the other, the change is irreversible and that strictly, "the overcoming force must always be greater than the force which it overcomes". Hence, "reversibility is a limit which in reality is never quite reached". Nonetheless, Clausius argued that we can still treat the limiting case as one which "really exists".

There have been far too many comments on reversibility to cite, but attention can usefully be drawn to the comments of Landsberg in his 1961 book [3] by way of illustration of the difficulties inherent in Clausius' definition. Clausius was referring to processes involving gases, but Landsberg was thinking more generally when he wrote:

1. Reversible processes are not really processes at all but sequences of equilibrium states;
2. The real criterion for reversibility is that no changes of any kind must remain in the surroundings of a system, when the given process is followed by the same process, taken in the opposite sense;
3. A process which is reversible must take place infinitely slowly.

Landsberg then went on to argue that as quasi-static processes do not in themselves need to be reversible and that simple mechanical systems such as a bouncing ball satisfy (2) but not (1) and (3); (1), (2) and (3) "must be regarded as referring to three different types of processes unless and until the contrary can be established".

An example of a thermodynamic process that appears to satisfy (1) and (3) but not (2) in Landsberg's list was presented by Sommerfeld [4]. Suppose a series of levels upon each of which rests a grain of sand or some other very small weight. The sand is slid sideways on to a piston which is depressed by some small amount to the next level. Another grain of sand is added to the piston which then moves down the next level and so on until the piston has moved a substantial distance. In order to return to the starting point the sequence of steps is reversed. Sommerfeld makes the point that no work is lost in this process as each weight is removed at the same level at which it was placed on the piston: the work done by gravity on the gas is recovered during the expansion. To quote Sommerfeld, the process is reversible "on condition that it has been carried out in *infinitely small steps* and *sufficiently slowly*".

Deeper analysis shows that even if this process maps a sequence of equilibrium states; this proposition will be examined later, it is not reversible. If a small weight w is added and the piston moves a distance dy, then over a succession of n such operations in the forward direction the work done over a total distance δy is:

$$W_F = w.dy_1 + 2w.dy_2 + 3w.dy_3 + ...nw.dy_n \\ = w\sum_{i=1}^{n} i.dy_i \qquad (1)$$

Strictly a term for the mass of the piston should be added, but as this is constant and is assumed to move through the same displacement in both directions it can be disregarded. In the reverse direction the work done is

$$W_R = -w\sum_{i=0}^{n-1} i.dy_i \qquad (2)$$

Equation (2) is not the same as equation (1). The last weight added to the piston remains at the lowest level as it has to be removed before the piston can begin its ascent, so the nett effect of this sequence of operations is to transport matter from the top of a series of levels to the bottom. The surroundings are therefore not left unaltered by a complete cycle of this process and by this criterion it is not reversible.

The argument adopted by Clausius and Sommerfeld, and accepted by generations of physicists, is that this is a practical consequence of trying to achieve a limiting case and that this difference can be ignored. In the mathematical limit, of course, the work done either way is simply the integral of pdV over the whole range of volumes.

It is not clear to this author that the difference can be dismissed so easily. The difference between the work done in the forward and reverse directions, that is, the work dissipated during the irreversible process, corresponds to the last term in the forward direction,

$$\delta W = nw.dy_n \qquad (3)$$

This is equivalent to the gravitational potential energy gained by the weight in moving through a distance $\delta y = n.dy$. The process is irreversible and the irreversibility cannot be reduced to zero no matter how small the weight, w, without reducing w to zero. Then, however, there is no driving force for the motion. This echoes Clausius' writings quoted earlier, but whereas Clausius argued that the limiting case "really exists", the present analysis indicates otherwise. Even in the limit that an infinitesimally small weight, w, were to be used there would still be a difference between the work done in the different directions. This excess work would have to be dissipated in some process, and although the effect might be very small over a single cycle, repeated operation of the cycle would amplify the effect. In the case of Sommerfeld's process, the operation would transport sand from the top to the bottom and the visible effect of repeating the cycle would grow with time.

There are two implicit assumptions and, in this author's view, one fundamental misconception in both Clausius' and Sommerfeld's arguments. The misconception arose with Clausius and has become the established view of thermodynamics. This is the confusion between process and state. The author has examined in great detail Clausius' writings on Aequivalenzwerth, the forerunner of entropy, and exposed a number of contradictions in his writings that have either not been recognised or not widely acknowledged. Not least among them, a reversible process should be performed quasi-statically. Clausius never used this word. He referred instead to the absence of "perceptible external motion", but the implication is the same and the contradiction arises because he was writing in the context of a heat engine. A heat engine works because a piston moves in and out repeatedly at speed, but for Clausius the ideal, or reversible engine, works infinitely slowly. In short, in an ideal engine designed to exploit the motive power of heat, to use Carnot's phrase, nothing moves.

In one respect Clausius was simply following Carnot, whose description of the operation of an ideal heat engine has been carried through into modern thermodynamics virtually unchanged. In one vital respect, though, Clausius diverged from Carnot. Whereas the latter was concerned with the reversibility of the cycle, which can be performed in the forward direction for a heat engine or in the reverse, with work being converted into heat, for a refrigerator, Clausius regarded a reversible cycle of being made up of separately reversible processes. In Carnot's view, an ideal reversible engine would extract as much heat from work operating as a refrigerator as an engine would use in doing the same amount of work. Operating in tandem, they would leave the environment unchanged. In Clausius' view, each separate process, such as an isothermal or adiabatic expansion, can be reversed.

The difference between these two views is quite profound, as illustrated by the previous discussion on Sommerfeld's model of a quasi-static process. A heat engine operates in a cycle and both Kelvin's statement of the Second Law and Clausius' first statement phrased in terms of the impossibility of transferring heat from a cool to a hot body were formulated on the understanding that processes were cyclic. Any degree of irreversibility, no matter how small and seemingly negligible over one cycle, will be amplified by repeated operation of the cycle. The heat engine will therefore not be ideal in the sense defined by Carnot. That is, it would not be possible to run a Carnot engine as a source of work powering another, identical Carnot engine operating as a refrigerator, or a source of heat, and leave the environment unchanged.

Although Carnot attempted to show how his cycle could be realised in practice, it is not necessary in fact to be able to execute a perfect Carnot cycle. The cycle is an idealisation which shows the maximum possible efficiency of a heat engine working between two temperatures, regardless of whether such an engine could ever be constructed or not. Arguably, the intermediate states that mark the transition between the isothermal and adiabatic stages are just points on a piston stroke. Clausius, on the other hand, emphasized the separate processes occurring between well-defined states and at one stage even replaced the isothermal expansion with a free expansion on the basis that the initial and final states are the same in the two processes. In the context of a heat engine this makes no sense, as the isothermal expansion is the stage that takes in heat whilst doing work. However, in Clausius' view, it was the states themselves that were important and it is evident from an early stage that he was interested in the microscopic structure of the working fluid and the processes causing the transitions between the different states of the cycle were of secondary importance.

As for the two implicit assumptions behind the Sommerfeld model mentioned earlier, these concern the effect of adding and removing a small weight. It is no more than an assumption that the addition of a weight causes the system to settle in a definite state: elementary mechanics tell us not only that without some mechanism of damping even the addition of a very small weight would cause the system to oscillate indefinitely, but also that if the displacement is small enough the oscillation will be simple harmonic. Secondly, even if the system settles in a different state upon addition of a weight, it is an assumption that removal of that weight will cause the system to return to the initial state. It should be apparent that the latter is necessary for the system to be considered reversible, as there is no plausible reason to suppose that the nett effect of a series of operations restores the original state if individual operations do not.

As far as this author is aware, the question as to whether an adiabatic system returns to the same state after the removal of the constraint has never been addressed, yet the well-known equation of state for an ideal gas contains three variables, P, V and T, only one of which, P, is determined by the external conditions in an adiabatic process. Both V and T will change upon the addition of the weight but on its removal the only requirement is that their ratio is fixed. There is no *a priori* reason, therefore, why the system should return to the same state.

This, then, is the question addressed in this paper. If, upon removal of the weight, the system returns to the original state, the system itself would appear to be reversible, even though the process by which the state is changed might be irreversible. As argued by Clausius and later by Sommerfeld, it is possible that in the limit of an infinitesimally small weight, w, the irreversibility can be disregarded. On the other hand, if the system were to settle in a state characterised by different values of V and T, but the same ratio of the two, not only would the process appear to be irreversible but the system would also appear to be intrinsically so. There is a good reason for supposing the latter. Previous work by the author on computer simulations of a hard sphere fluid showed that in fact a mechanism of damping intrinsic to the gas exists [5]. This would suggest that the work of compression is dissipated internally, making it possible that the work done on the gas is different from the work done by the gas during expansion. The assumption of intrinsic reversibility is examined in light of these considerations as well as theoretical investigation of the work done.

2. Intrinsic Reversibility

The method adopted here proceeds with an analysis of a system known to be mechanically reversible to establish the principles applicable to an adiabatic piston. Consider, then, a spring suspended vertically and loaded with a mass m so that it is extended from its equilibrium length by an amount y_1. A small mass, δm, is added and the spring is extended to a position y_2. Clearly there must be some damping, otherwise the spring would simply oscillate with the addition of the extra mass. Likewise, if the extra mass is removed the spring will return to its original position provided the motion is damped. In one respect, this is an analogue of the piston described by Sommerfeld: we could keep adding weights in the form of grains of sand to extend the spring and after some distance remove them sequentially to return the system to its original state. The process is irreversible in the sense that matter, in the form of a grain of sand, would be transported from the top to the bottom, thereby leaving the environment changed, but we have every reason to believe that the original state would be restored because the potential energy in the spring depends only on its extension from equilibrium.

Therefore, if $y_2 = y_1+\delta y$, then, from Hooke's law;

$$\Delta U = \tfrac{1}{2}ky_2^2 - \tfrac{1}{2}ky_1^2 = ky_1\delta y + \frac{1}{2}k\delta y^2 \qquad (4)$$

It doesn't matter which way this operation is performed, either from y_1 to y_2 or vice versa, the energy difference in the spring is identical. Although the process of extending the spring and returning it to its original state is clearly irreversible, the irreversibility is external to the system, as can be shown by the following argument.

Consider the addition and subsequent removal of a small weight. The weight acting on the spring can be equated with the restoring force at equilibrium, so the amount of work done on the spring during the extension is the difference between the work done by the weight and the work done overcoming friction, w_f:

$$(m+\delta m)g(y_2 - y_1) - w_f = ky_2\delta y - w_f \qquad (5)$$

Likewise, the work done by the spring after the mass is removed is

$$(m)g(y_2 - y_1) + w_f = ky_1\delta y + w_f \qquad (6)$$

There is no reason to suppose that the work expended overcoming friction should be the same in both directions, but if we assume it is we find,

$$2w_f = ky_1\delta y - ky_2\delta y = k\delta y^2 \qquad (7)$$

This gives the magnitude of the work done against friction as;

$$w_f = \frac{k\delta y^2}{2} \qquad (8)$$

Therefore, from equation (6) the work done by the spring is,

$$w_S = ky_1\delta y + \frac{k\delta y^2}{2} \qquad (9)$$

This is identical to equation (4). Likewise, substitution of equation (8) into (5) gives the work done on or by the spring as the same as the energy change between the two states. It follows, therefore, that the work done overcoming friction is the same in both directions. It is by no means obvious that in practice this should be so, but in so far as it leads directly to the work being done on or by the spring in both directions being the same and equal to the energy difference between the two states, it is both justified and central to the outcome.

The same technique can now be applied to a piston containing an ideal gas. In this case, though, removal of the weight is required for expansion. Therefore, we might imagine a system at internal pressure P_2 expanding by the removal of a small weight to a pressure P_1 and then restored to the original pressure by adding the weight. As before, the weight can be replaced by the force acting on the piston at the equilibrium position, so the total work done by the weight during compression is $P_2\delta V$. This work has to compress the gas and provide the energy to overcome friction, so the work done on the gas can be written as;

$$w_g = P_2\delta V - w_f \qquad (10)$$

On expansion, the work done on the weight is $P_1\delta V$. This work is done by the gas, which also has to provide the energy to overcome friction, Therefore, the total work done by the gas is,

$$w_g = P_1\delta V + w_f \qquad (11)$$

If the system is to return to the same state the work done by the gas has to be the same in both directions and as we have already established that there must exist a mechanism of damping that allows for an identical amount of energy to be dissipated in both directions, we can write,

$$w_f = \frac{(P_2 - P_1)\delta V}{2} = \frac{\delta P\delta V}{2} \qquad (12)$$

We can eliminate friction, as before, and

$$w_g = P_2\delta V - \frac{\delta P \delta V}{2} = \delta V(P_2 - \frac{\delta P}{2}) \quad (13)$$

It follows from either equation (10) or (11), that;

$$w_g = \frac{(P_1 + P_2)}{2}\delta V \quad (14)$$

Again, the same expression for work is obtained from either compression or expansion if the work dissipated externally is the same in both directions.

This reversible work can now be compared with the work that is actually done, which is given by the difference in internal energy between the two states:

$$\Delta U = \tfrac{3}{2}\left(P_2V_2 - P_1V_1\right) \quad (15)$$

If $P_1 = P_2 - \delta P$ and $V_1 = V_2 + \delta V$, then:

$$\Delta U = \tfrac{3}{2}\left(\delta PV_2 - P_2\delta V + \delta P\delta V\right) \quad (16)$$

The first quantity on the right, δPV_2, can be found as follows. As this is an adiabatic process, the states (P_1, V_1) and (P_2, V_2) are linked by the relationship,

$$P_2V_2^{\gamma} = P_1V_1^{\gamma} = \left(P_2 - \delta P\right)\left(V_2 + \delta V\right)^{\gamma} \quad (17)$$

The last term on the right can be re-written using the binomial expansion as;

$$\begin{aligned}\left(V_2 + \delta V\right)^{\gamma} &= \left[V_2\left(1 + \frac{\delta V}{V_2}\right)\right]^{\gamma} \\ &\approx V_2^{\gamma}\left(1 + \frac{\gamma\delta V}{V_2}\right)\end{aligned} \quad (18)$$

Substituting back into equation (17) and expanding the brackets yields;

$$\delta PV_2^{\gamma} = \gamma P_2V_2^{\gamma-1}\delta V - \gamma\delta PV_2^{\gamma-1}\delta V \quad (19)$$

Dividing through by $V_2^{\gamma-1}$ yields,

$$\delta PV_2 = \gamma P_2\delta V - \gamma\delta P\delta V \quad (20)$$

Equation (16) then becomes,

$$\Delta U = \tfrac{3}{2}\left[P_2\delta V\left(\gamma - 1\right) + \delta P\delta V\left(1 - \gamma\right)\right] \quad (21)$$

Therefore,

$$\Delta U = P_2\delta V - \delta P\delta V = P_1\delta V \quad (22)$$

At first sight, equation (22) agrees with the well-known result that the work done by an ideal gas in an infinitesimal change is *PdV*, but in fact equation (22) is asymmetric. In equation (17), P_1 is expressed in terms of P_2 and likewise with the volume, but had it been the other way around the work done would have come to:

$$\Delta U = P_1\delta V + \delta P\delta V = P_2\delta V \quad (23)$$

In the limit that $\delta V \to dV$, it is possible that P_1 and P_2 may be considered indistinguishable and the work expressed as *PdV*, but this is a mathematical abstraction. In any real system, the two states must have slightly different pressures and so some small, even infinitesimal, difference between the two expressions will always exist. Therefore, for finite, but small changes, this asymmetry is potentially a problem. It raises the immediate question of its interpretation: how can the work done be different depending on how the problem is expressed mathematically?

3. Discussion

In interpreting equations (22) and (23), it is important to note that the origin of the difference does not lie in any approximations made during the derivation other than the assumption that δV is small enough to allow the binomial expansion. The smaller δV is, the more accurate this approximation becomes. In the limit that *dV* tends to zero the two expressions for the work done could be considered to become equal, but this is a mathematical abstraction and leads right back to the objection raised earlier: if there is no differentiation between internal and external pressure there is no change in the system.

This aspect of reversibility has not only caused considerable confusion within the literature about the nature of work but has also led to the view that, even though in practice a real work process may be irreversible, we can nonetheless assume a process exists that will map out a series of equilibrium states provided the change is infinitesimally small or very slow. By way of example, Moore [6] described a system in which a weight attached to a pulley slowly draws out a piston. However, gravity is doing the work and it is not immediately clear how the gas particles lose energy. Becker [7] made a similar comment: "… we can easily see why a gas becomes colder during an adiabatic expansion. Such an expansion can be carried out by pulling out the piston … with a small velocity *w*". Again, the implication is that an external agency is doing the work. Even Fermi [8] is ambiguous on the subject: "… we can produce a reversible expansion … by … shifting the piston outward very slowly".

Becker's explanation for the loss of molecular energy is based on the well-known idea in mechanics that in an elastic collision between a light and very heavy object moving in the same direction the light particle rebounds with a velocity smaller in magnitude than its incident velocity. In other words, a molecule loses energy if it collides with a piston moving away from it and gains energy if the piston is moving towards it. However, this doesn't explain what happens to that energy. By definition, if the collision is elastic the energy of the molecule must be transferred to the piston, which must increase its speed as a result. The only way to ensure that the velocity of the piston remains small compared with the average speed of the molecules is to make the excess pressure driving the motion of the piston small, but this is not mentioned by either Moore or Becker. If the piston is constrained to move slowly by some mechanism, then it would seem likely that the excess energy lost by the molecules will go into heating the piston and during compression the energy gained by the molecules will cool the piston.

This model assumes that the collisions are elastic under these conditions, but this is a simplistic representation of what is, in reality, a complex stochastic process of adsorption and ejection. Indeed, collisions cannot in general be elastic, otherwise molecules would not be able to exchange energy with the walls of a chamber if they were at a different temperature from the gas. Becker's model of energy loss, which Sears and Salinger also use [9], is therefore flawed. It neither represents real physical processes of energy exchange with a moving piston nor accounts for what happens to the energy exchanged with the piston. Until that energy is accounted for, it is impossible to say anything about the final state of the system at the end of a work process.

Throughout the thermodynamic literature there is an assumption that a real process exists that will take a system from one state to another and back again. Sometimes this is implicit, at other times explicit. Callen [10], for example explains that during a work process the state of the system is ill-defined and he describes the system as disappearing from one point and reappearing at another. However, the two end-points are assumed to be connected by a two-way process. A succession of such processes constitutes a quasi-static process. It should be apparent, however, that these and other arguments for the existence of reversible processes are no more than supposition: there does not appear to be any experimental evidence for the existence of two-way processes linking states.

The assumption that reversible processes exist also involves an assumption about dissipation. As discussed, we know from elementary mechanics that a bound system perturbed from equilibrium will oscillate with SHM, so there must be some damping for the system to settle in a final state. However, equation (14) shows that external damping will not account for the difference between the external work done and the work done on the gas unless the internal work depends on both the initial and final states. This is not supported by the analysis leading to equations (22) and (23), which shows that for a definite, non-zero change in volume the work done depends on one or other of the initial or final pressures, depending on how the system is represented mathematically.

The fact that this asymmetry in the work done does not depend on any approximations other than the binomial expansion has already been mentioned, but it is also worth noting that it is not a consequence of including the product of the two small differences, $\delta V \delta P$. It is quite common in treatments employing finite differences to ignore terms of this nature, but this is in fact the work done by the excess pressure and is essential to the final result. Moreover, ignoring it would not remove the asymmetry; it would simply reverse the outcome, so that in equation (22) the work done would be $P_2 \delta V$ and in equation (23), $P_1 \delta V$. The asymmetry would still exist, therefore, with the two expressions for the work done being the same.

One obvious interpretation is that this expresses a direction, as these two work terms correspond to the total external work done in compression on the one hand and expansion on the other. If this is the case, not only must there be an internal mechanism of damping, but this is not a two-way process. For example, if the system starts in a state (P_1, V_1) and is compressed to (P_2, V_2) by the addition of a small weight, removal of the weight will cause the system to expand to a state (P_1, V_3), where, for a reversible, or two-way, process, V_3 should be the same as V_1. However, if the work of compression has a magnitude $P_2(V_1 - V_2)$ and the work of expansion a magnitude $P_1(V_3 - V_2)$, it is clear that V_3 cannot be the same as V_1 as the internal energy change is different in both directions. Reason suggests that $V_3 > V_1$. The work of expansion is less than the work of compression so the gas will have gained energy and its temperature increased in consequence. Hence the temperature of the gas must be higher than in the starting state and as the ratio of $T:V$ determines the pressure, the final volume must also be higher.

In this interpretation, then, equation (22) represents expansion and equation (23) compression, and the subscripts are simply labels identifying the states between which the separate transitions occur. These are not necessarily the same states in the two processes. This interpretation is supported by the recent discovery by the author of an internal mechanism of damping in computer simulations of an ideal gas within the compound piston [5]. If the particles are modelled as hard spheres that collide elastically with the piston the motion of the piston dies away because of equipartition of energy. If inter-particle collisions are switched off, which is possible in a computer simulation, the piston oscillates indefinitely. Inter-particle collisions randomise the velocities, which leads to damping.

The implications of this simple result have not been fully appreciated. External damping is a pre-requisite for internal reversibility; that is, the restoration of the initial state when the external work is not the same in both directions. If it is no longer necessary to invoke external damping, it is impossible for the initial state to be restored simply by reversing the change in pressure. In short, adiabatic work processes appear to be intrinsically irreversible.

It is worth summarising at this point the separate elements leading to this conclusion.

1. There is no a priori reason to suppose that simple adiabatic work processes are reversible because only one of the three variables that together determine the thermodynamic state is fixed by external constraints.
2. Sommerfeld's model of a reversible quasi-static process has been shown to be irreversible as matter is transported from the top of the stack to the bottom and the environment is not left unchanged.
3. For Sommerfeld's process to be internally reversible, that is, for the initial state to be restored upon removal of the small weight, the system must be damped externally and the work done on or by the gas must depend on both the initial and final states.
4. This requirement contradicts the outcome of a simple mathematical analysis that shows for a small, but non-zero change in external pressure the work done on or by the gas is equal in magnitude to the total external work.
5. This implies an internal mechanism of damping and such a mechanism has been established.
6. Internal damping means, therefore, that the system will come to rest in a final state upon a small change of constraint, but it will not return to the same state upon reversal of the constraint.

Taken together, these points suggest that adiabatic work processes, even those performed quasi-statically, are intrinsically irreversible.

This immediately raises a question as to the meaning of reversibility in thermodynamics. Mathematically, reversibility implies that the work function is integrable, as

$$\int_a^b PdV = -\int_b^a PdV \tag{24}$$

This is mathematically exact. If the consequence of imposing a small step-change in pressure is irreversible, this would imply that a sequence of such step changes cannot be integrated. If so, then, as per equations (1) and (2), the work done over a large interval will remain as a sum of a sequence of small changes.

This in turn raises a question over the interpretation of entropy. The idea that entropy is a state function is essentially mathematical: it is a consequence of combining the First Law with the Second Law to arrive at,

$$TdS = dU + PdV \tag{25}$$

As is well known, this can be integrated between any two states and the outcome is independent of the path. One consequence of this is that entropy is assumed to increase in an irreversible process. The essential argument is that it is possible to get from any state in thermodynamic phase space to any other state by a series of quasi-static process [11]. Some of them involve an exchange of heat and others adiabatic work, but all are assumed to be, by definition, reversible. Geometrically, this is equivalent to the following picture: thermos-dynamic phase space is split into a series of surfaces of constant entropy; movement along a surface is possible by a reversible adiabatic process and movement between adjacent surfaces is possible by an exchange of heat corresponding to *TdS*. An irreversible process will take the system from one isentropic surface to another, but always in a direction such that the nett flow of heat required to move back through the surfaces to the original state is outward. That is, there is a nett decrease in entropy in the system. As it would have been possible to make the irreversible transition by a series of reversible processes, the nett effect of which would have been to increase the entropy, it follows that entropy must have increased during the initial irreversible process.

The difficulty with this approach is that it focusses on the states of the system rather than processes. The states are defined by the properties of the system, such as temperature, pressure and volume, but processes are defined by changes in external constraints. Arguably, as all thermodynamic processes must involve a change in one or other of the constraints they are all irreversible. Consider, for example, a simple change of temperature at constant volume in an ideal gas. This can be brought about by placing the system in contact with a reservoir at the desired temperature. The change can be reversed simply by placing the system in contact with another reservoir at the original temperature. The quantity of heat flowing in the two directions is identical and within the system the change in entropy is well defined and given by $\frac{3}{2}Nk\ln[T_b - T_a]$. The two end states are well defined regardless of the magnitude of the change, but in the reservoirs the decrease in entropy on heating is clearly smaller than the increase in entropy on cooling, as the heat flow is occurring at different temperatures.

This irreversibility is intrinsic: no matter how small a change is envisaged, it cannot be achieved without a step change in the external constraint. Over a large change, however, the process can be made to appear reversible by dividing the process into a series of small steps. However, it is only an appearance of reversibility: the entropy change in a cooling stage will cancel with the change in entropy in a preceding heating stage, as shown, for example, in the following three-stage process. Starting from a temperature T, heating would occur through contact with reservoirs at temperatures $T+\delta T$, $T+2\delta T$ and $T+3\delta T$. Cooling would occur at $T+2\delta T$, $T+\delta T$ and T. Heating and cooling at $T+2\delta T$ and $T+\delta T$ cancel out, but this still leaves the entropy changes during the final heating stage and the final cooling stage as two that do not. Even though these would partially compensate for each other, by Landsberg's criteria there would be a nett entropy change in the environment that would render the process irreversible.

This difference between internal changes and external processes is more apparent in work processes. In isothermal work the additional constraint of a fixed temperature means that the original state will be restored on restoration of the original external pressure, but the process itself will not be reversible as the work of compression will exceed the work of expansion. If the work is divided into small steps, such as adding and removing weights, the analysis of the Sommerfeld model contained in equations (1) and (2) will apply. The work done during a change in one direction will cancel with the work done during a change in another, similar to the example of heating just considered, but whereas heating and cooling at the same temperature could be argued as being equivalent, it is clear that these work processes are not simply the reverse of the other: the magnitude of the work is the same, but the system is moving between different volumes. In consequence, the final stages do not cancel, leading to the transport of matter and excess work corresponding to the change in gravitational potential energy given by equation (3).

The mechanism of damping also plays a part. External damping is clearly a pre-requisite for internal reversibility: only if the work done on the gas in a change of constraint is equal to the work done by the gas in the reverse change will the heat flow from the reservoir into the system, and hence the nett entropy change, be the same in both directions. However, if the motion is damped internally, the internal work will match the external work and the entropy change associated with the heat flow will be different in both directions. This automatically raises a question about the association of a given value of entropy with a given state of a system.

Clearly, reversibility of the state, which applies whether the damping is external or internal, is not the same as reversibility of the process, but with adiabatic work neither the state nor the process are reversible. The work done on the gas in compression would appear to be different from the work done by the gas in expansion and heat would have to be extracted from the system at the end of a sequence of changes and their reverse in order to restore the initial state.

These considerations raise the question as to whether there is a distinction between the idea of entropy as a function of state and entropy changes in real processes. Mathematically, it is perfectly possible to define a thermodynamic state as a point in thermodynamic phase space with whatever coordinates are appropriate: internal energy, volume, particle number, etc. It is equally possible to consider transitions between the points in thermodynamic phase space as movement along a locus of points characterized by particular properties of the phase space: isotherms, adiabats, etc. By definition, movement in one direction is simply the reverse of movement in the other. Most importantly, differential calculus will describe the differences between states arbitrarily close to each other, with the consequence that equation like (25), for example, are valid. It follows that "pathways" between states can be represented as integrals. Clearly, entropy is a property of this mathematical structure and is therefore a property of state.

Whether this mathematical structure finds a correspondence in the physical world is entirely another matter. In particular, the question arises as to whether the "pathways" between states represented by the integration of exact differentials correspond to physical processes. This is the very essence of the argument that a quasi-static process is reversible. It is also the assumption that has been examined in this paper. The preceding arguments suggest that in fact all processes are irreversible and in the case of work processes there are consequences for entropy unless the damping is entirely external to the system. For isothermal work with internal damping in an ideal gas, the entropy changes within the gas differ according to the direction of the process, despite the same two states being involved. Overall, the entropy change is zero as the change in the reservoir offsets the change in the gas in either direction, but within the gas the entropy change in compression exceeds that for expansion. For adiabatic work there is, by definition, no entropy exchange with the exterior during the work processes, but there would appear to be an entropy change required to restore the initial state.

The irreversibility can be traced back to the need to change one of the external constraints by a finite, non-zero amount in order to change the state. No matter how small a change is envisaged, it is still a step change and the sum of a series of such changes corresponds to a sum of terms rather than an integral. The implication is that physical processes of this kind are not integrable and points to a real difference between the mathematical structure of thermodynamic phase space in which an integrating factor is known to exist and real systems subject to real, irreversible processes.

This distinction between the two is not trivial. If the mathematical notion of entropy as a state function has a counterpart in the physical notion of entropy as a property of body, there must exist real reversible processes that take a system from one state to another. Then, and only then, can there be any meaning to the idea that the integrating factor is anything more than a property of the mathematical construction of thermodynamic phase space and is in fact a real, physical property of thermodynamic systems. However, this would not overcome an essential difficulty with the notion of entropy that dates back to Clausius himself. When Clausius derived the function he later called the entropy of a body he did so by introducing an inequality into what was essentially the First Law [12]. This appears to have been overlooked within thermodynamics, but has the consequence that if entropy is a property of a body which is uniquely defined in a given thermodynamic state then it must increase during an irreversible adiabatic process. By definition, however, $dQ = 0$ and from the First Law $dU = dW$. The energy changes within the system are fully accounted for by the First Law, but as TdS also has the units of energy it would appear that some additional property of a body with the units of energy is increasing. This increase is inconsistent with the First Law.

This conclusion is inescapable: an irreversible adiabatic change implies a change from one isentropic level to another within thermodynamic phase space so if the entropy of thermodynamic phase space is also the entropy of a body it must increase even though the change in internal energy exactly matches the work done.

Perhaps the final word on this view should be left to Tatiana Ehrenfest-Afanassjewa, who seemed to express essentially the same view in her preface to the 1959 translation of the Ehrenfest's famous treatise on statistical mechanics [13]: "However, it became clear to me afterwards that the existence of an integrating factor has to do only with the mathematical expression of $\Delta Q = dU + dA$ in terms of the differentials … of the equilibrium parameters ... and is completely independent of the direction in time of the development of the natural processes." A few sentences later, she continued: "Nevertheless, even today many physicists are still following Clausius, and for them the second law of thermodynamics is still identical with the statement that the entropy can only increase."

4. Conclusion

The conditions for reversibility in a simple thermodynamic system undergoing a quasi-static adiabatic process of the kind envisaged by Sommerfeld have been examined. Starting from the assumption that the excess work performed when a weight moves through a small distance compared with the reverse operation when the weight is removed is dissipated externally, it has been shown that in order for the process to be intrinsically reversible the work done on or by the gas should depend on the arithmetic mean of the initial and final pressures. However, examination of the difference in energy between the two states suggests that the work done matches exactly the change in potential energy of the external load. In short, there is no excess work to overcome any external resistance. However, the system has to be damped in order to settle in the final state and it is suggested that the damping is internal. This then leads to the conclusion that the process is intrinsically irreversible because the work done during compression is greater than the work done during expansion.

The nature of irreversibility has been discussed at length and a distinction made between the mathematical structure of thermodynamics and the physical processes required to induce a change in a real thermodynamic system from one state to another. The mathematical structure is characterised by the use of exact differentials and their corresponding integrals to map out connections, or transitions, between states in thermodynamic phase space. It is argued, however, that real processes require discrete steps in one or other of the external constraints and in consequence there are no simple processes that match the differentials of the mathematical structure. In consequence, entropy is not a property of a body and there is no basis for associating the irreversible changes discussed in this paper with an increase in entropy.

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The Schrödinger Equation from the Viewpoint of the Theory of Hidden Variables

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The author of the present work is a follower of the deterministic (causal) interpretation of quantum mechanics. The deterministic interpretation agrees with quantum formalism only if "nonlocal hidden variables" are taken into consideration. In this work a critical analysis of Schrödinger's article: "Quantization as an Eigenvalue Problem" (1926) is given. It is proven from the laws of classical mechanics, that the time-independent Schrödinger equation extracts from solutions of the Hamilton-Jacobi equation only those solutions that satisfy a necessary condition of stability. It is demonstrated, on the basis of the deterministic interpretation of quantum mechanics, that an electron's spin in an atom precesses. The energy of the precessional motion is determined by the Rydberg's formula.

Keywords: Nonlocal hidden variables, Schrödinger equation, theory of stability

1. Interpretation Crisis of Quantum Mechanics

Currently, there exists an interpretation crisis in quantum mechanics that is, in essence, due to the lack of comprehension of the underlying physics hidden behind the equations. While the mathematical formalism of quantum mechanics describes many experiments well there are heated discussions among scientists about its physical interpretation. The various interpretations offer different approaches to the issues that arise, which include *the wave function collapse*, *paradoxes such as EPR*, and etc. However, at present time there is no physical interpretation of quantum formalism that would not have contradictions within it or with accepted ideas and theories.

To support our reasoning let us assess two interpretations: the Copenhagen and the causal (deterministic) interpretations. Followers of the Copenhagen interpretation insist on the point of view that physics is the science which rests solely with measurements. Thus, under this interpretation it is invalid to making any statements about the systems' properties prior to measurement. In the Copenhagen interpretation joint probability of noncommuting operators cannot be used because direct measurement experiments cannot be conducted. However, followers of the Copenhagen interpretation have difficulty explaining "the essentially quantum effects" (e.g., teleportation of polarization of the photon). In the context of these experiments the problem of interpretation of the quantum formalism is further aggravated: it is necessary to assume that, although some properties of reality exist before measurements only potentially (for example, polarization of each photon from the polarization-correlated photon pairs) however, there is a correlation between them. If one assumes that quantum objects have a priori properties corresponding to noncommuting operators, either negative probabilities, or «hidden variables» should be introduced in quantum mechanics.

The causal interpretation of quantum formalism centers about the existence of «hidden variables». Historically, the issue of incompleteness in the description of physical reality by quantum mechanics was put forward for the first time by Einstein, Podolsky, and Rosen in 1935 (*the EPR paradox*). They proposed the existence of «hidden variables», which uniquely characterize the given state of the system; thus, allowing a quantum system to be consistent with the deterministic theory. In 1964, however, John S. Bell advanced his famous inequalities. It followed from the violation thereof in quantum theory that any theory of «hidden variables» claiming to be able to describe experimental results, must be "nonlocal" only. "Nonlocality" means two possibilities: either existence of a physical field which allows for interactions to attain speeds greater than the speed of light {introduction of such a field in physics will obviously contradict the theory of relativity}, or propagation of "signals" of the changes of a particle's quantum state with an infinite speed, in essence, the possibility of a long-distance forces acting over free space.

A work of Von Neumann is often mentioned in this regards in which he proved the impossibility of any «hidden variables» in quantum mechanics. In 1935 Grete Hermann published an article in which she exposed an apparent mistake in the Von Neumann's prove. This article remained unnoticed by much of the scientific community for some time; however, the mistake was once again independently verified in 1966 by John S. Bell.

In order to "explain" the experiments with "the essentially quantum effects," while at the same time avoiding introducing long-distance forces or a field which allow speeds greater than the speed of light in quantum mechanics, physicists began to talk about non-separability of quantum mechanics (in other words about existence some type of an information "link", between remote quantum object). Don Howard wrote in his book [1] that, quantum mechanics is non-separable, local theory {separability means that spatially separated systems exist in independent states}. Locality assumes that the state of the system may only be modified through effects propagating at sublight speeds}. It is mistaken to believe that the ideas of non-separability of quantum mechanics and the existence of long-distance forces do not contradict the theory of relativity. Note that to measure the speed of light it is necessary to have a receiver and a transmitter that are not only separated in space, but are also autonomous in their behavior.

The author of the present work is a follower of the deterministic (causal) interpretation of quantum formalism that is the existence of "nonlocal hidden variables" like a physical field. Introduction of a new field in quantum mechanics allows deriving the Schrödinger equation strictly mathematically from the deterministic laws of classical mechanics. Such derivation of the Schrödinger Equation is given in this work. Since the approach presented below is similar to the approach that was used by Erwin Schrödinger himself (1926) [2] (with correction of his mistakes), we will start with the brief critical analysis of the Schrödinger's article.

2. Schrödinger's Work "Quantization as an Eigenvalue Problem"

In this article, the equation describing the quantum levels of energy of a non-relativistic hydrogen atom was presented for the first time. Schrödinger was inspired by the de Broglie's idea of "matter waves". Originally, he thought the "matter waves" to be real and envisioned a particle as an actual wave packet. In essence, Schrödinger was, as de Broglie, a supporter of the causal interpretation of quantum phenomena.

To explain the quantization procedure Schrödinger turned to classical mechanics. He started with the Hamilton-Jacobi equation (in the case when potential energy does not depend on time explicitly). This equation is valid, when a mechanical system is under action of a conservative force $\overline{F} = -gradU$. The usage of the Hamilton-Jacobi equation was not accidental: this equation allows reduction of a classical dynamics problem to a solution of a partial differential equation.

As the starting point of his study Schrödinger considered the Hamilton-Jacobi equation for the motion of an electron with mass m_e in a hydrogen atom. In this case the Hamilton-Jacobi has the form

$$\frac{1}{2m_e}\left[\left(\frac{\partial S}{\partial x}\right)^2+\left(\frac{\partial S}{\partial y}\right)^2+\left(\frac{\partial S}{\partial z}\right)^2\right]-\frac{e^2}{r}=\varepsilon \qquad (1)$$

where S is the Hamilton's principal function, and e is the charge of an electron, ε is a constant. Schrödinger next introduced a new function ψ in order to replace function S according to the following substitution

$$\psi(\overline{r}) = \exp(S/\kappa) \qquad (2)$$

After substituting Eq. (2) into Eq. (1) the latter becomes a quadratic form with respect to ψ and its derivatives. Schrödinger next searched for real, single-valued, bounded function ψ that would give extreme value to the integral of the quadratic form, taken over the entire configuration space

$$\delta\int\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2-\frac{2m_e}{\kappa^2}\left(\varepsilon+\frac{e^2}{r}\right)\psi^2\right]d\tau \qquad (3)$$

The Euler–Lagrange equation for this variational problem is equation

$$\frac{\kappa^2}{2m_e}\Delta\psi+(\varepsilon+\frac{e^2}{r})\psi=0,\ \Delta\text{-laplacian} \qquad (4)$$

where ψ satisfies an additional condition, which Schrödinger represented as

$$\int|\psi|^2\,d\tau=1. \qquad (5)$$

Setting $\kappa = \hbar$ in Eq. (4) Schrödinger then obtained, the well-known, Bohr's energy

$$\varepsilon_n = -\frac{e^4 m_e}{2 \hbar^2 n^2} \qquad n = 1, 2, 3, ... \tag{6}$$

For an arbitrary potential energy U and an arbitrary mass m of an elementary particle Eq. (4) takes the form

$$(\hbar^2 / 2m) \Delta\psi(\overline{r}) + (\varepsilon - U)\psi(\overline{r}) = 0 \tag{7}$$

which is currently known as the time-independent Schrödinger equation. Note, that Planck's constant $\hbar$ was introduced into the analysis 'artificially' to have an agreement with experiments

Eq. (7) was accepted by scientific community almost immediately. Schrödinger suggested a clear method of finding the quantum energy levels of an atom. However the derivation of the Eq. (4) from the laws of classical mechanics, presented by Schrödinger, seemed unclear and even erroneous to physicists for the following reasons.

a) *Substitution (2) assumes that* ψ *is a real function, while Eq. (4) has complex solutions.*

b) *The set of solutions of Eq. (7) with condition (5) in a general case is not a subset of solutions of the Hamilton-Jacobi equation with a potential energy U.*

c) *The physical meaning of the variational principle used in the derivation of the equation Eq. (4) was not clear.*

For the reasons listed above the Schrödinger Equation was taken as a postulate. Thus, mathematical ties with classical mechanics were broken and a new field of science for description of the microworld phenomena emerged - quantum mechanics. In the same year, 1926, M. Born proposed the probabilistic interpretation of wave function ψ.

3. The Schrödinger Equation as a Condition of Stability

Consider the variational principle (3) that was used by Schrödinger in his derivation. As early as 1929 N.G. Chetaev, a well-known expert in the theory of stability, worked in University of Göttingen where he must have become familiar with the Schrödinger's work. In 1931, when Chetaev returned to the USSR, he published an article [3] in which an attempt was made to clarify the physical meaning of the variational principle (3) that Schrödinger had used. He assumed that Schrödinger Eq. (7) under condition (5) extracted from all solutions of the Hamilton-Jacobi equation only those that satisfy the condition of stability.

In his derivations, instead of the substitution (2) Chetaev used the more correct one

$$\psi(\bar{r}) = A(\bar{r}) \exp\left(Si/\hbar\right) \tag{8}$$

However, Chetaev was unable to obtain the Schrödinger equation exactly because (like Schrödinger) he did not introduce "nonlocal hidden variables", like a physical field. Below we derive the Schrödinger equation using the Chetaev's approach, by introducing an unknown potential Φ, which is an operator dependent on a trajectory [4].

The Hamilton-Jacobi equation with a given potential energy U and additional potential energy Φ has a form

$$\frac{(\nabla S)^2}{2m} + U + \Phi = \varepsilon \tag{9}$$

where m is the mass of the particle. Eq. (9) was derived for a material particle; however, it is also valid for the motion of a center of mass of an extended object and also for the motion of center of mass with rotation about center of mass, if the energy of rotational motion is included in Φ.

Consider now the motion of a particle that it would have if small disturbing forces with potential energy W are present. Eq. (9) in this case takes the form:

$$\frac{(\nabla S)^2}{2m} + U + \Phi + W = \varepsilon \tag{10}$$

Of all the possible motions of a material system, we will consider only those for which the arbitrary constants of the complete integral of the Hamilton-Jacobi equation (integral Jacobi) have certain given values, and will call the collection of these motions a *packet*. Follow Chetaev's method, we assume that the influence of the perturbing forces on a packet at an arbitrary point is proportional to the density of trajectories A^2 at that point. For the packet consisting of stable trajectories this influence must be minimal. That is, the action of disturbing forces is relatively less for the packets for which

$$\int W \psi \psi^* d\tau \Rightarrow \min, \tag{11}$$

here ψ is determined from equation (8). While concurrently, taking the density A^2 to have the following condition

$$\int \psi \psi^* d\tau = \int A^2 d\tau = 1 \qquad (12)$$

The integrals are taken over the entire volume of configuration space. Substituting the expression for W from Eq. (10) into Eq. (11), we obtain the following variational problem

$$\delta \int F\, d\tau \equiv \delta \int \left(\frac{(\nabla S)^2}{2m} + U + \Phi - \varepsilon \right) A^2 d\tau = 0 \qquad (13)$$

Functions $S(\overline{r})$ and $A(\overline{r})$ realize extreme of the definite integral (13), if they satisfy the system of two Euler–Lagrange equations:

$$\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial S_x^{'}}\right) + \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial S_y^{'}}\right) + \frac{\partial}{\partial z}\left(\frac{\partial F}{\partial S_z^{'}}\right) - \frac{\partial F}{\partial S} = 0 \qquad (14)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial A_x^{'}}\right) + \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial A_y^{'}}\right) + \frac{\partial}{\partial z}\left(\frac{\partial F}{\partial A_z^{'}}\right) - \frac{\partial F}{\partial A} = 0 \qquad (15)$$

Assuming that operator Φ do not change under variation of S, we obtain Eq. (14) in the form

$$2(\nabla A)(\nabla S) + A\Delta S = 0 \qquad (16)$$

Assume that operator Φ depends on A and does not depends on derivatives $A_x^{'}, A_y^{'}, A_z^{'}$, then condition (15) can be represented in

$$\frac{(\nabla S)^2}{2m} + U + \Phi_o + \frac{\partial \Phi}{\partial A} - \varepsilon = 0\,. \qquad (17)$$

If on the trajectories that are solutions of the Variational problem (13) (that is satisfy the necessary condition of stability) we take

$$\frac{\partial \Phi}{\partial A} = 0 \text{ and } \Phi_0 \equiv U_Q = -\frac{\hbar^2}{2m}\frac{\Delta A}{A} \qquad (18)$$

then the system of equations (16) and (17) along with substitution (8) are equivalent to the time-independent Schrödinger equation. In order to prove this statement one should substitute expression (8) for ψ in the Schrödinger equation (7) and then separate the real and imaginary part of the equation.

Note, that conditions (18) allows the substantial nonlinear problem of calculating trajectory by means of operator Φ that itself depends on that trajectory to simply reduce to a linear Schrödinger equation.

This way, eigenvalues ε_n (6) obtained from the time-independent Schrödinger equation are the ones that extract from solutions of Eq. (9), only those solutions that satisfy the necessary condition of stability (13).

Among these solutions (even within the same packet) there can be "extra" solutions that despite satisfying the necessary condition are unstable, and, therefore, are not realized in nature. Theoretically speaking, in order to find these "extra" trajectories we need to clarify the form of the potential Φ. This mysterious potential Φ, which is responsible for stabilization of the election's motion along the orbits, is a mathematical representation of some physical properties of «hidden variables».

What can be said about the unknown potential Φ on the basis of the proposed above derivation of the time-independent Schrödinger equation from the laws of classical mechanics?

1) Potential Φ and all its derivatives are continuous functions in the vicinity of the trajectories satisfying the necessary conditions of stability.

2) The potential Φ depends on the form of the trajectory. It can be visualized on the following simple example: a body is moving uniformly an ideal incompressible fluid. In a stationary case the pressure field in fluid depends on the trajectory of the body. Note that nonlocality is present in this problem in the following way: the pressure in an ideal incompressible fluid is transmitted simultaneously over entire volume, that is, there is «*an instantaneous connection between distant particles*». However, no one talks about existing an information "link", between remote parts of incompressible fluid. Nonlocality here is merely a part of the model. In real fluids there exists a pressure wave – a "precursor" which propagates with speeds much greater than that of the moving body.

3) It can be proved that A is constant on the trajectories that satisfy the necessary condition of stability. Taking this into consideration the first of the conditions (18) can be interpreted as follows: the decomposition into series of the potential Φ in the direction perpendicular to trajectory has the form

$$\Phi = \Phi_0 + \frac{\partial^2 \Phi}{\partial A^2}(A - A_0)^2 + o\left\{(A - A_0)^2\right\} \qquad (19)$$

Following from (19) the force acting on the particle from an unknown field when the particle undergoes a small deflection from the given trajectory is analogues to Hooke's force. This force is the one that can stabilize the motion of a particle along the orbits corresponding to eigenvalues of energy. How can Hooke's law arise in this situation? It can be understood if we consider a model in which structures (quasi-particles) are formed in the physical vacuum associated with the particles motion in vacuum. To say anything more concrete about the form of these structures, one should has some hypothesis about the nature of the "hidden variables" first. However, such a hypothesis is an independent research topic, which goes out of the focus of the present work. We just would like to mention here that the simplest structure that forms in a fluid is a vortex.

4. Electron in the Field of an Atomic Nucleus Rydberg Formula

Now let us analyze the Schrödinger equation for the hydrogen atom {Eq. (4), $\kappa = \hbar$ } in the classical approach. Note here, that solutions of the Eq. (4) can be derived strictly mathematically without introducing any operators from quantum mechanics. The solutions can be written in the form $\psi(r,\vartheta,\varphi) = A_{nlk}(r,\vartheta)\exp(ik\varphi)$ (it can be found in textbooks). From these solutions one can obtain the velocity and the trajectory of the electron's motion, as well as the quantum potential U_Q. Indeed, the phase of the wave function (8) can be written as $S/\hbar = k\varphi$, where $k = 0, \pm1, \pm2, ..., \pm l$; $l \le n-1$. As known, ∇S in spherical coordinates has the form: $\nabla S = \frac{\partial S}{\partial r}\bar{i}_r + \frac{1}{r}\frac{\partial S}{\partial \vartheta}\bar{i}_\vartheta + \frac{1}{r\sin\vartheta}\frac{\partial S}{\partial \varphi}\bar{i}_\varphi$. The component of the velocity along the radius $\bar{i}_r$ is zero, because $\frac{\partial S}{\partial r} = 0$; the component of the velocity along $\bar{i}_\vartheta$ is also equal to zero because $\frac{\partial S}{\partial \vartheta} = 0$. The only nonzero component of the velocity is the component along $\bar{i}_\varphi$. By substituting the above expression into the formula for the velocity of the electron on the trajectories obtained from the Schrödinger equation we obtain

$$\bar{V} = \frac{1}{m_e}\nabla S = \frac{\hbar k}{m_e r \sin\vartheta}\bar{i}_\varphi \qquad (20)$$

From the formula (20) it can be seen that only two situations are possible: either the center of mass of the electron is at rest ($k = 0$) or is moving along a circular orbit lying in a plane parallel to the xy plane with the center on z axis (the above reasoning is applicable to any coordinate axis). Since the motion on these circular trajectories satisfies only the necessary condition of stability, among these trajectories there can be trajectories that are not stable.

Mathematical derivations presented below are valid for any circular orbits; however, to simplify our calculations without loss of generality we will consider the circular orbits with the center at the origin. It follows from Eq. (20) that the speed of an electron along the circular path with the centers at the origin can be represented as

$$\bar{V} = \frac{\nabla S}{m_e} = \frac{\hbar k}{m_e r}\bar{i}_\varphi \qquad (21)$$

If all of the above orbits with various radii exist in nature (that is the orbits would not only satisfy the necessary but also the sufficient conditions for stability), then from the Newton's second law it would follow directly that

$$\frac{m_e V^2}{r} = \frac{\hbar^2 k^2}{m_e r^3} = F, \qquad (22)$$

That is, there exists an unknown force F acting on the particle for any given radius; in this case the external potential U would be irrelevant. There are, however, certain orbits r_B at which force F coincides with the Coulomb force. It can be easily verified that these orbits are Bohr orbits

$$r_{B_k} = \hbar^2 k^2 / m e^2 \qquad (23)$$

Thus Bohr orbits are the solutions of the Schrödinger equation for the hydrogen atom under the classical approach. Moreover, from all of the possible solutions of the Schrödinger equation, the Bohr orbits are the ideal candidates for being stable.

5. The Precession of the Electron's Spin in an Atom

For Bohr orbits the following motion integral is valid

$$(1/2)\, m_e V_k^{\,2} - e^2 / r_{B_k} = \varepsilon_k \qquad (24)$$

However, the Schrödinger equation gives the following generalized motion integral for Bohr's orbits

$$\frac{m_e V_k^{\,2}}{2} - \frac{e^2}{r_{B_k}} + U_Q = \varepsilon_n \qquad (25)$$

where ε_n are the energies on the Bohr orbits, determined from Eq. (6).

Comparing the above two expressions, for Bohr's orbits we obtain

$$\Phi_0 \equiv U_Q = \varepsilon_n - \varepsilon_k\,. \qquad (26)$$

Thus, in Eq. (25) along with the term that characterizes the energy of the center of mass, there is another term, which determines $U_Q = \varepsilon_n - \varepsilon_k$. This brings up a question: under which physical assumptions are the motion integral (25) possible? The motion integral in form (25) is possible if the motion of an object can be represented as a superposition of two motions: the motion of its center of mass and the motion about the center of mass. The extra term U_Q in this case is the energy associated with the motion about the center of mass (for example, the precession of the electron's spin). We will discuss this in detail below.

The equations of motion of the object's center of mass and the equations of motion about the center of mass may not be independent, and because of that they cannot be analyzed separately. In such case the Hamilton's principal function, as well as generalized motion integral, depends on variables corresponding to both rotational and translational motion. However, there might be partial solutions in which generalized motion integral splits into two independent parts: the motion of center of mass and the motion about the center of mass. The motion integral (25) is an example of such case.

Suppose that the motion about the center of mass is the precessional motion of the electron's spin. We will describe spin precession as precession of a classical gyroscope, because there is a preferable direction in this problem (which is determined by the plane of an orbit and the normal to this plain). It follows from gyroscopic theory that for precessional motions of fast gyroscopes' which are limited to small precessional angles, the generalized motion integral contains energy E corresponding to the precessional motion only:

$$E = U_Q = \varepsilon_n - \varepsilon_k \qquad (27)$$

Thus potential U_Q equals to the energy of the precessional motion of the electron's spin. It can be seen from Eq. (26) that on the Bohr orbit the formula for the energy is, in fact, the Rydberg's formula.

Conclusion. The deterministic approach presented above gives us a strong mathematical base with which to suggest that the electron's spin in an atom is precessing. It also opens a new direction to the study of the nature of "hidden variables".

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From Nonlinear Quantum Physics to Eurhythmic Physics

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From the first quarter of the twentieth-century to the beginning of the new millennia the indeterminism, view promoted by Orthodox Quantum Mechanics, was accepted by the majority of the scientific community. This approach to understand Nature, is based mainly on the Cartesian linear method in which the whole is assumed to be the simple sum of the constituent parts. Now, a nonlinear quantum physics was proposed allowing the recovery of causality. Furthermore, in sequence of this work, a more general complex nonlinear physics, eurhythmic physics, was developed allowing the unification of the different domains of physics.

Keywords: Orthodox quantum mechanics, nonlinear quantum physics, eurhythmic physics, linearity, nonlinearity, principle of eurhythmy, complexity, unification of physics

1. Introduction

The Copenhagen indeterministic approach for understanding the quantum realm, proposed by Niels Bohr [1], was imposed in physics at the Solvay Conference of 1927.

As is well known, there were many thinkers, out of which we can point out de Broglie [2, 3], Einstein and Schrödinger, that always objected to this view of the word and tried to recover causality. Still, only after the important work of David Bohm [4, 5] and Jean-Pierre Vigier [5], in the early fifties of the last century the true recovering of causality took place.

Indeed, the School of Lisbon, initiated by Andrade e Silva [3], a disciple of de Broglie, was able to devise the first experiments that could answer the question about the true nature of the quantum waves. Are quantum waves real physical entities or, on the contrary, they are mere probability waves, thus devoid of any physical reality, as claimed by the orthodox view? Recent experiments, done in Germany [6], tell us that quantum waves do have, in fact, physical reality [7].

Furthermore, it was shown, not only theoretically but also experimentally, that it is possible to go much beyond the limits imposed by Heisenberg relations [8].

These works culminated with the publication of the book [9], Towards a Nonlinear Quantum Physics, in 2003. In the sequence of this effort and following de Broglie research program to its natural consequences, a proposal for true nonlinear field theory, which includes both classical and quantum realm, was published in 2015: Eurhythmic Physics or Hyperphysics the Unification of Physics [10].

Traditional physics is ontologically founded on the Cartesian linear method [11] where the whole is assumed to be equal to the sum of the constituent parts that mix without any modification and consequently the action is proportional to the reaction. The eurhythmic approach [12] to understand Nature, assumes the basic inner complexity of the physical entities. Furthermore, assumes that the whole is in general more than the simple linear composition of the constituent parts and that a small action may, under certain conditions, give rise to a huge reaction. This is a consequence of the fact that the parts that make the whole, due to the reciprocal interaction, change themselves in a greater or lesser degree. Only when this change may be neglected, at the scale of description we are interested in, the linear approach may prove to be adequate.

At the beginning of the twenty century, there occurred a great improvement in the making of experimental devices. These fantastic devices allowed the probing of Nature at very short scales, both in time and space. With the help of the interferometer, time intervals measurements of about fentosecond order became available. With spectroscopy, we had a window to probe Nature at the microphysical level. In these conditions, experimental evidence, coming from diverse experimental origins, namely in the fields of great velocities and at the micro-scale of observation, clearly show, that the traditional Cartesian method was not adequate for dealing with such discoveries.

Still the researchers of those times, full formatted in the Cartesian way of thinking, tried at all costs to interpret the new complex nonlinear phenomena into the traditional linear framework. From this gigantic effort, relativity and quantum mechanics were born.

Now, that we have the advantage of being more distant in time, so that we may have a clearer vision of what the physics of the XXth century really was – a brute-force attempt to linearize essentially nonlinear phenomena.

These facts lead us to the conclusion that it is necessary to develop a better approach to understand Nature that is, Physis. This new global complex inter-relational nonlinear process will, naturally, lead to a true unification of physics. The new unified, global physics, Eurhythmic Physics or Hyperphysics [10], will assume as basic starting point that the phenomena we want to describe, both at the quantum scale and in the domain of great velocities, are inter-relational nonlinear processes that require a basic integrated complex nonlinear approach based on the organizational principle of eurhythmy [12].

Naturally, these facts in any way belittle the great merits of the Cartesian linear method and consequently of traditional physics. Indeed, the Cartesian method was one of the greatest achievements of humankind that allowed the scientific revolution of the XVIIth century. Nevertheless, as expected, it is only a mere human construct. So, it is no surprise to verify that this usefully process of enquiring Nature has its inherent limits that at the end of the nineteenth-century have started to appear recurrently.

Up to now all, the theories with good, sound concrete applications have been essentially linear theories. Still, we know quite well, that most of the complex problems posed by everyday life are not subject to such a simplistic linear description. Everybody recognizes that in certain circumstances, a minute action may give origin to a huge reaction. The principal virtue of the linear approach lies mainly in its great operational simplicity.

Now, in order to further progress in the understanding of natural phenomena, that is, of the Physis, it is necessary, from the very beginning, to adopt a nonlinear, complex inter-relational way of thinking. At the same time, we need to bear in mind, that sometimes at the level of statistical averages, when the reciprocal interaction among the participants may be neglected at the scale of description we are interested on, some intrinsically nonlinear problems may adequately be approached using the simple linear Cartesian framework.

2. Basics of Nonlinear Quantum Physics

The great French physicist Louis de Broglie [3] was the first who dared to presented a nonlinear approach for solving the apparent mystery raised by the duality wave-corpuscle. Here we present only a brief sketch of nonlinear quantum physics [9] developed by Lisbon School under his research program.

Still, since the concept of nonlinearity may pose some questions. It is convenient, from the very beginning, to clarify its real meaning. Many researches, mainly those associated with the practical applications of physics may, rightly claim that most concrete problems in order to be solved in the real practice need a nonlinear approach. Even Schrödinger equation, which is basically a linear equation, could be made nonlinear by the introduction of an adequate potential. In such conditions, where lies then the necessity of this said nonlinear quantum physics? Why not stay with the traditional linear quantum mechanics and made the necessary ad hoc adaptations to make it nonlinear whenever necessary?

This necessity of introducing ad hoc terms, whenever necessary, results from the fact that we are dealing basically with nonlinear complex problems. In such conditions, when trying to solve the real concrete problems this inherent complexity needs to be dealt with. So, this fundamental complexity is introduced largely in a disguised way, as a nuisance, as a noise, as a friction, as an imperfection before the supposed true right Laws of Nature that absolutely Rule phenomena. It is, implicitly or explicitly, assumed that the physical laws, a mere human construct, govern the natural phenomena we observe.

In our approach, the intrinsic complex nature of the quantum phenomena is assumed ad initio. Still, whenever, at the scale of description of Nature we are dealing with, the interaction among the constituent parts of the whole may be neglected, the simple linear Cartesian approach is to be applied and thus we recover traditional quantum mechanics, and consequently his dear principle of linear superposition, at least at the predicting level.

In nonlinear quantum physics, it is assumed that a quantum particle is much more than a single point-like entity. A quantum particle is a very complex entity, composed of two inter-related parts:

1 - An extended, yet finite, region, the theta wave, of relatively minute intensity.

2 - A kind of very small kernel of relatively high intensity named by acron.

Inside the theta wave field there is a kind of a very small localized and complex structure, the acron. Mathematically, the quantum particle may be expressed,

$$\phi = \phi(\theta, \xi) \tag{1}$$

or, assuming the simplest linear approach, as made by de Broglie,

$$\phi = \theta + \xi, \tag{2}$$

where ξ stands for the acron and θ for the theta wave.

In previous works, following de Broglie, this very small high energetic region of the complex particle was called, singularity or even corpuscle. Still due to the confusion with the abstract concept of mathematical singularity and from the fact that this part of the particle has an inner very complex structure it is now named by the Greek word acron [13]. This word comes from the Greek, άκρον meaning the higher pike just like acropolis, standing for the higher city.

Next drawing, Fig. 1, tries, roughly, to picture the real part of the quantum complex particle.

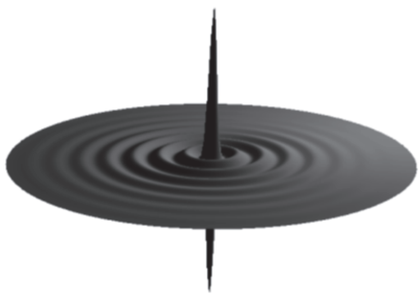

Fig. 1. Graphic sketch of a complex particle.

As stated, the energy of the theta wave is relatively very small. Indeed, a rough estimation [14] for the ratio between the energy of the photonic acron and that of its associated theta wave field gives

$$E_\xi / E_\theta \approx 10^{54} \tag{3}$$

A most important assumption of nonlinear quantum physics is the principle of eurhythmy [12]. This principle concretely states that the acron being immersed in its theta wave moves in a stochastic way preferentially to the regions where the intensity of the theta wave field is greater. This means that the probability of finding the acron is proportional the intensity of the global wave in which the acron is immersed

$$P(x) \propto |\theta|^2 dx. \tag{4}$$

This principle was introduced early in the first quarter of the XXth by Louis de Broglie [2] to describe quantum phenomena. In order to explain the single particle double slit interference de Broglie introduced this principle initially called guiding principle.

The physical reality, of the de Broglie waves known also by quantum waves, guiding waves, empty waves, theta waves, quantum vacuum states or subquantum waves, has recently been confirmed by the experiments done by German group of Menzel [6].

2.1. *Master Nonlinear Equation*

The master nonlinear equation may be derived [9] from the two basic equations of classical physics, which read:

1 - The equation of conservation of energy,

$$E = E_c + V \tag{5}$$

This equation, states that the total energy E is equal to the kinetic energy E_c plus the potential energy V.

2 - The equation of conservation of fluids,

$$\frac{\partial J}{\partial x} = -\frac{\partial \rho}{\partial t} \tag{6}$$

Writing the solution θ in the generic exponential form

$$\theta(\vec{r}, t) = a(\vec{r}, t) e^{i\frac{1}{\hbar}\varphi(\vec{r}, t)}, \tag{7}$$

and applying the traditional classical relations

$$\nabla\varphi = P;\ v = \frac{\nabla\varphi}{m};\ J = \rho v;\ \rho = a^2;\ J = a^2 \frac{\nabla\varphi}{m}; \tag{8a}$$

$$E_c = \frac{1}{2} m v^2 = \frac{1}{2m} (\nabla\varphi)^2;\ E = -\frac{\partial\varphi}{\partial t} = -\varphi_t. \tag{8b}$$

After some calculations one gets

$$\begin{cases} \frac{1}{2m} \varphi_x^2 + V = -\varphi_t \\ \frac{1}{2m} (2a_x \varphi_x + a\varphi_{xx}) = -a_t \end{cases} \tag{9}$$

in which, $a_x = \partial a / \partial x$, and successively.

By fusion of the two equations and after some mathematical manipulations is possible to arrive at the master nonlinear equation

$$-\frac{\hbar^2}{2m}\theta_{xx} + \frac{\hbar^2}{2m}\frac{|\theta|_{xx}}{|\theta|}\theta + V\theta = i\hbar\theta_t. \qquad (10)$$

It is easy to see that if $\theta_1, \theta_2, \dots, \theta_n$ are solutions to the nonlinear master equations then the general solution $\theta = \theta(\theta_1, \theta_2, \dots, \theta_n)$ is not, in general obtained, by traditional additive Cartesian rule for composition of functions

$$\theta = \theta(\theta_1, \theta_2, \dots, \theta_n) \neq \theta_1 + \theta_2 + \cdots + \theta_n.$$

Only in very special cases, the usual additive rule of composition may prove to be adequate.

The master nonlinear equation transforms formally into the habitual linear Schrödinger equation

$$-\frac{\hbar^2}{2\mu}\theta_{xx} + V'\theta = i\hbar\theta_t, \qquad (11)$$

when the nonlinear term is null or constant

$$\frac{\hbar^2}{2\mu}\frac{|\theta|_{xx}}{|\theta|} = \text{const.}$$

Once having the basic master equation, it is possible to obtain the particular solutions for each physical case. Whenever the linear approach is adequate we are in the framework of the traditional quantum mechanics.

2.2. *Beyond Fourier Ontology*

What is called Fourier ontology [15] corresponds, in reality, to a hidden additional postulate of quantum mechanics.

This extra postulate, of orthodox quantum mechanics, claims that the only waves that have a perfect, a pure single frequency, both temporal and spatial, are the physically inexistent, infinite in time and space, harmonic plane waves. All other possible finite waves, describing real physical situations, are no more than a mere linear composition of these highly idealized and abstract infinite waves.

In addition to this strong and unphysical statement the next step is to deeply connected, the infinite pattern repetition of these waves, corresponding to the temporal and spatial frequencies, with the most basic phenomenological formulas upon which the whole quantum physics is based,

$$E = \hbar\,\omega, \qquad (12)$$

$$p = \hbar\,k, \qquad (13)$$

Planck and de Broglie formulas. These expressions relate the energy and the momentum of the quantum particle with the spatial and temporal frequency of the physically inexistent infinite harmonic plane wave.

An immediate consequence of this ontology is that if a particle has one single perfect value for the energy or momentum then, the particle becomes fuzzy and somehow occupies all space and time. In such conditions, no longer it can be considered a real physical particle. In such conditions, if we wish to have real physical particles endowed with a single energy, with a single frequency, then it is absolutely necessary to reject Fourier ontology. We ought to accept that, a finite wave may have a single frequency.

Recent developments in mathematics, fortunately allow us now to make this fundamental step. Indeed, an important and innovative mathematical tool, now named by wavelet local analysis [16] was devised, in the early eighties of the last century, by Jean Morlet. In the sequence of this work, Grossmann, Meyer and many others developed local analysis by wavelets. Now, local analyses by wavelets as turned into a very powerful mathematical tool that, naturally, includes Fourier nonlocal analysis as a particular case!

Fourier nonlocal and temporal analysis uses as basic units, harmonic plane waves. In such conditions, in this ontology, any wave, any finite wave is, in last instance a simple composition of these infinite mathematical waves.

In local analysis by wavelets, on the contrary, since there are many possible finite waves, the decomposition that is, the analysis of a given function may be done in many different ways. This means that there are many kinds of finite waves which may, in principle, be used to make the decomposition or composition a given function. According to the concrete situation, it is possible select the most adequate basic wavelet. Essentially we have,

$$f(\xi) \xrightarrow{\text{Basic wavelet choice}} \theta$$

$$\theta \xrightarrow{\text{Decomposition}} g(a,b)$$

$$g(a,b) \xrightarrow{\text{Composition}} f(\xi)$$

In which the function to be analyzed is $f(\xi)$ and the coefficient function is $g(a,b)$. Here the coefficient function is a function of two variables, while in nonlocal and nontemporal Fourier analysis it is only function of one single variable $g(k)$.

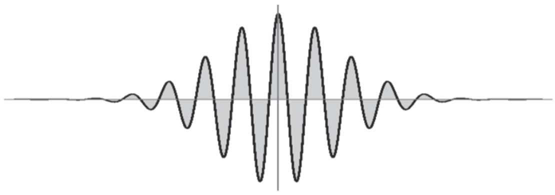

Fig. 2. Morlet wavelet's real part.

One of these wavelets, which was proposed by Morlet and since then named after him, is the Gaussian or Morlet wavelet which reads,

$$\theta(r) = e^{-\frac{r^2}{2\sigma^2}+ikr}. \quad (14)$$

The plot of the real part of this wave is shown in Fig. 2.

This wavelet is assumed to have, naturally, a single pure spatial angular frequency k, such that $\Delta k = 0$ and with $\Delta r = \sigma$ finite. In these conditions, this essentially finite wave may eventually be employed to approximately describe the real physical extended part of the complex finite particle.

In addition, a great advantage for the utilization of Morlet wavelets as basic wavelets results from the following reasons:

The first reason, is related with the simplicity and beauty criteria. Definitely, Morlet wavelets have one of the simplest forms for the analytically expressible wavelets. This formal simplicity allows us to do most of the calculations thoroughly without any need of undesirable approximations.

The second reason, comes out from the simplest practical fact. When the width, that is, the size of the basic wavelet σ starts increasing, this finite wave approaches the infinite harmonic plane wave. Therefore, in the limit, Morlet wavelet approaches the kernel of Fourier analysis the infinite harmonic plane wave,

$$\sigma \longrightarrow \infty \Rightarrow \theta(r) = e^{-\frac{r^2}{2\sigma^2}+ikr} \longrightarrow \theta(r) \cong e^{ikr}. \quad (15)$$

Furthermore, by following a process much similar to the one of Bohr, with infinite harmonic plane waves, and using instead Morlet wavelets, it is possible [12] to derive a more general set of dispersion relations.

$$\Delta r^2 = \frac{1}{\Delta k^2 + \frac{1}{\sigma_0{}^2}}. \quad (16)$$

Now, by relating this expression with the phenomenological formulas of Planck and de Broglie we able to arrive at the more general set of uncertainty relations, which read,

$$\Delta r^2 = \frac{h^2}{(\Delta p)^2 + \frac{\hbar^2\ k_0{}^2}{4\pi^2\ M^2}}. \quad (17)$$

The constant M, in the expression, links the width and the wavelength of the wavelet,

$$\sigma = M\lambda. \quad (18)$$

The more general uncertainty relations, naturally, contain formally, as a particular case, Heisenberg relations. This situation occurs whenever the width of the mother wavelet is very large $\sigma_0 \to \infty$.

So, when

$$\sigma_0 \to \infty, \qquad \Delta r^2 = \frac{1}{\Delta k^2 + \frac{1}{\sigma_0{}^2}} \to \Delta r \Delta k = 1,$$

or

$$\Delta r \Delta p = \hbar.$$

Predictions [9] for actual measurements done with optical near field super-resolution microscopes are far beyond the realm of Heisenberg relation. These measurements naturally fall in the range of description of the more general uncertainty relation derived from the complex nonlinear physics.

3. Eurhythmic Physics

Eurhythmic Physics [10] is a natural extension of nonlinear quantum physics. Furthermore, the new physics of the complex, allows us to integrate into a single and beautiful conceptual whole traditional physics: classical physics, quantum physics and relativity. At the same time, by "looking" at Nature with "other eyes" it opens an entirely new universe of experimental and technological possibilities ...

In this paper only the fundamentals of the nonlinear physics of the complex shall presented. Eurhythmic Physics was developed based on five basic assumptions. They are called mere assumptions not postulates, as is commonly expressed relative to other theories, because we are full aware they are no more than that: simple assumptions! Nevertheless, the advantage of these assumptions lies in the concrete fact that they will allow the development of a new more general and global unified physics.

First assumption:

There is an objective **Reality**. This reality is observer independent. Still, the observer reciprocally interacts with the very same reality being modified and of course modifying **It** in a greater or lesser degree.

Second assumption:

There is a basic physical natural chaotic medium named the **subquantum medium**. All physical processes do occur that is, are emergences of this natural chaotic medium. By chaotic medium, it is understood a medium in which, in general, it is not possible to make predictions. This subquantum medium is, in some way, alike to the Apeiron, the indefinite medium of Anaximander. The subquantum medium is the true real being the one which exists by itself.

Third assumption:

What are named physical entities, such as, particles, fields and so… are more or less stable organized states of the basic chaotic subquantum medium.

Fourth assumption:

In general, the complex physical entities, the particles, are very complex relatively stable organized states of the subquantum medium. They are composed of an extended, yet finite, region **the theta wave** and inside there is a kind of a relatively very small localized structure **the acron.**

Fifth assumption:

The principle of eurhythmy. This basic organizing principle, allows the making of mathematical predictions. It states that the acron moves chaotically in the theta wave field following a stochastic path that in average leads it to the regions were the intensity of the field has greater intensity.

The principle of eurhythmy [12], comes from the Greek ***euritmia***, which is the composition of the root *eu* plus *rhythmy*. With *eu* standing for the right, the good, the adequate, and *rhythmy*, for the way, the path, the harmonic motion. The composed word meaning: the adequate path, the good path, the good way, the right way, the golden path, and so on.

As may be easily understood, the principle of eurhythmy is only meaningful in the context of complex nonlinear systems. It is convenient to recall that even the fundamental acron is already a very complex organized structure of the subquantum medium. The principle tells us that the complex entity we call acron transits from one state of the theta wave field to another not in a deterministic way. In such conditions, it is not possible to predict the future state of the acron. The impossibility of predicting, even in principle, the future state of the acron results from the chaotic interaction between the acron and the surrounding theta wave field. Yet, and here is the crucial point, even if there is an inherently practical impossibility of predicting the next state of the acron, it is nevertheless possible to establish an overall statistical tendency or propensity for the acron to reach the next stage. This fact leads naturally to the concrete practical mathematical formulation of the principle of eurhythmy. Thus, in this sense, as we have previously seen, and under the recognized approximations, the principle of eurhythmy says that the acron transits from a previous state to the next state in such a way that the transition probability is proportional the intensity of the theta wave field. In such conditions the probability of finding an acron in a theta wave may be expressed as being proportional to the intensity of the field, see, expression (4).

On the other hand, it is also convenient to keep in mind that, due to the highly complex nonlinear nature of the phenomena, from one scale of observation and description of the Physics to the next, it may happen that the new emergent complex entities, even if they are a composition of parts, nonetheless their properties cannot, in general, cannot be derived from the properties of the building elements. This statement is a simple consequence of the fact that the composite parts interact and therefore modify themselves reciprocally in a greater or lesser degree. In such conditions, the whole, the resulting emergent entity, has properties of its own. The best we may aim is to predict, in certain particular given conditions, the emergence of a new physical entity and, if we are fortunate enough, some of their general broad properties.

The principle of eurhythmy is a most basic key we have for understanding Nature. Indeed, this principle has been generalized by some authors [17, 18, 19] to include other sciences like for instance biology and others. In addition, as pointed out previously, the new physics, based on the principle of eurhythmy, eurhythmic physics, contains at the predicting level classical physics, quantum physics and relativity. In this sense, eurhythmic physics promotes a true unification of physics.

From those assumptions, it is possible to develop the Eurhythmic Physics, the interested reader may see reference [10] for particulars. In this paper, we shall limit ourselves only to some few consequences of this new way of looking at Nature.

Now, we shall see that some very important concepts of traditional physics such as: force, masse and charge are indeed not basic fundamental concepts. They are simple derivable useful notions adequate only at their proper scale of application and description of Physics.

3.1. *Gravitation*

Gravitic phenomena, may easily be integrated in a natural way into the more general framework of the nonlinear physics of the complex, the Eurhythmic Physics.

It is known [20, 21] that even before the time of Newton many efforts were made with in order to explain and understand gravity and gravitic interaction.

From these early efforts we may recall Descartes, who assumed that planets were carried out in their trajectories by vortex (tourbillons) of a celestial fluid. Also, Huygens proposed a kind mechanism supported by calculation to explain gravitation. In 1690, Nicholas Fatio, a friend of Newton, proposed, to the Royal Society of London, a corpuscular theory to explain of gravity. Still, the most important contribution was given by Georges-Louis Le Sage (1724-1803).

Now, it is convenient to recall that the laws for describing **how** the gravity forces act were known largely due to the work of Newton. Nevertheless, the problem was to know the **why** of these forces. What gave origin to observed behaviour of such natural forces?

Le Sage tried to explain the why of the of gravitational forces in terms of what he called ultramundane corpuscles. In these circumstances, these ultramundane corpuscles filled all space, are going to strike the gravitic bodies. The net result of this stinking action was a pushing force. In such situations, an isolated body in space would not move, see Fig. 3,

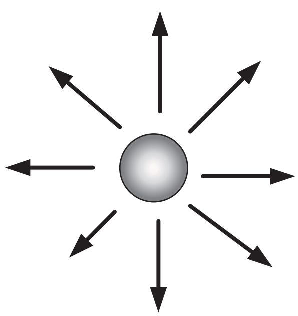

Fig. 3. An isolated body does not move due to the conjugated action of the radial pushing force.

The symmetric conjugated action of equal and opposite pushing forces of the ultramundane corpuscles lead the gravitic body to be still.

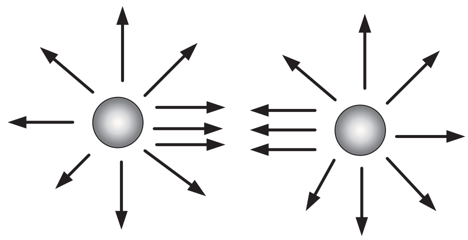

Fig. 4. Two bodies approach due to the mutual shielding action.

Nevertheless, when we have two bodies in space, Fig. 4 each one makes a kind of shield to the other. In such circumstances, they approach each other. It may be shown that this overall pushing force acts according to Newton attraction law of the inverse of the square of the distance.

The theory of Le Sage knew a certain amount of success till the late XIXth century. Then, due to the development of statistical physics, it was shown that this theory had a problem related with the principle of conservation of energy.

There were made various attempts, in the second half of the twentieth century, to recover Le Sage pushing theory using either corpuscles or waves. Nonetheless, all those late attempts had problems with the energy conservation principle. In reality, none of those gravitational theories, based on the dear classical principle of equality between action and reaction, worked well. To make things even worse, all those ad hoc theories, based on the simplistic linear Cartesian approach, missed a basic ontology for understanding physical reality.

For understanding in a clear and intuitive way the real nature of the gravitic phenomena it is essential to reject the linear Cartesian approach. It is necessary to adopt the physics of the complex, the Eurhythmic Physics. In this necessary to assume from the very beginning, nonlinearity and onto interdependency among the physical beings.

3.2. *The Concept of Mass and Charge*

In traditional physics, the concepts of mass and charge also play a major role. In fact, they have been assumed to be the most basic fundamental properties of the particles. Now in the more general framework of nonlinear physics, these concepts no longer have the same status. Indeed, it as was shown [10] they are only secondary derivable concepts more or less adequate according to the physical situation we want to describe. In order to understand this last statement, it is convenient first to analyze briefly how the complex particle moves when injected in a relatively large theta wave field of approximated constant intensity.

As a consequence of the organizing principle of eurhythmy the motion of acron is always relative to the surrounding theta wave field. So when a complex particle enters a large theta wave field, as can be seen in the sketch shown in Fig. 5

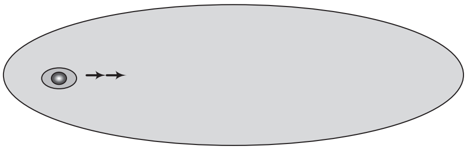

Fig. 5. The complex particle, a small theta wave with an acron enters a large theta wave field.

Two extreme cases may happen:
a) The relative intensity of the entering theta wave is much greater than the one of the extended theta wave field, Fig. 6.

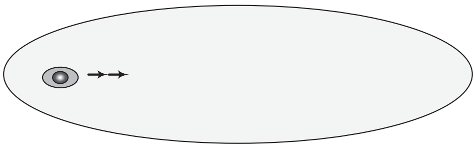

Fig. 6. The large theta wave field with relative feeble intensity.

In this situation, for all practical purposes, the acron ignores completely the extended theta wave field and sees only its initial own theta wave. Suppose now that it happens that the small theta wave field of the particle, a photon for instance, enters is a large gravitic field. In this case, this very phenomenon, of the acron ignoring the large field in which is immersed may be interpreted by saying that the particle, the photon is massless. Meaning, that in this situation, the photon is not subject to gravitic interaction. The same conclusion could be drawn if the same photon enters an electromagnetic field. Either in this case we would be lead to say that the photon is a chargeless particle in the sense that it does not respond, interacts that is, do not depend on the electromagnetic field.

b) The relative intensity of the entering theta wave is much less than that of the large theta wave field, (Fig. 7).

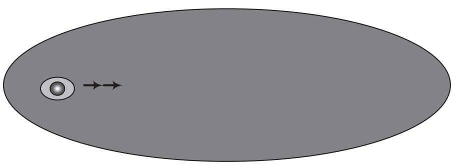

Fig. 7. The large theta wave field with relative high intensity.

Since the large theta wave field is much more intense than the one of the entering particle, the acron is full sensitive to the extended field. In such conditions, the average motion of the acron results practically form the interaction with the large extended theta wave field. If we are dealing with a gravitic field, the conclusion to draw is that the entering particle has mass. In the case of electromagnetic field the particle would be said have charge.

Nevertheless, and here is where the crucial point is: the same particle could be said to have mass when immersed in a very intense gravitic field or, on the contrary, if placed in a relatively feeble gravitic field the same particle would not be considered to have mass. So, according to the situation, the very same particle could be said to have or not have mass or charge. Since the concepts of mass or charge depend on the specific interacting situation it follows naturally, that they do not enjoy a fundamental basic status.

3.3. *The Concept of Force*

The concept of force plays a capital role in the traditional physics. Indeed, forces, be it gravitic, electromagnetic, or any other are assumed to be the most basic and fundamental interactions in Nature. Now, in the physics of the complex, the concept of force has a much less import role.

Due to the nature of the complex particle, composed of a wave plus the acron, the traditional concept of force, be it attractive or repulsive, may be understood in a very easy and intuitive way. The theta wave field of a relatively isolated particle has approximately a radial symmetry. In these circumstances, the acron immersed in this field moves in a random way according to the principle of eurhythmy. Since the probability of moving in each direction is the same the acron remains in average still.

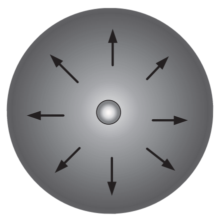

Fig. 8. Due to the radial symmetry of the field intensity the acron in average does not move.

When there happens to be two particles placed in such a way that the two theta wave fields overlap. In these conditions the resulting field intensity increases in the overlapping region and consequently the radial symmetry of the intensity previously "seen" by the acra is broken, see Fig. 9. In this situation, according to the principle of eurhythmy, the acra tend naturally to approach each other. This is what is commonly called attraction or, attractive force.

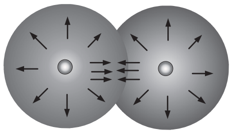

Fig. 9. The symmetry of the theta wave field intensity is broken and the acra tend to approach.

It is possible to show [10] that when not very near to the central positions the average motion of the acra is approximately described by the common attraction force law varying with the inverse square of the distance.

The repulsion, or repulsive force happens when, due to a nonlinear interacting complex process, the waves overlap in phase opposition. In this case, the global intensity instead of increasing decreases. In such conditions, the acra tend, according to the principle of eurhythmy, to draw apart from each other.

3.4. *Relativity*

In this section we shall only be concerned with the fundamentals of relativity. Once we accept that in certain particular interacting conditions between the photon and the surrounding medium, the average velocity of the photonic acron is constant no matter the velocity of the source relative to the measuring apparatus we are logically conduced to Lorentz transformations and consequently we are in the realm of relativity [22, 23, 24].

In order to better understand the situation, let us suppose, as happens in the great majority of experimental situations, the photonic acron enters a large theta wave field. This large field, may approximately be described by a uniform homogeneous referential at the scale of the experiment we are dealing with. In this situation two extreme situations may occur:

a) The relative intensity of the mother theta wave of the entering photon is much less the one of the medium in which measurements are made, Fig. 10.

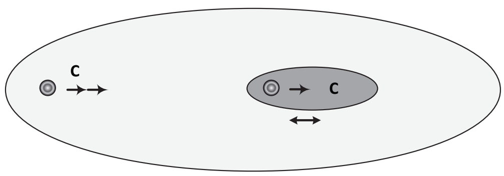

Fig. 10. The intensity of the measuring medium is much greater than entering photon theta wave.

In this case the photonic acron is only sensitive to the measuring medium and everything happens as if the photon forgets its initial velocity and travels in the medium with the maximal possible velocity, called saturation velocity. In this case, the most common, the measured velocity is constant and is always c no matter if the measuring system moves or not. The measuring system behaves like a real physical independent device consequently the average velocity of the photonic acron stabilizes relative to it

b) Now, let us consider the opposite extreme case. The relative intensity of the mother theta wave of the entering photon is much greater than the one of the medium in which measurements are made, Fig. 11.

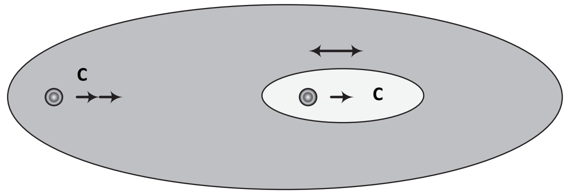

Fig. 11. The intensity of the measuring medium is much smaller than entering photon theta wave.

In this situation, the photonic acron ignores completely the field, the measuring medium in which is immersed, and consequently the habitual Galilean law of addition of velocity is quite adequate to describe the situation. In such a case we may have $v > c$. Indeed, results obtained in experiments done with the Sagnac interferometer just correspond to this extreme case.

Since the realization of this experiment, which has been done with photons [25], electrons [26] and neutrons [27], many trials have been made to interpret the observed results see, for instance, Selleri [28]. Indeed, Sagnac utilized the habitual linear additive rule and with that he was able to correctly predict the observed results. Still, since his prediction lead to velocities greater than c and consequently are against relativity which claims that the maximal possible velocity is c this raised a large amount of arguing. In fact, many authors tried to explain the results of the experiment in the framework of relativity which assumes that the maximal possible velocity is c. As can be seen in the literature[7], there are almost as many explanations as the authors that have tried to explain the results in the framework of relativity. In some cases, the same author [29] presents even more than one possible explanation. The complexity of the problem stems mainly from the fact that the experiment is done in a rotating platform. In such case, there may occur a possible accelerating effect leading the explanation of the experiment to fall in the framework of general relativity.

This controversy, whether Sagnac experiment is against or in accordance with relativity, was settled recently by R. Wang et al. [30] with a very interesting experimental setup they called linear Sagnac interferometer. In this case the platform is still, what moves is a single mode optical fiber coil, Fig. 12.

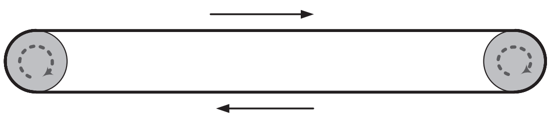

Fig. 12. Linear Sagnac interferometer.

They did the experiment with a 50 meter length linear interferometer with wheels of 30 cm. The observed relative phase shift difference for the two beams of light following in opposite directions along the optical fiber was indeed dependent only on the length of the interferometer and consequently independent of the angular velocity of the wheels.

From the experimental results obtained with the linear Sagnac interferometer one is lead to conclude that in this particular case the linear additive rule applies consequently we may have velocities greater than *c*, which clearly shows that relativity is not adequate to describe this specific physical process.

As a final note, I would like to stress that these observed facts in any way deny the usefulness of relativity. Relativity is a good approach to describe reality at its proper scale of applicability. What is quite wrong is to claim that relativity is the last, the complete and final theory ever devised by mankind.

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A Particle Model Explaining Mass and Relativity in a Physical Way

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Physicists' understanding of relativity and the way it is handled is up to present days dominated by the interpretation of Albert Einstein, who related relativity to specific properties of space and time. The principal alternative to Einstein's interpretation is based on a concept proposed by Hendrik A. Lorentz, which uses knowledge of classical physics alone to explain relativistic phenomena. In this paper, we will show that on the one hand the Lorentz-based interpretation provides a simpler mathematical way of arriving at the known results for both Special and General Relativity. On the other hand, it is able to solve problems which have remained open to this day. Furthermore, a particle model will be presented, based on Lorentzian relativity and the quantum mechanical concept of Louis de Broglie, which explains the origin of mass without the use of the Higgs mechanism. It is based on the finiteness of the speed of light and provides classical results for particle properties which are currently only accessible through quantum mechanics. NOTE: This paper is a corrected update of the proceedings of the Vigier meeting in Baltimore 2014.

Keywords: Relativity, Lorentzian relativity, Gravity, Gravitation, Particle mass, Dark matter, Dark energy

1. Introduction

The current state of physics is characterized by many open problems:

- Dark Matter
- Dark Energy
- Cosmological Inflation
- The Weakness of Gravity
- The Fact that Gravity Only Attracts
- Quantum Gravity
- Origin of Mass
- Supersymmetric Particles
- Leptoquarks.

We will here restrict ourselves to the following (open) problems:

- The origin of mass – as the Higgs theory is not able to explain mass quantitatively
- The complexity of relativity; relativity will be based on known physical facts, not on specially introduced new principles. The Lorentzian interpretation of relativity followed here is not only much simpler to handle with similar results, but is also able to solve open questions like dark matter and dark energy
- The extreme mathematical effort necessary for the quantum mechanical mass model of Higgs and for the operation of Special and General Relativity has so an easy alternative

In our view, these problems constitute a general crisis in present-day physics, as they seem to be interrelated; they share a common cause. This common cause is, in our opinion, an incorrect paradigm in the sense of Thomas Kuhn [13]. This current paradigm is composed of two parts:

- Elementary particles are point-like – and have no internal structure
- Relativity is caused by the properties of space-time.

We believe that these assumptions are incorrect. The reality is as follows:

- Elementary particles are extended and they do have an internal structure
- Relativity is caused by the properties of fields and by the internal structure of elementary particles.

In the following, we will demonstrate the consequences of this altered paradigm for the following areas:

- Special relativity
- Particle physics, particularly the origin of mass, and
- General relativity, i.e. gravity.

2. A Brief Look at the History of Relativity

The basic experiment, which triggered all the discussions about relativity, was the Michelson-Morley experiment. Several explanations for the observed null result were offered by physicists at the time. We shall only describe two of them here, which continue to be relevant to this day.

An early explanation was presented by the Dutch physicist Hendrik A. Lorentz [3]. He used the results of Oliver Heaviside, who found in the theory of Maxwell that a field contracts in motion. A consequence is the contraction of objects in motion. This contraction causes the null-result of the MM experiment.

However, most physicists later adopted the position put forward by Einstein that the speed of light is ontologically the same for every inertial system, not just as a result of its measurement. This position went along with the view that no ether of any kind existed. According to Einstein, the constancy of the speed of light is a result of the nature of space and time.

Einstein later extended his idea of a contraction of space to the more general idea of a curvature of space-time and thus created a mathematical model with which to describe gravitation.

3. Special Relativity

3.1. *Special Relativity, the Way of Einstein*

The goal of the MM experiment to determine the speed between an assumed ether and the Earth failed. This has misled physicists to the conclusion that an ether – at least as an absolute reference system – does not exist.

Einstein, who was a positivist during that time (but later not) did not want to have an unmeasurable ether in his theory. So he developed a theory which did not refer to some fixed reference system. To do this, he had, in order to cope correctly with the speed of light c, to solve the relation, $c + v = c$ for any speed $v \neq 0$. He was, as we know, mathematically successful by introducing the 4-dimensional space-time. (In the context of General Relativity, this 4-dimensional mathematical space has to be curved. This needs Riemannian geometry, which is a great challenge to be handled.)

3.2. *Special Relativity, the Lorentzian Way*

The Lorentzian way of relativity is based on the existence of a fixed reference system. If we accept the existence of a fixed reference system, even if not measurable, relativity can be operated using the well-known Euclidian geometry and is in this way comparatively easy to operate.

The basic phenomena of special relativity – dilation and contraction – can in fact be explained by means of classical physics. Furthermore, it can be shown that the apparent constancy of the speed of light follows from these two phenomena.

3.3. *Use of Parameters in the Lorentzian Way*

There is a fundamental difference regarding the two basic parameters space and time. Einstein has used these parameters by the direct, every-day understanding. However, we should be aware of the fact that both parameters are abstractions rather than quantities accessible by a direct measurement.

This is best visible for the notion of "space". It is physically impossible to perform measurement on space. One would have to firmly connect a measurement tool to points in space; this cannot be done. The physical equivalence to space is the extension of fields. It is a similar problem about "time". When we measure time, we count oscillations in order to refer the result to a temporal process. On the other hand, any temporal process is internally controlled by oscillations.

So, in order to go from Einstein's physics to Lorentz, we will replace the extension of space by the extension of fields and any consideration about time by the observation of oscillations.

3.3.1. CONTRACTION OF EXTENDED OBJECTS

Fields in motion contract. Historically this was discovered by O. Heaviside in 1888 as a consequence of Maxwell's theory of electromagnetism. The contraction is expressed mathematically by the relationship: $d \rightarrow d' = d/\gamma$ where $\gamma = 1/\sqrt{1-v^2/c^2}$, later called the Lorentz factor.

We then have to take into account that electric fields determine the size and the shape of macroscopic objects. Those objects are made up of atoms and molecules, which are bound together by electric multi-pole forces. Since fields contract when in motion, all extended objects in our world also contract when in motion. The Michelson-Morley apparatus therefore contracts when the laboratory moves with respect to some frame of reference, fully explaining the null-result. These conclusions were originally put forward by FitzGerald and Lorentz. – Later it was shown that every type of field contracts when in motion.

3.3.2. DILATION

The dilation of all time-related processes in physics is the consequence of the internal motion within elementary particles.

3.3.2.1. The Indications for Internal Oscillations

As early as 1909, J. Ziegler mentioned that "in the smallest objects", i.e. in elementary particles, a permanent motion occurs at the speed of light *c*. When Louis de Broglie detected the wave-particle phenomenon in 1923 [4], it became obvious that there was an oscillation associated with every particle.

This detection initiated the quantum mechanical description of a particle by means of a wave function. Paul Dirac developed a relativistic wave function of the electron in 1928 [5]. When Erwin Schrödinger analysed this wave function, he concluded in his famous paper of 1930 [6] that a constant motion occurs in the electron at the speed of light, *c*, which he gave the German name '*zitterbewegung*'.

In the context of the particle model presented here, it is assumed that not only the electron, but all leptons as well as all quarks exhibit this internal motion at *c*; this is simply an extension of what we know about the electron.

This assumption not only explains time dilation but also some further properties of those particles, such as their magnetic moment and spin. Despite the contrary conviction held by conventional quantum mechanics, these phenomena can in fact be explained by classical means.

3.3.2.2. The Internal Oscillation

In anticipation of chapter 4, we will assume here that the oscillation takes on the form of two sub-particles performing circular motion. This motion takes place exclusively at the speed of light *c*, in accordance with the particle model that will be presented.

3.3.2.2.1. Dilation Caused by Circular Motion

Any periodic motion occurring at the speed of light, *c*, must necessarily cause an extension of the period and a corresponding reduction in the frequency when the entire configuration undergoes linear motion. This will be shown here for the simple case of circular motion.

Let us take the simplest case. The elementary particle shall move in an axial direction at a velocity v. This causes the circular motion to become a helical motion. The extension of the period of oscillation follows from the Pythagorean Theorem:

As the speed along the helix must be *c* and the forward motion (with respect to an observer at rest) is v, the speed along the projected circuit *q* is given by $q^2 = c^2 - v^2$.

If the radius of the orbit is *R*, then the period *T* of the configuration at rest is $T = 2\pi \cdot R/c$.

When in motion, the period *T'* of one revolution is given by $T' = 2\pi \cdot R/q$.

Combining these two equations, we arrive at the following expression for the two periods: $T'/T = c/q = 1/\sqrt{1-v^2/c^2} = \gamma$ which is the Lorentz factor.

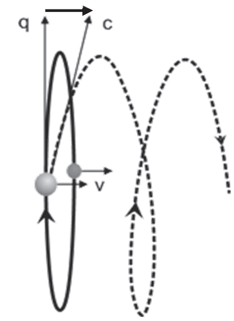

Figure 3.1. Dilation in an elementary particle

This extension of the period of rotation means that a moving clock, the speed of which is related to this period, will run more slowly and will so display a shorter time interval. The time indicated by the moving clock is conventionally called the "proper time" and denoted by the symbol τ. Using this we arrive at the conventional form of the temporal part of the Lorentz Transformation for an object moving at a velocity, v:

$$\tau = t \cdot \sqrt{1 - v^2/c^2} \, . \quad (3.1)$$

3.3.2.2.2. Dilation in the General Case

In the more general case, the motion of the elementary particle will not be in the direction of the axis but in some other direction. In this case, the result is the same; however the calculation is more complex and will not be presented here.

3.4. *Constancy of c*

Whereas Einstein treats it as an ontological fact that the value of *c* is a true constant in every inertial system, in the physics-based relativity presented here, only the *measured* value of *c* is a constant. This is caused by the contraction of the gauges and by the retardation of clocks, which also causes a de-synchronization of clocks in different positions.

4. The Basic Particle Model

The particle model presented in this paper is inevitable for the understanding of physical phenomena in two ways. On the one hand, it is a consequence of the facts of relativity. Particularly General Relativity can be understood very easily on the basis of this particle model, whereas this is extremely complex by the method chosen by Einstein. On the other hand, it explains the cause of mass and of other properties of particles without the use of quantum mechanics.

4.1. *Structure of an Elementary Particle*

An elementary particle is composed of two sub-particles, which are called Basic Particles in the scope of this concept. The main arguments that are necessary in order to understand the structure of elementary particles in this way are the following:

From the quantum mechanical analyses of Dirac and Schrödinger it follows that a permanent motion at *c* occurs within the electron. Its frequency represents the energy state of the particle. From the dilation it follows that this internal oscillation at *c* applies to all particles, since dilation is a general phenomenon and not restricted to a specific type of particle.

In addition, we have to take into account the fact that there are further restrictions on the possible structure of an elementary particle:

- An oscillation is only possible if there are at least two sub-particles in the elementary particle. Otherwise the oscillation would violate the law of the conservation of momentum
- If those sub-particles are constantly moving at the speed of light, c, then they cannot have any rest mass. It is a general fact of relativity, independent of the specific interpretation of relativity, that only a massless object can travel at a velocity of c.

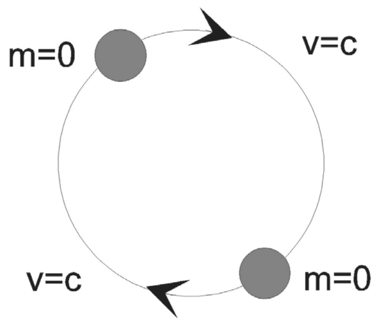

Figure 4.1. Structure of an elementary particle

In the following, this particle model will be called the '**Basic Particle Model**'.

These assumptions define most of our general particle structure. In addition, they raise a new question. If the sub-particles of an elementary particle have no mass, whereas on the other hand the particle as a whole does have a mass, what is the origin of that mass?

4.2. *The Mass of an Elementary Particle*

The inertia observed in physics is a direct consequence of the fact that the speed of light *c* is finite. As explained above, according to this model the elementary particle is made up of two basic particles. The bond between these basic particles has to be of such a nature that both particles remain a certain distance apart; otherwise the elementary particle would have no extension. It must, however, have an extension in order to have a spin and a magnetic moment.

4.2.1. THE BOND WITHIN AN ELEMENTARY PARTICLE

The bond between the two basic particles can only be a multi-pole bond, in that a multi-pole field has a potential minimum, which defines the equilibrium distance between the two particles and hence the extension of the particle as a whole. A planetary model is not applicable as the basic particles have no mass.

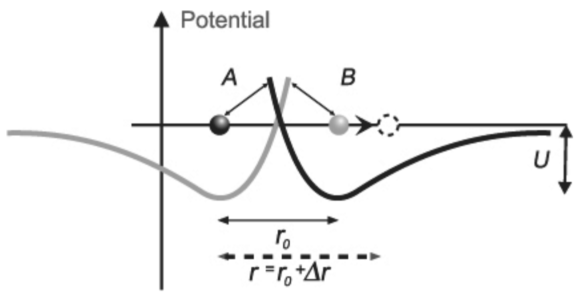

Figure 4.2. The Binding Field

The potential of the multi-pole field is assumed to be shaped such as to produce a force, *F*, given by:

$$F = S \cdot \frac{\Delta r}{r^3} \tag{4.1}$$

where *S* is the field constant of the binding field, Δr the offset from the equilibrium position, and *r* the distance between the sub-particles.

The following consideration suggests that this binding force is in fact the strong force. The multi-pole configuration is achieved by an appropriate arrangement of monopole charges of different signs. The bond produced by this arrangement must be strong enough to compensate for an additional - repulsive - electric charge in the case of a charged elementary particle such as an electron. For this reason, no force other than the strong force can produce a stable bond.

4.2.2. THE BEHAVIOUR IN MOTION

The binding field between the basic particles has this specific shape in order to create a bond which keeps a constant distance between the two. If a particle is now set in motion, the field follows the changing position with a certain delay which is caused by the finiteness of the speed of light. As a consequence, the other basic particle remains in its current position for a short time. And as a further consequence, the field due to this other particle will not change at all for a short time. The displacement of the basic particle in question therefore requires a force to be applied for a short time.

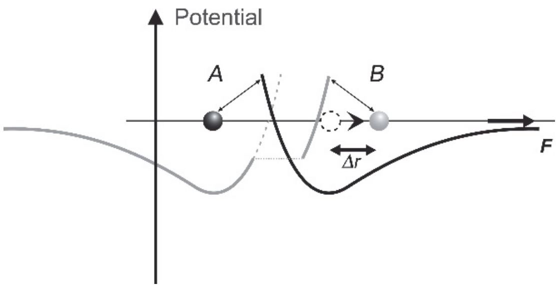

Figure 4.3. Binding field in motion

After a period of time given by the distance between the two basic particles, the change in the field due to the displacement of particle B arrives at the other particle, A, which is then repositioned. After a further period of time, the field due to the repositioned particle A will reach particle B, and no force will be necessary any longer. This fact, that a change in the state of motion requires an intermediate force, accounts for the physical phenomenon of inertia.

Note: This displacement of particle B is presented here as a *step-like* motion. This is done to make the process and the spatial field change easier to visualise. The reality is of course different. The motion of B is a smooth process. In the course of this smooth change in position the field changes continuously, but with the delay shown here.

4.2.3. THE FORCE IN THE CASE OF CONSTANT ACCELERATION

For the quantitative determination of the inertial force, we shall assume that one of the basic particles, B, is accelerated by an external agent. This causes the position of this basic particle to be displaced relative to the other one, A. The quantity of the initial displacement results from the time which the change in the field caused by particle B needs to propagate to particle A at a speed of c: $\Delta t_1 = r/c$.

With a constant acceleration of a, the displacement occurring during this time is given by: $\Delta r = \frac{1}{2}\cdot a \cdot \Delta t^2 = \frac{1}{2}\cdot a \cdot \left(2\cdot\frac{r}{c}\right)^2$ for $\Delta t = 2\cdot\Delta t_1 = 2\cdot r/c$ before particle A can react and the reaction of A propagates back to particle B. This displacement requires a force, given by eq. (4.1). $F = 2\cdot S\cdot\frac{1}{r}\cdot a\cdot\frac{1}{c^2}$. Correspondingly the inertial mass is given by

$$m = \frac{F}{a} = 2\cdot S\cdot\frac{1}{r}\cdot\frac{1}{c^2},\quad m = S\cdot\frac{1}{R}\cdot\frac{1}{c^2}, \tag{4.2}$$

where the diameter r of the particle is replaced by the radius $R = r/2$.

The unknown parameter S can be determined using the magnetic moment of the electron. First, we recall the classical equation for the magnetic moment μ of a particle. $\mu = i\cdot\pi\cdot R^2$.

The loop current i within a particle with an elementary charge e_0 at frequency ν is simply: $i = \nu\cdot e_0$ with $\nu = c/2\pi R$. So, it follows that:

$$\mu = c\cdot e_0\cdot R/2\,. \tag{4.3}$$

If we now combine eqs. (4.2) and (4.3) by eliminating R we get $\mu = \frac{1}{2}\cdot\frac{S}{c}\cdot\frac{e_0}{m}$. This is in fact the Bohr magneton which is in standard physics given as $\mu = \frac{1}{2}\cdot\hbar\cdot\frac{e_0}{m}$. This demonstrates that $\hbar$ replaces the middle term, so we have

$$S = \hbar\cdot c\,. \tag{4.4}$$

Now inserting eq. (4.4) into eq. (4.2) we end up with the formula

$$\boxed{m = \frac{\hbar}{R\cdot c}} \tag{4.5}$$

for the mass of an elementary particle made up of two basic particles.

This is now the inertial mass of an object deduced from the delay with which field forces between charges are propagated.

This result has the following remarkable properties:

1. It yields the fact that the quotient of force and acceleration is a constant at non-relativistic velocities. Therefore, this is a deduction of Newton's law of motion. For Newton, this law had the property of an axiom (or a principle of nature).
2. The result shows that the mass of an elementary particle is inversely proportional to its size, *R*.

Please note: Physics textbooks state that the Bohr Magneton can only be derived using quantum mechanics. The preceding, however, shows that this equation can be derived classically using the Basic Particle Model.

4.3. *The Relativistic Mass*

4.3.1. THE INCREASE IN THE MASS DURING MOTION

According to eq. (4.5), the mass of a particle at rest is given by

$$m_0 = \hbar/(R \cdot c) \tag{4.6}$$

When in motion, the radius *R* shrinks by the Lorentz factor $\gamma = 1/\sqrt{1-v^2/c^2}$ So, when in motion the mass changes according to

$$m_0 \to m = m_0 \cdot \gamma \,. \tag{4.7}$$

4.3.2. THE RELATIONSHIP BETWEEN MASS AND ENERGY

From the preceding section: $m = m_0 \big/ \sqrt{1-v^2/c^2}$ or equivalently $m = m_0\sqrt{c^2/(c^2-v^2)}$ where m_0 is the rest mass of the particle. It follows that an increase in the velocity of an object will increase its mass. On the other hand, an increase in velocity means an increase in its energy. The relationship between mass and energy, which is the most famous equation attributed to Einstein, can easily be deduced by expressing the increase in mass in terms of the kinetic energy. A fairly simple calculation leads to the result:

$$\boxed{E = mc^2}. \tag{4.8}$$

4.3.3. THE EXPERIMENTAL SITUATION OF THE ELECTRON

There is an apparent conflict between the model presented here and the experiment. Present-day physics regards the electron, for instance, as a particle which is point-like and has no internal structure. This is deduced from scattering experiments. The conclusion is based on the premise that, if an electron had sub-particles, these sub-particles would have some mass. Such a conception of the electron would in fact contradict the measurements.

The Basic Particle Model, on the other hand, assumes that basic particles do not have any mass. When this assumption is made, no conflict with the experiment exists.

4.3.4. THE COMPARISON WITH THE HIGGS MODEL

The Higgs model is understood by present-day physics to explain the phenomenon of mass. However, the following problems occur with its use:

1. Higgs theory does not provide a method for independently determining the masses of known particles. For every particle, a specific parameter – the Yukawa coupling – is required, which is not provided by the Higgs theory.
2. There is a discrepancy between the necessary Higgs field in the vacuum and the vacuum field measured in the universe [9]. This discrepancy is of an order of at least 10^{58}.

This means that on the basis of present data, the Higgs theory does not provide a satisfactory answer to the question of mass. The Basic Particle Model, by contrast, does not need any additional parameters as long as the size or the magnetic moment of a particle is known and provides a precisely correct result, i.e. it has an uncertainty of $< 10^{-5}$.

5. General Relativity

The general theory of relativity (GRT) is Einstein's theory to explain gravitation. Einstein does so using his geometrical model of space-time. In contrast to Einstein, the Basic Particle Model combined with Lorentzian relativity explains gravitation on the basis of physical processes.

5.1. *Gravitation According to Einstein*

According to Einstein, the concept of space-time also provides the explanation for gravitation. Objects move along geodesics in four-dimensional space-time. In the vicinity of a mass, or according to Einstein equivalently in the vicinity of an occurrence of energy, space-time is curved, and so the geodesic is the natural path of an object.

In order to determine the motion of objects in a gravitational field, it is necessary to determine the shape of the geodesics in question. This requires the use of multi-dimensional Riemannian geometry, which is a very challenging task.

The calculations using Einstein's approach are so complex that in usual cases the more specialized calculation of Schwarzschild, the so-called Schwarzschild Solution, is used.

5.2. *Gravitation as a Physical Process*

Gravitation based on physics rather than geometry makes use of the fact that in a physical interpretation, which refers directly to the measurement, the speed of light c is not always constant but varies in the vicinity of matter. This variation of c causes in general the refraction of light-like particles. This refraction also influences the movements within an elementary particle and causes the particle to accelerate. This process explains gravitation and produces quantitatively correct results for all the phenomena treated by general relativity.

Below we will show that general relativity based on the Basic Particle Model is equivalent to Einstein's version.

5.2.1. SPEED OF LIGHT IN A GRAVITATIONAL FIELD

The speed of light varies in the gravitational field in the vicinity of an object. As a result, photons and light-like particles are refracted in this field.

The dependency of c on position for a spherically symmetric object is known to be

$$c_{gr}(r) = c_0 \cdot (1 - 2 \cdot \frac{G \cdot M}{r \cdot c_0^{\ 2}})^p \qquad (5.1)$$

where c_{gr} is the reduced speed of light in a gravitational field, c_0 is the speed of light in gravitation-free space, G is the gravitational constant and M is the mass of the object, which is traditionally said to cause the gravitational potential; r is the distance from the centre of gravity. The power p is ½ or 1 depending on the direction of motion with respect to the centre of gravity, i.e. tangential or radial respectively.

Eq. (5.1) is initially used here as an experimental result. Although this dependency is also asserted by Einstein's theory, we do not need to make any reference to Einstein. We will later explain how this dependency follows from the model presented.

This dependency was first measured by I. I. Shapiro around the year 1970 using radar ranging between Earth and Venus. The measurement was later repeated by others with increasing accuracy.

5.2.2. GRAVITATIONAL LENSING

Gravitational lensing follows from the fact that the path of a photon is deflected by a gravitational field (Fig. 5.1).

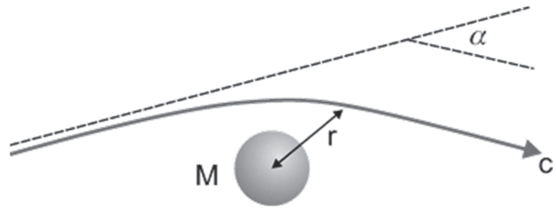

Figure 5.1. Deflection by the sun

Using eq. (5.1) and applying classical geometry and the laws of refraction to calculate the angle of deflection and the acceleration, yields the following result:

$$\alpha = 4 \cdot GM/(c^2 y) \qquad (5.2)$$

where y is the distance from the vertex of the path to the centre of the sun and α is the angle of deflection. For the acceleration, a at the vertex of the path, the result is

$$a_{vertx} = GM/r^2 \, . \qquad (5.3)$$

For details of the derivation, see [11] and [12].

After inserting the values applicable for the sun, we get the correct, known result of 1.75 arc-sec.

This number corresponds to twice the normal gravitational acceleration and conforms to observations. This numerical result, as well as the analytical result, eq. (5.2), also agrees with the predictions of Einstein's General Relativity – however without any use of Einstein's General Relativity.

5.2.3. GRAVITATIONAL ACCELERATION FOR A PARTICLE AT REST

When an elementary particle is placed in a gravitational field, its orbiting basic particles are subject to refraction as explained in section 5.2.2. This refraction causes the basic particles inside the elementary particle to deviate from their circular path. This in turn will cause the entire elementary particle to move.

Taking the case where the elementary particle is oriented such that its orbital axis points towards the source of gravity, the refraction causes the basic particles to spiral towards the source of gravity. So the entire elementary particle will move in the direction of the source. Fig. 5.2 shows the accelerated downward motion. Due to refraction, the pitch angle of the basic particles, α in this Fig. 5.2, will steadily increase. This causes the elementary particle to perform an accelerated motion towards the gravitational source. (Please note that for the sake of simplicity only the path of one of the two basic particles is shown in Fig. 5.2.)

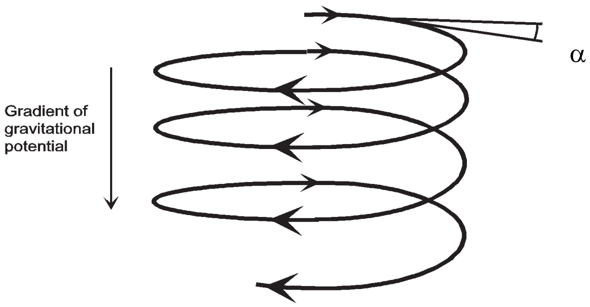

Figure 5.2. Progressive downward spiral

In this case, the acceleration of the (composite) elementary particle is similar to the acceleration given by eq. (5.3): $a = \frac{G \cdot M}{r^2}$ which is the Newtonian acceleration.

If the elementary particle has an arbitrary orientation, the process is mathematically more complicated but yields the same result.

5.2.4. THE EQUIVALENCE PRINCIPLE

Looking at Fig. 5.2, the deflection of the path of the basic particles is independent of the radius of the particle and, because of eq. (4.5), independent of the mass of the particle. This means that the independence of gravitational acceleration from mass has a very natural cause. No assumptions about any equivalence are necessary.

5.2.5. THE LORENTZIAN PATH TO GENERAL RELATIVITY

Next, we will show that the general experimental proofs of Einstein's GRT can be deduced using the concept presented in the preceding sections. We will now show that this concept is able to explain these observations in the same way as Einstein's theory. Hence it can be shown that the known proofs using Einstein's theory of relativity are also proofs of the Lorentzian interpretation of relativity.

The following list shows well known proofs of General Relativity:

- The Shapiro Effect (Reduction of *c*)
- Gravitational Lensing
- Geodetic Effect - with the Lense-Thirring Effect
- Time Dilation in a Gravitational Field
- Black Holes
- Perihelion Advance.

The Shapiro effect and gravitational lensing have been explained above. The other proofs use the Schwarzschild solution for general relativity which will be treated in the following.

5.2.6. THE SCHWARZSCHILD SOLUTION

To work with Einstein's field equations is an extremely challenging task. A short time after Einstein published his theory of general relativity, Karl Schwarzschild presented a solution for the simplified, less general solution of a spherically symmetric field, such as that of the sun, which is a frequent situation in astronomy. The experiments and observations cited in the literature as proofs of Einstein's general relativity usually refer to the results of the Schwarzschild solution.

The Schwarzschild solution is normally deduced by starting with Einstein's field equations and using Riemannian geometry, and then restricting these to the special situation. Here we will present a different deduction. We will start with the physical version of relativity (following Lorentz) and the Basic Particle Model and demonstrate how easily this solution can be deduced from these physical foundations.

According to the Basic Particle Model, an elementary particle is made up of two sub-particles orbiting each other. Their temporal behaviour, the proper time of an object in motion, is described by eq. (3.1) as: $\tau = t \cdot \left(1 - v^2/c^2\right)^{1/2}$.

This equation is now differentiated with respect to *t*, squared, and rearranged:

$$c^2 \left(d\tau/dt\right)^2 = c^2 - v^2 . \tag{5.4}$$

In a gravitational field the speed of light *c* will change and so this time behaviour as well. Understanding this change guides us directly to the Schwarzschild Solution. We first split the speed parameters *c* and v into a radial and a tangential component, since the Schwarzschild Solution is normally given in terms of polar coordinates (here still in the absence of a gravitational field):

$$c^2 \cdot (d\tau/dt)^2 = c_{rad}{}^2 + c_{\tan}{}^2 - v_{rad}{}^2 - v_{\tan}{}^2 \qquad (5.5)$$

where v_{rad} and v_{tan} are the radial und the tangential component of the velocity of the particle, c_{rad} and c_{tan} are the radial und the tangential component of the velocity of light, which is here the velocity of the basic particles. In this context they are still identical.

Now inside a gravitational field: If we consider the influence of a gravitational field, we have to take into account the fact that c changes in a gravitational field to c_{gr}, according to eq. (5.1), so:

$$c_{rad} \rightarrow c_{rad,gr} = c_{rad} \cdot k^2 \qquad (5.6)$$

$$c_{\tan} \rightarrow c_{\tan,gr} = c_{\tan} \cdot k \qquad (5.7)$$

where here, in order to simplify the equations, we have used the following abbreviation:

$$k = \left(1 - 2 \cdot \frac{G \cdot M}{r \cdot c^2}\right)^{1/2}. \qquad (5.8)$$

(It is helpful to keep in mind that there is always $k \leq 1$.)

Now we apply eqs. (5.6) and (5.7) to eq. (5.5) and get

$$c^2 (d\tau/dt)^2 = c_{rad}{}^2 k^4 + c_{\tan}{}^2 k^2 - v_{rad}{}^2 - v_{\tan}{}^2. \qquad (5.9)$$

As a consequence of the direction-dependent change in c, a field contracts in the radial direction and so also contracts the size Δr of the particle in the radial direction: $\Delta r \rightarrow \Delta r_{gr} = \Delta r \cdot k$.

So, in a gravitational field an elementary particle, even when at rest, changes its shape from a circular to an ellipsoidal one. This complicates the mathematical treatment. We solve this easily as the size of the particle and the orbital speed of the basic particle are both reduced by the same factor k.

We can formally choose a modified coordinate system so that the radial components are extended by the factor $1/k$. This extension is so that the shape of the elementary particle is again circular and the basic particles move at constant speed c irrespective of the direction.

Please note, that this is a purely geometric change in order to determine the change of the length of the orbit and so the period of the orbit of a particle when it is at motion in a gravitational field. The strength of the gravitational field at the position of the particle is not affected by this change. This transformation does not alter the calculated period of the internal oscillation and so not the temporal behaviour of the particle, which is the topic of our calculation.

For the radial component of the speed of light it follows now that

$$c_{rad,gr} \rightarrow \hat{c}_{rad,gr} = c_{rad,gr}/k = c_{rad}\, k/k^2 = c_{rad}/k \qquad (5.10)$$

and for the radial component of the particle's speed $v_{rad} \rightarrow \hat{v}_{rad} = v_{rad}/k$. The affix "^" indicates the reference to the altered coordinate system, i.e. the extension into the radial direction.

By making this replacement, i.e. referring the motion to the changed coordinate system, we can use eq. (5.7) for both directions, as the general reduction of c is now independent of the direction. The result of the temporal equation eq. (5.5), now in a gravitational field is not changed by making this replacement.

$$c^2 (d\tau/dt)^2 = \hat{c}_{rad,gr}{}^2 + c_{\tan,gr}{}^2 - \hat{v}_{rad}{}^2 - v_{\tan}{}^2$$

$$c^2 (d\tau/dt)^2 = c_{rad}{}^2 k^2 + c_{\tan}{}^2 k^2 - \hat{v}_{rad}{}^2 - v_{\tan}{}^2. \qquad (5.11)$$

Now, inserting $\hat{v}_{rad} = dr/dt \cdot 1/k = r'/k$ and $v_{\tan} = d\varphi/dt \cdot r = \varphi' \cdot r$ and using $c^2 = c_{rad}{}^2 + c_{\tan}{}^2$ into eq. (5.11) we get for the temporal behaviour of the elementary particle $c^2 \left(\frac{d\tau}{dt}\right)^2 = k^2 \cdot c^2 - k^{-2} r'^2 - \varphi'^2 \cdot r^2$.

Next, we multiply both sides by $\dot{t}^2 =: \left(\frac{dt}{d\tau}\right)^2$ and replace k back according to eq. (5.8). So we get finally:

$$\boxed{c^2 = c^2 \dot{t}^2 \left(1 - 2\frac{GM}{rc_0{}^2}\right) - \dot{r}^2 \left(1 - 2\frac{GM}{rc_0{}^2}\right)^{-1} - \dot{\varphi}^2 r^2} \qquad (5.12)$$

which is a common form of the Schwarzschild solution.

5.2.7. THE CAUSE OF GRAVITATION

We have seen that gravity is in fact *not a force but a refraction process*. And the cause of this refraction is the varying speed of light c in the vicinity of matter.

5.2.7.1. Varying Speed of Light

Eq. (5.1) is the basis for explaining all phenomena attributed to gravitation. The next question to be answered is, why c is reduced in the vicinity of matter. The answer in the scope of this model is that the reduction of c is caused by the effect of the exchange particles, which produce the binding field between the basic particles.

According to the Basic Particle Model, the binding field is the field of the strong interaction, which is – also according to the model – the universal force in our world affecting all existing particles (ref. to chapter 4).

These exchange particles, which are emitted by a multi-pole compound and cause attraction and repulsion in a random sequence, also interact with every light-like particle. They cause such a particle to be deflected towards the origin of the exchange particle (i.e. the basic particle) or away from it. So the light-like particle performs a random walk as depicted in Fig. 5.3. As a result, the average speed of the light-like particle is reduced, even though the microscopic speed is still the speed of light *c*.

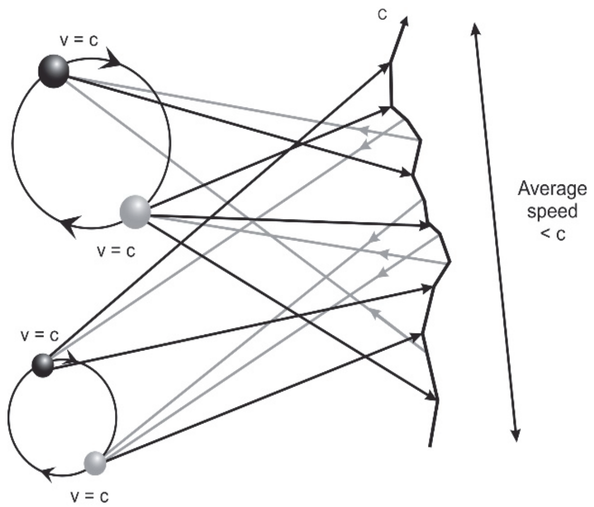

Figure 5.3. Disturbed path of a light-like particle.

5.2.7.2. Determination of the Reduction in Speed

We will now present the formula for the reduction in the speed of light, *c,* in a gravitational field. However, we will not deduce it here but refer to [11] for the details.

First of all, according to the model the flow of exchange particles and so the gravitational influence is independent of the size of the particle and according to eq. (4.5) independent of the mass. It is an astonishing fact, but from the model it follows that mass is not the cause of gravity. Every elementary particle provides the same contribution to the gravitational field.

If we define the number of elementary particles in the gravitational source as being *N*, then the resulting deflection of a light-like particle passing by depends on whether the motion is in radial or in tangential direction. The resulting reduction in the speed of light, *c,* depends on *N* in the following way: $c_{eff} = c \cdot \left(1 - gN/c^2 r\right)^p$.

The parameter *g* is the proportionality factor for the influence of the flow related to *N* particles, replacing the gravitational constant *G*, and again is p=1 for radial motion and p=1/2 for tangential motion. The dependency of the extension of multi-pole fields in a gravitational field works analogously to the contraction of fields at motion. The result for the reduced distance is $r_{red} = r \cdot \left(1 - gN/c^2 r\right)^{(p-1/2)}$, with *p* defined as above.

6. Cosmology

This chapter deals with open problems in astronomy and cosmology, including dark matter, inflation, and dark energy.

6.1. *Dark Matter*

Some decades ago, it was noticed that the rotational speed within and around big galaxies is in conflict with the equilibrium speed determined on the basis of standard gravitation. Fig. 6.1 shows this discrepancy. As a solution, present-day physics assumes a specific type of matter, which is invisible and has almost no interaction with known matter, but must have (according to standard physics) a high mass to explain the missing mass of the calculations. This missing mass has been given the name “dark matter”.

In Fig. 6.1, the solid curve labelled “disk” is the rotational speed as a function of the radius based on a normal gravitational calculation. The uppermost single values are measurements of the real speed; a curve (also solid) is fitted through these measurements. The dark solid line labelled “halo” describes the required distribution of the proposed “dark matter” in order to explain the measured values.

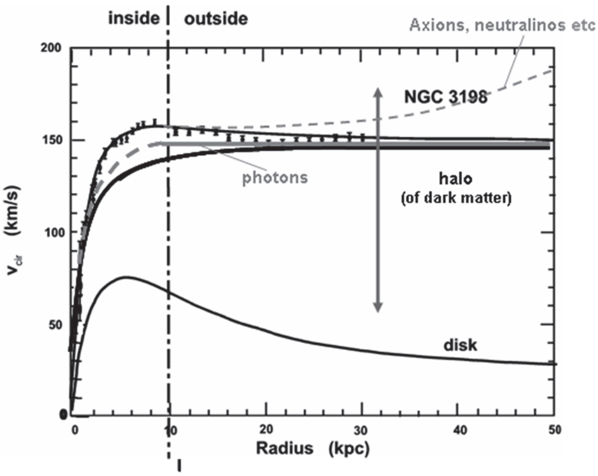

Figure 6.1. Equilibrium conflict in the galaxy NGC 3198 (The radius of the galaxy is 10 kpc)

The horizontal grey line, which is very close to the "halo" curve, follows from the assumption, described above, that every elementary particle contributes equally to the gravitational field. It represents the contribution of light particles, i.e. neutrinos and photons. In the drawing, the height of this line has been adjusted to fit this diagram, however it fits the known data within a tolerance factor of 2-3. Its curvature, however, is given by the natural distribution of the light particles and is not parameterised.

Of the light particles mentioned, the photons are mainly generated by the hot, shining stars in the centre of the galaxy. The neutrinos are similarly generated by the nuclear processes within the stars, the sources of which are also mostly in or close to the centre of the galaxy. These particles produce a continuous flux away from the centre at the speed of light *c* (or almost this speed).

6.2. *The Horizon Problem*

From the temperature distribution of the Cold Microwave Background (CMB) it is concluded that there must have been a correlation between separate regions of the universe a short time after the Big Bang, when the universe became transparent. On the other hand, those regions have already moved away from each other before the phase of transparency at such a high speed that, in the face of the limited speed of light, no causal relationship can exist. This conflict has been named the "horizon problem".

6.2.1. INFLATION ACCORDING TO (EINSTEIN)

Following Einstein's interpretation, present-day physics assumes a change in space as a solution, postulating that space was approx. 10^{60} times smaller than today and then expanded, initially very rapidly, later slowly until the present. This purported process was given the name 'inflation'.

Present-day physics does not have a proper explanation for the process of inflation. As an ad-hoc assumption, a new field produced by so-called "inflatons" is thought to cause it.

6.2.2. THE HORIZON PROBLEM EXPLAINED BY THE VARYING SPEED OF LIGHT

From a logical point of view, the problem with this correlation is the conflict between spatial extension and the speed of light. So instead of assuming a change in 'space', it can equally well be assumed that the speed of light changed, namely that it was extremely large during a short period close to the Big Bang. Afterwards the speed of light decreased rapidly at first, and later more slowly to its present value.

The assumption that the speed of light changed during the evolution of the universe is attractive anyway, since it would not only solve this causal problem. As a further benefit, it could also solve the fine-tuning of basic physical parameters, which is not understood at present. (See, for example, the work of A. Albrecht and J. Magueijo [7].) Hence, aside from other problems, it avoids the necessity of a 'landscape' of 10^{100} uni-(multi-)verses.

6.3. *Dark Energy*

The observations of type 1a supernovae have recently led to the conclusion that the objects in our universe are accelerating. The results of Riess et al. [8] are presented in Fig. 6.2.

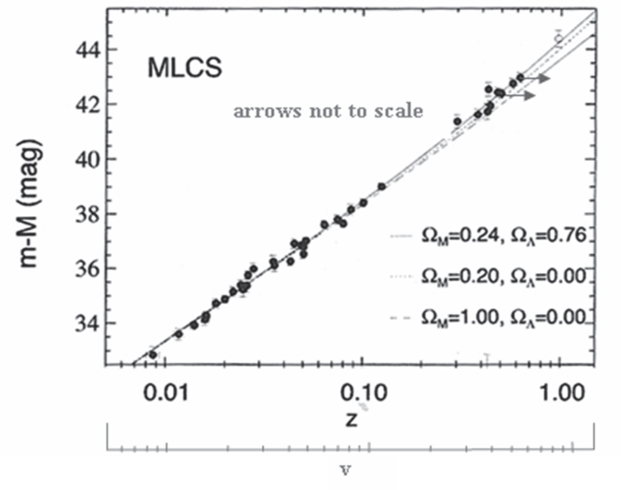

Figure 6.2. Supernova 1a, Hubble diagram – with possible correction factor

Fig. 6.2 shows the apparent magnitude of the observed supernovae (as the ordinate) versus the redshift *z* (the abscissa), which is identified with the recessional velocity of the stars. The redshift *z* is defined as

$$z = \Delta\nu/\nu_{ob} \tag{6.1}$$

where ν_{ob} is the *observed* frequency and $\Delta\nu$ is the frequency shift. From the Doppler Effect, it follows that the velocity *V* is:

$$V = c \cdot z/(z+1) \tag{6.2}$$

which is used for the evaluation according to Fig. 6.2, but conventionally with the assumption that *c* is a constant over all times and that space is unchanged during the time investigated.

According to Hubble's Law, all stars, and hence also the supernovae investigated, should be located on a straight line, represented in Fig. 6.2 by the dotted line for the most probable assumption. This means that the recessional velocity of these objects is proportional to their distance from the observer. However, the measurements in the upper part can be understood as lying too far to the left, which means that the redshift of the older supernovae is too small compared with the younger stars in the lower (left-hand) region. (Please disregard the arrows for the moment). This is commonly interpreted as meaning that the younger supernovae are too fast compared with the older ones. They are assumed to be accelerated.

However, the assumption described above, that the speed of light *c* has changed, is able to explain the acceleration as an evaluation effect. From eqs. (6.1) and (6.2), it follows for the velocity *V* in the case of a Doppler shift $\Delta\nu$, that

$$V = c \cdot \Delta\nu/(\Delta\nu + \nu_{ob}). \tag{6.3}$$

If it is assumed that the speed of light *c* was higher at an earlier time, then *c* must be replaced by a larger value and the resulting *V* will be larger.

The arrows in Fig. 6.2 show qualitatively the effect of this additional speed. This means that with reference to the speed scale the supernovae can now be positioned on the dotted line without any conflict with observation. This is indicated as an example by the arrows in the figure. In physical terms, this means that the alleged acceleration vanishes.

7. Conclusions

We have shown that relativity can be derived from physical processes, i.e. the properties of particles and fields. We believe there are compelling arguments that the speed of light is only constant with respect to an absolute frame of reference, not with reference to any arbitrary inertial system. Furthermore, we see good arguments that the speed of light has changed during the history of the universe – as assumed by other physicists. This means that the speed of light is only constant over a limited time interval.

This approach has extraordinary benefits:

- Relativity now fits seamlessly into physics as a whole. The theory becomes far more comprehensible and its formalism is much easier to understand. When accounted for in this way, relativity can even be taught at a high-school level; yet the results still conform to those derived using Einstein's approach, at least to the extent that these can be proven by experiments and observations.

- Important unresolved questions of present-day physics are resolved with surprising ease:

(1) The *dark energy problem* is resolved as a result of the changing speed of light during the evolution of the universe. The change in *c* can be deduced from a physically plausible process.

(2) The *dark matter problem* vanishes, as it follows from the model that every elementary particle contributes the same amount to the gravitational field - irrespective of its mass. In the quantitative calculations, photons are able to constitute dark matter particles. Furthermore, the spatial distribution of dark matter in the universe, which

could not otherwise be explained, fits the model.
(3) *Inertial mass* is explained, including the dynamic aspects, i.e. the relativistic increase in mass and the mass-energy equation. In present-day physics, the origin of mass is still an open issue. The Higgs theory does not constitute a working explanation.
(4) *Quantum gravity* is no longer an open issue. On the one hand, gravity is shown to be a side effect of the strong force and, on the other hand, the strong force is fully covered by quantum mechanics. Hence the alleged conflict between relativity and quantum mechanics disappears.

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Advanced Waves, Absorber Theory, Quantum Equations and Negative Mass

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Although advanced waves have never been observed in experimental devices, different theories have been proposed by several authors because advanced waves are relevant in time-symmetric theories as quantum theory with its CPT theorem. We recall and precise the definition of advanced waves, we criticize the Wheeler-Feynman absorber theory, we show that computational discrete derivative equations lead to negative mass, we discuss the problem to negative energy states and predict gravitational properties of negative mass particles.

Keywords: Advanced waves, Absorber theory, Negative mass, Space-time shifts, Discrete derivatives

1. Introduction

The topic of advanced waves has been considered of very great importance by Richard P. Feynman [1]. The most known attempt to explain why we cannot observe advanced waves in experimental devices is the so-called "absorber theory" of Wheeler-Feynman [2]. It has been considered as a fundamental and coherent theory by several authors who have extended or generalized the absorber theory in different ways: in the action-at-a-distance electrodynamics [3], in the electromagnetic time arrow [4], in the transactional interpretation of quantum correlations [5], in the quantum theory of scattering [6] and emission [7] processes, in cosmological properties of the universe [8], in new cosmological models [9], and in superluminal radiation fields [10] generated by negative mass-square in a refractive and absorptive spacetime. On the contrary some authors have stated that quantum theory requires no absorber at all as "the apparent nonlocality could be conveniently interpreted in terms of advanced waves" [11], as a correlated "*two-photons field a priori does not possess a definite time structure*" [12] and "*the notion of retarded and advanced waves is an essentially classical concept*" [13].

After this short overview and from the point of view of the several time-symmetric equations, two questions can be raised:

1. Is there always an efficient absorber in the vicinity of every field source ?

2. Why would advanced waves preferentially absorbed ?

Therefore it is clear that the well known absorber theory should be discussed. Preliminarily we will recall the definition of advanced waves from the general solution of the wave equation.

2. About the Solutions of the Wave Equation

2.1. *Solutions in a 1-Dimensional Space*

The scalar wave equation in a 1-dimensional space has been first discovered and solved by the French mathematician Jean-Baptiste le Rond d'Alembert [14, 15, 16]. A wave (or a wave packet representing a free particle) which travels along a rectilinear axis has a position x depending on the time t according the one-dimensional equation below:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = V^2 \frac{\partial^2 u(x,t)}{\partial x^2} \quad (1)$$

where V is a the constant velocity of the wave. The derivative operator can be factored as:

$$\left[\frac{\partial}{\partial t} - V\frac{\partial}{\partial x}\right]\left[\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right]u(x,t) = 0 \quad (2)$$

Besides the d'Alembert's formula which matches the boundary conditions of vibrating strings:

$$u(x,t) = \frac{f(x-Vt)+f(x+Vt)}{2} + \frac{1}{2\mathrm{V}}\int_{x-Vt}^{x+Vt} g(s)ds \quad (3)$$

the general solution appears to be a linear combination of two functions u_1 and u_2:

$$u(x,t) = u_1(x - Vt) + u_2(x + Vt) \quad (4)$$

of the new variables

$$\xi = x - Vt; \eta = x + Vt \quad (5)$$

Remarks: In the plane $\{x, t\}$ the equations $\xi = 0$; $\eta = 0$ define the two converging axis $\{\xi\}$ and $\{\eta\}$ and the generalization to a 3D space $\{x_1, x_2, x_3\}$ defines similarly the six converging axis $\{\xi_1\}$, $\{\eta_1\}$, $\{\xi_2\}$, $\{\eta_2\}$, $\{\xi_3\}$, $\{\eta_3\}$. These axis generate a hypercone. For a constant velocity V, we can introduce a time coordinate

$$x_0 = Vt \quad (6)$$

and define a hyperbolic metric of signature (+ + + –), then all vectors on any axis defined above are null vectors.

In the case of light in vacuum we have $V = c$ and this hypercone is the light-cone.

For a higher degree of generality we include a refractive medium of index $n = c/V$ and we keep the velocity V in our equations.

2.2. *The Arrow of Time in Classical Kinematics*

From the point of view of the classical kinematics we have to consider the time arrow, and the axis $\{t\}$ is oriented in the increasing direction $dt > 0$. So we can see that the waves u_1 and u_2 propagate on the axis $\{x\}$ in opposite directions. The orientation of the space axis $\{x\}$ is arbitrary, and if we choose the opposite direction with:

$$x' = -x \quad (7)$$

the general solution (4) shall be written:

$$u(x,t) = u_1(-x' - Vt) + u_2(-x' + Vt) \quad (8)$$

To simplify our explanation let's consider the emission of the two waves from the initial position:

$$t = 0 : x' = x = 0 \quad (9)$$

thus the position x or x' at time t of the two waves are:

$$\begin{aligned} &\text{For} u_1 : x = +Vt; x' = -Vt \\ &\text{For} u_2 : x = -Vt; x' = +Vt \end{aligned} \quad (10)$$

So we see that both waves u_1 and u_2 are retarded waves which propagate in opposite directions.

2.3. *Solutions in a 3-Dimensional Space*

The wave equation in a 3-dimensional space has been first discovered by the Swiss mathematician Leonhard Euler [17]. This differential wave equation is well known as:

$$\left(\nabla^2 - \frac{1}{V^2}\frac{\partial^2}{\partial t^2}\right)u(r,t) = 0 \quad (11)$$

There are several types of solutions and different expressions of each solution for plane waves, spherical waves, periodic and non periodic waves (including wave pulses), monochromatic or not, expressed with Cartesian, cylindrincal or spherical space-time coordinates. They can be expressed as a sum of eigenmodes with the angular momentum ω and the wave number k, or as a sum of spherical Hankel functions.

In our purpose we study the propagation of waves independently of their wave form, so we consider isotropic spherical waves in spherical coordinates. We introduce the quantity $r\,u(r,t)$ to eliminate Euler angles, the previous equation (11) can be written as:

$$\frac{\partial^2[ru(r,t)]}{\partial t^2} - V^2\frac{\partial^2[ru(r,t)]}{\partial r^2} = 0 \quad (12)$$

and resolved into:

$$u(r,t) = \frac{1}{r}u_1(r - Vt) + \frac{1}{r}u_2(r + Vt) \qquad (13)$$

The scalar equation (13) is similar to equation (4) and thus it shows also that both waves u_1 and u_2 are retarded waves which propagate in opposite directions, but with a difference: r is the radial distance of the wave from the point O defined by $r = 0$. The radius r at time t of the two spherical waves are:

$$\begin{array}{l} \text{For} u_1: r = +Vt \\ \text{For} u_2: r = -Vt \end{array} \qquad (14)$$

For the wave u_1 we have $dt > 0 \Rightarrow dr > 0$. Starting from the source point O the wave u_1 is a diverging retarded spherical wave.

For the wave u_2 we have $dt > 0 \Rightarrow dr < 0$. Starting at time $t = 0$ from a radius $\rho > 0$ the wave u_2 is a retarded spherical wave converging to the point O and reaching O after the time $\tau = \rho/V$. But if there is no absorber at the point O, the spherical wave will continue to be propagating from O with an increasing radius r, i.e. as a wave of type u_1.

So we can conclude that waves u_1 and u_2 are both retarded waves which propagate as described below:

- The wave u_1 is a diverging retarded spherical wave,
- The wave u2 is a retarded spherical wave converging to the point O and then it becomes a wave u1 diverging from O.

Converging spherical waves are very highly improbable in nature, but they can occur in a specially built experiment: e.g. a metallic sphere can reflect HF electromagnetic waves towards the centre of the sphere where the emitter is located, a spherical mirror can reflect light to the centered source, a photon can be trapped in a spherical cavity.

Such macroscopic experiments can show electromagnetic waves are retarded when they are emitted and also after they are being reflected.

However a confusion between advanced waves and converging spherical waves has been made by some authors as Jonathan D.H. Smith [18]. He stated that there are two basic types “expanding or retarded waves” (as u_1), and “contracting or advanced waves” (as u_2) and he concluded that advanced waves cannot exist as the second law of thermodynamics excludes coherent waves converging to a symmetry center.

We will see further that the concept of advanced waves has to be related to time inversion.

2.4. *The Point of View of Special Relativity*

In the framework of the special Relativity the light-cone is oriented from past to future with the same requirement of the time arrow and the above demonstration holds. According to the Zeeman theorem [19] the time order must respect the (macro-)causality principle held up by A. Einstein [20]. So only retarded waves are allowed by this principle although the most general Lorentz group includes antichronous transformations.

Advanced waves are excluded by principle, although antichronous transformations exists.

3. Time Inversion in the Framework of Quantum Theory

Time symmetric quantum equations do not allow us to define a priviledged direction of time as the usual time arrow, as the transformation ($+t \rightarrow -t$) called time inversion is allowed in quantum theory.

Moreover the reinterpretation principle of Stückelberg [21] and Feyman [22] allows us to consider the positron e^+ as a negative electron e^- travelling backwards in time: it is the prototype of an advanced wave. The time inversion, the inversion of the electric charge and the change of parity belong to the CPT transformation group.

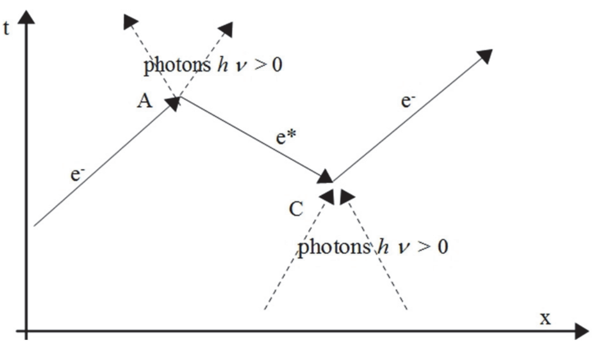

Figure 1.

Figure 1 represents:

C: entry of 2 photons, creation of a pair of electron (e^-) and a positron,

A: annihilation of the positron with an other electron (e^-), output of two photons,

e*: the positron (going from C to A) is shown as a negative electron (e^-) going backwards in time from A to C,

photons with positive frequency ν and positive energy $h\nu$.

With the time inversion the 1-dimensional equation (4) can be understood as representing two waves propagating in the same space direction of the oriented axis $\{x\}$ but with no priviledged time arrow, as:

$$u(x,t) = u_1(x - V(+t)) + u_2(x - V(-t)) \quad (15)$$

so u_1 is a retarded wave and u_2 is an advanced wave, both traveling in the same space direction $\{x\}$, and the advanced wave is traveling backwards in time.

4. Discussion of the Absorber Theory and its Experimental Test

In their paper [2] Wheeler and Feynman describe a mechanism of radiation of a field source (here an accelerated electric charge). Similar works have been previously proposed by Ritz [23] and Tetrode [24]: they considered that the radiation of a source is related to an absorber.

The field source emits both retarded waves and advanced waves, as much as 50% each. The supposed absorber is composed of all possible absorbing particles (also electric charges) which should be present all around the source. The absorber response to the waves received from the source is a radiative reaction which cancels all advanced waves emitted by the source.

We will not discuss the 9 hypothesis[i] on which the absorber theory is built and we will not recall with details the 4 steps of development of the theory[ii]. We just wish to make 3 fundamental remarks:

1. The absorber is not well defined. All along the paper it is the surrounding absorbing medium[i], all other particles, and finally an ideal sphere[iii] of radius *r*.

2. The advanced waves produced by the spherical absorber are converging to the initial field source, and after they "collapse" on the source these waves are diverging as originated from the source, but they still are advanced waves as we explained it in the above chapter. The authors do not explain how this "collapse" transform the advanced waves into a second (50%) retarded field which appear to be owned by the source.

3. To be able to cancel each other, the advanced waves produced by the source and the advanced waves produced by the spherical absorber must have a phase difference of 180°, and obviously it requires a very accurate sphere radius as:

$$r = (\tfrac{1}{2} + k)\lambda_k; k \in \mathbb{N} \quad (16)$$

where the λ_k are the wave lengths of cancelled advanced waves. Consequently all advanced waves with $\lambda \neq \lambda_k$ cannot be cancelled at all: the efficiency of the absorber depends on the source spectrum.

Nevertheless an experimental test of the absorber theory has been built with a microwave source alternately radiating into free space and into a local absober [25].

5. About Computational Discrete Derivative Equations

From the point of view of quantum theory, the successive positions of a particle (discontinuous trajectories) or its localization at the detection has to be quantified, and it is usually done with the momentum representation – the lost paradigm of relativist quantum theories [26] – where the position operator:

$$\hat{x}(p,E) \quad (17)$$

which is a function of the energy *E* and momentum *p* is used in differential equations as the Schrödinger equation in the momentum representation [27].

An other way to consider discontinuous trajectories (or discontinuous localizations) is the computation of the wave function at every successive space-time shift of a system of particles. This can be done with computational backward/forward discrete derivatives of a wave function with space-time shifts as it was proposed by Daniel M. Dubois [28]. The forward and backward discrete time derivatives of a function *F* are defined as:

$$\frac{\Delta_f F(t)}{\Delta t} = \frac{F(t+\Delta t)-F(t)}{\Delta t} \quad (18)$$

$$\frac{\Delta_b F(t)}{\Delta t} = \frac{F(t)-F(t-\Delta t)}{\Delta t} \quad (19)$$

where Δt is a discrete time interval. The definition is similar for forward and backward discrete space

[i]See pg 160 of reference [2].
[ii]"The radiative reaction: derivation" I to IV of reference [2].
[iii]See pg 166 and figure 1 in pg 173 of reference [2].

derivatives with a space shift Δx. The author introduced a generalized complex continuous time derivative defined as a complex combination of backward and forward derivatives:

$$\frac{\Delta_w F}{\Delta t} = w\frac{\Delta_f F}{\Delta t} + (1-w)\frac{\Delta_b F}{\Delta t} \quad (20)$$

with the weight:

$$w = \frac{1 \pm i}{2} \quad (21)$$

Considering the discrete derivative propagation equation[iv]:

$$\frac{\Delta\Phi(x,t)}{\Delta t} = v\frac{\Delta\Phi(x,t)}{\Delta x} \quad (22)$$

where $v = \Delta x / \Delta t$ is a velocity, in the case of a monochromatic plane wave function Φ:

$$\Phi = e^{(i\omega t + ikx)} \quad (23)$$

he obtained the phase velocity v_p and the group velocity v_g: as:

$$v_p = v = \frac{\omega}{k} v_g = \frac{\Delta\omega}{\Delta k} \quad (24)$$

Introducing the space and time intervals λ, τ and using the generalized forward/backward continuous derivatives, he obtained the following equation[v]:

$$\frac{\partial\Phi(x,t)}{\partial t} \pm i\frac{\tau}{2}\frac{\partial^2\Phi(x,t)}{\partial t^2} = v\left(\frac{\partial\Phi(x,t)}{\partial x} \pm i\frac{\lambda}{2}\frac{\partial^2\Phi(x,t)}{\partial x^2}\right) \quad (25)$$

With the generalized wave function:

$$\Phi(x,t) = \phi(x,t)e^{(\pm i\frac{t}{\tau} \pm i\frac{x}{\lambda})} \quad (26)$$

he obtained the following equation[vi]:

$$\frac{\partial^2\phi(x,t)}{\partial t^2} = vu\frac{\partial^2\phi(x,t)}{\partial x^2} - \frac{1}{\tau^2}\left(1 - \frac{v}{u}\right)\phi(x,t) \quad (27)$$

with $u = \lambda / \tau$.

5.1. *Deduction of the Klein-Gordon Quantum Relativist Equation*

From the equation (27) above with $v\,u = c^2$ he deduced the second order Klein-Gordon quantum relativist equation[vii] for bosons:

$$-\hbar^2\frac{\partial^2\phi(x,t)}{\partial t^2} = -\hbar^2 c^2\frac{\partial^2\phi(x,t)}{\partial x^2} + m_0^2 c^4\phi(x,t) \quad (28)$$

where the relativist mass is related to the time shift τ with:

$$m = \pm\frac{\hbar}{c^2\tau} \quad (29)$$

Remark: with $v = u = \pm$ c the Klein-Gordon equation gives the wave equation for photons with a null rest mass $m_0 = 0$:

$$\frac{\partial^2\phi(x,t)}{\partial t^2} = c^2\frac{\partial^2\phi(x,t)}{\partial x^2} \quad (30)$$

5.2. *Deduction of the Schrödinger Quantum Equation*

Taking a null time shift $\tau = 0$ in the equation (25) with the general wave function:

$$\Phi(x,t) = \phi(x,t)e^{\pm i\frac{x}{\lambda}} \quad (31)$$

and taking the rest mass related to the space shift λ as:

$$m_0 = \frac{\hbar}{v\lambda} \quad (32)$$

he deduced the equation[viii]:

$$i\hbar\frac{\partial\phi(x,t)}{\partial t} = \pm\left(\frac{-\hbar^2}{2m_0}\frac{\partial^2\phi(x,t)}{\partial x^2} - \frac{\hbar^2}{2m_0\lambda^2}\phi(x,t)\right) \quad (33)$$

where the plus/minus symbol corresponds respectively to positive and negative mass.

For a positive rest mass and the particular wave equation:

[iv]Equation (56) in op. cit.
[v]Equation (58) in op. cit.
[vi]Equation (60) in op. cit.
[vii]Equation (76) in op. cit.
[viii]Equation (91b) in op. cit.

$$\phi = \phi_1 e^{\pm i\frac{vt}{2\lambda}} \quad (34)$$

he obtained the Schrödinger equation for a free positive mass:

$$i\hbar\frac{\partial\phi_1(x,t)}{\partial t} = \frac{-\hbar^2}{2m_0}\frac{\partial^2\phi_1(x,t)}{\partial x^2} \quad (35)$$

6. About a Possible Negative Rest Mass

6.1. *Negative Energies Predicted by the Relativist Quantum Theory*

The French physicist Jean-Marie Souriau has shown [29, 30], from the complete Poincaré [31] group (defined by Minkowski [32, 33]), that reversing the energy of a particle is equivalent to inversing its time direction.

In the framework of quantum theory this result appear clearly in the plane wave equation associated to a free particle

$$\Psi = Ae^{\frac{i}{\hbar}(p\cdot x - Et)} \quad (36)$$

where A is a matrix depending on the type of particle. It can be written with a negative time as:

$$\Psi = Ae^{\frac{i}{\hbar}[p\cdot x + (+E)(\dot{-t})]} \quad (37)$$

or with a negative energy as:

$$\Psi = Ae^{\frac{i}{\hbar}[p\cdot x + (-E)(\dot{+t})]} \quad (38)$$

For mass particles the negative energy $E < 0$ implies a negative relativist mass:

$$m = \frac{E}{c^2} < 0 \text{and} m_0 = m\sqrt{1 - \frac{v^2}{c^2}} < 0 \quad (39)$$

and thus a negative rest mass m_0.

Moreover the negative energy E of a mass particle with the usual relation:

$$E = \hbar\omega \quad (40)$$

implies the negative frequency $\omega < 0$ of its associated wave function. In the equation (36) the exponent remains imaginary and Ψ is still a complex function which is valid for quantum theory. Consequently a photon with a negative frequency should be interpreted as a photon going backward in time, i.e. in the opposite time direction of the time arrow.

In the reinterpretation of Feynman diagrams (e.g. the creation of an electron positron pair followed by the annihilation of the positron by an electron) the time inversion is applied to half spin (i.e. electrically charged) particle but it is never applied to photons: input photons and output photons remain unchanged. So photons with negative frequencies are never considered.

6.2. *Interpretations of Negative Solutions of Dirac Equation*

The free Dirac equation [34]:

$$i\hbar\frac{\partial\Psi}{\partial t} = (c\hat{\alpha}\cdot\hat{p} + mc^2\hat{\beta})\Psi \quad (41)$$

is time symmetric, and so it includes negative solutions in its energy spectrum below:

$$]-\infty, -m_0c^2] \cup [m_0c^{2,} + \infty[\quad (42)$$

These negative energy states may be wave functions with negative frequencies or might be mass particles with a negative energy and a negative mass.

To explain anomalous negative energy quantum states predicted by his equation, Dirac conceived a model of vacuum, called the Dirac sea, populated of negative energy particles. In Dirac "theory of holes" a electron of positive energy correspond to a hole in the Dirac vacuum and it has a non null probability to fall in a negative energy state and thus the hydrogen atom would not be stable. This theory has an other failure: negative energy electrons of the Dirac sea do not interact despite of their electric charge. This theory has been abandonned when the antielectron (positron) was discovered.

Antiparticles (positrons, antiprotons) have been said to match negative energy states predicted by the Dirac equation because of their wave function, although they have a positive inertial mass as it is shown by Wilson chambers experiments.

As we already know [35][ix] the charge conjugation of the wave function change the sign of the Hamiltonian operator H and of the momentum operator $\boldsymbol{P}$ as:

[ix]op. cit. ch. XX, § 36, p. 818.

$$\langle P\rangle_C = -\langle P\rangle$$
$$\langle H(-e)\rangle_C = -\langle H(e)\rangle \quad (43)$$

but it does not change the rest mass so the relativist energy remains positive.

So the wave packet associated to a charged particle can contain negative frequencies. If the wave packet contains only positive frequencies (or only negative frequencies) the particle has a classical uniform rectilinear motion, but if the wave packet contains both positive and negative frequencies, the particle has a complex motion with quick oscillations[x] known as "*Zitterbewegung*".

6.3. *Study of Negative Mass Properties*

In the wave equation (36) and in the relativist equation (39) the energy E is the kinetic energy and so the mass m is the inertial mass, not the gravitational mass. So the possibility of a negative inertial mass raises a very important question: does the inertial and gravitational mass identity holds for any negative mass? In other words which sign should we postulate for the gravitational mass?

$$m_{\text{gravitation}} = |m_{\text{inertia}}| \quad (44)$$

or

$$m_{\text{gravitation}} = m_{\text{inertia}} \quad (45)$$

With the postulate (44) the gravitational mass is always positive, but with the postulate (45) gravitational mass and inertial mass are always identical, with the same sign.

With a negative inertial mass the second Newton law appear to be amazing with opposite directions of the force and the acceleration:

$$m_{\text{inertia}} < 0 => \vec{F} = -|m_{\text{inertia}}|\vec{\gamma} \quad (46)$$

a) Negative mass properties according to the mass identity principle

Let's consider the usual gravitation law:

$$|\vec{F}| = G\frac{m_1 m_2}{r^2} \quad (47)$$

where G > 0 is the gravitational constant, m_1 and m_2 two masses. Two positive masses attract each other and so the forces $\vec{F_1}$ and $\vec{F_2}$ applied respectively on m_1 and m_2 have opposite directions, as:

$$\vec{F_1} = -\vec{F_2} = G\frac{m_1 m_2}{r^2}\vec{u} \quad (48)$$

where $\vec{u}$ is a unit vector oriented from m_1 to m_2. Then the resulting accelerations $\vec{\gamma_1}$ and $\vec{\gamma_2}$ of m_1 and m_2 are defined as following equations:

$$\vec{\gamma_1} = +G\frac{m_2}{r^2}\vec{u} \quad (49)$$

$$\vec{\gamma_2} = -G\frac{m_1}{r^2}\vec{u} \quad (50)$$

As we can see on equations (49, 50) with positive masses the accelerations have opposite directions, so two positive masses attracts each other as it is well known.

With $m_1 > 0$ and $m_2 < 0$, substituting $m_1 = |m_1|$ and $m_2 = -|m_2|$ in equations (49, 50) we obtain the following accelerations:

$$\vec{\gamma_1} = -G\frac{|m_2|}{r^2}\vec{u} \quad (51)$$

$$\vec{\gamma_2} = -G\frac{|m_1|}{r^2}\vec{u} \quad (52)$$

So the two masses m_1 and m_2 are accelerated in the same direction $-\vec{u}$. Comparing equations (51, 52) with equations (49, 50) we see that m_1 attracts m_2 but m_2 repels m_1. If $|m_1| = |m_2|$ their distance remains constant, thus the accelerations are equal and constant: it is the "run away motion" which has been disregarded by William B. Bonnor [36].

If $|m_1|$ is several orders greater than $|m_2|$ (e.g. in the case of a negative mass electron or proton falling down onto the Earth) the acceleration $\vec{\gamma_1}$ can be neglected and therefore the negative mass m_2 is accelerated with $\vec{\gamma_2}$ exactly as if it were a positive mass.

With $m_1 < 0$ and $m_2 < 0$, substituting $m_1 = -|m_1|$ and $m^2 = -|m_2|$ in equations (49, 50) we obtain the acceleration $\vec{\gamma_1}$ as equation (51) and $\vec{\gamma_2}$ as:

$$\vec{\gamma_2} = +G\frac{|m_1|}{r^2}\vec{u} \quad (53)$$

so m_1 is accelerated in the direction $-\vec{u}$ and m_2 in the

[x] op. cit. ch. XX, § 37, p. 819.

direction $+\vec{u}$, thus two negative masses repel each other as it has been demonstrated primarily by Hermann Bondi [37] from Einstein field equation and by Robert L. Forward [38] from the point of view of propulsion.

b) Properties of negative inertial mass with always positive gravitational masses

According to the new principle (44) gravitational masses m_1 and m_2 would always be positive, then the gravitation law (48) would produce other effects with negative inertial masses. Instead of the equations (49, 50) we would have:

$$\overrightarrow{\gamma_1} = +G \frac{m_1}{m_1{}'} \frac{m_2}{r^2} \vec{u} \quad (54)$$

$$\overrightarrow{\gamma_2} = -G \frac{m_2}{m_2{}'} \frac{m_1}{r^2} \vec{u} \quad (55)$$

where prime symbols indicate inertial masses.

With inertial masses $m'_1 > 0$ and $m'_2 < 0$, substituting $m_1{}' = m_1$ and $m_2{}' = -m_2$ in equations (54, 55) we obtain:

$$\overrightarrow{\gamma_1} = +G \frac{m_2}{r^2} \vec{u} \quad (56)$$

$$\overrightarrow{\gamma_2} = +G \frac{m_1}{r^2} \vec{u} \quad (57)$$

So the two masses m_1 and m_2 are accelerated in the same direction $+\vec{u}$. So m_1 attracts m_2 but m_2 repels m_1. If $|m_1| = |m_2|$ there is again the "run away motion" (see previous section). In that case a negative mass electron or proton would not fall down onto the Earth but would be ejected from the Earth.

With inertial masses $m'_1 < 0$ and $m'_2 < 0$, substituting $m_1{}' = -m_1$ and $m_2{}' = -m_2$ in equations (54, 55) we obtain:

$$\overrightarrow{\gamma_1} = -G \frac{m_2}{r^2} \vec{u} \quad (58)$$

$$\overrightarrow{\gamma_2} = +G \frac{m_1}{r^2} \vec{u} \quad (59)$$

so m_1 is accelerated in the direction $-\vec{u}$ and m_2 in the direction $+\vec{u}$, and again two negative masses repel each other.

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Super-Coherent Quantum Dynamics of Zero-Point Field and Superluminal Interactions in Matter

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We recently suggested the idea according to which inertia and matter could be ultimately originated by a superradiant phase transition of quantum vacuum or zero-point field (ZPF), including all gauge fields, occurring from the scale of elementary particles to that of macroscopic bodies. We have shown this process leads to the formation of spatial quantum domains, called "coherent domains" (CD), in which a matter-wave field oscillates in phase with a "tuning" auto-generated radiation field "condensed" from quantum vacuum. These synchronized oscillations link a very large number of elementary matter and radiation quantum oscillators belonging to the considered physical system giving rise to their collective motion in which each of them loses its physical individuality to become a part of a whole "entangled" (macroscopic) object. In this paper we'll discuss, by considering the dynamics of coherent and incoherent quantum fluctuations occurring in a single coherent domain, the generation of the so-called "supercoherence", namely the phenomenon consisting in the tuned quantum interaction between two or more coherent domains in matter. We'll also prove this mechanism is able to originate a novel and so far, unrecognized type of long-range superfast synchronized interaction between material objects involving the exchange of, even superluminal, photons.

Keywords: Coherence, Superluminal interactions, Quantum dynamics, Zero-point field

1. Introduction

In a recent paper [1] we suggested the idea according to which the inertial mass of an elementary particle or body could be considered as the result of a superradiant phase transition (SPT) of Quantum Vacuum (QV), including the Zero-Point-Field (ZPF) oscillations of all gauge fields, from a perturbative ground state (PGS) to the coherent ground state (CGS), representing the "true" ground state of the system.

According to this model, we named CMH (Caligiuri-Mass-Matter-Hypothesis), matter could be considered as arising from the coupling amplification between the quantum Zero-Point matter and gauge fields, due to their coherent (phased) interaction. By means of this process, that we can interpret as a spontaneous phase transition of physical space itself (pictured as a Bose condensate, described by a classical-like density function [1]), the Zero-Point matter-wave and radiation field fluctuations will acquire, within a precise region of space determined by their tuned oscillation, a strong and stable amplitude sustained by their coherent mutual interaction, leading to the "condensation" of both matter and radiation fields out from quantum vacuum.

One of the most important consequences of this dynamics is the formation of the so-called quantum "coherent domains" (CDs), namely some volumes of space having a definite extension (of the order of magnitude of the wavelength characterizing the common oscillation of matter and radiation fields) whose dimension can vary from the scale of elementary particles up to that of macroscopic ensemble of atoms/molecules and an associated tuning radiation field, that coherently interacts with the matter field, "composed" of superradiant quanta. In the case of matter aggregates (condensates), i.e. on a spatial scale much greater than the range of nuclear interactions, this radiation field substantially concides with the e.m. field.

As we have seen, a necessary condition for the coherent evolution to occur is the quantized nature of the energy spectrum characterizing the matter system that couples to the gauge field(s) of quantum vacuum. The short time behaviour of such coupled system is then characterized by a "runaway" (more or less pronounced according to the coherence degree) towards the CGS (the real ground state) and the consequent formation of CDs.

For a many-levels matter system, among all the possible quantum transitions, the coherent evolution "selects" a given couple of levels E_1, E_0 (for which the transition strenght is the highest) with which the matter-e.m. field coupling occurs, neglegting all the other possible transitions.

The transition from PGS to CGS corresponds to a classical-like path in the quantum transition amplitude for which the coupled system behaves as a macroscopic quantum object whose elementary systems (the quanta of matter and e.m. field it "contains") loose their individuality to become parts of a whole macroscopic quantum system. The process of CD formation is then accompanied by the presence of quantum fluctuations of the matter and e.m. field amplitudes around the classical-like path. These fluctuations dissipate the energy of the initial PGS into the "outside" environment in order to allow the transition of the system towards the less energetic CGS, namely the real ground state. They can be then considered as the manifestation of the dynamics of excited states of CGS. It has been shown [1] the dynamics of these fluctuations is such that it allows to consider every CD, as a whole, like a (quantum) - macroscopic system characterized by a multi-level discrete energy spectrum.

The space-structure of the coherente states in matter is then characterized by the formation of an array of coherent domains, whose size is a function of the matter-wave oscillation frequency $\omega_0 = \left| E_1 - E_0 \right|$ (in the following we'll use the system of natural units where $\hbar = c = 1$) namely (r_{CD} is the "radius" of a single CD, supposed to be spherically symmetric)

$$r_{CD} \simeq \frac{3\pi}{4\omega_0} \tag{1}$$

As already shown [1], the above dynamics determines the generation, inside the CDs, of a coherent field of superradiant photons characterized by a "coherence" frequency ω_{coh} whose value is always less than the corresponding one for a "free" photon, associated to the same transition $E_1 \leftrightarrow E_0$, namely ω_0 .

The first physical consequence of such frequnecy "rescaling" is that the superradiant photon "mass" has, inside the CD, an immaginary value, determining the "trapping" of the coherent radiation field inside the CD itself, due to the total reflection, at the matter-vacuum interface between the CD's boundary and its outside, preventing its dissolution by radiating the coherent field inside them. The second one is the presence of an evanescent tail of coherent e.m. field extending outside CD, far away from its "border". As previously explained [1], for $r > r_{CD}$, the electromagnetic vetor potential $\vec{A}\left(\vec{r}\right)$ is given by the expression

$$A\left(r\right) \sim \frac{\exp\left[-\sqrt{\omega_0^2 - \omega_{coh}^2}\left(r - r_{CD}\right)\right]}{\omega_0 r} \tag{2}$$

According to (2) the e.m. field is then composed by a coherent component inside the CD (for $r < r_{CD}$) and an evanescent component (for $r > r_{CD}$) extending outside it, whose spatial depth makes it able to overlap the radiation field associated to the neighborhood coherent domains.

As we'll discuss in this paper, this very meaningful feature makes it possible the coherent interaction between different spatially separated CDs, through a mechanism quite similar to that giving rise to the formation of a single CD, with the difference that, now, each single CD (that can be viewed as an "elementary" quantum oscillator) couples with each other through the evanescent tail of its own coherent e.m. radiation field "condensed" from quantum vacuum.

As we'll see in the following, this coherent interaction between coherent domains, a process we can name "*supercoherence*", could be able to originate a very interesting phenomenon, namely the creation, when the CDs associated to two or more matter aggregates are sufficiently close each other, of a superluminal "environment" for the tunneling of virtual superradiant photons belonging to the overlapping evanescent radiation field generated by the interacting CDs. We'll finally discuss the possible existence, within such environment, of a new and so far unrecognized kind of long-range, even superluminal, interaction, so suggesting an interesting novel interpretation of superluminal phenomena in terms of Coherent QFT.

2. Dynamical Evolution of a Generic Matter and Electromagnetic Field Coupled System from the Standpoint of Coherent QED and the Origin of Quantum Fluctuations

As already discussed [1], the general dynamics of a quantized matter system interacting with a radiation field can be described by the Hamiltonian

$$H_{tot} = H_K + H_{\text{int}} + H_{field} + H_{SR} \tag{3}$$

where H_K is the "kinetic" component, H_{int} is the term describing the interaction between matter-quantum wave field and the free quantized field associated to that interaction (i.e. the gauge field mediating it) having the same formal structure of the free quantized e.m. field given by H_{field} and H_{SR} is the "short-range" Hamiltonian, usually in the form of two-body interaction

$$H_{SR} = \frac{1}{2}\iint \Psi^\dagger\left(\vec{x},\delta,t\right)\Psi\left(\vec{x},\delta,t\right)\times \\ \times V\left(\vec{x}-\vec{y}\right)\Psi\left(\vec{x},\sigma,t\right)\Psi\left(\vec{x},\sigma,t\right)d^3x\, d^3y\, d\delta\, d\sigma \tag{4}$$

where $\Psi\left(\vec{x},\delta,t\right)$ is the matter-wave field wavefunction $\vec{x}$ are the spatial coordinates and δ a set of internal variables describing the quantum state of the elementary oscillator) and V a potential function. As well known, within the QFT framework, the matter quantum field can be expressed, in terms of creation and annihilation operators of the Fock space $a_n^\dagger\left(t\right)$ and $a_n\left(t\right)$ obeying the equal-time relation

$$\left[a_n\left(t\right),a_m^\dagger\left(t\right)\right]_\pm = \delta_{nm} \tag{5}$$

as

$$\Psi\left(\vec{x},\delta,t\right) = \sum_n a_n\left(t\right)\varphi_n\left(\vec{x},\delta\right) \tag{6}$$

where $\left\{\varphi_n\left(\vec{x},\delta\right)\right\}$ is a complete set of orthonormal functions that diagonalizes the single-particle Hamiltonian and $\left\{E_n\right\}$ are their relative eigenvalues.

The transition amplitude of such a system from a initial state $\left|i,t_i\right\rangle$ to a final state $\left|f,t_f\right\rangle$ can be obtained by the path-integral representation

$$\left\langle f,t_f\left|i,t_i\right.\right\rangle = \\ = \int\left[d\Psi^\dagger\left(\vec{x},\delta,t\right)\right]\left[d\Psi\left(\vec{x},\delta,t\right)\right]\left[da_{\vec{k}r}{}^\dagger\left(t\right)\right]\left[da_{\vec{k}r}\left(t\right)\right]\cdot \\ \cdot\exp\left\{\frac{i}{\hbar}\int_{t_i}^{t_f} dt\, L_{tot}\left(\Psi,\Psi^\dagger,a_{\vec{k}r},a_{\vec{k}r}^\dagger\right)\right\} \tag{7}$$

where $L_{tot} = L_m + L_{field}$ and the calculation is made considering only the paths for which the number of matter(-quantum) systems in interaction with the field contained inside the volume V is fixed and equal to N (the conserved quantum operator).

As discussed in great detail in [1], in the "classical" limit $N \to \infty$, the amplitudes of matter wave field and the e.m field can be written as

$$\overline{\Psi}\left(\vec{x},\delta,t\right) = \varphi\left(\vec{x},\delta,t\right) + \frac{1}{\sqrt{N}}Q\left(\vec{x},\delta,t\right) \tag{8}$$

$$\overline{a}_{\vec{k}r}\left(t\right) = \alpha_{\vec{k}r}\left(t\right) + \frac{1}{\sqrt{N}}q_{\vec{k}r}\left(\vec{x},\delta,t\right) \tag{9}$$

where the functions $Q\left(\vec{x},\delta,t\right)$ and $q\left(\vec{x},\delta,t\right)$ respectively represent the quantum fluctuations of the matter and e.m. field around the their "classical" paths $\varphi\left(\vec{x},\delta,t\right)$ and $\alpha_{\vec{k}r}\left(t\right)$ that can be determined by the principle of minimal action

$$\delta\int_{t_i}^{t_f}\left(\overline{L}_m + \overline{L}_{em}\right)dt = 0 \tag{10}$$

where we assume a Langrangian function of the form

$$\overline{L}_{em} = \sum_{\vec{k}r}\left[\frac{i}{2}\left(\overline{a}_{\vec{k}r}^\dagger\dot{\overline{a}}_{\vec{k}r} - \dot{\overline{a}}_{\vec{k}r}^\dagger\overline{a}_{\vec{k}r}\right) + \frac{1}{2\omega_{\vec{k}}}\dot{\overline{a}}_{\vec{k}r}^\dagger\dot{\overline{a}}_{\vec{k}r}\right] \tag{11}$$

By using (10) we can obtain [2], respectively by varying it with respect to φ^* and $\alpha_{\vec{k}r}^*$, the Euler-Lagrange equations for the "classical" amplitudes φ and $\alpha_{\vec{k}r}$. The expansion of the action S around these stationary paths can be written as

$$S\left[\Psi,\Psi^\dagger,a_{\vec{k}r},a_{\vec{k}r}^\dagger\right] \cong S\left[\varphi,\varphi^*,\alpha_{\vec{k}r},\alpha_{\vec{k}r}^*\right] + \\ + \frac{1}{N}\int\frac{\delta^2 S}{\delta\Psi\delta\Psi^\dagger}\left[\varphi,\varphi^*,\alpha_{\vec{k}r},\alpha_{\vec{k}r}^*\right]Q^\dagger Q + \\ + \frac{1}{N}\sum_{\vec{k}r}\sum_{\vec{h}t}\int\frac{\delta^2 S}{\delta a_{\vec{k}r}^\dagger\,\delta a_{\vec{h}t}}\left[\varphi,\varphi^*,\alpha_{\vec{k}r},\alpha_{\vec{k}r}^*\right]q_{\vec{k}r}^\dagger q_{\vec{k}r} \tag{12}$$

in which we have neglected the higher order terms.

3. The Dynamics of Coherent and Incoherent Matter-Electromagnetic Field Coupled Quantum Fluctuations

As already shown [1, 2], the Lagrangian function obtained by applying the principle of minimal action to

(12) is able to describe the dynamics of the quantum fluctuations generated during the superradiant phase transition from PGS to CGS. Also they can be subdivided into two categories: the *coherent* and the *incoherent* fluctuations.

In this way the quantum matter field can be written as

$$\overline{\Psi}\left(\vec{x},\delta,t\right)=\varphi+\delta\varphi_{coh}+\delta\varphi_{inc} \tag{13}$$

where $\delta\varphi_{coh}$ and $\delta\varphi_{inc}$ respectively represent the coherent and iuncoherent quantum fluctuations. The coherent fluctuations can be interpreted as excited states of the CGS whose modes belong to the coherent state and keep a definite phase relation with it. They are described by the Lagrangian

$$L_{coh}=\sum_{\vec{k}}a_{\vec{k}}^{*}\left(i\frac{\partial}{\partial t}-\frac{\left|\vec{k}\right|^{2}}{2m}\right)a_{\vec{k}} \tag{14}$$

where the coefficients $a_{\vec{k}}$ define the Fourier expansion of $\delta\varphi_{coh}$. The energy spectrum of the coherent fluctuations (excitations) $\left\{e_{\vec{k}}\right\}$ can be then simply obtained as

$$e_{\vec{k}}=\frac{\left|\vec{k}\right|^{2}}{2m} \tag{15}$$

and is characterized by a “pass-high” cut-off, to ensure a sufficient density of modes inside the quantization volume, given by

$$\left|\vec{k}\right|\geq k_{0}\sim\sqrt{\frac{N}{V}} \tag{16}$$

The incoherent fluctuations instead represent the field modes able to excite energy levels belonging to PGS. The Langrangian function describing such fluctuations can be easily obtained in a similar fashion as

$$L_{inc}=\sum_{i}\sum_{\vec{k}}b_{i\vec{k}}^{*}\left[i\frac{\partial}{\partial t}-\left(\varepsilon_{i}+\frac{\left|\vec{k}\right|^{2}}{2m}+\Delta E\right)\right]b_{i\vec{k}} \tag{17}$$

where the coefficients $b_{i\vec{k}}$ define the Fourier expansion of incoherent fluctuations $\delta\varphi_{inc}$ associated to the level i whose energy is ε_{i} and the quantity $\Delta E=$ $\Delta E=E_{CGS}-E_{PGS}$ is the energy difference between the CGS and PCG per elementary mattter oscillator constituting the system. The energy spectrum of incoeherent quantum fluctuation is then characterized by the presence of a gap ΔE, representing the energy required to excite a quasi-particle quantum oscillator making it to escape from CGS to PGS.

4. The Spectrum of Quantum Fluctuations

We’ll now discuss the question of time-evolution of the overall quantum fluctuations during the transition from PGS to CGS, calculating the energy spectrum of such oscillation.

The CGS represents the real ground state of a condensed matter system and, for this reason, its energy results lower than the energy of PGS of the quantity ΔE whose value depends on the features of the specific physical system to be considered.

In a closed system, the evolution from PGS to CGS would then be forbidden due to the energy conservation that would prevent any stable transition to a less energetic state. Nevertheless. this kind of transition is allowed in the case of open quantum systems (for which we distinguish an “outside” environment that, in our case, could be also identified with the ZPF itself).

The quantum fluctuations we are going to describe can be expressed like amplitude perturbations with respect to those characterizing the stationary values associated to CGS. So we can write, fow a generic two-levels quantum system ($i=1,2$)

$$\beta_{i}\left(\tau\right)=\beta_{i,CGS}+\delta\beta_{i}\left(\tau\right) \tag{18}$$

$$A\left(\tau\right)=A_{CGS}+\delta A\left(\tau\right) \tag{19}$$

where β_{i} and A respectively represent the generic matter and e.m. fields while $\beta_{i,CGS}$ and A_{CGS} their values in the CGS, and $\tau=\omega_{0}t$. The time evolution of such fields is dscribed the so-called “coherent equations” [1, 2], namely

$$i\dot{\beta}_{0}\left(\tau\right)=gA^{*}\left(\tau\right)\beta_{1}\left(\tau\right) \tag{20}$$

$$i\dot{\beta}_{1}\left(\tau\right)=gA\left(\tau\right)\beta_{0}\left(\tau\right) \tag{21}$$

$$-\frac{1}{2}\ddot{A}\left(\tau\right)+i\dot{A}\left(\tau\right)=g\beta_{0}^{*}\left(\tau\right)\beta_{1}\left(\tau\right) \tag{22}$$

where, for simplicity, we have neglected the e.m. field "mass-term" and the term g represents the coupling constant between matter and radiation field.

The behaviour of quantum fluctuations can be studied by considering the variation of the fields from the state that makes minimum the action (12). By varying the system (20)-(22) we obtain, inside a CD, the new set of equations

$$\delta\dot{\beta}_0 = -ig\left(\beta_1\delta A^* + A^*\delta\beta_1\right) \tag{23}$$

$$\delta\dot{\beta}_1 = -ig\left(\beta_0\delta A + A\delta\beta_0\right) \tag{24}$$

$$\frac{i}{2}\delta\ddot{A} + \delta\dot{A} = -ig\left(\beta_1\delta\beta_0^* + \beta_0^*\delta\beta_1\right) \tag{25}$$

where we have assumed $\delta\left[\dot{f}\left(\tau\right)\right] = \partial_\tau\left[\delta f\left(\tau\right)\right] \equiv \delta\dot{f}$ and $\delta\left[f^*\left(\tau\right)\right] = \left[\delta f\left(\tau\right)\right]^* \equiv \delta f^*$.

The non-linear system (23)-(25) completely describes the time-evolution of quantum fluctuations $\delta\beta_i\left(\tau\right)$ and $\delta A\left(\tau\right)$ provided the functions $\beta_i\left(\tau\right)$ and $A\left(\tau\right)$ are known from the (20)-(22), and must be generally solved numerically. Nevertheless, a simple solution can be obtained if we limit ourselves to short-time behaviour of its solutions (that is, on the other hand, the most interesting one in the general case). By time-derivating the (25) we have

$$\begin{aligned}&\frac{i}{2}\delta\dddot{A} + \delta\ddot{A} = \\ &= -ig\left(\dot{\beta}_1\delta\beta_0^* + \beta_1\delta\dot{\beta}_0^* + \beta_0^*\delta\dot{\beta}_1 + \dot{\beta}_0^*\delta\beta_1\right)\end{aligned} \tag{26}$$

that becomes, by using (20) and (21) in the right side

$$\begin{aligned}&\frac{i}{2}\delta\dddot{A} + \delta\ddot{A} = \\ &= g^2A\beta_1^*\delta\beta_1 - g^2A\beta_0\delta\beta_0^* - ig\beta_1\delta\dot{\beta}_0^* - ig\beta_0^*\delta\dot{\beta}_1\end{aligned} \tag{27}$$

If we now consider the initial conditions characterizing the "runaway" of the quantum system towards the CGS [1, 2], namely the matter and radiation fields having neglegible amplitudes, we can write, at $\tau \sim 0$, $A\left(\tau\right) \sim O\left(0\right)$, $\beta_0\left(\tau\right) \sim O\left(1\right)$ and $\beta_1\left(\tau\right) \sim O\left(0\right)$ and the (27) becomes

$$\frac{i}{2}\delta\dddot{A} + \delta\ddot{A} \sim -ig\delta\dot{\beta}_1 \tag{28}$$

Using the same assumptions in the (24), we have $\delta\dot{\beta}_1 \sim -ig\delta A$ that, inserted in (28), gives

$$\frac{i}{2}\delta\dddot{A} + \delta\ddot{A} + g^2\delta A = 0 \tag{29}$$

The differential equation (29) can be reduced to an algeabric one by assuming $\delta A \sim e^{i\lambda}$, namely

$$\frac{1}{2}\lambda^3 - \lambda^2 + g^2 = 0 \tag{30}$$

whose three solutions determine the short time behaviour of quantum fluctuations of e.m. field. Similar expressions can be obtained for matter-field flucutations $\delta\beta_i$ [1]. As known from the general theory, in order to ensure the stability of CGS (namely the oscillatory behaviour of quantum fluctuations) the three solutions of (30) must be all real.

The e.m. field associated to quantum fluctuations interacts with the coherent e.m. field $A_{CGS}\left(\tau\right)$, condesed from quantum vacuum, generating amplitude modulation of the coherent field [1, 2]. By writing the coherent and fluctuating fields respectively as $A_{CGS}\left(t\right) \sim e^{i\dot{\phi}\omega_0 t}$ (in which the time derivative is taken with respect to τ) and $\delta A\left(t\right) \sim e^{i\lambda\omega_0 t}$, we can deduce the frequency spectrum of the modulated field as [1, 2] (i = 1,2,3)

$$\Omega_i = \omega_0\left(\lambda_i - \dot{\chi}\right) \tag{31}$$

Equation (31) has a very important meaning since it shows the CD itself can be considered, until the coherent quantum fluctuations will excede the incoherent ones, as a coherent "macroscopic" quantum system characterized by a discrete energy levels spectrum (composed by three frequencies in the short time behaviour) and then itself potentially able to coherently interact (through the same mechanism leading to the formation of a single CD) with other CDs.

In the CGS, the functions describing the field phases θ_i and χ must be linear functions of time t (or τ) so the value of frequencies calculated through (31) are constants. In order to obtain such values we can use the coherent equations (20)-(22) for the matter and e.m. fields respectively given by

$$\beta_{i,CGS}\left(\tau\right) = b_i e^{i\theta_i\left(\tau\right)} \tag{32}$$

$$A_{CGS}\left(\tau\right) = ae^{i\chi(\tau)} \quad (33)$$

where a and b_i don't depend on time.

It has been shown [1, 2] the system (20)-(22) admits the solution (corresponding to the minimun energy level)

$$\dot{\theta}_1 = ga\frac{b_2}{b_1} \quad (34)$$

$$\dot{\theta}_2 = ga\frac{b_1}{b_2} \quad (35)$$

$$\dot{\chi} = 1 \pm \sqrt{1 - \frac{2gb_1b_2}{a}} \quad (36)$$

with the constraints [1]:

$$b_1^2 + b_2^2 = 1 \quad (37)$$

$$a^2\left(1 - \dot{\chi}\right) + b_1^2 - b_2^2 = Q\left(0\right) \quad (38)$$

$$\dot{\theta}_1 - \dot{\theta}_2 - \dot{\chi} = 0 \quad (39)$$

where $Q\left(0\right)$ is a constant of motion (corresponding to the "initial momentum" of the system composed by elementary quantum oscillators). The requirement (37) allows us to assume

$$b_1 \equiv \cos\left(\frac{\vartheta}{2}\right); \quad b_2 \equiv \sin\left(\frac{\vartheta}{2}\right) \quad (40)$$

with $0 \le \vartheta \le \pi$. We can also suppose the e.m. field amplitude in the CGS to be proportional to the coupling constant g namely we assume

$$a = \alpha \cdot g \quad (41)$$

where, in the superradiant case, $\alpha > 1$.

We can now search for the solutions of the system (34)-(36) in the variables ϑ and α. This involves some simple mathematics.

In particular, by using the (39) in the (36), we have

$$\dot{\theta}_1 - \dot{\theta}_2 - 1 = \pm\sqrt{1 - \frac{2gb_1b_2}{a}} \quad (42)$$

that, inserting the (34) and (35) in the left side, becomes, remembering the (40) and (41), the identity $\sin 2x = 2\sin x \cos x$ and the duplication formulas for $\tan x$ and $\cot x$,

$$\alpha g^2\left(\tan\frac{\vartheta}{2} - \cot\frac{\vartheta}{2}\right) - 1 = \pm\sqrt{1 - \frac{\sin\vartheta}{\alpha}} \quad (43)$$

Now, after some simple manipulations, we can write the identity

$$\tan\frac{\vartheta}{2} - \cot\frac{\vartheta}{2} = -2\cot\left(2\frac{\vartheta}{2}\right) = -2\cot\vartheta \quad (44)$$

that, used in (43), finally gives

$$1 + 2\alpha g^2 \cot\vartheta = \pm\sqrt{1 - \frac{\sin\vartheta}{\alpha}} \quad (45)$$

representing the first equation relating ϑ and α. In order to write the second equation, needed to solve the coherent system (34)-(36), we use the (36) in the first member of (38) giving

$$\mp a^2\sqrt{1 - \frac{2g}{a}\sin\frac{\vartheta}{2}\cos\frac{\vartheta}{2}} + \cos^2\frac{\vartheta}{2} - \sin^2\frac{\vartheta}{2} = = Q\left(0\right) \quad (46)$$

that, using the (41) and the trigonometric identity $\cos^2\left(\vartheta/2\right) - \sin^2\left(\vartheta/2\right) = \cos\vartheta$, becomes

$$\mp\alpha^2 g^2\sqrt{1 - \frac{\sin\vartheta}{\alpha}} + \cos\vartheta = Q\left(0\right) \quad (47)$$

that is the second equation needed.

The coherent system (34)-(36) is then equivalent to the following one

$$\begin{aligned} 1 + 2\alpha g^2 \cot\vartheta &= \pm\sqrt{1 - \frac{\sin\vartheta}{\alpha}} \\ \frac{Q\left(0\right) - \cos\vartheta}{\alpha^2 g^2} &= \pm\sqrt{1 - \frac{\sin\vartheta}{\alpha}} \end{aligned} \quad (48)$$

The equations (48) must be generally solved numerically but, in the "full superradiant" case $g^2 \gg 1$, we can find a simplified solution as follows.

In this case we can assume $b_1 \simeq b_2$, namely $\vartheta \simeq \pi/2$ and then $\sin\vartheta \sim 1$ and $\cos\vartheta \sim 0$. By squaring the second equation of (48) we have

$$1-\frac{1}{\alpha}\simeq\frac{Q^2\left(0\right)}{\alpha^4 g^4} \tag{49}$$

but, in superradiant case, we also have $a \gg 1$ and then $\alpha = a/g \simeq 1$ and, consequently, $\alpha^n \simeq \alpha$ $(n > 1)$. From (49) we can now easily get the value of α, namely

$$\alpha \simeq 1+\frac{Q^2\left(0\right)}{g^4} \tag{50}$$

We can now calculate the value of ϑ by squaring the first equation of (48)

$$\left(1+2\alpha g^2\cot\vartheta\right)^2 = 1-\frac{\sin\vartheta}{\alpha} \tag{51}$$

now recalling the assumptions $\sin\vartheta \sim 1$ and $\alpha \sim 1$ we can deduce, due to the vanishing of the right side of (51)

$$1+2g^2\cot\vartheta \simeq 0 \tag{52}$$

Under the assumption $\vartheta \sim \pi/2$ we can write $\cot\vartheta \simeq \pi/2-\vartheta$ so that the (52) becomes

$$1+2g^2\left(\frac{\pi}{2}-\vartheta\right)\simeq 0 \tag{53}$$

and then, finally

$$\vartheta \simeq \frac{1}{g^2}+\frac{\pi}{2} \tag{54}$$

The previous results allow us to find, in the "full superradiant" case, the value of the matter and e.m. field amplitudes of the CGS and then, through the equation (31), the spectrum of coherent quantum fluctuations.

5. Small Oscillations Regime

We'll now consider the dynamics of small oscillations of matter and e.m. field amplitude around the equilibrium state, namely the CGS. This will also allow us to relate the Rabi frequency of such oscillations to the specific features of the coherent system under examination.

We start again from the coherent equations (20)-(22) considering the small oscillations around the stationary path, given by the field amplitudes (34)-(36). From the previous discussion we know the field phases θ_i and χ have to be, in the CGS, linear functions of time, namely we have $\theta_i \propto \tau$ and $\chi \propto \tau$. This means we can rewrite (32) and (33) as

$$\beta_{i,CGS}\left(\tau\right) = b_i e^{i\dot{\theta}_i\tau} \tag{55}$$

$$A_{CGS}\left(\tau\right) = a e^{i\dot{\chi}\tau} \tag{56}$$

in which we can think of $\dot{\theta}_i$ and $\dot{\chi}$ as respectively the angular frequencies ω_i and ω of the matter and radiation fields oscillations. We now write (20)-(21) by considering the time derivative with respect to variable t, recalling that for a generic function of time f $\partial_\tau f\left(\tau\right) = \omega_0^{-1}\partial_t f\left(t\right)$,

$$\frac{d\beta_1}{dt} = -ig\omega_0 A^*\beta_2 \tag{57}$$

$$\frac{d\beta_2}{dt} = -ig\omega_0 A\beta_1 \tag{58}$$

By taking the time derivative of (58)

$$\frac{d^2\beta_2}{dt^2} = -ig\omega_0\left(\frac{dA}{dt}\beta_1 + A\frac{d\beta_1}{dt}\right) \tag{59}$$

and substituting in the right side the (57) we have

$$\frac{d^2\beta_2}{dt^2} = -ig\omega_0\frac{dA}{dt}\beta_1 - \omega_0^2 g^2\left|A\right|^2\beta_2 \tag{60}$$

Equations (55) and (56) can be respectively written as

$$\beta_1 = b_1 e^{i\dot{\theta}_1\omega_0 t} \tag{61}$$

$$\frac{dA}{dt} = a\left(i\omega_0\dot{\chi}\right)e^{i\dot{\chi}\omega_0 t} \tag{62}$$

that, inserted in (60), gives

$$\frac{d^2\beta_2}{dt^2} = g\omega_0^2 a\dot{\chi}e^{i\omega_0\left(\dot{\chi}+\dot{\theta}_1\right)t} - \omega_0^2 g^2\left|A\right|^2\beta_2 \tag{63}$$

Appplying the "rotating-wave approximation" we can neglect, in (63), the first fast oscillating term $e^{i\omega_0(\dot{\chi}+\dot{\theta}_1)t}$ with respect the second one, oscillating as $e^{i\omega_0\dot{\theta}_2 t}$ and obain

$$\frac{d^2\beta_2}{dt^2} \simeq -\omega_0^2 g^2 \left|A\right|^2 \beta_2 \tag{64}$$

that, using the (41), becomes in the CGS ($\alpha \sim 1$)

$$\frac{d^2\beta_2}{dt^2} + \omega_0^2 g^4 \beta_2 = 0 \tag{65}$$

and the frequency of the related oscillations is given by

$$\bar{\omega} = \omega_0 g^2 \equiv \frac{\Omega_0}{2} \tag{66}$$

where Ω_0 is the frequency of the Rabi oscillations given in (31) for the ground state of CGS.

6. The Emergence of Super-Coherence and of Superluminal Interactions in Matter

The above discussion leads to a very meaningfull physics result namely the possibility to write, for an esemble of several CDs in matter, a set of quantum field equations in all similar to the "coherent equations" (20)-(22) already obtained for a single CD. This proves, providing that suitable conditions for the density of CDs (i.e. the number of the coherent domains in a given volume) and their temperature are satisfied [1], that the same type of coherent dynamics, leading to the formation of a single CD, can arise, on a wider scale, for a very large ensemble of CDs, so generating a sort of "super-coherent" domain (SCD), whose dimension would be of the order of

$$L_{SCD} \sim \frac{2\pi}{\Omega_0} \tag{67}$$

where Ω_0 is the lower Rabi frequency, belonging to the excited energy spectrum of the single CD given by (31) in the short-time behaviour, whose value is given through (66).

Such a phenomenon, called "*Supercoherence*", then generalizes the concept of the coherent domain by considering the tuned interaction between two or more CDs, each of one considered a single discrete energy levels system.

As already discussed in [2] the interaction between two individual CDs can occur through the overlapping of the evanescent components of the coherent e.m. field associated to each of them. Due to supercoherence such overlapping is then able to reduce the enery of the real ground state (a sort of CGS involving all the interacting CDs) of the system and eventually minimize it when the CDs are close packed at the inter-center distance $\Delta d = 2r_{CD}$.

Furthermore, due to the frequency rescaling of the e.m. field inside the CD according to which the common frequency of oscillation ω_{coh} of the e.m. field and matter inside it is lower than the value ω_0 characterizing the perturbative (incoherent) state, we can write

$$\omega_{coh} = \left|1 - \left(\dot{\theta}_1 - \dot{\theta}_2\right)\right|\omega_0 < \omega_0 \tag{68}$$

Equation (68) implies that the superradiant photon "mass" acquires an imaginary value inside the coherent e.m. field as can be easily shown by considering the Einstein equation for a photon (with, in our case, $\omega_{coh} < \omega_0$ and $r_{CD} < \lambda$)

$$m^2 = \hbar\left(\omega_{coh}^2 - \frac{4\pi^2}{r_{CD}^2}\right) < \hbar\left(\omega_0^2 - \frac{4\pi^2}{\lambda^2}\right) = 0 \tag{69}$$

that gives $m = i \cdot m_0$, m being the mass acquired by the photon inside the CD.

The superradiant e.m. field, resulting from the tuned interaction between matter and electromagnetic field inside the CD, can then spread across classically forbidden regions (here represented by the outside of CDs) in the form of evanescent modes.

As we have seen, equation (69) assigns a purely imaginary rest mass to the photons belonging to coherent e.m. field generated inside CDs. As pointed out by Recami [3, 4], tunneling photons associated to evanescent field can be characterized by a superluminal group velocity or, equivalently, by a negative square mass of the photons belonging to it. This can easily shown by considering that a quantum evanescent photon satisfies the Klein-Fock-Gordon equation for a wave function ψ, namely (in one dimension):

$$\left[-\frac{\partial^2}{\partial t^2} + \nabla^2 - \frac{m_0^2}{\hbar^2}\right]\psi\left(x,t\right) = 0 \tag{70}$$

where c is the velocity of light in vacuum, m_0 the absolute value of the proper mass of the evanescent photon. A solution of (70) is given by

$$\psi(x,t) = A \cdot \exp\left[-\frac{p \cdot x + E \cdot t}{\hbar}\right] \qquad (71)$$

where A is the wave amplitude and corresponds to a particle of imaginary rest mass $m_r = i \cdot m_0$ moving at a superluminal velocity $v > c$ (in some inertial frame) and satisfying the relativistic relation

$$E^2 = p^2 - m_0^2 \qquad (72)$$

E and p being, as usual, the total energy and the momentum of the particle associated to the wave ψ .

According to this picture it is then possible to consider the superradiant coherent photons belonging to the coherent e.m. field arising inside the CDs in matter as the result of the quantum coherent dynamics of quantum vacuum, as superluminal photons consituting a macroscopic "tachyon" field [1, 2]. In fact, despite the general (and in the most part of cases unfounded) skepticism of the international scientific community about the possible existence of superluminal phenomena (namely a process in which the propagation velocity of some field or particle is, apparently or actually, greater than the value of velocity of light in vacuum c), there exist several theoretical and experimental evidences of the real occurrence of such kind of processes [2].

For example, Nimtz shown [5], by considering the quantum tunneling of signals and particles through different types of opaque barriers, the fundamental causality principle, according to which the effect must always follows its cause, is not violated in this case and this kind of superluminal fields are able to carry physical information [5].

More recently, similar results have been also obtained by L. M. Caligiuri [2] by considering alternatives formulations of Einstein's Special Theory of Relativity and by Ziolkowski [6], in relation to the study of electromagnetic metamaterials.

As shown by L.M. Caligiuri [1, 2, 7] the CD can be considered as a "waveguide" for the coerent e.m. field generated inside it, while its evanescent tail extending "outside" it like the spreading of e.m. waves across "impenetrable" opaque optical barriers, in our case represented by theCDs boundaries.

This phenomenon just corresponds to the optical total reflection of an e.m. wave at the interface between two media characterized by a different refraction index.

A purely classical treatment of this phenomenon, based on Maxwell's equations, shows, for example, that the electrical field intensity of the evanescent wave transmitted in the second medium is given by an expression of the type

$$E_\perp(x) = f(\alpha, n) E_{in} e^{-\frac{x}{d_p}} \qquad (73)$$

where E_{in} is the intensity of the incident field, $n = n_2/n_1$, $f(\alpha, n)$ is an analytic function whose form depend on the considered component of $E_\perp$ and d_p is the "penetration" depth, representing a rough estimation of the spatial "confinement" of the decaying evanescent field in the second medium.

This process is also analogous to the phenomenon of quantum tunneling experienced by particles through a potential barrier. This physical analogy has been suggested by several earlier studies [8] based on the mathematical similarities between the solutions of Helmotz equation and that of Schrodinger equation.

According to this picture, the tunneling of evanescent modes of e.m. waves is equivalent, from a quantum viewpoint, to the tunneling of virtual photons [5] through the corresponding potential barrier.

Some researchers [9] criticized this interpretation claiming that such mathematical analogy is not sufficient to state the validity of the correponding physical model. Nevertheless, in the case of QED coherent dynamics, this objection is surely unfounded since the coherent e.m. field actually behaves as a macroscopic quantum entity obeying a "classical" like wave equation [1, 2].

The virtual photons belonging to superradiant coherent field are characterized by a non-local behavior and the related evanescent modes result not directly observable inside the barrier [8], nevertheless they can interact with matter inside the tunnel [7, 10], in a similar fashion to "ordinary" homogeneous plane-wave modes.

Evanescent e.m. modes can be interpreted [10] as free e.m. field basing on the idea of refractive index of a passive, macroscopically continuous media. In this sense they can be considered as the result of the spatial phase modulation, at the interface of two media, between the incident and the reflected wave. The evanescent e.m. modes, considered as classical c-number fields, can then interact with matter no differently from ordinary homogeneous e.m. waves.

As we have seen from the above discussion, when two CDs are sufficiently close each other, their respective e.m. evanescent fields, given by (2), could overlap interacting each other.

The novel mechanism discussed in this paper to explain such interaction is based on the idea, already proposed by L. M. Caligiuri [1, 2] within the framework of QED coherence in matter, and constists in considering the CDs overlapping zone as a region of space through which the tunneling of virtual photons, belonging to the evancescent e.m field associated to the two interacting CDs, can take place.

The spatial extension of such region will obviously depend on the distance between the two CDs and on the spreading of their associated evanescent fields, described by their respective tails outside the CDs, whose "extension" is in turn a function of ω_{coh} .

This situation is schematically shown in Fig. 1.

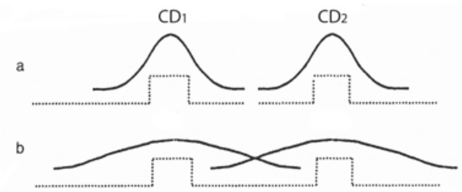

Figure 1. Overlapping between the evanescent fields associated to two close coherent domains.

If the "distance" d between two CDs is much greater than the average value of d_p associated to the coherent evanescent e.m. fields "escaping" from their respective CDs (see Fig. 4a), we can assume there is no overlapping between such fields and consequently a negligible probability of interaction between them.

On the other hand, when

$$d \leq d^* = 2\left(r_{CD} + d_p\right) \tag{74}$$

the overlap between the two evanescent e.m. fields takes place so allowing a "non-local" interaction between the CDs (see Fig. 4b). As we have seen in the above discussion, the (74) admits the limiting case $d = 2r_{CD}$ corresponding to the minimum energy level and, consequentially, to the maximum interaction amplitude.

Generally, the value of d_p can be estimated (at least as order of magnitude) by observing that the quantum interaction between CDs and their evanescent coupling so far described (due to the total reflection and evanescent wave behavior of coherent e.m. field at the CD - vacuum interface), can be considered quite analogous to that of the tunneling of e.m. waves through opaque optical barriers (in turn explainable, as already seen, in terms of tunneling of virtual photons).

Furthermore, being the tunneling related to the presence of an evanescent field, the probability of traversing the barrier is inversely proportional to its length. For these reasons it must be considered as near-field phenomenon whose "detection" is then limited to a near-field zone. The experimental evidences so far available about this process [5, 8, 11, 12] suggest the extension D of this near field region could be given by

$$D \simeq 10^3 \lambda_0 \tag{75}$$

where λ_0 is the wavelength of the incident field at the interface where the total reflection occurs.

Equation (75) can be generalized to our model by writing

$$d_p \simeq 10^3 \lambda_0 = 10^3 \left(\frac{2\pi}{\omega_0}\right) \tag{76}$$

Furthermore, as shown in [5], a generally valid relation between the tunneling time τ of the evanescent wave (or the tunneling photons) and the frequency f of the signal (or the associated quantum particle) undergoing the process can be established, namely

$$\tau \simeq \frac{1}{f} = \frac{2\pi}{\omega} \tag{77}$$

that results substantially independent from the barrier length (for not too long barriers otherwise the evanescent field attenuation makes no interaction possible).

Equation (77) allows us to write the tunneling time, in our case, as

$$\tau \simeq \frac{2\pi}{\omega_{coh}} \tag{78}$$

since, due to the frequency rescaling originated by the coherent dynamics inside CD, the frequency associated to the superradiant field doesn't correspond to the wavelength $\lambda_0 = 2\pi / \omega_0$ but to rescaled value ω_{coh}.

The above considerations allow us to estimate the maximum tunneling "velocity" (namely the group velocity of evanescent waves or, equivalently, of the virtual photons undergoing tunneling) as

$$\begin{aligned} v_{\max} &= \frac{d^*}{\tau} = \\ &= \frac{\omega_{coh}}{\pi}\left[r_{coh} + 10^3\left(\frac{2\pi}{\omega_0}\right)\right] \simeq 2 \cdot 10^3 \left(\frac{\omega_{coh}}{\omega_0}\right) \end{aligned} \tag{79}$$

where, in the last equality, we have used the (1) and noted that $r_{coh} \ll 10^3 \left(\omega_{coh}/\omega_0\right)$. By using (79) we see this maximum velocity is superluminal when

$$\left(\frac{\omega_{coh}}{\omega_0}\right) > 5 \cdot 10^{-4} \tag{80}$$

a condition that can be easily satisfied inside a coherent CD. More generally we can calculate the tunneling velocity as a function of the spatial separation between two CDs, by considering the tunneling time τ given by (78) and its features [8], as

$$v = \frac{\left|\Delta\vec{x}\right|}{\tau} \simeq \frac{\left|\Delta\vec{x}\right|\omega_{coh}}{2\pi} \tag{81}$$

with $\Delta\vec{x} = \vec{x}_{CD_1} - \vec{x}_{CD_2}$ (where $\vec{x}_{CD}$ is the position vector of a CD), showing the tunneling velocity is a function of the coherent domains separation and the frequency of the tuned coherent oscillation of matter-wave and e.m. field inside CDs.

The model just discussed then suggests the existence of new and so far unrecognized type of interaction in matter, originated by the coupling between the coherent evanescent e.m. fields generated inside the CDs when they are "sufficiently" close each other.

This coupling induces an e.m. energy exchange from one CD to another, establishing a sort of mutual communication channel between them that acts like a so-called "evanescent-field coupler".

This phenomenon has been already studied from the standpoint of classical physics and also applied in the construction of many devices, mainly but not only involving the use of optical fibers, just called "evanescent-field-optic-couplers" [2, 13].

On the other hand, from the standpoint of quantum physics, since the evanescent e.m. field is physically equivalent to virtual photons, this energy transfer will correspond to the exchange of tunneling photons between the two coupled CDs, that realizes a nearly instantaneous communication between them.

This is further supported by the non-local feature of evanescent e.m. modes generated inside a barrier that, as shown by Nimtz et al. [8], would be present at the same time all over the tunneling barrier, namely at its beginning and the end simultaneously.

According to the proposed model, the tunneling velocity (corresponding to the overall duration of the process including the spreading time across the barrier) is directly proportional to the CDs mutual distance $\left|\Delta\vec{x}\right|$. Furthermore, by equation (80) we can expect the presence of a spatial region in which the tunneling velocity is superluminal and another in which it is not.

In particular, the minimum value $L_{\min}$ of $\left|\Delta\vec{x}\right|$ required for the superluminal propagation to occur can be easily found by imposing the condition

$$v = \frac{\left|\Delta\vec{x}\right|\omega_{coh}}{2\pi} > 1 \quad \Rightarrow \left|\Delta\vec{x}\right| > \frac{2\pi}{\omega_{coh}} \equiv L_{\min} \tag{82}$$

while its upper limit $L_{\max}$ can be simply assumed, by physical considerations, equal to

$$L_{\max} \sim 2d_p = 2 \cdot 10^3 \left(\frac{2\pi}{\omega_0}\right) \tag{83}$$

since for $d \geq 2d_p$ we can assume there is no evanescent-field coupling.

The set of values of $\left|\Delta\vec{x}\right|$ for which superluminal interaction can take place are then given by

$$\frac{2\pi}{\omega_{coh}} < \left|\Delta\vec{x}\right| < 2 \cdot 10^3 \cdot \frac{2\pi}{\omega_0} \tag{84}$$

We are then in position to explain the mechanism according to which such a coherent (even superluminal) interaction can take place.

When the evanescent e.m. field is generated (through the quantum mechanism previously described) at the boundary of the first CD, it instantaneously spreads across the barrier constituted by the physical space separating it from the second CD.

From the quantum standpoint this corresponds to the tunneling of virtual photons belonging to the evanescent e.m. field through the physical space barrier between the two CDs.

The entire process is view, from the inside of each interacting CD, as having a finite (but extremely short) duration (corresponding to the very high velocity of tunneling photons or, equivalently, of the evanescent e. m. wave, given by (81)).

Virtual photons crossing the first CD interface can then simultaneously "appear" at the other side of the barrier where they can interact with the other CD. A similar process could also occur in the opposite "direction", namely starting from the second CD towards the first one, giving rise to a sort of mutual "communication" between the two CDs.

This process would not be limited to a single couple of nearby CDs but will involve all the CDs placed, in a given volume of matter or space, at a distance $\left|\Delta\vec{x}\right| \leq 2d_p$ from a given CD. In this way every CD included in a sphere of radius $2d_p$ can be virtually considered as the center of another sphere containing some other CDs interacting with it and so on.

Furthermore, such communication process could occur at different "speeds": subluminal in the CDs separation range $2r_{coh} < \left|\Delta\vec{x}\right| < L_{\min}$, superluminal if $L_{\min} < \left|\Delta\vec{x}\right| < 2d_p$.

Finally, for $\left|\Delta\vec{x}\right| > 2d_p$ we can assume there is no interaction due to the rapid "decay" of the evanescent e.m. fields.

It is very important to note that, for a given CD placed in a given volume of matter, all the interactions, occuring through the described "channels", with the other CDs included in the sphere of "radius" $2d_p$, can occur almost simultaneously within the same time interval τ (namely the tunneling time of virtual photons or the spreading time of e.m. evanescent wave through the "barrier").

Furthermore, the consequent interactions between the second and third CD (placed in the sphere or radius $2d_p$, centered on the second CD) and so on will occur within the same time τ, so creating a network of tuned interactions between distant couples of CDs whose distance could be much higher than d_p in this way just realizing the long-range of "*supercoherent*" interaction.

7. Conclusion and Outlook

The transition from PGS to CGS, that leads to the formation of coherent domains in condensed matter, have been analyzed in terms of the short-time behaviour of its quantum coherent and incoherent fluctuations of matter and e.m. fields.

By applying the principle of "least action", a set of differential equations describying the time behaviour of the matter and e.m. fluctuations has been written. By assuming the presence of small amplitude fluctuations, and considering the short-time limit of the solutions of the stationary state (i.e. the matter and e.m. fields describing the CGS), a simplified third-order differential equation for the e.m. field quantum fluctuations has been achieved, whose solutions in the time domain give the frequency spectrum of these fluctuations.

Due to the dynamical stability of CGS, this spectrum results necessarily discrete and it is characterized by three real-value oscillation frequencies, originated from the superposition of the e.m. quantum fluctuations and the coherent e.m. field condensed from quantum vacuum inside the CD and at its neighbourhood (in the form of evanescent field), that physically represent the frequencies of the excited states of CGS.

In the case of small oscillations we have also obtained a simplified solution of the coherent equations, able to give us the values of the discrete energy levels of the excited states of CGS as a function of the coupling constant g (between matter and e.m. field).

This important result allows us to consider every CD like a macroscopic quantum object, characterized by a discret energy spectrum and able to coherently interact with other CDs, according to a dynamics in all similar to that leading to the formation of the single CD but where the role of the elementary quantum-mechanical oscillators is now played by the interacting CDs themselves. By means of such process, we can name "*Supercoherence*", provided that suitable boundary conditions (for density and temperature of the CDs ensemble) are satisfied, every CD is able to coherently couple with every other one up to a distance several times greater than the extension of a single CD, according to a network of long range interactions that tunes the oscillations of several CDs with an e.m. field condensed from ZPF.

We have also proven that CDs act, with regard to such field, like optical waveguides allowing the evanescent tail of coherent e.m. field to spread across the "boundary" of the CD itself, causing the overlap between evanescent fields belonging to different "sufficiently" close CDs and the tunneling of the superradiant (virtual) photons of which they are made through them.

On the other hand, several founded experimental studies have shown that tunneling time of e.m. evanescent waves or virtual photons, through optical barriers of various types, could be characterized by a superluminal velocity of propagation through the barriers, irrespective of the type of barrier or wave frequency and amplitude.

Basing on these results we have finally shown these tunneling photons, originating inside different "close" CDs, can move at superluminal velocity through their respective coherent evanescent e.m. fields, so realizing a novel and so far unrecognized type of physical long-range superfast interaction in matter, directly arising from the coherent dynamics of ZPF.

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Did the Kabbalah Anticipate Heisenberg's Uncertainty Principle?

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Evidence is adduced from the works of Rabbis Shimon Bar Yochai (c. 100-160 CE), Yitzhak Luria (1534-1572) and Moshe Chaim Luzzatto (1707-1746), that Heisenberg's uncertainty principle (1927), a pillar of quantum mechanics, is anticipated in a Kabbalistic construct known as the *Radla* (Unknowable Head). Homologies are demonstrated as they relate to the fabric of reality, the intrinsic incomprehensibility and paradoxical nature of the universe, the translation of indeterminacy into experiential reality, worlds *in potentia*, and the grand scale unicity of the universe. Possible implications of these homologies for modern physics, human knowledge boundaries, free will and prophecy are discussed.

Keywords: Cosmology, Heisenberg's uncertainty principle, Jewish mysticism, Kabbalah, quantum physics, *Radla*

1. Introduction

The last quarter century has witnessed a burgeoning literature relating contemporary science—and in particular, quantum physics—to the Jewish mystical tradition or Kabbalah [1-7]. In some instances, scientists have extracted from the latter metaphors that clarify, add perspective or lend vividness to scientific concepts, many of which hinge on arcane mathematical formulae, that are often counter-intuitive and difficult to convey in ordinary parlance [3]. On a more profound level, and germane to the thesis of this paper, is the possibility that modern physics and ancient mysticism display unprecedented degrees of confluence because both disciplines—one founded on empirical research, the other anti-empirical and revelation-based—may provide legitimate and complimentary insights into the nature of reality.

The current work contributes to this ongoing dialogue by adducing evidence that a particular Kabbalistic construct known as the *Radla* is strikingly parallel to, and may coincide conceptually with, mainstream interpretations of Heisenberg's uncertainty principle (UP). To develop this theme, we begin by briefly outlining the ontology of quantum mechanics (QM), with emphasis on Heisenberg's principle. We then discuss the *Radla* concept in the context of broader Kabbalistic doctrine as elucidated in the classical mystical texts (mainly the *Zohar* and *Etz Chaim*) and the writings of the 18th century scholar, Rabbi (R') Moshe Chaim Luzzatto. In the main body of the work, we juxtapose citations from leading physicists and Kabbalistic sources in an attempt to underscore a concordance of ideas regarding the fundamentals of reality which transcends the radically different lexicons naturally invoked by these disparate disciplines. In the concluding sections, we describe several epistemological ramifications of this juxtaposition and reflect on the value of mining further the interface between physics and metaphysics for the enrichment of both science and religion.

2. Clarifications and Disclaimers

The main objective of this exercise is to test an hypothesis that the UP and attendant quantum formulations of physical reality are intrinsic to the mystical worldview of the Kabbalists. Our goal is not to provide sweeping generalizations concerning perceived parallelisms between Jewish mysticism and contemporary physics. Rather, we focus here on several specific features of the Kabbalah and quantum mechanics (QM), *viz*., the *Radla* and the UP, which we believe may unite both disciplines within a common conceptual framework. Nor do we argue that prescient insight into the underpinnings of physical reality is unique to the Jewish mystical tradition. Indeed, non-Jewish metaphysical systems, e.g. Plotinus's *Enneads* 5 and various Eastern philosophies, contain motifs that resonate with current scientific thinking. To control for potential biases inherent in the translation of texts evinced to buttress our assertions, critical statements from the Kabbalistic literature are reproduced in their original Hebrew and readers are encouraged to render their own interpretations. The Kabbalah frequently employs the term, "light" (*Ohr*, in Hebrew) metaphorically to connote spiritual forces which emanate from, and mediate the Will of, the Creator. When used in this context throughout the manuscript, "Light" is capitalized to distinguish it from conventional, physical light.

3. Quantum Mechanics

3.1. *Historical Perspective*

QM is an enormously successful branch of physics that builds upon and transcends classical (Newtonian) notions of physical existence [8, 9]. Many have identified its origins with the discovery of "blackbody radiation" (the delivery of energy in discrete packets, or "quanta") by Max Planck in 1900. A quantum mechanical understanding of matter, energy, space and time unfolded apace with the seminal contributions of Ernest Rutherford, Niels Bohr, Albert Einstein, Erwin Schrodinger and others in the first half of the 20th century. During the last sixty years, input from pioneers such as Murray Gell-Mann, Richard Feynman, Steven Weinberg and Eugene Wigner have enabled further refinements of quantum theory, a marriage of particle physics and cosmology, and the advent of numerous 'disruptive' technologies based on this knowledge. Interested readers are referred elsewhere for further details concerning the history of quantum physics and timeline of key discoveries which have punctuated the field [8, 9].

The remarkable accomplishments of QM has led to the commonly expressed belief that the fundamental forces governing physical existence are now largely understood and that future efforts in the field will mainly be directed towards achieving more precise measurements of the phenomena already disclosed. Notwithstanding the justification for this claim, the latter in and of itself is no trivial task. As discussed below, the very act of measurement, when conducted on an infinitesimally small quantum scale, necessarily perturbs, and is inextricably linked with, the system undergoing observation.

3.2. *The Heisenberg Uncertainty Principle*

In 1927, Werner Heisenberg published his groundbreaking paper on the "uncertainty principle" in *Zeitschrift fur Physik* [10]. According to the UP, paired physical properties of a system cannot both be measured to arbitrary precision; the more accurately one property is known, the less precisely the other can be known. Importantly, this imprecision is not contingent upon the skills of the observer or the resolution of the measuring apparatus, but is an inherent attribute of physical systems as dictated by the equations of QM. While it is true that the very act of measurement affects the physical properties of particles (e.g., its position or momentum), the UP makes a more profound claim—that we *cannot* know, as a matter of principle, the present in all its details [10].

In classical physics, it is theoretically possible to ascertain the position and momentum of every particle in the universe and thereby accurately determine the future. In contemporary quantum physics, it is fundamentally impossible to predict future events because one can never attain full knowledge of the position and momentum of even a *single* particle. In the standard (Copenhagen) interpretation of QM (e.g., the results of the classical "2-slit experiment" [8]), every possible outcome for an event, represented mathematically as a statistical wavefunction, exists in the unobserved state. The act of observation elicits a "collapse of the wavefunction," whereby one of these many potential outcomes is "selected" as the reality actually experienced.

The Copenhagen interpretation of QM was bolstered after attempts to refute it failed. Examples of such investigations include the "*gedanken* (thought)" experiments of the famous Einstein-Bohr debate of the 1920's [11] and, more tellingly, resolution of the EPR (Einstein-Podolsky-Rosen) paradox [12] which resulted from repeated experimental violation (1972-1982) of "Bell's inequality" (1964) in support of quantum theory. Moreover, the results of these experiments (especially those of Alain Aspect in 1982) implied that (i) all particles emerging from the Big Bang singularity maintain an indefinite 'connectedness' with one another, (ii) each particle therefore 'knows' about the existence of every other particle, and (iii) due to preserved complementarity intrinsic to the Copenhagen interpretation, the properties of one particle (e.g. position, momentum, spin, etc.) change instantaneously and commensurate with changes in a 'partner' particle regardless of the extent of their physical separation (Einstein's "spooky action at a distance"). For the latter to arise by classical causal interaction, information would need to pass from particle A to particle B at impossible supraluminal speeds. Quantum theory dictates that the particles' shared history forever "locks" them in a reciprocal dance ("quantum entanglement") that does not obligate the transmission of new information between them ("acausality").

In this article, we will refer primarily to the classical Copenhagen interpretation of QM, although there exist competing variations of this interpretation (e.g., Bohr vs. Von Neumann) as well as several non-Copenhagen formulations. Prominent among the latter are Heisenberg's Ghost Reality, Einstein's Neo-Realism, David Bohm's Undivided Wholeness (cited in section 5.6), David Finkelstein's New Quantum Logic, Hugh Everett's Many-Worlds interpretation, and Information Theory [13]. As one illustration of a distinctly non-Copenhagen interpretation, Everett's model of QM states that all statistically feasible outcomes actually do become manifest in some version of reality. In this model, observations do not "collapse" the wavefunction into a singular reality but generate a multiverse of innumerable parallel, non-intersecting worlds [14].

4. The Kabbalah

The Kabbalah teaches about a hierarchy of interlocking spiritual domains which progressively 'descend' in holiness, beginning with the unfathomable Godhead, traversing fractal-like through a system of 'coarsening' immaterial worlds, and culminating in the Creation of the physical universe. In addition to elaborating an ontogeny for all existence, the Kabbalah explains, often allegorically, the hidden ways by which God continuously guides the unfolding universe and the dynamic systems that are in place to interact with Nature and Man. As depicted in the Kabbalah, the universe is governed by a complex system of "Lights" or "forces" which, through myriad interactions, provoke chain reactions that ultimately impact humans and their physical surroundings [15]. Central to the Kabbalistic viewpoint is the absolute unity of the Creation at its core, with all semblances of separateness and differentiation becoming apparent only after "filtration" of the one Infinite Light (*Ohr Ein Sof; Heb.)* through the various *Sefirot* (defined below).

The primary Kabbalistic texts we have consulted are the *Zohar* (R' Shimon Bar-Yochai; c. 100-160 CE), the teachings of R' Yitzhak Luria (the *Arizal*; 1534-1572) as transmitted by his student R' Chaim Vital (1543-1620), and the works of R' Moshe Chaim Luzzatto (*Ramchal*; 1707-1746). The *Arizal* elaborated all the main concepts of the Kabbalah and provided innovative explanations of the *Sefirot* and *Partzufim* ("configurations" – see below). The corpus *Etz Chaim*, compiled by R' Vital, encompasses the teachings of the *Arizal* and remains the major reference text of Lurianic Kabbalah. In eighteenth century Europe, *Ramchal* greatly facilitated the contemporary understanding of the Kabbalah by re-organizing and explicating many cryptic passages of the *Zohar* and *Etz Chaim* [15].

4.1. *The Sefirot and Partzufim*

Although the Light (emanation) of the Infinite is a unified whole, each of ten *Sefirot* (pl.) represents a "filter" that holds and transforms a certain part of this Light into a particular force, attribute, or action. Each *Sefirah* (sing.) is composed of a vessel (*Keli)* which holds its part of Light (*Ohr)*. There is no differentiation of the *Ohr* within the *Keli* itself, as it is part and parcel of the original, undivided Light; differences emerge from the particularity or position of the *Sefirah's Keli*. According to the Kabbalah, arrangements of ten *Sefirot* are the blueprint of all things created, as everything that exists is ultimately comprised of these ten forces.

A *Partzuf* (face, visage, or countenance) is a configuration of one or more *Sefirot* acting in coordination or towards a defined purpose. Some *Partzufim* are masculine, while others are feminine. The masculine correspond to kindness/expansiveness—*Chesed*— and are manifestations of the Divine name of *MaH* (value of 45, derived from the numerical rendering – *Gematriyah* in Hebrew - of a specific configuration of the primary 4-letter name of God or Tetragrammaton). The feminine correspond to *Gevurah*—connoting a combination of rigor and limitation—and are manifestations of the name of *BaN* (numerical value of 52). Different permutations of the unions (combinations) of *MaH* and *BaN* (*Chesed* and *Gevurah*) are responsible for bringing into existence and guiding the Creation in all its particulars. The *Partzufim* exist in a dynamic state of action, illumination, and interaction referred to as *Tikkunim* of the *Partzufim.* The *Tikkunim* transduce the Higher Will into specific effects for the guidance of the universe; certain manifestations fluctuate with time and are influenced by the actions and moral behavior of Man.

The six main *Partzufim* (in order of spiritual "descent") are:

Atik Yomin—Ancient of Days
Arich Anpin—Long Countenance
Abba—Father
Imma—Mother
Ze'ir Anpin—Small Countenance
Nukva—Feminine

4.2. *The Radla*

The configuration *Atik Yomin* is superior to all the configurations and is itself composed of ten *Sefirot*. Its manifestation of the name *MaH* (45) corresponds to its masculine principle; its manifestation of the name of *BaN* (52) corresponds to its feminine principle. It is the most concealed of the configurations, the leading force and the source of all the others.

The first three *Sefirot* of the *Nukva/*feminine aspect of the *Partzuf, Atik Yomin* is the guiding force for all the "lower" *Partzufim* and, inevitably, the physical Cosmos. Together, these three *Sefirot* constitute the *Radla*— רישא דלא אתידע - the "Unknowable Head." The term is first mentioned in the Zohar [16] and elaborated in the writings of the *Ramchal* (see Section 5). The *Radla* encompasses all possible realities; everything that came or will come into existence has its roots in it. The *Radla* is called 'unknowable' because the outcomes of its actions (in unfolding the Creation) are in no way graspable by our understanding or imagination. All possibilities exist within her, but in our perceived reality, they manifest themselves in all manner of uncertainty in which what appears to be is and is not at the same time. These counterintuitive notions of multiple and contrary realities are inherently paradoxical and unique to the *Radla*.

5. The Radla and Quantum Uncertainty

In this section, key descriptions of quantum uncertainty and the *Radla* are juxtaposed in order to underscore their similarity of meaning. The sources regarding the latter are mostly *Ramchal*'s writings because of his tendency to elucidate these concepts in a more systematic fashion than earlier sources.

In drawing these comparisons, we are sensitive to a distinction between two categories of uncertainty: 'epistemological uncertainty' and 'ontological uncertainty'. The former, which captures the term's ordinary usage, focuses on the inability of human beings to know the ultimate reality due to limitations of their cognitive faculties; the latter connotes the inherent unknowability of ultimate reality due to its intrinsic character. Although many statements in the Kabbalistic literature reflect the thesis of epistemological uncertainty, several of the passages we cite allude to ontological uncertainty of the sort encountered in contemporary physics.

5.1. *The Fabric of Reality*

Leading physicists have underscored the inadequacy of classical physics in fully explaining physical existence, the quantum worldview, and the essential significance of the "Uncertainty Principle." Kabbalistic sources similarly implicate the *Radla* construct as the ultimate source or progenitor of physical reality.

A. Quantum Physics:

"*The great extension of our experience in recent years has brought light to the insufficiency of our simple mechanical conceptions and, as a consequence, has shaken the foundation on which the customary interpretation of observation was based* [17]."

"*Uncertainty is perhaps the central feature of quantum theory* [18]."

"*The quantum is the crack in the armor that covers the secret of existence* [19]."

"*I would conclude that extra dimensions really exist. They're part of nature* [20]."

B. The Kabbalah:

"*In the Radla is the secret of the union of MaH and BaN at their highest level, to become the origin of the total governance [of the Universe]* [21]."

(רדל"א הוא סוד חיבורי מ"ה וב"ן למעלה מקור לכל ההנהגה.)

"[The *Radla*] *is the source; from it issues forth all uncertainty at the outset* [22]."

(שהיא הראשונה, שבה נולדים הספיקות בתחלה.)

5.2. *The Intrinsically Incomprehensible Universe*

The overarching opinion of leading quantum physicists is that essential spacetime and the fabric of the Universe are unknowable *by their very nature*, and not on account of the imprecision of our measuring devices. Similarly, the Kabbalah in places indicates that the workings of the *Radla* are fundamentally opaque to human reason, not because of any limitations in our understanding *per se*, but as a consequence of the *Radla*'s inherent unknowability. The extent to which explication of this aspect of unknowability remained consistent across the centuries, spanning the writings of the *Zohar*, the *Arizal*, and the *Ramchal*, attests to the authors' conviction and fidelity to the *Zohar*'s intended meaning.

A. Quantum Physics:

"*Science cannot solve the ultimate mystery of nature* [23]."

"*We cannot know, as a matter of principle, the present in all its details* [10]."

"*In more than forty years, physicists have not been able to provide a clear metaphysical model [of quantum reality*] [24]."

"*It is safe to say that nobody understands quantum mechanics* [25]."

"*The creation lies outside the scope of the known laws of physics* [26]."

"*The very concept of spacetime…isn't precisely defined* [27]."

B. The Kabbalah:

"Ancient of all the ancients, concealed of all the concealed, acting and not acting, it [*Radla*] acts to sustain all. Not acting [from our perspective] because it is not in any way graspable [28]."

(עתיקא דכל עתיקין, סתימא דכל סתימין, אתתקן ולא אתתקן, אתתקן בגין לקיימא כלא, ולא אתתקן בגין דלא שכיח.)

"*It [the Radla] is called the superior, concealed wisdom; a wisdom that may not be graspable or manifest; no one can understand it* [28]."

(ואקרי מוחא עלאה, מוחא סתימא, מוחא דשכיך ושקיט, ולית דידע ליה.)

"*Atika Kadisha*, *most concealed of all that is concealed, Head of all heads [the construct "above" Arich in all Worlds], a head which is not a beginning [there exist still higher realities than the Radla], presently not understandable and will never be understood* [28]."

(עתיקא קדישא סתימא דכל סתימין, רישא דכל רישא רישא דלאו רישא, ולא ידע, ולא אתידע.)

"*It is called Radla because of all the uncertainties that are in it* [29]."

(נקרא רדל"א ע"ש הספיקות שיש בה.)

"*We cannot imagine or know anything [of the Radla]. This is the concept of 'unknowable' (by its very nature, and not merely unknown?)* [30]."

(אינם מושגים ונודעים כלל, זהו ענין דלא אתידע.)

"*All of this [the interaction of MaH and BaN in Radla] is the matter of uncertainty* [30]."

(כל זה הוא ענין הספיקות.)

5.3. *Translation of Indeterminacy into Experiential Reality*

Quantum mechanics and the Kabbalah concur that the principles governing the existence of our universe at its most primal level operate according to "laws" that differ radically from those mediating the day-to-day reality we experience. This paradoxical "disconnect" between the micro- and macro-worlds is amply acknowledged as the interface between quantum uncertainty and Newtonian mechanics in physics, and in the relation of the *Radla* to *Arich Anpin* and "lower" manifestations within the Kabbalah's hierarchical cosmology. Both disciplines strive to delineate precisely what occurs at this critical interface, aptly described by R' Moshe Schatz as the enigmatic "great bridge" between the quantum and familiar worlds [31].

In the final analysis, contemporary physics and the Kabbalah intimate that a thorough understanding of the mechanism responsible for transducing quantum/*Radla* indeterminacy into experiential reality may never be achievable by dint of the former's inherent unknowability.

A. Quantum Physics:

"*Everything we call real is made of things that cannot be regarded as real* [32]."

"*All we know about [the world] are the results of experiments [observations];" i.e. we have no knowledge about the complete state of even a single particle in the quantum realm which gives rise to the reality we perceive* [8].

B. The Kabbalah:

"*The seven lower Sefirot [of Atik Yomin] are enclothed in the configuration Arich Anpin and, as I indicated, are expressed within [the governance of] time* [33]." *(Arich Anpin/Atik Yomin* is the transition point between the unknowable thought of the Creator and the familiar concept of time. Likewise, quantum uncertainty is translated into familiar spacetime.

(כי הז"ת המתפשטין בא"א, כבר אמרתי שהם לפי הנהגת הזמן.)

"*The Radla is above Arich Anpin and is the secret of the union of the eternal and temporal guidance* [33]."

(רדל"א היא עומדת למעלה מא"א, והוא סוד חיבור הנהגה הנצחיות בהנהגת הזמן.)

"*And from what occurs in it [the Radla], emanates the main guidance conveyed by the Partzufim. From the Radla to its outcomes* [in our experiential reality], *we can grasp and understand nothing* [30]."

(ולפי מה שמתנהג בה - נולד הנהגה גדולה בפרצופים אך בין היא ובין תולדותיה אינם מושגים ונודעים כלל.)

"*The origin of the governance according to the amalgamations of MaH and BaN is in the Radla. And according to this governance, the main governance of the Partzufim arises [30].*" (*This and the next quotation state that events—combinations of MaH and BaN—within the unknowable Radla give rise to the familiar emanations of the "lower" Partzufim*).

(מקום ההנהגה לפי ענין התחברות של מ"ה עם ב"ן הוא ברישא דלא אתידע. ולפי מה שמתנהג בה -- נולד הנהגה גדולה בפרצופים.)

"*All actions performed in this world come about according to these amalgamations [of MaH and BaN]. Nothing that is not rooted there [in the Radla] can occur* [34]." (Here we have a homology to the translation of quantum uncertainty, which is represented as a probability curve, into defined realities.)

(לפי כל החיבורים האלה נמצאים כל המעשים שנעשו ושנעשים בעולם. כי לא יהיה מה שלא הושרש כאן.)

5.4. *Worlds in Potentia*

From the Copenhagen (and other) interpretations of QM arises the counterintuitive notion that all possible outcomes of an event, as determined by the statistical wavefunction, indeed exist as potential states capable of exerting detectable influences within the familiar world [8]. This spectacular concept is similarly embodied in Kabbalistic accounts of the *Radla*.

A. Quantum Physics:

"*In QM, every possible outcome for an event exists in the unobserved state prior to collapse of the wavefunction* [8]."

B. The Kabbalah:

"*Every combination of MaH and BaN [Reality] that could possibly be found was, in fact, made* [30]."

(כל מיני החיבורים שהיה אפשר להמצא בין מ"ה וב"ן - באמת נעשו.)

5.5. *The Inherently Paradoxical Universe*

The tenets of QM and the *Radla* invoke a definition of "paradox" that deviates from other conventional usages of the term. In general, we regard paradox as an incomplete comprehension of an event or state. We assume that the paradox would spontaneously dissipate upon disclosure of all its relevant components, relying on an intuition that Nature is inherently rational. Both QM and the Kabbalah teach that at its deepest level, the observable universe obeys laws that are *fundamentally* paradoxical. Far from merely representing a manifestation of the incompleteness of our knowledge, paradox is the warp and woof of physical reality.

A. Quantum Physics:

"*It's not that we can't simultaneously specify the position and motion of an electron, but that it does not have a simultaneous specific position and motion* [35]." (According to the Copenhagen interpretation of QM, the superposition of states comprises many possible, even mutually-exclusive outcomes, e.g., a cat in a box that is both dead and alive in the famous Schrodinger thought experiment; or a single (unobserved) photon that passes simultaneously through slit A and B in the 2-slit experiment. Each time the cat (or photon) is observed, the wavefunction collapses, with repeated observations yielding one result or another in seemingly random order. Although counter-intuitive, the "real" (macro-) world as we perceive devolves from this quantum uncertainty [9, 36]).

B. The Kabbalah:

"*There are combinations [outcomes of MaH and BaN] which are opposites; still, both are there, because Partzufim [which transmit the outcomes of the Radla] are constructed in that way and from these two [opposites] derive the qualities of the Partzufim. According to the dictates above [in the Radla], actions* *are carried out by the Partzufim; however, this is in no way evident or comprehensible* [30]."

(ויש חיבורים הפכיים, ואף על פי כן שניהם נמצאים, כי הפרצופים מורכבים כך ומשניהם נמצאים איכויות בפרצופים. ולפי שליטתם למעלה - כך נעשה מה שנעשה בפרצופים, אך אינו מושג ונראה כלל.)

"*This Head [Radla], from what we understand of it, causes all the uncertainties. One moment it appears that [the outcome is] one thing, in another moment it looks like something else… If we look into this matter more deeply, it appears not this way, but in a changed manner*" [22].

(מה שהרישא הזאת עצמה, כפי מה שמשיגים בה, גורמת הספיקות האלה כי פעם אחת נראה שיש בה כך, ופעם אחת יש בה כך..... ואם מסתכלים באותו הענין יותר - נראה שאינו כך, אלא בדרך אחר מתחלף ממנו.)

"*These uncertainties [of the Radla] are unlike the uncertainties of the [familiar] world. In the latter, we may be uncertain whether a thing exists or not; whereas, in truth, all things perceived as 'uncertain' are present in her* [the *Radla*] [22]." (This description of the inherent paradox of reality may be the most supportive Kabbalistic statement in favor of ontological, as opposed to epistemological, uncertainty.)

(שאין הספיקות ההם כמו ספיקות דעלמא, שאנו בספק אם יש דבר אחד, או אם אינו, אלא האמת הוא, כל מה שאנו מזכירים בספיקות - כל אותם הדברים ישנם באמת בה)

5.6. *Unicity on a Grand Scale*

In virtually all of its iterations over the many centuries, a "prime directive" of the Kabbalah remains attestation of the absolute oneness of the Creator and His Creation in the face of apparent separateness and individuation, with the *Radla* representing the crucial nexus between the whole and its parts. Contemporary quantum physics similarly points to a blatant interconnectedness of all forces and particles comprising the observable universe.

A. Quantum Physics:

"*Quantum physics reveals a basic oneness of the universe* [24]."

"*The world acts more like a single indivisible unit, in which even the 'intrinsic' nature of each part (wave or particle) depends … on its relationship to its surroundings* [37]." (Experiments refuting challenges to QM, especially those of Alain Aspect in 1982, imply that particles sharing common origins maintain an indefinite "connectedness" with one another notwithstanding their separation in space and time [8]).

"*The inseparable quantum interconnectedness of the whole universe is the fundamental reality, and [the] relatively independent behaving parts are merely particular and contingent forms within this whole* [38]."

B. The Kabbalah:

"*Everything is connected to it [the Radla] and it is connected to all; it encompasses all* [28]."

(דהא כלא ביה מתדבקן והוא מתדבק בכלא, הוא כלא.)

"*All reality is [fundamentally] governed by a single Light [force]. The [forces comprised by the] Radla is [are] in actuality a part of this encompassing Light*" [22].

(שכל ההנהגה היא אור אחד. והנה רדל"א הוא מין אור אחד

"*So that the entire guidance [of the Universe] is contained within each of its parts*" [34].

(להיות כל ההנהגה בכל חלק.)

6. Implications

Until the period known as the *Haskalah* (Jewish Emancipation; c. 1770-1880) and for reasons of religion, language, politics and demographics, the mainstream world and Jewish literatures were, in large measure, distinct and non-intersecting. Early Biblical literature notwithstanding, the West (including many secular Jews) to this day remains unaware of a considerable body of philosophical/metaphysical thought contained within a vast assembly of (mainly Hebrew and Aramaic) extra-Biblical Jewish manuscripts.

In this article, we attempted to show that fundamental tenets of 20th century quantum physics are reflected in antecedent esoteric Jewish writings. We specifically addressed a novel hypothesis that manifestations of the *Radla*, as elucidated in ancient, medieval and 18th century Kabbalistic texts, are strikingly similar, if not identical, to concepts embodied by Heisenberg's UP. To garner support for this thesis, we first presented a brief scientific ontology of QM, with emphasis on the formulation of the UP. After introducing some general principles of the Kabbalah germane to this topic, we explicated the *Radla* construct and proceeded to illustrate an homology between the latter and the UP by drawing on published statements from leading 20th century physicists and relevant Kabbalistic sources. We documented that in both systems, the reality of the familiar macro-world is entirely contingent upon (and flows from) more fundamental and highly counterintuitive phenomena. Within these implicate realms, there is no causality as we intuit it, but rather a non-deterministic universe in which multiple, even mutually exclusive, possibilities co-exist for every possible outcome or observation. Virtually nothing is known concerning the mechanism whereby "events" in the quantum/*Radla* domain translate into phenomena of the experiential world. Seemingly random, uncaused fluctuations inherent to this realm limit what can be predicted about all future events. The *Zohar* writes that the *Radla* "is not attainable by wisdom or knowledge; a Head which is not understandable and will never be understood." These words are an equally apt description of the quantum world. Precisely how the ancients invoked knowledge which so closely approximates the empirical findings of contemporary physics is an open question. The Kabbalists certainly did not conduct quantum experiments. Many Orthodox Jews assume that they derived these truths from contemplation of the Torah's esoteric tradition (termed *Sod* or *Nistar* in Hebrew) which, as part of the Oral Law, was transmitted via Moses at Sinai or by direct mystical experience (*Ruach Ha'Kodesh*).

To the extent that key aspects of reality addressed by the *Radla* and the UP are one and the same, it may be possible to integrate Kabbalah and QM within a common and mutually-informative conceptual framework. Fig. 1 illustrates how such a synthesis might proceed. In this schema, the evolution and boundaries of human insight into the fabric and workings of the universe are represented by a set of three stacked cubes: a small Classical (Newtonian) box contained within an intermediate Quantum box, which, in turn, is encompassed by a large Kabbalah box. The perimeters of the cubes indicate the theoretical limits of fundamental knowledge about the universe attainable by each discipline. In the classical (pre-quantum) era, Newtonian physics sufficed to resolve, with relative precision, numerous queries concerning the mechanical operations of the universe (line 1). Deeper, more nuanced insights into the nature of reality could only be roughly approximated by (or were entirely opaque to) Newtonian thought and required the advent of quantum theory for their satisfactory resolution (line 2). The tenets of QM dictate that it is impossible to predict future events with any degree of certainty because one can never attain full knowledge of the position and momentum of even a single particle. But this statement may be true only within the Quantum box which, restricted by the UP, establishes a barrier beyond which science cannot probe. By contrast, there are named constructs, such as the masculine aspect of *Atik Yomin* (mentioned above) and *Adam Kadmon*, which are situated *"above"* the *Radla* in the Kabbalistic hierarchy. These Divine manifestations lie beyond the reach of modern science (bounded as it were by the UP/*Radla*) but may potentially still be available to human reason (line 3).

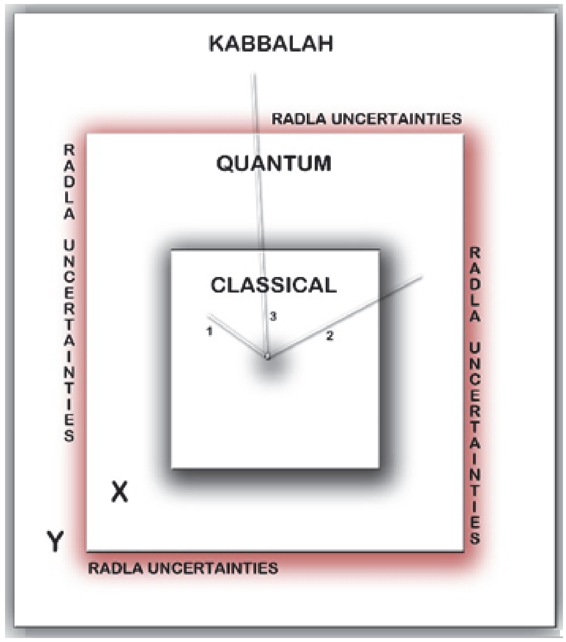

Figure 1. A model depicting boundaries of human insight into the fabric and workings of the universe imposed by Newtonian (classical) physics, quantum mechanics and the Kabbalah. See text for details. From Afilalo R, Schipper HM. The Torah u-Madda Journal (16: 134-152, 2012-13), with permission.

The Judaeo-Christian tradition delineates a pathway whereby realms beyond the *Radla* may be accessed: prophecy (*Nevu'ah*). On rare occasions, the Creator confers upon select persons the capacity for prophetic vision. From the current perspective, one may conclude that, in these instances, God wills individual minds to transcend spacetime and the indeterminacy of the *Radla*/UP (point Y in Fig. 1) in order to glimpse the singular reality of the Divine plan. This would entail, of necessity, the suspension of the randomness of quantum uncertainty for as long as the *Radla* barrier is rendered permeable to the prophet's thought. Scripture suggests further (e.g., Num. 12:6-8) that this anomalous peek behind the *Radla* curtain and the ensuing awareness of Divine Intent varies in duration and lucidity commensurate with the stature of the individual prophet.

Consider the Torah's account of the story of Balaam (Num. 22:2-25:25). The Moabite king Balak is aware of Balaam's prowess as a master conjurer and employs him to curse the nation of Israel. In accord with the current thesis, when not receiving *Nevu'ah*, Balaam's consciousness is confined by the *Radla* (point X in Fig. 1), on par with the rest of humanity. As such, his option to curse (or bless) Israel may be exercised as he sees fit. Not so when Balaam is made recipient of Divine prophecy; throughout the narrative the Torah indicates (and Balaam himself acknowledges) that his power to choose a course of action is abrogated for the duration of the prophetic experience, and his behavior is compelled to conform to the Divine plan. Mechanistically, permeation of the *Radla* membrane (point Y in Fig. 1), when it is enabling prophetic instruction, interfaces with and subjugates Balaam's will to the singular design accruing from a Divinely-inspired "collapse of the universal wavefunction"—a state incompatible with autonomy and personal agendas.

7. Conclusions

We have attempted in this essay to show that the operations of the *Radla* as described in the Zohar and in the major works of the *Arizal, Ramchal*, and other Kabbalistic luminaries bear suggestive and thought-provoking similarities to Heisenberg's UP, a pillar of quantum mechanics. Viewing these homologies in juxtaposition, we illustrated how they may inform our understanding of several key cosmological principles, including the very fabric of Creation, the translation of indeterminacy into experiential reality and the intrinsically paradoxical, but ultimately unified, nature of the physical universe. Possible implications of the *Radla*/QM paradigm for epistemology and prophecy were considered.

Two highly counterintuitive systems—one rooted in ancient mystical thought and the other in cutting-edge scientific experimentation—exhibit a striking convergence in their description of certain fundamental aspects of existence. Such intriguing parallelisms have inspired the physicist, Joel Primack and the science historian, Nancy Ellen Abrams to proclaim the following:

"*We will turn to Kabbalah, medieval Jewish mysticism, as a possible source of language and metaphor, because certain Kabbalistic concepts fit our picture amazingly well. Moreover, Kabbalah's cosmology gave meaning and purpose to the everyday lives of its adherents, which we hope may become possible with the scientific cosmology emerging today*" [3].

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Kabbalah and the Physics of David Bohm

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The development of quantum physics over the last century has stimulated a *rapprochement* between contemporary science and the Jewish mystical tradition or Kabbalah. Here, evidence is adduced from the classical mystical texts (e.g. the *Zohar*) and the works of leading Kabbalists of the16th-18th centuries supporting an hypothesis that pivotal concepts elaborated by the influential physicist, David Bohm (1917-1992) are intrinsic to the Jewish mystical worldview. Specifically, we will demonstrate striking parallelisms between the "implicate-explicate orders" and "holographic universe" of Bohmian mechanics and the Kabbalistic principles of *Hitlabshut* (ensheathment), *Hitkallelut* (interinclusion) and *Hitkashrut* (interpenetration). Possible implications of these homologies for epistemology, religion and modern physics are discussed.

Keywords: Bohm, Consciousness, Enfoldment, Explicate order, Fractal, Hidden variables, Holographic universe, Implicate order, Interinclusion, Interpenetration, Jewish mysticism, Kabbalah, Quantum physics, Unfoldment

1. Introduction

The last quarter century has witnessed a burgeoning literature relating contemporary science—and in particular, quantum physics—to the Jewish mystical tradition or Kabbalah [1-7]. In some instances, scientists have extracted from the latter metaphors that clarify, add perspective or lend vividness to scientific concepts, many of which hinge on arcane mathematical formulae, which are often counter-intuitive and difficult to convey in ordinary parlance [3]. On a more profound level, and germane to the thesis of this paper, is the possibility that modern physics and ancient mysticism display unprecedented degrees of confluence because both disciplines—one founded on empirical research, the other anti-empirical and revelation-based—may provide legitimate and complimentary insights into the nature of reality.

The current work contributes to this ongoing dialogue by bringing to light several remarkable parallelisms between the Kabbalah and the science of the 20th century physicist, David Bohm. To develop this theme, we begin by briefly recapitulating the ontology of quantum mechanics (QM), with emphasis on the contributions of Bohm. We then discuss the concepts of *Hitlabshut*, *Hitkallelut* and *Hitkashrut* in the context of broader Kabbalistic doctrine as elucidated in the classical mystical texts (mainly the *Zohar* and *Etz Chaim*) and the writings of Rabbi (R') Moshe Chaim Luzzatto (1707-1746) and R' Shalom Sharabi (1720-1777). We attempt to demonstrate, notwithstanding the radically different lexicons naturally invoked by these disciplines, a concordance of perspective between the Kabbalistic ideas of *Hitlabshut*, *Hitkallelut* and *Hitkashrut* (defined below) and the 'implicate order' and 'holographic universe' of David Bohm (1917-1992). We conclude by reflecting on the value of such exercises for the enrichment of both science and religion.

2. Clarifications and Disclaimers

The main objective of this exercise is to test an hypothesis that features central to the physics of David Bohm are intrinsic to the mystical worldview of the Kabbalists. Our goal is not to provide sweeping generalizations concerning perceived similarities between Jewish mysticism and contemporary physics. Rather, we focus here on several specific aspects of Bohmian mechanics (the 'implicate order' and 'holographic universe') and the Kabbalah (*Hitlabshut*, *Hitkallelut* and *Hitkashrut*) which we believe may cohere both disciplines within a common conceptual framework. Nor do we argue that prescient insight into the underpinnings of physical reality is unique to the Jewish mystical tradition. Indeed, non-Jewish metaphysical systems, e.g. Plotinus's *Enneads* 5, the writings of Thomas Aquinas and various Eastern philosophies, contain motifs that resonate with current scientific thinking. The Kabbalah frequently employs the term, "light" (*Ohr*, in Hebrew) metaphorically to connote spiritual forces which emanate from, and mediate the Will of, the Creator.

Table 1

Abba: Father; a *Partzuf*

Adam Kadmon (A"K): Primordial Man; a World

Arich Anpin (A"A): Long Countenance; *a Partzuf*

Asiyah: the World of Action

Atik Yomin (A"Y): Ancient of Days; a *Partzuf*

Atzilut: World of Emanation

B'chira: free will

Binah: Understanding; *a Sefirah*

Briah: the World of Creation

Chesed: Lovingkindness/Expansiveness; *a Sefirah*

Chalal: empty "space" (also *Makom Panoy)*

Chochmah: Wisdom; a *Sefirah*

Da'at: Knowledge; a *Sefirah*

Ein Sof: the Infinite (Godhead)

Etz Ha'chaim: Tree of Life; Kabbalistic superstructure

Gematriyah: sum of numerical values of letters comprising a Hebrew word or phrase

Gevurah: Strength/Restriction; a *Sefirah*

Gilui: revelation

Havdalah: concluding Sabbath blessings (Jewish liturgy)

He'elam: concealment

Hitkallelut: interinclusion

Hitkashrut: interpenetration

Hitlabshut: enclothment; ensheathment; overlap

Hod: Splendor; a *Sefirah*

Imma: Mother; a *Partzuf*

Kav: line or ray (of the *Ohr Ein Sof*)

Keli: "vessel" of a *Sefirah* that holds its share of *Ohr* (Light)

Keter: Crown/Divine Will; a *Sefirah*

Klal: whole/wholeness; generality

L'vush: "clothing"

Makom Panoy: empty "space" (also *Chalal*)

When used in this context throughout the manuscript, "Light" is capitalized to distinguish it from conventional, physical light. Similar uppercase lettering or italics is employed for "Space", "Time", "Before", "After", "Above" "Below", etc. when purely metaphysical constructs best conveyed by such terms are intended. Key passages cited from the Hebrew and Aramaic literatures are reproduced (in transliteration) to allow interested readers to render their own interpretations. A glossary of the relevant Kabbalistic terms is provided in Table 1. The author was granted a *Heter* (Rabbinic assent) from the *Posek* (adjudicator), Rabbi Ephraim Goldstein (Brooklyn, NY) to proceed with this initiative.

Table 1

Malchut: Kingship; a *Sefirah*

Mashpiah: donor; connotes Masculine influence

Midrashic: homiletic

Mispar Katan: numerical diminutive of a *Gematriyah*

Mitzvah (Mitzvot, pl.*)*: positive deed

Miyut Ha'yareach: diminishment of the Moon

Mochin: "brains" (*Sefirot* of a higher *Partzuf* which 'animate/control' a lower *Partzuf*)

Netzach: Victory/Eternity; a *Sefirah*

Nukva: Feminine; a *Partzuf*

Ohr: Light (metaphysical)

Oivi: "horizontality", often implying 'ascent' into greater wholeness

Orech: "verticality", often implying 'descent' into greater separateness

Partzuf (Partzufim, pl.*):* face/visage/countenance; a configuration of *Sefirot*

Pirud: divisiveness/separateness

Prat (Pratim; pl.*):* part/particular

Raisha D'Lo Ityadah (*Radla*)*:* The Unknowable Head (aspect of *A"Y*)

Reshimu: residue (of the *Ohr Ein Sof*)

Seder Hishtalshelut: Causal hierarchy within the Kabbalistic superstructure

Sefirah (*Sefirot*, pl.): metaphysical Force or Attribute of the *Ein Sof*

Shlaymut: wholeness

Tiferet: Beauty/Harmony/Truth; a *Sefirah*

Tikkun (Tikkunim, pl.*):* rectification

Tzimtzum: retraction (of the *Ohr Ein Sof*)

Yesod: Foundation; a *Sefirah*

Yetzirah: the World of Formation

Ze'ir Anpin (Z"A): Small Countenance; a *Partzuf*

Zivug: union

3. Quantum Physics

An historical account of key developments in quantum physics, with particular emphasis on Heisenberg's Uncertainty Principle (UP), was previously published in the *Torah u-Madda Journal* [8]. Quantum mechanics (QM) is an enormously insightful branch of physics that builds upon and transcends classical (Newtonian) notions of material existence [9, 10]. Many have identified the origins of QM with the discovery of "blackbody radiation" (the delivery of energy in discrete packets, or "quanta") by Max Planck in 1900. A quantum mechanical understanding of matter, energy, space and time unfolded apace with the seminal contributions of Ernest Rutherford, Niels Bohr, Albert Einstein, Werner Heisenberg, Erwin Schrodinger and others in the first half of the 20th century. During the last sixty years, input from pioneers such as Murray Gell-Mann, Richard Feynman, Steven Weinberg and Eugene Wigner have enabled further refinements of quantum theory, a marriage of particle physics and cosmology, and the advent of numerous 'disruptive' technologies based on this knowledge. Interested readers are referred elsewhere for further details concerning the history of quantum physics and a timeline of key discoveries which have punctuated the field [9-11].

The tenets of QM differ profoundly from those of classical physics in ways that often appear paradoxical and highly counter-intuitive. In classical physics, it is theoretically possible to ascertain the position and momentum of every particle in the universe and thereby accurately determine the future. In contemporary QM, it is fundamentally impossible to predict future events because one can never attain full knowledge of the position and momentum of even a *single* particle. In the standard (Copenhagen) interpretation of QM, every possible outcome for an event, represented mathematically as a statistical wavefunction, exists in the unobserved state. The act of observation elicits a "collapse of the wavefunction," whereby one of these many potential outcomes is "selected" as the reality actually experienced [12].

Germane to the current thesis, a considerable body of quantum theory and experimental evidence implies that (i) all particles emerging from the Big Bang singularity maintain an indefinite 'connectedness' with one another, (ii) each particle therefore 'knows' about the existence of every other particle, and (iii) due to preserved complementarity, the properties of one particle (e.g. position, momentum, spin, etc.) change instantaneously and commensurate with changes in a 'partner' particle regardless of the extent of their physical separation (Einstein's whimsical "spooky action at a distance"). For the latter to arise by classical causal interaction, information would need to pass from particle A to particle B at impossible supraluminal speeds. Quantum theory dictates that the particles' shared history forever "locks" them in a reciprocal dance ("quantum entanglement") that does not obligate the transmission of new information between them ("acausality"). Citing examples from the physical and biological sciences, Edgar Mitchell maintains that "the non-local attribute of nature is much more than just a curious artifact of subatomic particle interactions, but rather is a more fundamental phenomenon that appears at all scale sizes" and that "any waves reverberating through the universe remain coherent with the waves at the source, and are thus sufficient to serve as the reference to decode the holographic information of any quantum hologram emanating from remote locations [13]." Bohm brings the notion of the Universe's interconnectedness to an entirely new level by injecting into QM the concepts of an 'implicate order' and a 'holographic' design, as described below.

4. The Physics of David Bohm

Bohm's physics cannot be considered 'mainstream' in so far as it deviated from the classical Copenhagen interpretation of quantum mechanics. Yet, Einstein openly acknowledged Bohm as one of his intellectual successors [14]. Indeed, Bohm's imprint not only regarding physics but on many fields of science, philosophy and sociology has endured and even gained in popularity with the passage of time [15]. In this section, we present a brief overview of Bohm's life and describe his seminal contributions to physics with emphasis on his 'implicate order' and 'holographic universe'. In Section 5-6, we argue that these Bohmian themes are highly concordant with a world-view ensconced in earlier Kabbalistic literature.

4.1. *Biosketch*

David's father, Shmuel (Sam) was raised in an Orthodox Jewish (Chassidic) home in Munkacs, Hungary and immigrated to America with his family towards the end of the First World War. David Joseph Bohm (Fig. 1) was born in Wilkes-Barre, a small Pennsylvania mining town, in 1917. Although displaying no particular intellectual proclivities in his childhood years, his imagination for the physical sciences was purportedly fired by a science fiction article on inter-planetary travel he read at age ten. He became obsessed with astronomy, the harmonious motion of celestial bodies, hidden dimensions, and the nature of light. An introverted and physically awkward boy, David cultivated his leisure wandering in the forests and hills surrounding his town after school while his classmates played baseball [14].

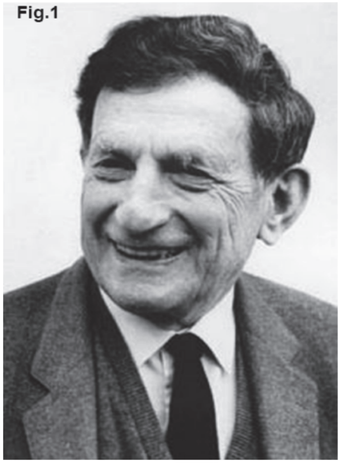

Figure 1. David Joseph Bohm (1917-1992). [©Mark Edwards/Still Pictures]

In his later teenage years, Bohm became fascinated with the logic of mathematical proofs as applied to geometry and algebra. He spent tranquil college years at Penn State contemplating a 4-dimensional cosmology which differed substantially from Einstein's Theory of Relativity. In college, he also developed an interest in politics with strong socialist leanings. His political views were destined to become curiously interwoven with his burgeoning scientific conjectures, and also bring him unwelcome scrutiny from the American government. Bohm went on to study physics at the prestigious California Institute of Technology. Despite receiving kudos for displays of mathematical brilliance, he regarded the ambience at CalTech as competitive, stifling and uninspiring. Bohm therefore abandoned CalTech in mid-curriculum to join a theoretical physics group headed by the renowned J. Robert Oppenheimer at the University of California (Berkeley) which he found liberating. There, his scientific creativity blossomed in areas ranging from the physics of particle collisions to high-energy plasmas. But it was afterwards, at Princeton and beyond, that the iconoclastic Bohm withdrew from mainstream physics to develop his theory of 'hidden variables' and the 'implicate order'. He was particularly disappointed in the way Niels Bohr and other leading physicists dealt with matters of interconnectedness and causality [16]. More and more, his conceptualization of Nature adopted a holism more reminiscent of Eastern religious philosophies than the prevailing science of his time. The physicist's rich and protracted correspondence with the Indian teacher and mystic, Jiddu Krishnamurti, whom Bohm revered, undoubtedly helped shape the emerging Bohmian *umwelt*. Bohm's perspective on Wholeness and of the universe as Hologram represented a startling departure from conventional physics with profound implications for the neurosciences, psychology, consciousness and religion. The following sections elaborate on several key themes of Bohmian mechanics which, we believe, have compelling homologues in the Kabbalistic literature.

4.2. *Hidden Variables and the Implicate Order*

Bohm's theorising and mathematical platform led him to consider the Cosmos and its myriad contents and processes as an emergent property of an indivisible Wholeness which he termed the "holomovement". Bohm conceptualized the holomovement as manifesting two major incarnations: (i) a familiar reality or 'explicate order' consisting of all things and events which are amenable to our senses directly or via instrumentation and (ii) an 'implicate order' comprising layer upon layer of 'hidden variables' beyond our perception. He viewed each deeper layer as more abstract than, but ultimately causative for, the dimension mapping immediately superficial to it, with the most proximate hidden layer giving rise to the explicate order. Bohm construed every perceptible object and event as rooted in a vast, possibly infinitely regressing series of causal matrices that ultimately originate from a state of absolute and inconceivable Wholeness. Bohm envisioned a highly dynamic interaction between the implicate and explicate orders. In Bohmian mechanics, shifts designated 'enfoldments' periodically make implicate that which was previously explicate – while 'unfoldment' of certain hidden variables renders them explicate and within the purview of human awareness [17, 18].

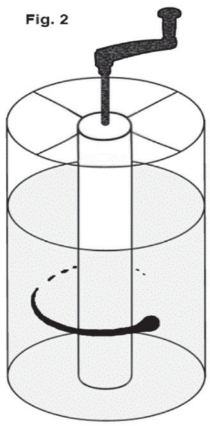

Figure 2. The implicate and explicate orders. Represented by Bohm as an ink drop in a rotating, glycerin-filled cylinder (see text for details). [From http://forumserver.twoplustwo.com/137/religion-godtheology/alpha-omega-gravity-order-899479, with modifications.]

To illustrate these points and their implications, Bohm invoked relatively simple analogies derived from everyday physical phenomena: 1) *Explicate and implicate orders*: Picture two concentric cylinders separated by a translucent viscous material such as glycerin (Fig. 2) [18]. Add a drop of black ink to the glycerin. While the cylinders are stationary, the ink is clearly visible as a dark spot within the glycerin (explicate order). Rapidly rotate the inner cylinder about its long axis. The black dot first stretches into a thin dark filament (still explicate) but soon disappears from view entirely (implicate order). Although the ink is now implicate, the information 'coding' for the original black spot is not lost. Indeed, if the motion of the cylinder ceases and is then resumed in the opposite direction, the dispersed, imperceptible particles of ink coalesce to re-form the dark filaments and eventually the original ink spot itself (explicate order). Bohm would refer to the initial disappearance of the ink spot/filament as 'enfoldment' within the holomovement, and its re-emergence as 'unfoldment'. 2) *Motion*: The standard interpretation of motion is that of an object moving from point A to B within the experiential (Bohm's explicate) realm. In Bohm's model, again drawing on the glycerin cylinder analogy, two drops of ink, A and B, are added to the rotating glycerin separated by time and space, e.g. 5 seconds and 5 millimeters apart. The inner cylinder is rotated until both spots become implicate, with Spot B disappearing 5 seconds after Spot A. At this juncture, the myriad particles belonging to Spots A and B are extensively inter-mingled, although the 'memory' of each particle's trajectory from its original ink spot is conserved as described above. The cylinder is then immobilized and spun at the same rate in the opposite direction. After a defined number of turns, spot B materializes (becomes explicate), followed 5 seconds later by the appearance of Spot A. As the cylinder rotates further, Spot B now becomes implicate and Spot A remains visible for an additional 5 seconds until it, too, enfolds. If this experiment is repeated with the reverse rotation conducted at much greater speeds, it will seem as if a single ink spot emerges and moves 5 millimeters from position B to position A before disappearing. Thus, according to Bohm, the linear motion of objects (be they electrons or elephants) we perceive in the experiential world is an illusion resulting from complex cycles of unfoldment-enfoldment between the implicate and explicate orders. [The latter should not be confused with the motion-like illusion provoked by a row of neon lights blinking in rapid succession as such phenomenon, in Bohm's terminology, requires no enfoldment and is entirely manifest within the explicate order.] In the foreword to Michael Talbot's popular book *The Holographic Universe* [19], Lynne McTaggart, citing Talbot, writes: "Bohm believes the reason subatomic particles are able to remain in contact with one another (see Section 3) is not because they are sending some sort of mysterious signal back and forth, but because their separateness is an illusion…at some deeper level of reality such particles are not individual entities, but are actually extensions of the same fundamental something." McTaggart goes on to state that "Bohm considered the universe a giant information headquarters of 'unbroken wholeness', in which everything in the universe is already present in some invisible domain beyond time and space – a field of all possibility – there to be called forth and made 'explicate', or manifest, when necessary."

4.3. *The Holographic Universe*

Holography was discovered in the 1940s by the Hungarian-Jewish mathematician, Dennis Gabor for which he was awarded the 1971 Nobel Prize in Physics [20,21]. The Merriam-Webster dictionary defines 'hologram' as "a 3-dimensional image reproduced from a pattern of interference produced by a split coherent beam of radiation (as a laser) [22]". Essentially, a hologram is a 2-dimensional recording of an interference pattern within a light field which, when appropriately illuminated (e.g. by laser), reconstructs a 3-dimensional image of the object originally captured within that field (Fig. 3). Unlike conventional photographs, holograms exhibit parallax and other visual depth cues that vary in a realistic manner with changes in the vantage point of the observer. Holograms also differ from the former in another, rather dramatic way: If one cuts out the left, bottom quarter of a standard photograph depicting a woman, the excised segment may contain an image of only her right leg. Subject the latter to the same procedure and perhaps only her right ankle will be visible in the smaller fragment. In the case of a hologram, however, the isolated portions reconstitute an image of the *entire* woman, albeit in miniature. This pattern repeats itself unendingly, producing smaller and smaller – but intact – women (or as per the case depicted in Fig. 3, globes), as the holographic image is progressively dissected. Simply put, in holograms the whole is recapitulated in each of its parts. According to the principles of quantum physics, this counter-intuitive phenomenon is based on the *non-local* nature of the interference pattern of light (see Section 3) which conveys the information needed to re-assemble the holographic image.

In addition to, and incorporating the concept of implicate and explicate orders, Bohmian mechanics posits that the entire Cosmos is based on a grand holographic design – with each part containing (enfolding) a miniature replica of the entire universe!

Bohm considered the existence of each component to hinge upon its intimate relationship to the whole, implying that individuality is only feasible if it unfolds from wholeness. In Bohm's words: "Quantum physics reveals a basic oneness of the universe" [23]; "The world acts more like a single indivisible unit, in which even the 'intrinsic' nature of each part (wave or particle) depends . . . on its relationship to its surroundings" [24]; and "The inseparable quantum interconnectedness of the whole universe is the fundamental reality, and [the] relatively independent behaving parts are merely particular and contingent forms within this whole" [25]. According to Bohm, our conventional notions of space, time, distance and separation apply only to the 'surface' of things as they are revealed within the explicate order. Akin to a hologram, two physical objects may be separated by vast expanses of space and time in the linear, explicate order, while little or no such separation may exist between their hidden components enfolded within the implicate order [26]. Evidence in support of Bohm's 'holographic universe' has been adduced in fields as disparate as astrophysics, molecular biology and the neurosciences [13, 16, 27-29]. One such intriguing report was published in 2007 by Jacob Bekenstein in *Scientific American* based on a theoretical analysis of 'black holes'. A black hole is a region of spacetime, thought to arise from the collapse of a star, with matter so dense and gravitational forces so powerful that nothing—not even light—can escape from inside it. According to Bekenstein, the mathematics underpinning certain behaviors of black holes suggest that all information in the universe, as in a hologram, is encoded on 2-dimensional (flat) surfaces and then transduced ('read out') by our minds as 4-dimensional spacetime [30]. Supported by experimentation in humans, animals and isolated nerve cells in culture [31-33], the Stanford neuroscientist Karl Pribram concluded, independently of Bohm (whom he later consulted), that aspects of the human brain may operate holographically ("holonomic brain theory"). Pribram's findings led him to dispute vigorously theories of topographically-discrete localization of brain functions favored by Wilder Penfield and others. Pribram argued that many functions of the central nervous system, e.g. memory storage/retrieval, sensory perception and consciousness, are at least partly non-localizing and better understood as enfoldments/unfoldments within a complex implicate neural order [14, 16, 18, 34-37]. Citing the work of Marcer & Schempp [28], Mitchell hypothesized that in the act of perception the brain behaves as a "quantum computer which utilizes both quantum and space/time information [13]". Some have even conjectured that human intuition, paranormal phenomena such as telepathy, clairvoyance and telekinesis, and certain neuropsychiatric states (e.g. schizophrenia) may be products of nonlocal quantum neuroholography [13, 16, 38-40].

5. The Kabbalah

The Kabbalah teaches about a hierarchy of interlocking spiritual domains which progressively 'descend' in holiness, beginning with the unfathomable Godhead, traversing fractal-like through a system of 'coarsening' immaterial worlds, and culminating in the Creation of the physical universe. In addition to elaborating an ontogeny for all existence, the Kabbalah explains, often allegorically, the hidden ways by which God continuously guides the unfolding universe and the dynamic systems that are in place to interact with Nature and humanity. As depicted in the Kabbalah, the universe is governed by a complex system of "Lights" or forces which, through myriad interactions, provoke chain reactions that ultimately impact humans and their physical surroundings [8, 41]. Central to the Kabbalistic viewpoint is the absolute unity of the Creation at its core, with all semblances of separateness and differentiation becoming apparent only after "filtration" of the one Infinite Light (*Ohr Ein Sof)* through the various *Sefirot* (defined below).

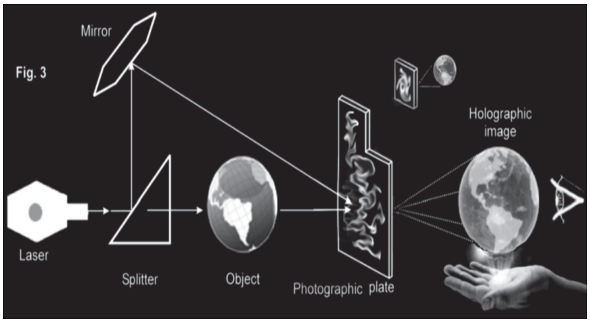

Figure 3. Laser holography. See text for details.

The primary Kabbalistic texts we have consulted are the *Zohar* (Book of Radiance), the teachings of R' Yitzhak Luria (the *Arizal*; 1534-1572) as transmitted by his student R' Chaim Vital (1543-1620), and the works of R' Moshe Chaim Luzzatto (*Ramchal*; 1707-1746) and R' Shalom Sharabi (*Rashash*: 1720-1777). The *Arizal* elaborated all the main concepts of the Kabbalah and provided innovative explanations of the *Sefirot* and *Partzufim* ("configurations" – see below). The corpus *Etz Chaim* (Tree of Life), compiled by R' Vital, encompasses the teachings of the *Arizal* and remains the major reference text of Lurianic Kabbalah. In eighteenth century Europe, the *Ramchal* and *Rashash* greatly facilitated the contemporary understanding of the Kabbalah by re-organizing and explicating many cryptic passages of the *Zohar* and *Etz Chaim* [8, 41]. The *Rashash*, in his major texts *Nahar Hashalom* (River of Peace) and *Rechovot Hanahar* (Roads of the River), was particularly instrumental in developing the themes of *Hitlabshut*, *Hitkallelut* and *Hitkashrut* (see below) which we submit resonate closely with the physics of Bohm.

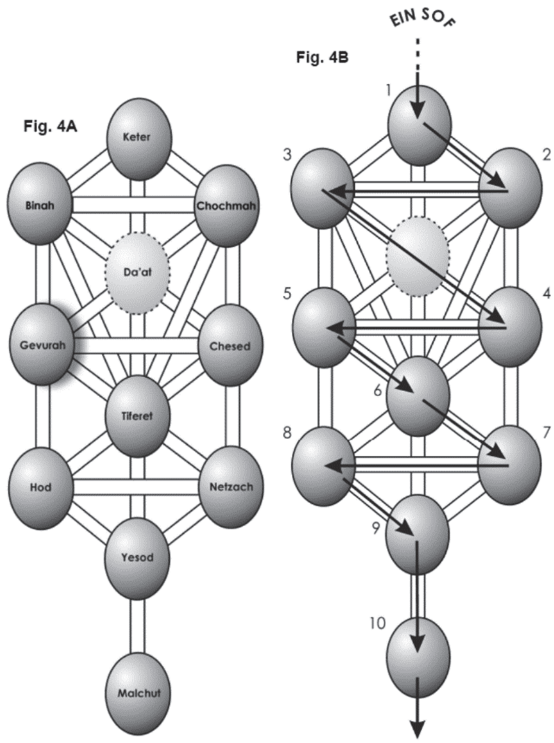

Figure 4. A. The ten *Sefirot* (*Da'at* is included when *Keter* is not). B. *Seder Hishtalshelut* (Kabbalistic causal hierarchy). Bars indicate potential interactions among the *Sefirot*. Arrows denote standard pathway for the 'descent' of Divine influence.

5.1. *Sefirot, Partzufim and Worlds*

Although the Light (emanation) of the Infinite is a unified whole, each of ten *Sefirot* (Fig. 4) represents a "filter" that holds and transforms a certain part of this Light into a particular force, attribute or action. The ten *Sefirot* are: *Keter* (crown/Divine Will), *Chochmah* (wisdom), *Binah (understanding)* - alternatively, *Chochmah, Binah, Da'at (knowledge) - Chesed* (lovingkindness/expansiveness), *Gevurah* (strength/restriction), *Tiferet* (beauty/harmony/truth), *Netzach* (victory/eternity), *Hod* (splendor), *Yesod* (foundation) and *Malchut* (kingship). Each *Sefirah* is composed of a vessel (*Keli)* which retains its part of Light (*Ohr)*. There is no differentiation of the *Ohr* within the *Keli* itself, as it is part and parcel of the original, undivided Light; differences emerge from the particularity or position of the *Sefirah's Keli*. According to the Kabbalah, arrangements of ten *Sefirot* are the blueprint of all things created, and everything that exists is ultimately comprised of these ten "forces".

A *Partzuf* (face, visage, or countenance) is a configuration of *Sefirot* acting in coordination or towards a defined purpose. The six main *Partzufim* (in "descending" spiritual order) are:

Atik Yomin (A"Y)—Ancient of Days
Arich Anpin (A"A)—Long Countenance
Abba—Father
Imma—Mother
Ze'ir Anpin (Z"A)—Small Countenance
Nukva—Feminine

To allow for the Creation and its spiritual and material contents, the *Ohr Ein Sof* "retracted" in a process known as *Tzimtzum*, thereby establishing a *Chalal* or *Makom Panoy* ("empty space").

The Kabbalah teaches that, in actuality, a faint glimmer of residual Holiness, deemed the *Reshimu,* lined the "interior" of the *Makom Panoy* and served as the primordial *Malchut*/*Nukvah*/Feminine component of all things destined to be created. A "ray" of Divine Light (*Kav*), emanating from the surrounding *Ohr Ein Sof*, penetrated the *Makom Panoy* to unite with the *Reshimu.* From this union (*Zivug*) was created all *Sephirot, Partzufim* and Worlds. The first, most lofty and therefore cognitively least accessible World created within the *Makom Panoy* is termed *Adam Kadmon* (*A"K*; Primordial Man). "Below" *A"K*, and growing progressively more remote from God's Essence, is *Atzilut* (the World of Emanation), *Briah* (the World of Creation), *Yetzirah* (the World of Formation) and *Asiyah* (the World of Action), commonly abbreviated as *ABY"A*. Each World possesses unique qualities which are beyond the scope of this essay. What is important here are the following concepts: (i) Each World comprises the six aforementioned *Partzufim*; each *Partzuf* is composed of the ten *Sefirot;* and each individual *Sefirah* is itself made of 10 "miniature" *Sefirot* in a recursive, fractal-like manner ("Their measure is ten, yet infinite [42]") (ii) The *Partzufim* overlap one another such that the three lower *Sefirot* (*Netzach-Hod-Yesod*) of the "Higher" *Partzuf* (e.g. *Imma*) constitute the *Mochin* ("brains" or *Chochmah-Binah-Da'at*) of the Immediately "subjacent" *Partzuf* (i.e. *Z"A*). The *Mochin* animate the *Partzuf* (analogous to the relationship of brain/mind to body) and transmit Divine Guidance from "higher" spiritual realms. (iii) *A"Y* can be construed as the "top" *Partzuf* of a given World (e.g. *Briah*) or the "bottom" *Partzuf* of the World immediately "above" (*Atzilut*). As such, it serves to "connect" Worlds akin to the bridging role of the *Mochin* between "adjacent" *Partzufim*. (iv) Physical reality (i.e the entire observable universe with all its space, matter and energy) comes into being at the very "bottom" (*Malchut of Malchut*) of *Asiyah*. Everything in Creation "above" this level is represented by a complex hierarchy of purely spiritual domains that, via intricate chains of cause-and-effect (*Seder Hishtalshelut*), ultimately impact the affairs of material existence [8, 43-45].

5.2. *Hitlabshut, Hitkallelut and Hitkashrut*

Hitlabshut, *Hitkallelut* and *Hitkashrut* are three vital and inter-related principles at the heart of Kabbalistic doctrine. They are intrinsic to the mystical topography of the classical texts, such as the *Sefer Yetzirah, Sefer Ha'Bahir,* the *Zohar and the Etz Chaim*; the writings of more recent Kabbalistic masters, including *Rashash, Ramchal*, and the *Leshem* (R' Shlomo Eliashiv, 1841-1926); and the prolific Chassidic literature (e.g. the compilation *B'shaa Shehikdimu* of the Rebbe Shalom DovBer (*Rashab)* of Lubavitch, 1860-1920). Familiarity with these basic concepts is paramount to appreciating the Kabbalah's understanding of (i) the myriad relationships among all the particulars (*Pratim*) of the Creation and the Forces which gave rise to and govern them, and (ii) the critical nexus between the Cosmos as a unified whole and its individual parts.

Hitlabshut (**התלבשות**), from the Hebrew root *L'vush* ("clothing"), denotes a system whereby the 'bottom' aspect of a World, *Partzuf* or *Sefirah* is 'enclothed by' or 'dressed within' the superior aspects of its immediately subjacent counterpart. We have already encountered a prime example of *Hitlabshut* in the case of the *Mochin* (Section 5.1). One way to visualize this relationship is to imagine an incompletely extended telescope pointing downwards (Fig. 5a): The higher rungs, representing more refined levels of spiritual reality closer to the Godhead (*Ein Sof*), are interior to and partially overlapped by the lower, progressively more 'mundane' rungs. The region of overlap serves as a conduit by which Divine Guidance originating in the upper strata 'descends' to influence events within the lower realms. Note that each lower stratum, by virtue of the overlap, serves to conceal from our direct perception (*He'elam*) the higher, 'inner' domain, while revealing (*Gilui*) by inference the latter's existence and functionality.

By analogy, the movements and touch of a gloved hand reveal much about the hand itself despite its 'hidden' nature. Note also that the degree of overlap among Worlds, *Partzufim* and *Sefirot*, indicating the propensity for Divine influence/blessing to flow from one level to the next, can vary with time and position within the Kabbalistic superstructure (*Etz Ha'chaim*, literally Tree of Life). Generally, the extent of overlap, i.e. flow of Divine Light, is least where divisiveness (*Pirud*) within the Creation is maximal. This occurs with increasing 'distance' from the Godhead, e.g. among the *Partzufim* of *Atzilut* relative to those of *A"K*; or whenever *Pirud* is exacerbated by the sins of Mankind.

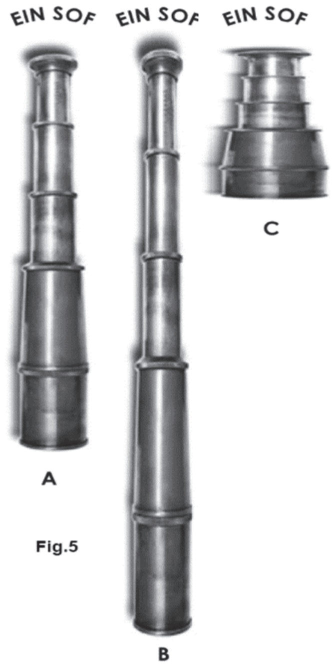

Figure 5. *Hitlabshut* (enclothment). Metaphorised by a set of extendable telescopes. A. Reference configuration of the Kabbalistic superstructure. Joints of the telescope symbolize degree of 'overlap' (enclothment) among *Sefirot*, *Partzufim* and Worlds. B. 'Descent' of Creation into *Orech* (increasing apparent disunity and 'distance' from *Ein Sof*). C. 'Ascent' into *Oivi* (progressive revelation of wholeness and the indivisible Light of *Ein Sof*), http://www.gilai.com/images/items/1498_big.jpg, (with modifications).

The *Rashash* [46] and others [47] refer to this telescopic extension downward into greater disunity as movement into *Orech* ('vertical' descent; depicted by the stretching of the telescope in Fig. 5b). This is the 'top-down' direction that the Creation naturally unfolds into to permit manifestations of apparent separateness, Evil and Free Will (*B'chira*). Contrariwise, the extent of

Hitlabshut ('overlap') is progressively augmented as one moves 'up' the *Etz Ha'chaim* or on account of the *Mitzvot/Tikkunim* (positive deeds or rectifications) performed by Man. This is tantamount to moving from *Orech* (*Pirud*) into states of increasing unification (*Shlaymut*) or *Oivi* ('horizontality'), as illustrated by retraction of the telescope in Fig. 5c. In extreme states of *Oivi*, in contradistinction to *Orech*, the hierarchical relationships among the created particulars dissolves and all things are perceived as spiritually equidistant from the Godhead (*Ein Sof*). Several examples may help flesh out this concept: (i) From the ladder-like perspective of *Orech*, we would ordinarily attribute greater value to humans than to gnats or pebbles. However, inasmuch as the three fulfil the mandates of the Creator, they are, when viewed from the perspective of *Oivi*, equally 'proximate' to the supernal *Sefirah* of *Keter*/Divine Will and thus equally vital to God's Plan. (ii) On *Orech's* vertical scale, a seminary student who used to learn 12 hours a day but now only manages to put in 10 hours is still held in higher esteem than a peer who increased his daily learning from one to two hours. Not so in *Oivi* – by shifting the ladder 'horizontally', student A has lost ground and receded from God's Will (*Keter*) into increasing disunity/*Pirud*, whereas student B has entered a more profound state of wholeness/*Shlaymut* [47]. (iii) In times of despair, the common Hebrew expression *Gam Zu L'tovah* ("even this is for the good") is transformed from a hopeful utterance into a proclamation of truth when perceived through the lens of *Oivi,* for, according to the Kabbalah, all circumstances are ultimately decreed by the Benevolent One for the benefit of humanity. (iv) Using the symbolism of mathematics, we shift from *Orech* to *Oivi* (and from *Prat* to *Klal*) whenever we collapse a *Gematriyah* (sum of numerical values of the letters comprising a Hebrew word) to its numerical diminutive (*Mispar Katan*). Thus may the 613 *Mitzvot* (commandments/duties) be regarded as 'branches' of the more fundamental Ten Commandments (6+1+3=10); and the latter as manifestations of the Will of the One God (1+0=1). (v) According to Jewish tradition, the orbs of the sun and moon were initially created equal in stature (state of *Oivi*). God subsequently diminished the Moon (*Miyut Ha'yareach*) and rendered it a passive recipient (*Keli*) for the light of the Sun. This *Miyut Ha'yareach* is tantamount to a 'vertical' descent from *Oivi* into *Orech*. In the Messianic era, the Moon will regain its original position of prominence (*V'kayma Siharah B'ashlamutah* – 'the moon will be established in its completeness' [42]), a movement into *Oivi,* and function in harmony with the sun as *Shnay Malachim Mishtamshim B'Keter Echad* – 'two kings sharing a single crown' [48]. [This dynamic tension between the Sun and Moon is but one special case of the pervasive Kabbalistic doctrine concerning the relationship of Masculine (*Mashpiah*-donor) and Feminine (*Keli*-recipient) which informs all aspects of the Creation [49]]. We contend that *Hitlabshut* and Bohm's 'Implicate Order' (Section 4.2) are identical theoretical constructs.

Hitkallelut (**התכללות**) stems from the root, *Klal* which connotes 'wholeness', 'cohesiveness' or 'generality', the antonym of *Prat* ('part' or 'specific'). *Hitkallelut* is commensurate with the notion that the Whole is recapitulated or contained within each of its parts (*Hakol Ma She'yesh Ba'klal Yesh Ba'prat*). This interinclusion is embodied in the mathematics of fractal geometry and in the perpetually-recurring images of the Mandelbrot set (https://www.youtube.com/watch?v=0jGaio87u3A). By the same token, each *Sefirah* contains within it all 10 *Sefirot* (Fig. 6). This is exemplified in the Jewish liturgy by the 'Counting of the *Omer*' ritual between the festivals of Pesach (Passover) and Shavuot (Weeks): *Chesed of Chesed, Gevurah of Chesed, Tiferet of Chesed,* etc. until *Malchut* of *Malchut* on the 49th day. Similarly, every *Partzuf* and World contains within it *all* the *Partzufim* and Worlds. The concept is underscored by a homiletic indicating that the bush wherein God revealed Himself to Moses on Mt. Sinai (*Klal*) was also present, in miniature, within each stone (*Prat*) hewn from the mountain [50, 51].

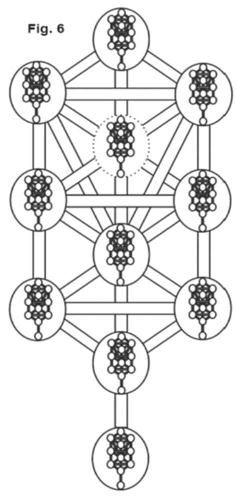

Figure 6. *Hitkallelut* (interinclusion). Represented as a deca-*Sefirotic* fractal.

Hitkallelut subsumes the mind-bending idea that the entire physical universe is reconstituted within each atom; that each interval of time embodies the entire Past, Present and Future (see *Rashi* comment to *Va'yetzeh* [52]: "All things currently in existence have always existed and will continue to exist in the future (*She'kol Davar Ha'hoveh Tamid K'var Hayah V'atid L'hiyot*))"; and that each Soul is replete with *all* Souls extant and pending! Viewed from this vantage point, it is understandable why one who observes a single *Mitzvah* to full capacity is rewarded as if s/he fulfilled all 613 *Mitzvot* [53]; why the Sabbath and its concluding (*Havdalah*) blessings inadvertently uttered by a disoriented traveler on a weekday are, by dint of the mini-Sabbath enfolded within it, not pronounced in vain [54, 55]; why punishment of each and every transgression effects atonement for the singular sin of the Golden Calf [56, 57]; why "All of Israel are connected one to the other" (*Kol Yisrael Areivim Zeh La'zeh*) [58-60] is not merely a moral imperative but a metaphysical fact; and why the saving of a single life is tantamount to rescuing the entire world [61]. *Hitkallelut* is highly reminiscent of, if not synonymous with, Bohm's 'Holographic Universe'. Bohm conceptualized the property of interinclusion as being mandated by the relationship of the part to the indivisible Whole. In a similar vein, drawing on the Kabbalah, the Rebbe *Rashab* of Lubavitch wrote [42]: "And this is the concept of interinclusion (e.g. of *Chochmah* and *Binah*) which is contingent on the revelation of the Unlimited *Ein Sof*." (*V'hu Inion Hitkallelut [Chochmah U'binah] She'zehu Al Yadei Ha'gilui D'Ein Sof Ha'bilti Gvul)*. Inasmuch as it reflects a deep, underlying Unity, the realization of *Hitkallelut* in Nature is, in the eyes of the Kabbalah, the quintessence of grace and fulfilment of the verse in Song of Songs: "Your entirety is beautiful…and you have no blemish [62, 63]".

Hitkashrut (**התקשרות**) derives from the root, *Kesher* and connotes 'binding', 'connection' or 'amalgamation'. It is a mechanism of interpenetration which promotes a grand underlying unification of the Creation and operates in conjunction with the principles of *Hitlabshut* and *Hitkallelut*. By way of example, let's consider the four worlds, *ABY"A* arranged as a vertical stack of four blocks, with *Atzilut* on top 'nearest' to the Godhead. We can consider each World as a particular (*Prat*) composed of 10 *Sefirot*. Dynamic interactions among the latter are necessary for the establishment and proper governance of each World. Generally, for the *Ohr* of the *Sefirah Chesed* to 'radiate' in the 'bottom' World of *Asiyah,* a top-down 'chain of command' (*Seder Hishtalshelut*) is brought into play (Fig. 4): Influences 'descending' from the *Ein Sof* via *A"K* 'activate' in serial fashion the 10 *Sefirot* of *Atzilut; Malchut* of *Atzilut* serves as *Keter* of *Briah* to mobilize sequentially the 10 *Sefirot* of that World. This pattern of 'descending' influence continues through *Yetzirah* and *Asiyah* (and eventually extends to us, if we're worthy, via the final *Sefirah* of *Malchut* of *Asiyah*).

The principle of *Hitkashrut* dictates that intimate bonds exist not merely among the *Sefirot* comprising any given *Prat*, but among 'like' *Sefirot,* e.g. *Chesed,* across *all* the *Pratim* (pl.) of Creation. *Hitkashrut* enables the *Chesed* (or any *Sefirah*) component of each and every part of the Creation to be "mobilized" concurrently (Fig. 7) when so decreed from Above, by-passing the 'domino effect' structure of the *Seder Hishtalshelut* (Fig. 4). The *Rashash* [46] would construe this shift from the sequential, 'series-like' actualization of *Chesed* within each branch and leaf of the *Etz Ha'chaim* (Kabbalistic superstructure) to the simultaneous 'parallel processing' of *Chesed* throughout the entire Creation as another instance of movement from *Orech* into *Oivi* and, hence, a greater expression of Wholeness *(Shlaymut).* Biblical literature is replete with examples of *Hitkashrut*. One famous instantiation of the principle, adduced from the inanimate domain, is the miraculous partition of all bodies of water concomitant with the splitting of the Red (Reed) Sea at the Exodus from Egypt [64].

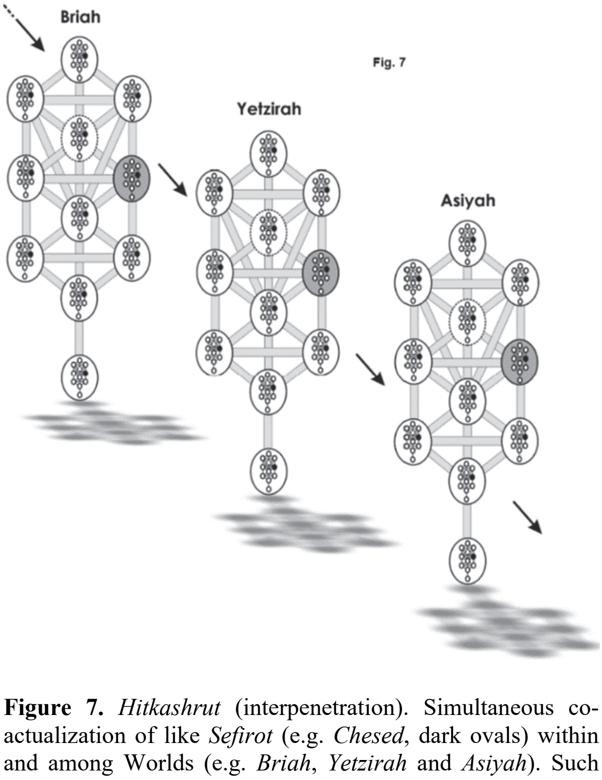

Figure 7. *Hitkashrut* (interpenetration). Simultaneous co-actualization of like *Sefirot* (e.g. *Chesed*, dark ovals) within and among Worlds (e.g. *Briah*, *Yetzirah* and *Asiyah*). Such 'parallel processing' of homologous parts circumvents the linear, hierarchical flow of Divine influence illustrated in Fig. 4B and represents a greater manifestation of *Shlaymut* (wholeness).

A study of basic biology affords numerous examples of *Hitlabshut*, *Hitkallelut* and *Hitkashrut*. The human body is a hugely complex system of discrete organs and tissues, each discharging unique duties for the health and welfare of the organism as a unified whole. Brain cells express proteins indispensable for the regulation of diverse physiological functions, sensory perception, movement and cognition; liver cells synthesize very different sets of proteins for the maintenance of the body's energy requirements and detoxification of harmful substances. Yet, in accord with the principle of *Hitkallelut,* each brain cell contains within its nucleus all the DNA required to generate the full gamut of liver (and indeed all other human) proteins, and *vice versa* for liver cells: *Hakol Ma Sh'yesh Ba'klal* ('everything contained within the whole…' - in this case, the body) *Yesh Ba'prat* ('…is recapitulated in each of its parts' - brain, liver, etc.). Our ability to clone an entire organism from a single cell is a pragmatic realization of this principle. In the example invoked, neuronal genes (DNA) coding for liver and other non-brain proteins, albeit present in latent form (*Hitkallelut*), are repressed (*He'elam/*concealed or made implicate in Bohm-*sprache*) and only those proteins necessary for the maintenance of normal neurological function are actually produced (*Gilui/*revealed or rendered explicate).

In his *Sparks of the Hidden Light*, R' Moshe Schatz broadens the anatomical analogy further to implicate the principle of *Hitkashrut*. He intimates that achievement of absolute biological integrity and optimal component performance presupposes a functional 'bonding' (*Hitkashrut*) of, say, the right eye with some aspect of "right eyeness" inherent to every limb and tissue [65]. Along similar lines, but now operating inter-personally, *Hitkashrut* would explain the *Midrashic* (homiletic) account of sudden and widespread fecundity among hitherto childless women that coincided with the birth of a child to the previously barren matriarch, Sarah [66]. In the examples cited, *Hitkashrut* would imply, respectively, that the right eye *per se* is but the fullest expression of an attribute distributed throughout the organism as a whole, and that Sarah's abrupt fertility is microcosmic of a property permeating the community at large. Such top-down organization and regulation of biological systems is in harmony with an emerging anti-reductionist viewpoint which maintains that, to intuit deepest levels of 'meaning' (a concept dismissed *a priori* by most contemporary molecular biologists but gaining in respectability in quantum mechanics circles [67]), living and conscious processes are more profitably understood in their own right rather than in terms of any deconstructing physics or chemistry [68]. In essence, these natural examples of *Hitkashrut* are no different from the aforementioned ubiquitous surge of Lovingkindness accruing from the simultaneous activation of *Chesed* within the innumerable deca-*Sefirotic* components of the Cosmos (*vide supra*). It is noteworthy that although Bohm's account of the holographic universe employed terms highly reminiscent of *Hitlabshut* and *Hitkallelut,* he did not explicitly enunciate a term homologous with the principle of *Hitkashrut*. Several possible explanations for this 'omission' are presented in Section 6.

6. A Synthesis

The advent of quantum mechanics during the last century has heralded an unprecedented convergence of scientific and Jewish mystical interpretations of physical reality. In a previous article published in *The Torah u-Madda Journal* [8], we garnered evidence from the *Zohar*, the *Etz Chaim* and the 18th century writings of R' Moshe Chaim Luzzatto, that Heisenberg's Uncertainty Principle (1927) [69], a pillar of quantum mechanics, is strikingly similar to the Kabbalistic construct known as the *Raisha D'Lo Ityadah* (*Radla;* The Unknowable Head). Homologies were demonstrated as they relate to the fabric of reality, the intrinsic incomprehensibility and paradoxical nature of the universe, the translation of indeterminacy into experiential reality, worlds *in potentia*, and the grand scale unicity of the universe. Possible implications of these parallelisms for modern physics, epistemology and prophecy were discussed [8].

The present work builds on this theme by demonstrating a provocative dovetailing of insight between the Kabbalah and the scientific theories promulgated by a leading 20th century physicist, David Bohm. We have attempted to show in Sections 4-5 that there is no sacrifice of intended meaning when the lexicon invoked by Bohm to elaborate his innovations in quantum physics is interchanged with homologous Kabbalistic terminology. Where Bohm speaks of a 'holomovement' to describe Reality's absolute Wholeness from which all particulars spring and remain inextricably linked, the Kabbalah employs the corresponding concepts, *Ohr Ein Sof*, *Shlaymut, Klal and Oivi*. Bohm's 'implicate order' can be readily understood as all domains at and 'above' *Malchut* of *Malchut* (the 10th and lowest *Sefirah*) of *Asiah,* the World of Action situated at the 'bottom' of the *Seder Hishtalshelut* (Kabbalistic hierarchy). Similarly, Bohm's 'explicate order' is tantamount to the physical domain amenable to our perception 'below' and transduced by *Malchut* of *Malchut* of *Asiyah.* Bohmian mechanics mandate that each domain of the holomovement arises from, ensheaths, and is causally influenced by the layer immediately 'implicate' to it. Similarly, the concept of *Hitlabshut* dictates that each component (*Sefirah, Partzuf*, World, etc.) of the *Etz*

Ha'chaim (Tree of Life) 'dresses' and is controlled by the component immediately 'above/interior' to it. Were Bohm cognizant of the *Seder Hishtalshelut*, he might naturally have construed it as a cascading structure of interacting implicate and explicate orders. The Kabbalah teaches further that the elements comprising the *Seder Hishtalshelut* are in flux among various states of *He'elam* (concealment) and *Gilui* (revelation). We submit that Bohm invoked the terms 'enfoldment' and 'unfoldment' to capture precisely this dynamic, with enfoldment connoting the 'upward/inward' movement into *Oivi* (hidden unification) and unfoldment a 'descent' into *Orech* (apparent disunity). Perhaps the most revolutionary idea that Bohm injected into contemporary quantum theory – one that continues to impact scientific disciplines beyond physics and fire the public imagination - is the Universe's holographic design. The theoretical and practical implications of a Cosmos wherein each and every part enfolds (recapitulates) the entire Whole can only be dimly appreciated at this juncture. Yet, this singular concept, termed *Hitkallelut* in Hebrew, is a fundamental feature of the ancient Kabbalistic landscape and an essential aspect of the intrinsic interinclusiveness of God's Creation. Thus, according to both Jewish mystical tradition and Bohmian mechanics, each particle and wave contains enfolded within it all of the matter and energy in the Universe; within every present moment - the distant past and remote future; within each thought - the sum of all human cognition and consciousness. By linking the principle of *Hitkashrut* to *Hitkallelut*, the Kabbalah takes the indivisibility of the holographic universe a step further. *Hitkashrut* reinforces the unicity of the Creation by establishing a functional connection between a specific part of one deca-*Sefirotic* system (or mini-hologram) and its doppelgangers within the entire created network of fractal sub-structures. Examples of how *Hitkashrut* may operate metaphysically and within the human organism were provided in Section 5.2. Interestingly, as alluded to in Section 5.2, Bohm did not describe a construct equivalent to the principle of *Hitkashrut* in the elaboration of his physics. One possible explanation for this is that a process akin to *Hitkashrut* may have been implicit to Bohm's formulation of the holomovement and its seamless relationship to its myriad parts. He may have regarded as axiomatic the notion that any change in item *I* within a single mini-hologram would reverberate instantaneously to affect *all* item *I*'s throughout the universe's entire fractal architecture – for if not, how might holographic symmetry be preserved? If this indeed was Bohm's reasoning, qualifiers, examples and mathematical proofs to support the principle may have been superfluous. Perhaps he deemed statements such as "each part is in a fundamental sense internally related in its basic activities…to all the other parts [17]" as sufficient to convey the gist of *Hitkashrut*–like phenomena. Alternatively, Bohm may have eschewed the idea of a holomovement-wide, concerted "co-activation" of replicate constituents or forces lest this might hint at the deliberate actions of a Supreme Consciousness, a position he exhibited some ambivalence towards (see Section 7). Finally, Bohmian mechanics, ostensibly uninformed by direct Kabbalistic influence (Section 7), may simply not have matured to the point of acknowledging the existence of the Universe's *Hitkashrut*-like properties.

7. Bohm on Religion

Is the conflation of Bohmian mechanics and the Kabbalah - one system based on reason and experimentation, the other a product of mystical thought and revelation - mere coincidence? Or was there something unique to Bohm's personality, intellect and environment that predisposed him to think about physics in the 'unorthodox' manner in which he did? Although it remains difficult to address this query with any degree of certitude, a modicum of conjecture may be warranted. Around the time of his Bar-Mitzvah (age 13), Bohm confessed to his community rabbi that science was his overarching passion and that he no longer felt connected to Judaism and its traditions [14]. As he expressed no overt statements to belie this sentiment throughout his professional life, it is unlikely that Bohm deliberately drew inspiration for his maturing scientific insights from the Kabbalah. He did, however, harbor certain views on theology and mysticism. In an interview conducted in 1986 by Renée Weber, a Harvard philosophy professor, Bohm opined, albeit somewhat evasively, on mysticism and the nature of God [70]. When asked whether the ultimate or super-implicate order is a euphemism for God, Bohm cryptically replied: "It's not a euphemism for God because [even] it [the super-implicate order] is limited". Weber reminded Bohm of a comment he had once made affirming the existence of a 'super-intelligence that is benevolent and compassionate, not neutral', to which Bohm tepidly responded: "We can propose that". Weber pressed on with the following: "What you have been saying sounds like mysticism – that we are grounded in something infinite. How does it differ from what the great mystics have said?" To this Bohm admitted: "I don't know that there's necessarily any difference" and, invoking Kierkegaard's definition of 'true religion', intimated that both legitimate physics and mystical insight must be 'grounded transparently in the power that constitutes one'.

So while denying traditional Judaism *per se* and falling short of actually declaring the existence of God, Bohm fathered a novel and compelling branch of physics which bears an astonishing resemblance to mainstream Kabbalistic philosophy. Referring specifically to Bohm's model, the physicist-philosopher, Bernard d'Espagnat stated that "present day physics forces us to take seriously conceptions lying so far apart from our usual experience – the scientific one included – that…the epithet 'meta-physical' naturally comes to mind" [71]. It remains possible that exposure in his youth to Chassidic lore and customs, a tradition steeped in Kabbalistic influence, may have unwittingly sensitized Bohm to formulate scientific theory along mystical lines. Moreover, as alluded to in section 4.1, Bohm enjoyed an intense and long-lasting intellectual discourse with Jiddu Krishnamurti. Krishnamurti, a master of Eastern philosophy and mysticism, may have consciously or subliminally channeled Bohm's nascent thought processes, already primed by latent religious indoctrination during childhood, along pathways ostensibly trodden by the ancient Kabbalists.

8. Concluding Remarks

Since its inception millennia ago, Jewish mystical thought has steadfastly attested to the absolute oneness of the Creator and His Creation in the face of apparent separateness and individuation. This perspective is at face value counterintuitive and outside the purview of classical (Newtonian) physics. The advent of quantum mechanics in the 20th century provided a novel conceptual framework for resolution of this great paradox, thereby breathing fresh life into the dialogue between Torah and science. That all particles and forces comprising the observable universe are blatantly interconnected ("entangled") was the inescapable conclusion which followed a series of intriguing '*gedanken*' (thought) experiments and the confirmatory bench work of Alain Aspect and colleagues at the University of Paris in 1982 [72, 73]. In the current article, we attempted to underscore further the growing reconciliation of Torah and contemporary science by juxtaposing several fundamental Kabbalistic principles with David Bohm's unique approach to quantum mechanics. Specifically, we adduced evidence from the respective literatures that Bohm's implicate and explicate orders, enfoldment/unfoldment and holographic universe are mathematically-valid descriptions of Reality long intuited by the Kabbalists as *Hitlabshut*, *Hitkallelut* and *Hitkashrut.* Indeed, the following statement by Talbot [19] is an accurate rendition of the world-view professed by both Bohm and the ancient mystics: "Everything interpenetrates everything…all apportionments are necessarily artificial and all of nature is ultimately a seamless web…the universe is at heart a phantasm, a gigantic and splendidly detailed hologram". To the extent that quantum physics and the Kabbalah address the self-same characteristics of natural law, ongoing exchange between these disparate disciplines could prove mutually reinforcing. Science may arm mystical traditions such as the Kabbalah with compelling analogies and vocabulary to open the wellsprings of the latter's (hitherto arcane) wisdom to modern societies. Reciprocally, the richly imaginative Kabbalistic doctrine could potentially demarcate novel directions and enlighten the enterprise of scientific inquiry. Along these lines, Cambridge's 1973 Nobel laureate, Brian Josephson suggested that Bohm's implicate order may one day allow for the assimilation of Mind or God within the framework of science [74]. The following comments by physicist Joel Primack and the historian of science Nancy Ellen Abrams were cited in our earlier work describing other parallelisms between quantum mechanics and the Kabbalah [8] but are worth repeating here: "We will turn to Kabbalah, medieval Jewish mysticism, as a possible source of language and metaphor, because certain Kabbalistic concepts fit our picture amazingly well. Moreover, Kabbalah's cosmology gave meaning and purpose to the everyday lives of its adherents, which we hope may become possible with the scientific cosmology emerging today [3]". While these words may pertain to many aspects of contemporary quantum mechanics and cosmology, nowhere do they resonate more cogently than with the physics of David Bohm.

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Lorentz Violation and CMBR Anisotropy

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This paper is dedicated to the comparison of the results of two terrestrial scale experiments with the cosmological evidence of the anomalies/asymmetries of the Cosmic Microwave Background Radiation (CMBR). The first terrestrial experiment is about the violation of Local Lorentz Invariance (LLI) and the second one is about the emission of neutrons from a steel bar subjected to ultrasounds. Both LLI violation and the neutron emission show a peculiar asymmetry and anisotropy which appear to have an interesting correlation with the direction of some anomalies/anisotropies of CMBR. It is put forward the idea that this correlation hints at an underlying fundamental anisotropy and asymmetry of the laws of Nature and that in this sense the well grounded 'Copernican Principle', stating that Earth is not a privileged observation point in the Universe, needs a deep revision. Earth would be neither a privileged point of observation in the Universe nor a non-privileged one. Due to the underlying asymmetry, it turns out that Earth would be just a point of observation conditioned by natural laws as they appear to be in the area of the universe where Earth is. In this sense, one does not need a spontaneous symmetry breaking in order to describe Nature since the asymmetry is already a fundamental and intrinsic part of it.

Keywords: CMRB anisotropy, Local Lorentz Invariance Violation (LLI), LLI electromagnetic tests, neutron emission

1. Local Lorentz Invariance Violation

One can summarize the tests of the validity of Local Lorentz Invariance (LLI) by the graph showed in Fig. 1 presenting its upper limits of violation. Several types of experiments are reported: Michelson–Morley experiment; isotropy of the speed of light; tests of time-dilation; NMR test of nuclear level.

Two levels of LLI violation upper-bound are evident between 10^{-10}–10^{-2} and between 10^{-22}–10^{-14}.

One question rises: are these two levels, energy-range controlled or interaction controlled? The issue is still open.

2. Helmholtz Coil Experiments Lorentz Invariance Violation

The lower limits of Local Lorentz invariance have been tested by electromagnetic experiments in which it was studied the induction produced by a static magnetic field generated by a fixed Helmholtz magnetic coil on a small dipole antenna belonging to the same reference frame of the coil.

In these experiments, schematically reported in Fig. 2 and Fig. 3 and described in details in [1-3], the voltage inducted in the antenna by the static magnetic field is compatible with the drift velocity of 0.06 m/s which means compatible with zero considering the velocity of Earth and/or that of the Galaxy. Therefore, there is no kinetic violation of LLI, i.e. there is no a preferred reference frame. However, the LLI violation that does exist, because there is an induced voltage, appear to be only a directional one indicating the existence of an asymmetry and anisotropy of space [1-3].

The graphs Figs. 2 and 3 show the existence of two directions along which LLI is violated:

xy plane – **$\alpha=\pi/4$** with respect to the direction of the north magnetic field of Earth
xz plane – **$\alpha=5\pi/4$** with respect to the direction of the north magnetic field of Earth
yz plane – no violation
where the angle α refers to the direction of the antenna with respect to the earth magnetic field.

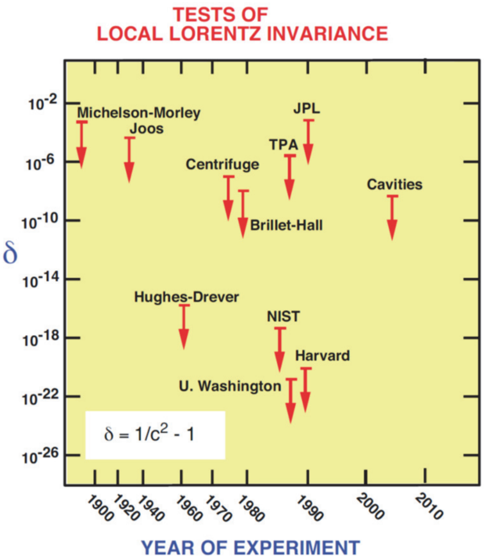

Fig. 1. Test of Local Lorentz Invariance.

3. CMBR Anisotropy and LLI Asymmetry

Since LLI violation appears to have an asymmetry, i.e. there are some privileged directions in space, we wanted to compare these directions with those existing for the CMBR. In order to do this, we transformed the CMBR privileged directions, see Fig. 4, from galactic coordinates into the altazimuth coordinates of the geographical position of the site where the coil experiment was performed (L'Aquila-Italy), in the days when it was carried out (beginning of June 1998), and at the intermediate times of each data taking run.

The direction of the antenna in the coil experiment are referred to the same altazimuth coordinates. Once in the same spatial reference system, we compared the alignment level of the vectors, CMBR anomalies/anisotropies and LLI violation directions, by taking their scalar product.

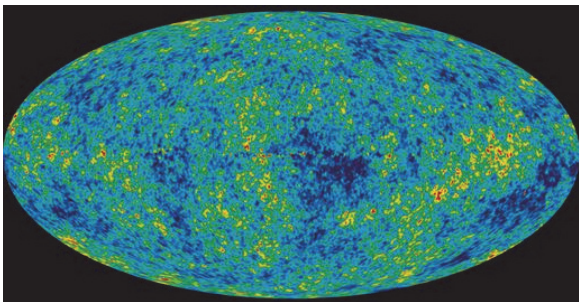

Fig. 2. Anisotropy of the CMBR showing galactic coordinates the presence of anomalies in the CMB temperature.

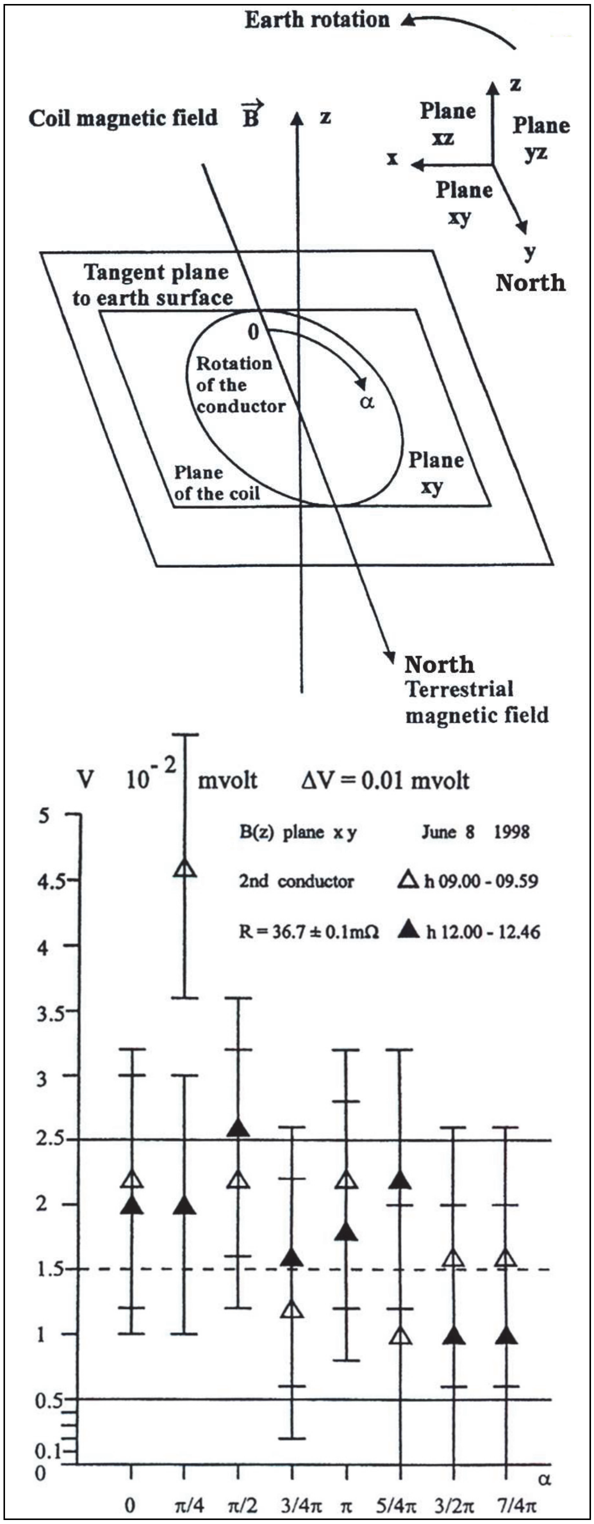

Fig. 3. LLI violation measurement with the apparatus in the xy plane.

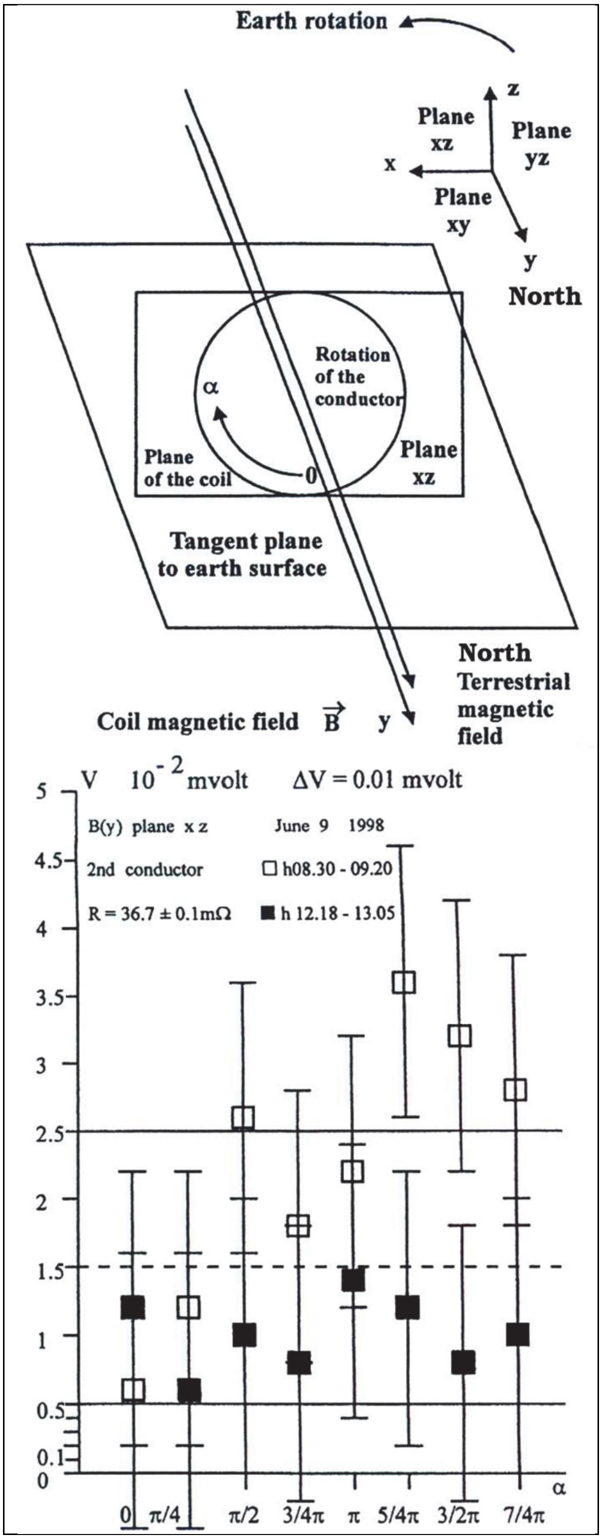

Fig. 4. LLI violation measurement with the apparatus in the xz plane.

4. Correlation between CMBR Anisotropies and LLI Violation Directions

The hypothesis of an underlying anisotropy appearing simultaneously at the two experimental levels, LLI experiment and the CMBR anisotropies, can be tested by analyzing the correlation of the time series from the LLI experiment and the corresponding values of the scalar products, see Tbl. 1 and Tbl. 2 [4].

These tables show the Spearman correlation indices (r) and the corresponding levels of significance (p_val) for the two cases: orientation and direction of the scalar product, and only direction, i.e. the absolute value of the scalar products.

Table 1.

	Orientation and direction		Direction	
CMBR's anisotropy	r	p_val	r	p_val
QOA	−0.346	0.016	0.020	0.892
HA	−0.190	0.196	0.272	0.061
PA	−0.128	0.386	−0.004	0.980
DM	−0.183	0.212	0.243	0.097
DB	−0.412	0.004	−0.120	0.417
MP	−0.240	0.100	0.095	0.521
CS	0.079	0.593	0.307	0.034

Table 2.

	Confidence interval	
CMBR's anisotropy	r_min	r_max
QOA (orientation and direction)	−0.646	−0.041
DB (orientation and direction)	−0.688	−0.135
CS (only direction)	0.035	0.586

The obtained results point towards a correlation, of negative sign, between the direction and the orientation of the LLI experiment measurements and the CMBR's anisotropies for the QOA (Quadrupole and Octupole alignements) and DB (Doppler boost) anomalies.

On the other hand, there is evidence of a positive correlation between the LLI experiment measurements and the direction of the CMBR anomaly/anisotropy CS (Cold Spot). We consider this second correlation as fundamental from a physical perspective, since the Cold Spot is not accountable so far in term of any model. This correlation points towards a possible existence of a common underlying asymmetry of the CMBR preferred directions and the directions of LLI violation [4].

5. Anisotropic Emissions of DST Neutrons and LLI Violation Directions

Our studies of the possible violation of LLI extended also to the nuclear interaction. In particular, it is possible to generate deformed Space-Time conditions in matter by locally concentrating a suitable amount of energy in Space and Time, i.e. reaching a suitable energy density in Space (J/m^3) and in Time (W). In Fig. 4 it is shown the ultrasonic device generating these conditions inside a steel bar from which neutron emissions in deformed Space-Time conditions (LLI violation) was recorded by a set of Polycarbonate detectors screened by boric acid [5-7]. We obtained the emitted neutron pattern shown in Fig. 5.

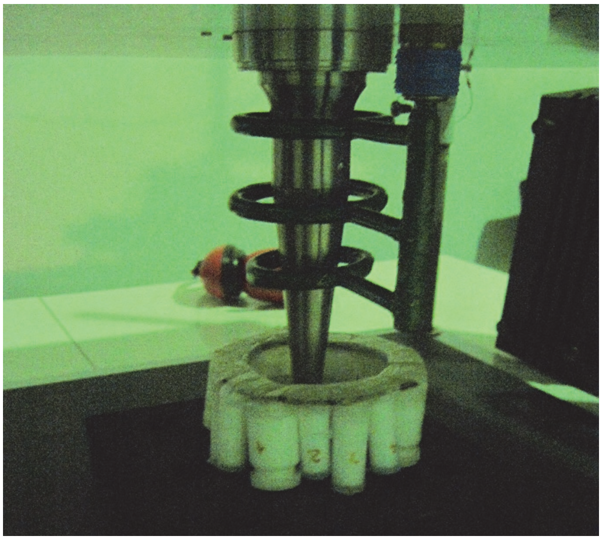

Fig. 5. Sonotrode, cooling pipieline and the 16 white containers with polycarbonate detectors immersed in boric acid.

In the same figure it is also reported the coincidence of the direction of LLI violation and the maximum of the anisotropic emission of neutrons

xy plane – **α=π/4** with respect to the direction of the north magnetic field of Earth

xz plane – **α=5π/4** with respect to the direction of the north magnetic field of Earth

6. CMBR Anisotropy and DST Neutrons Asymmetry

The same type of investigation about the possible correlation between the directions of LLI violation and the directions of the anomalies/anisotropies of the CMBR has also been performed between the direction of DST neutron emission and the CMBR.

In order to investigate the existence of identical preferred directions emerging from both phenomena it is necessary to describe the preferred directions found for the CMBR in the same spatial reference system of the Neutron experiment.

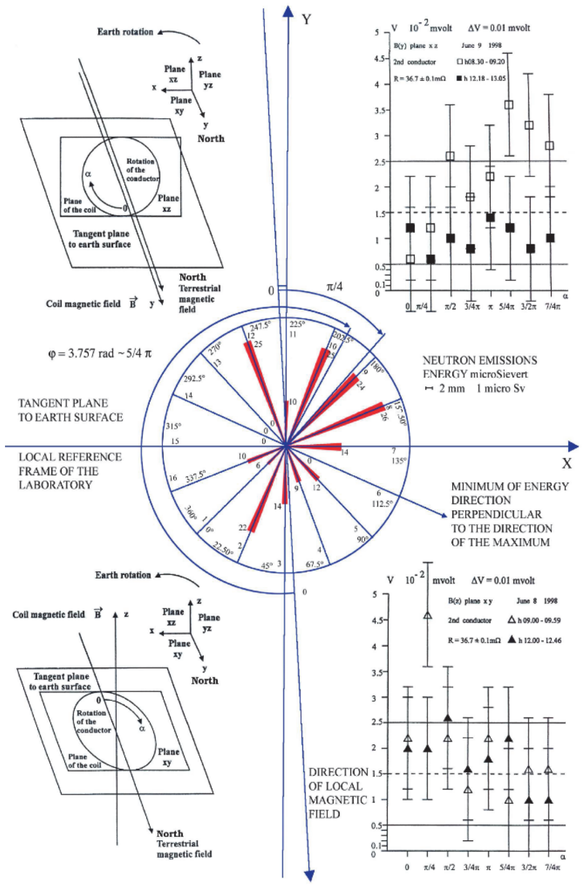

Fig. 6. Star of neutrons. The emitted neutron pattern at the centre of the figure is highly anisotropic and antisymmetric. The four drawings/diagrams on the four corners, represent the violation of LLI in the electromagnetic experiment of Fig. 3.

The spatial reference system, used to describe the directions of the neutron emissions, is an altazimuth system with the North represented by the magnetic North.

The geographical North differs very slightly from the magnetic North. We evaluated that, at the site and the date of the experiment, this difference is approximately 1°. Since the precision of the preferred directions of CMBR is equal to 1° of galactic latitude and longitude, we can consider the reference system of the experiments as an altazimuth system with the north given by the geographical North. We also considered fixed in the sky each CMBR privileged direction within

3 minute duration of the experiment (the angular displacements in the sky of these directions during this time is negligible). Therefore, we transformed the CMBR privileged directions from galactic coordinates in altazimuth coordinates for the geographical coordinates of the site, at the day and at the time of the experiment data recording.

7. Correlation Between CMBR Anisotropies and DST Neutron Emission Asymmetry

Once the directions of neutron emission and of the anomalies of CMBR were referred to the same reference system, we calculated the scalar product of the angles between the direction of anomalies and the 16 directions of neutron emission. A Spearman correlation test was carried out between the values of the scalar product and those of the micro Sievert of emitted neutron. In Tbl. 3 the indices (r) of the Spearman test and the corresponding levels of significance are reported for the two cases: orientation and direction of the scalar product, and only direction, i.e. the absolute value of the scalar products.

Beside PA (Power Asymmetry) and DM (Dipole Modulation), there is evidence of a (negative) correlation between the DST-Neutron experiment measurements and the direction of the CMBR anisotropies MP (Mirror Parity) and CS (Cold Spot).

This evidence is slightly stronger for the Cold Spot (CS) anisotropy only for direction, where both the significance of the statistical tests and the correlation index are higher, moreover the confidence interval is further shifted with respect to the zero value, see Tbl. 3.

Table 3. Confidence intervals from r-min and r-max for the statistically significant correlation indices of the Spearman correlation.

CMBR anisotropy	Confidence Interval	
	r_min	r_max
PA (orientation and direction)	-0.81	-0.04
DM (orientation and direction)	-0.82	0
MP (only direction)	-0.86	-0.03
CS (only direction)	-0.86	-0.12

Therefore, these evidences suggest, as in the case of the LLI experiment, that there exist a common alignment between the directions of the CMBR anisotropies and those of the nuclear emissions of DST neutrons.

Moreover, it appears that the correlation between the CMBR anisotropies and those of the DST neutron emission is stronger than the correlation with the LLI experiments [8].

8. The Spearman Correlation Index

For the sake of completeness, we used the Sprearman coefficient since we had to deal with non-Gaussian data having different variance, as in DST neutrons and Helmholtz coil experiments.

In fact, this index is well suitable because it does not depend on adjustable parameters. Moreover, it does not search for the linearity of the response among the variables under test but it looks only for the common variation since it is more general not to consider the measured variables as possessing a linear inter-dependence. Anyway, we used also the Pearson index and obtained comparable results. For this reason, we rely on the robustness of the statistical tests performed.

9. Terrestrial Scale Experiments and Cosmological Scale Phenomena Common Asymmetry

Therefore, it appears to exist a strong correlation between terrestrial scale experiments and the properties of cosmological scale phenomena:

- Asymmetry in LLI violation
- Anisotropy in DST neutron emission
- Anisotropy in CMBR.

10. Conclusions

Despite the well grounded ‘Copernican Principle’, stating that Earth is not a privileged observation point in the Universe, the evidences presented so far seem to point towards the need of a deep revision. Earth is neither a privileged point nor a non-privileged point of observation in the Universe, rather, it is a point of observation conditioned by natural laws themselves, which appear to possess a common asymmetry.

In this sense, the mathematical description of Nature needs no longer a spontaneous symmetry breaking, it does not need any symmetry at all despite its usefulness in solving the mathematics of physics (Noether theorem since 1918).

All it is needed is Lorentz violation and to understand the underlying asymmetry in the natural laws of interactions. In a more cautious way we can simply state that the laws of Nature are already endowed with their own asymmetry.

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Advanced Response of the Baikal Macroscopic Nonlocal Correlation Detector to the Heliogeophysical Processes*

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Macroscopic quantum entanglement is the manifestation of nonlocality, the consistent theory of which has been undeveloped yet. The heuristic consideration of the problem leads to the conclusion that the advanced nonlocal correlations are present in the dissipative random processes, which have been really observed in the previous experiments. The key experimental idea is to establish correlation between entropy productions in the source-process and perfectly protected against any local impacts probe-process in the detector. The strongest macroscopic nonlocal correlations are observed at extremely low frequencies; therefore, the long-terms experiments therewith under very stable conditions are necessary. Since 2012, a new experiment is carried out at the Baikal Deep-Sea Neutrino Observatory. Two nonlocal-correlation detectors, measuring spontaneous variations of potential difference of weakly polarized electrode pairs, were installed at near-surface and near-bottom horizons; there can be no classical correlations between them. The data processing has revealed the correlations between the signals of the bottom and top detectors and the 4200 km distant land detector located in Troitsk. The detectors respond nonlocally to the external (heliogeophysical) random processes; their signal correlation, determined by the causal analysis, is directed downwards: from the detector on the earth's surface to the detector near the Baikal floor. However, this correlation obeys the quantum weak causality principle: the bottom detector responds earlier than for the top one and the top detector responds earlier than the surface one. The retarded response component is presented too, but it is always less than the advanced one. The main source-process is solar activity. The highest signal-to-noise ratio in relation to this process appears at the top Baikal detector. By data of this detector, time-reversal causal connections of its signal with the random components of solar radio and X-ray fluxes have been revealed. The possibility of the forecast of solar activity by nonlocal correlation with several months' advancement has been demonstrated. According to entanglement monogamy property, great nonlocal correlation with one subsystem implies only small correlations with other ones. Indeed, nonlocal correlations of detector signal with other natural (geophysical) random dissipative processes turned out smaller. Nevertheless, they are also of interest; in particular, nonlocal nature of observed correlations has been verified by violation of the steering inequality with combination of solar and geomagnetic source-processes. The main results of the Baikal experiment are demonstration of macroscopic entanglement on the solar system scale, and demonstration of validity of the quantum weak causality principle, which, in turn, allows observing the random future.

Keywords: Entanglement, Heliogeophysical processes, Nonlocal correlations

1. Introduction

Macroscopic quantum entanglement is intriguing phenomenon studied in different special cases e.g. [1, 2], although the consistent theory of such a macroscopic manifestation of nonlocality has been undeveloped yet. However, based on the experimental data and theoretical considerations the heuristic equation of macroscopic

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entanglement was suggested [3-6]. This equation relates the entropy production in a probe random process (that is a detector) with entropy production in the sources and shows that transaction occurs with symmetrical retardation and advancement. The propagation velocity for diffusion entanglement swapping can be very small. Accordingly, the retardation and advancement can be very large.

The symmetry of the retarded and advanced correlation components is broken by a shielding medium between source and probe processes: the retarded response of a probe process is to some extent less than the advanced one (this is just manifestation of fundamental time asymmetry) [3, 6].

The predicted advanced correlation is in agreement with the principle of weak causality [7]: for the uncontrolled entangled states (or, in other terms, for the random processes) advanced nonlocal correlations through a timelike interval and hence time reversal causality are possible. Such a time reversal approach provides uniform interpretation of the experiments on observation of the advanced correlation in the case of usual microscopic entanglement [8-11]. We develop this approach in experimental context for the case of macroscopic entanglement.

Macroscopic nonlocal correlations with considerable advanced component were detected and studied (up to forecasting application) in the extensive series of experiments with large-scale natural random dissipative source-processes [3-6, 12-21]. The key experimental idea was to establish correlation between entropy productions in the source-process and perfectly protected against any local impacts probe-process in the detector. Theory of the detector relates the entropy variation in it with the measurable signal. Similarly, it is possible to relate entropy variation in the source with the measurable activity index. Thus, the macroscopic entanglement equation was tested.

However, the strongest macroscopic nonlocal correlations are observed at extremely low frequencies; therefore, the long-terms experiments therewith under very stable conditions are necessary. It is extremely difficult to maintain such conditions in a usual laboratory at the Earth surface. As a result, accuracy of above mentioned experiments and reliability of their interpretation were limited. To overcome that difficulty the detectors should be placed into naturally stable environment. There is just such an environment in the Baikal Lake. Baikal is the deepest lake in the World and its thick and quiet water layer is an excellent shield against any classical local impacts. In particular, the temperature near the floor is constant up to 0.01 K. By this reason since 2012 a new experiment is carried out at the Baikal Deep-Sea Neutrino Observatory. Two nonlocal-correlation detectors, measuring spontaneous variations of potential difference of weakly polarized electrode pairs (that is spontaneous variations of nanoscale double electric layer potentials) were installed at near-surface and near-bottom horizons. There can be no classical correlations between them. The data processing has revealed the correlations between the signals of the bottom and top detectors and the 4200 km distant land detector located in Troitsk. The detectors respond nonlocally to the external (heliogeophysical) random processes; their signal correlation, determined by the causal analysis, is directed downwards: from the detector on the earth’s surface to the detector near the Baikal floor. However, this correlation obeys the quantum weak causality principle: the bottom detector responds earlier than for the top one and the top detector responds earlier than the surface one. The retarded response component is presented too, but it is always less than the advanced one.

By data of all the previous laboratory experiments, the main source-process is solar activity. However, this fact should be confirmed directly by data of a deep-sea detector. It has become possible now since search of optimal setup configuration was finished and enough long (two years) continuous time series were obtained. Study of advanced Baikal detector response to solar activity is the purpose of the present work.

Although advanced correlation is entanglement witness, it is possible to test nonlocal nature of the observed correlation in in more conventional manner: by violation of Bell-like inequality. It was done before by data of three laboratory detectors of different types [3-6, 12, 14, 16-19, 21]. We do it again by high quality data of the Baikal detector with combination of two dependent sources: solar and geomagnetic activity.

In Sec. 2 we discuss the specific causal properties of nonlocal correlations in order to distinguish them from the classical local ones. The experimental setup is briefly described in Sec. 3. The results are presented and discussed in Sec. 4. We conclude in Sec. 4.

2. Causality and Nonlocality

The principle of weak causality admits for the entangled states both forward and reverse time order of a cause and an effect. Therewith time reversal causality is allowed only for a randomly generated cause (that is a cause not resulting deterministically from evolution). Thus, time reversal causal links do not constitute any causal chains. Such causality looks strange, but it does not imply the well-known classical paradoxes. An observer may get some information about the random future, but she/he cannot change it. Although the principle of weak causality was introduced by Cramer along the way of his transactional interpretation of nonlocality [7], it is applicable to other approaches to the emergence of advanced quantum nonlocal correlations [8, 11, 22].

To implement this idea strictly we need a definition of causality (and its quantitative measure) not appealing to time order. The appropriate mathematical tool for such an implementation is causal analysis, which in turn has classical and (of course, more complicated) quantum version (e.g. [6, 23]). Although we consider quantum phenomena, in the experiment we deal with only classical output of measuring device and may use simpler classical version (just as in the experimental verification of inequality using classical correlations or Shannon entropies).

Recall the essence of classical causal analysis. For any variables *X* and *Y*, in terms of Shannon marginal *S(X)*, *S(Y)* and conditional *S(X|Y)*, *S(Y|X)* entropies the independence functions can be defined:

$$i_{Y|X} = \frac{S(Y\mid X)}{S(Y)},\ i_{X|Y} = \frac{S(X\mid Y)}{S(X)},\ 0 \le i \le 1. \qquad (1)$$

Roughly saying, the independence functions behave inversely to module of correlation one. However, they characterize one-way correlations, which are asymmetric for causally related variables. In addition, they are well suited for any linear and nonlinear relationships. Next, the causality function γ is considered:

$$\gamma = \frac{i_{Y|X}}{i_{X|Y}},\ 0 \le \gamma < \infty. \qquad (2)$$

By definition *X* is the cause and *Y* is the effect if $\gamma < 1$. And inversely, *Y* is the cause and *X* is the effect if $\gamma > 1$. On theoretical and plenty of experimental examples, it had been shown that such a formal approach to causality did not contradict its intuitive understanding.

In terms of γ the principle of classical causality is formulated as follows:

$$\gamma < 1 \Rightarrow \tau > 0,\ \gamma > 1 \Rightarrow \tau < 0,\ \gamma \to 1 \Rightarrow \tau \to 0, \qquad (3)$$

where τ is time shift of *Y* relative to *X*. Only in case of nonlocal correlation, one can observe violation of this principle. It is just the case of weak causality, which does not obey the combination of inequalities of axiom (3).

It should be noted that strict dichotomy of the weak causality (violating chronology) and its counterpart, the strong one (respecting chronology), can be done only in the framework of quantum causal analysis, where another, more complicated measure of causality is used (the course of time c_2 instead of the causality function γ, because the latter loses sense at a negative conditional entropy) [6, 23]. However, if both conditional entropies are non-negative, in particular for device classical output (even measuring an entangled state), both principles, the classical and strong causality are equivalent. Finite causality, which does not obey these principles, is weak causality. It may occur only in the entangled states. Thus, violation of (3), say, $\gamma > 1$ at $\tau > 0$, is entanglement evidence that is a sufficient condition of nonlocality.

There is a way to test a necessary condition of nonlocality in usual manner, not appealing to time shifts, as it is done through the violation of Bell-like and steering inequalities. Suppose some process C acts upon a distant process A by means of any local interaction along the causal chain $C \to B \to A$. The intermediate process B is situated so that local carriers of interaction cannot come A avoiding B (e.g. B occupies a spherical layer around A). Then the claim of locality implies the steering inequality (derived in Refs. [6, 17, 19]):

$$i_{A|C} \ge \max\left(i_{A|B}, i_{B|C}\right) \qquad (4)$$

Violation of Ineq. (4) is necessary condition of entanglement. This is much like Bell-CHSH inequalities. The difference is that the latter verifies local realism, while Ineq. (4) verifies only locality.

3. Experiment

The Baikal deep-sea experimental setup is described in detail in Refs. [24] (primary configuration) and [25] (final configuration); so we present here only short description.

The experimental setup is deployed in the Southwestern part of the lake. The site depth is 1367 *m*. The bottom detector is set at the depth 1337 *m*, the top one is set at the depth 47 *m*. For comparison, synchronous measurements with the detector on the Earth surface (located in the laboratory in Troitsk spaced at 4200 *km*) are conducted. All the detectors are of electrode type, which by previous experience turned out the most reliable among other types (e.g. [6]). Every detector represents a couple of weakly polarized *AgClAg* electrodes HD-5.519.00 with practically zero separation. These electrodes are best in the World by their self-potential insensitivity to the environmental conditions.

The detector signals are measured and stored in the electronics unit set at the depth 20 *m*. The sampling rate is 10 *s*. The relative error of measurements is less than 0.01%. In addition, the electronics unit contains the temperature and acceleration sensors. The setup is fixed by the heavy anchor on the floor and by the submersed buoy at the depth 15 *m*. At a few kilometers from the setup, the auxiliary measurements of the sea currents and magnetic are also conducted continuously.

The deep-sea setup is designed to be operated autonomically for a year. Every March the electronic unit is lifted on the ice for data reading and battery changing and then it is installed again for the next year. This operation takes a few hours and at long time scale, the observation series can be considered as continuous.

Data were processed by the methods of spectral, causal and usual correlation analysis.

Below we consider results of data processing of the longest, biannual time series 2013/2015.

4. Results

Since detailed study of nonlocal correlations between all three detectors: bottom one *Ub*, top one *Ut* and land (laboratory) one *Ul* was undertaken before [25], in present work we are concentrated on nonlocal correlation with solar (and partly geomagnetic) activity.

The main source-process is solar activity. According to entanglement monogamy property, great nonlocal correlation with one subsystem implies only small correlations with other ones. Indeed, nonlocal correlations of detector signal with other natural (geophysical) random dissipative processes by data of all previous experiments turned out smaller. We will confirm this fact below.

The detector *Ut* proved to be optimal for the signal-to-noise ratio in the study of external heliogeophysical processes (the greatest noise is contained in the detector on the Earth surface *Ul*, the smallest – in the bottom one *Ub*, but in the latter the signal from the external processes have significantly shielded the water column). Therefore, we use for further analysis biannual *Ut* series.

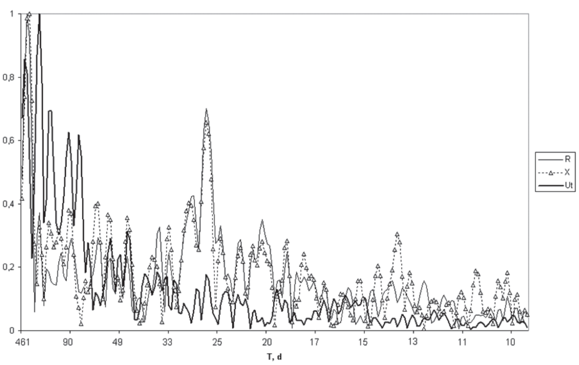

Figure 1. Normalized amplitude spectra of the signals of detector *Ut*, solar radio wave flux *R* and *X*-ray flux.

In Figure 1 the normalized amplitude spectra of the detector *Ut,* solar radio wave flux *R* (2800 *MHz*) and *X*-ray flux (0.5-4 *Å*) at the period range is from 10 to 460 days (*d*). As expected *Ut* practically does not respond to deterministic variations of the solar activity, which is represent in *R* and *X* by split 27-day variation and its harmonics. Over longer periods, where the random component (in particular, Reiger intermittent variation [26]) is dominated, the response in *Ut* is clearly visible. Detailed analysis showed the greatest similarity of the spectra of *Ut* with *R* in the band of periods 365 > T > 59 *d*, and *X* in the band 365 > T > 77 *d*.

Next to causal and correlation analysis data from such band-pass filtration. were used. The computation results are presented in Figures 2 and 3. Their stability was tested by alternate noising of the time series by the flicker noise (21% power).

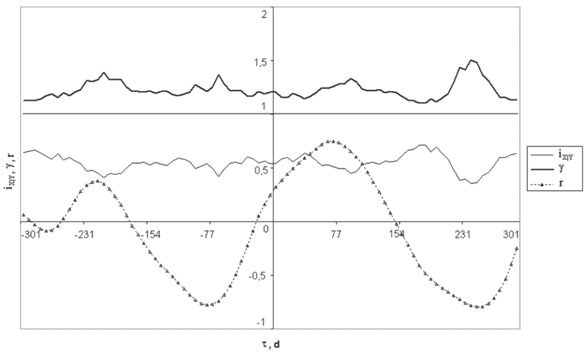

Figure 2. Causal and correlation analysis of *Ut* (X) and *R* (Y). τ < 0 corresponds to retardation of *Ut* relative to *R,* τ > 0 – to advancement.

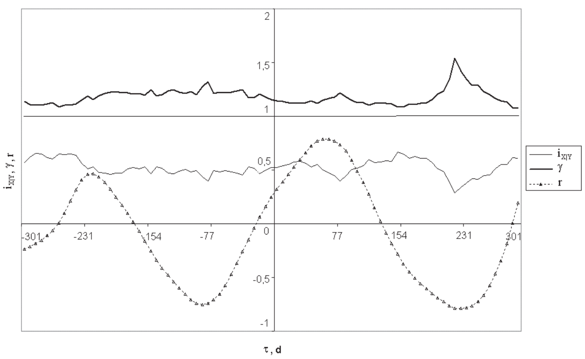

Figure 3. Causal and correlation analysis of *Ut* (X) and *X* (Y). τ < 0 corresponds to retardation of *Ut* relative to *X,* τ > 0 – to advancement.

In a couple of *Ut* – *R* it is seen that solar activity is a cause with respect to detector signal ($\gamma > 1$) and the highest maximum of causality $\gamma = 1.5^{+0.0}_{-0.1}$ is observed at advancement of *Ut* with respect to *R* for 250 days. At the same advancement, the deepest minimum of independence $i_{Ut|R} = 0.35^{+0.05}_{-0.00}$ and maximal correlation $r = -0.79^{+0.02}_{-0.01}$ are observed. In a couple of $Ut - X$ the results are close: the extreme $\gamma = 1.5^{+0.1}_{-0.0}$, $i_{Ut|R} = 0.28^{+0.02}_{-0.00}$, $r = -0.79^{+0.02}_{-0.01}$ are at advancement 230 days. Thus, the characteristic for nonlocal connection of the random processes time reversal causality is observed.

The level of advanced correlation is sufficient to forecast solar activity with optimal advancement 250 days. However, the large value of optimal advancement with dramatically reduced, due to the above-mentioned filtration, the length of *Ut* series make impossible yet to use forecasting algorithm of sliding regression (as it was done in the similar problem in Ref. [25]). But the very possibility of solar activity forecast is easily demonstrated by the series shift. It is shown in Figure 4 by the example of *R*.

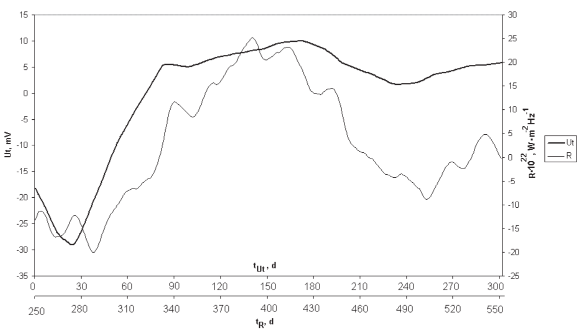

Figure 4. *Ut* approximately forecasts *R* with advancement 250 days.

In agreement with the results of all previous experiments of this kind, there are also weaker advanced connection of the detector signal with the global geomagnetic activity characterized by the *Dst*-index. With filtration 365 > *T* > 59 days in the couple *Ut* – *Dst* at advancement of *Ut* with respect to *Dst* for 110 days the extreme values $\gamma = 1.3^{+0.0}_{-0.1}$, $i_{Ut|Dst} = 0.51^{+0.08}_{-0.00}$, $r = -0.65^{+0.01}_{-0.00}$ are observed; with filtration 365 > *T* > 77 days extreme values are observed at the same advancement and amount $\gamma = 1.2^{+0.1}_{-0.0}$, $i_{Ut|Dst} = 0.47^{+0.00}_{-0.01}$, $r = -0.68^{+0.03}_{-0.00}$. The smaller value of advancement compared to solar activity is also consistent with the observed in all previous experiments a trend to a direct relation of this quantity with the scale of the source.

Although the nonlocal nature of the correlations follows from the observed violation of the principle of strong causality ($\gamma > 1$ at $\tau > 0$), the combination of connections of *Ut* with geomagnetic and solar activities presents the opportunity to another, independent proof of nonlocality, not appealing to time shifts: through violation of Ineq. (4).

In our case $A = Ut$, $B = Dst$, $C = R$ or X. Classical local influence of solar activity, indexed by *R* or *X* is well known. However, the electrode detector is insensitive to the local magnetic field (at least, up to 10^{-3} *T*), as well as it is insensitive to, mediated by the cosmic ray flux, classical local impact of solar activity [6]. But even assuming that these ideas are incomplete and some classical correlations *R* or *X* with *Ut* are available, Ineq. (4) must be satisfied. Let us check it by substitution of experimental values of independence functions under the same filtration of the series $A = Ut$, $B = Dst$, $C = R$ or X.

With filtration 365 > *T* > 59 days: for $C = R$: $i_{Ut|R} = 0.35^{+0.05}_{-0.00}$, $i_{Ut|Dst} = 0.51^{+0.08}_{-0.00}$, $i_{Dst|R} = 0.57^{+0.00}_{-0.01}$; for $C = X$: $i_{Ut|X} = 0.36^{+0.14}_{-0.00}$, $i_{Ut|Dst} = 0.51^{+0.08}_{-0.00}$, $i_{Dst|X} = 0.55^{+0.00}_{-0.00}$. With filtration 365 > *T* > 77 days: for $C = R$: $i_{Ut|R} = 0.34^{+0.00}_{-0.05}$, $i_{Ut|Dst} = 0.47^{+0.00}_{-0.01}$, $i_{Dst|R} = 0.54^{+0.00}_{-0.00}$; for $C = X$: $i_{Ut|X} = 0.28^{+0.02}_{-0.00}$, $i_{Ut|Dst} = 0.47^{+0.00}_{-0.01}$, $i_{Dst|X} = 0.55^{+0.01}_{-0.01}$.

In all four variants Ineq. (4) is reliably violated. Thus, the nonlocal (quantum) nature of the observed in the Baikal experiment correlations is proved also through the violation of the steering inequality without resorting to arguments of time shifts.

5. Conclusion

There is a time reversal causality causal connection of the signal of Baikal nonlocal correlation detector at a depth of about 50 *m* with a random component of solar activity (in the indexes to the radio- and X-ray fluxes). The possibility of the forecast of this component with advancement of more than 200 days has been demonstrated.

Using a combination of solar and geomagnetic sources, nonlocal character of the correlations has been proved by violation of the steering inequality.

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Interpretation of LIGO Results Using an Extended Form of Boscovich's Unified Field Theory

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Using an analysis from a physical and phenomenological viewpoint employing the renowned and recognized continuity of the Boscovich force curve, a new paradigm is formulated to explicate various physical phenomena in both the micro-world and the macro-world. Within this paradigm, an algorithm is established which produced a functional representation of the various atomic line spectra of hydrogen and the temperature dependent black-body energy distribution of radiation which compared very favorably with the experimental data. The Boscovichian points which are assumed to be endowed with certain characteristics move under the action of a force (acceleration) field that varies inversely proportional to the cube of the radius from the center of force which leads to an orbit described by an equiangular (logarithmic) spiral. This spiral consists of intercepts that correspond to stable and unstable points on the Boscovich curve. These intercepts are the roots of the equations employed and are described in the Pavia paper. Further representations also produced very favorable results for the photoelectric effect, (to be published). In addition, utilizing the shape of Boscovich's "extended" curve of force, the prospect of the interpretation of the mysterious attractive and repulsive forces beyond the visible Newtonian region of space, often described in terms of "black holes", "dark energy", etc. is proposed. The recent LIGO experiments provides a means of using this extended Boscovich's to analyze these results and is presented herein.

Keywords: Boscovich, LIGO, unified field theory

1. Introduction

In Sept. 2011, this author presented a paper at a conference on Boscovich held at Pavia Univ., Italy. Its title was "An Analytical Form of Boscovich's Curve with Applications". Boscovich published his first edition (1758) of his most work, "Theory of Natural Philosophy Derived to a Single Law of Forces Which Exist in Nature". A third edition was published in 1764 and subsequently translated into English by Child (1922).

Boscovich's idea of a single law of force is continuous and simple was used to produce excellent agreement with such microscopic data such as atomic line spectra and blackbody radiation and recently the photoelectric effect (to be published). Since it is assumed that any unified field theory should include not only microscopic data, but also macroscopic data, it was important to see if Boscovich provided such a means. He did wherein he introduced an extended form, which is rarely mentioned. This is the form that is used in the LIGO analysis. The following pages are taken from the Pavia paper.

2. Overview

Boscovich, in *Theoria* notes 10-11, presents his curve of forces as single continuous curves shown in Fig. (1)

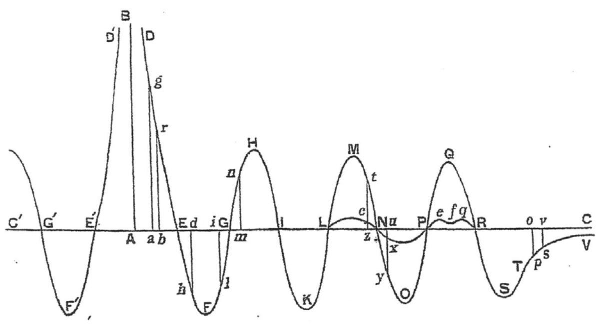

Fig. (1) from Boscovich, *Theoria philosophiae naturalis* (Venice, 1763). Note 1 – Hereafter referred to as "*Theoria*".

This paper uses the Latin-English edition, prepared by J.M. Child (1922). Boscovich states that this graphical representation does not require knowledge of geometry to set it forth (note 11 ibid) but only to glance at it as a portrait.

3. Description

A detailed description is presented in *Theoria* with the following six conditions:

> 117. The investigation of the equation, by which a curve of the form that will represent my law of forces can be expressed, requires a deeper knowledge of analysis itself. Wherefore I will here do no more than set out the necessary requirements that the curve must fulfill & those that the equation thereby discovered must satisfy. It is the subject of Art. 75 (2) of the dissertation *De Lege Virium*, where the following problem is proposed. Required to find the nature of the curve, whose abscissae represents distances & whose ordinates represent forces that are changed as the distances are changed in any manner, & pass from attractive forces to repulsive, & from repulsive to attractive, at any given number of limit-points: further the forces are repulsive at extremely small distances and increase in such a manner that they are capable of destroying any velocity, however great it may be.
>
> 118. In addition to what is proposed in this Art. 75, I set forth in the article that follows it the following six conditions; these are the necessary and sufficient conditions for determining the curve that is required.
>
> (i) The curve is regular, & simple, & not compounded of a number of arcs of different curves
>
> (ii) It shall cut the axis C'AC of Fig. (1), only in certain given points, whose distances AE', AE, AG', AG, and so on are equal pairs on each side of A.
>
> (iii) To each abscissa there shall correspond one ordinate & one only.
>
> (iv) To equal abscissa, taken one on each side of A, there shall correspond equal ordinates.
>
> (v) The straight line AB shall be an asymptote, and the asymptotic area BAED shall be infinite.
>
> (vi) The arcs lying between any two intersections may vary to any extent, may recede to any distances whatever from the axis C'AC, and approximate to any arcs of any curves to any degree of closeness, cutting them, or touching them, or osculating them, at any points and in any manner (Note 117-118).

4. Interpretation

Boscovich describes the arcs & areas & then offers an interpretation of the various sections of his curve.
He states, in *Theoria*:

> 167. With regard to the curve, there are three points that are especially to be considered; namely, the arcs of the curve, the area included between the axis & the curve swept out by the ordinate by its continuous motion, & those points in which the curve cuts the axis.
>
> 168. As regards the arcs, some may be called repulsive & others attractive, according indeed as they lie on the same side of the axis as the asymptotic branch ED or on the opposite side, & terminate ordinates that represent repulsive or attractive forces. The first art ED must certainly be asymptotic on the repulsive side of the axis, & continued indefinitely. The last arc TV, if gravity extends to indefinite distances according to a law of forces in the inverse ration of the squares of the distances, must also be asymptotic on the attractive side of the axis, & by its nature also continued indefinitely. All the remaining arcs are represented in Fig. (1) as finite (Notes 167-168).

Boscovich continues to describe the curve's analytical features in *Theoria:*

> 179. So much for the arcs & the areas; now we must consider in rather more careful manner those points of the axis to which the curve approaches. These points are either such that the curve cuts the axis in them, for instance, the points E, G, I &c. in Fig. (1); or such that the curve only touches the axis at the points. Points of the first kind are those in which there is a transition from repulsions to attractions, or vice-versa; & these I call limit-points or boundaries, since indeed they are boundaries between the forces acting in opposite directions. Moreover these limit points are twofold in kind; in some, when the distance is increased, there is a transition from repulsion to attraction; in others, on the contrary, there is a transition from attraction to repulsion. The Points E, I, N, R are the first kind, and G, L, P are of the second kind. Now, since at one intersection, the curve passes from the repulsive part to the attractive part, at the next following intersection it is bound to pass from the attractive to the repulsive part and vice-versa. It is clear that then the limit points will be alternately of the first & second kinds.
>
> 180. Further, there is a distinction between limit points of the first and those of the second kind. The former kind have this property in common;

namely that, if two points are situated at a distance from one another equal to the distance of any one of these limit-points from the origin, they will have no mutual force; & thus, if they are relatively at rest with regard to one another, they will continue to be relatively at rest. Also, if they are moved apart from this position of relative rest, then, for a limit-point of the first kind, they will resist further separation & will strive to recover the original distance, & will attain to it if left to themselves; but, in a limit-point of the second kind, however small the separation, they will of themselves seek to get away from one another & will immediately depart from the original distance still more. For, if the distance is diminished, they will have, in a limit point of the first kind, a repulsive force, which will impede further approach & will impel the points to mutual recession, & this they will acquire if left to themselves; thus they will endeavor to maintain the original distance apart. But in a limit-point of the second kind they will have an attraction, on account of which they will approach one another still more; & thus they will seek to depart still further from the original distance, which was a greater one. Similarly, if the distance is increased, in limit-points of the first kind, due to the attractive force which is immediately obtained at this greater distance; there will be a resistance to further recession & an endeavor to diminish the distance; & they will seek to recover the original distance if left to themselves by approaching one another. But, in limit-points of the second class a repulsion is produced, owing to which they try to get away from one another still further; & thus of themselves they will depart still more from the original distance, which was less. On this account indeed I have called those limit-points of the first kind, which are tenacious of mutual position, limit-points of cohesion, & I have termed limit-points of the second kind limit-points of non-cohesion (Notes 179-180).

5. Analytic Interpretation of Boscovich's Force Curve

The next step in the development of a quantitative analysis of the Boscovich curve will entail the magnitude of the various cycles. We now investigate the significance of an inverse cube type force. Boscovich mentions a force curve having the form given by: $\frac{a}{x^2} + \frac{b}{x^3}$ (Theorem note 121 in Supplement III notes 74) in which he cites Newton and Kepler. In particular he emphasizes the significance of the fact that the inverse cube force $\frac{\mu}{r^m} - \frac{\nu}{r^{m+n}}$ is very important at very small distances. This is the region of the oscillating characteristic of his force curve and this is the region that will be of importance in the description of various microscopic phenomena.

Boscovich is not alone in this inverse curve assessment. Heilbron (1982, pp. 52-86) mentions several attempts to modify Newton's law of gravitation. This includes names such as Lambert, Calendrini and Clairaut. Mention should also be made of Lewis (1989, pp. 652-653) who referred to the influence of Boscovich on Bertrand Russell who argued with Hannequin's criticism of both Boscovich and Kant. Russell defends Boscovich wherein he investigates the stability of a system based on Boscovich.

Following Boscovich, Russell (1897, App II.1) determines that stability of four equidistance points acting according to an attractive-repulsive force may be given by:

$$\frac{\mu}{r^m} - \frac{\nu}{r^{m+n}} \qquad (1)$$

For m = 2 and n = 1, this becomes:

$$\frac{\mu}{r^2} - \frac{\nu}{r^3} \qquad (2)$$

All of the aforementioned concentrated their efforts in the macroscopic region, however, like Boscovich, there are some in the modern era who examined the significance at an inverse cube law in the microscopic (small distance) region. Foremost among these were Thomson (1902, p. 160) where his interpretation of the structure of the atom assumed a radial repulsive force varying as the inverse cube of the distance from the center of the atom. Combined with a radial attractive force varying as an inverse square of the distance from the center. According to Gill (1941):

> J. J. Thomson mentions Boscovich in his theory of electrons. In this connection, H. Strache's book, *Die Einheit der Materie, des Weltaters und der Naturkrafte*, 1909, is also worth consulting. For similar reasons of those of Boscovich's, Strache rectifies the law of gravitation for small distances. As an example of such a rectification he gives the following formula:
>
> $$y = k\frac{mm_1}{x^2}(1 - \frac{b}{x}) \qquad (3)$$
>
> For great distances $\frac{b}{x}$ is very small so that the

formula passes into the law of gravitation. For x > b then y positive; for x = b we have y = 0, whereas y is negative for x < b. [Note: Gill's typographical errors corrected in last sentence.] Then the attraction goes over into repulsion. For x = b both masses are in equilibrium. With respect to the radii of action of the atoms and corpuscles there must be several points of equilibrium. On p. 6 the reproduction of Boscovich's curve of forces with five neutral points (p. xiv).

Following Boscovich's idea of the fact that at small distances, the inverse cube term predominates, leads to the assumption that this is the region of the oscillating position of the curve. With this in mind, an analytic function is given by:

$$F(x) = \frac{Be^{(-kx)}\sin\left\{\frac{1}{5}\pi\left(e^{(kx)}-e^{(-kx)}\cos(\pi x)\right)\sqrt{5}\right\}}{x^3} \qquad (4)$$

Eq. (4) may be referred to as an analytic description of Boscovich's curve, which represents its oscillating features.

Boscovich curve. Note as x → 0, F(x) → ∞ in accordance with Boscovich's curve shown in Fig. (1) (*Theoria*).

Any attempt to investigate or describe phenomena using the form of Boscovich's curve shown in Fig. (1) is usually given only to qualitative terms. To this writer's knowledge the analytical form shown by Eq. (4) and used in the Pavia paper, is the first time that an analytical expression has been described. It was employed in the calculation of microscopic phenomena such as line spectra series of hydrogen, and subsequently blackbody radiation. Also as mention in the Pavia paper, an attempt would be made to use Boscovich's curve in the calculation of the photoelectric effect. This has been completed and will be published.

In addition it was mentioned that the possibility of using Boscovich's ideas in the cosmological region would also be undertaken. This brings us to the macroscopic region and will address the discovery of the apparent discovery of gravity waves reported by Abbott et al. (2016) from the LIGO observations.

6. The Investigation of the LIGO Observations

To analyze the cosmological region demonstrated in LIGO, it was necessary to go to what is referred to as an extended version of Boscovich's curve. In Fig. (14) of the *Theoria*, Boscovich, discusses his curve thusly, as shown in our Fig. (2):

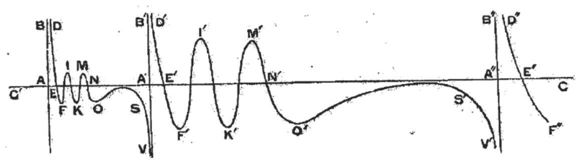

Fig. (2)

171. If, in Fig. (14), there are any number of segments AA', A', A'A", of which each that follows is great with regard to the one that precedes it; & if through each point there passes an asymptote, such as AB, A'B', A"B", perpendicular to the axis; then between any two of these asymptotes there may be curves of the form given in Fig. (1). These are represented in Fig. (14) by DEFI &c, D'E'F'I' &c; and in these the first arm E would be asymptotic & repulsive, & the last SV attractive. In each the interval EN, where the arc of the curve is winding, is exceedingly small compared with the interval near S, where the arc for a very long time continues closely approximating to the form of the hyperbola having its ordinates in the inverse ratio of the squares of the distances; & then, either goes off straightway into an asymptotic and attractive arm, or once more winds about the axis until it becomes an asymptotic attractive arc of this kind, the area corresponding to either asymptotic arc being infinite. In such a case if a number of points are assembled between any pair of asymptotes, or between any number of pairs you please, & correctly arranged, there can, so to speak, arise from them any number of universes, each of them being similar to the other, or dissimilar, according as the arcs EF...N, E'F'...N' are similar to one another, or dissimilar; & this too in such a way that no one of them has any communication with any other, since indeed no point can possibly move out of the space included between these two arcs, one repulsive and the other attractive; & such that all the universes of smaller dimensions taken together would act merely as a single point compared with the next greater universe, which would consist of little point-masses, so to speak, of the same kind compared with itself, that is to say, every dimension of each of them, compared with that universe & with respect to the distances to which each can attain within it, would be practically nothing. From this it would also follow that any one of these universes would not be appreciably influenced in any way by the motions & forces of that greater universe; but in any given time, however great, the whole inferior universe would

experience forces, from any point of matter placed without itself, that approach as near as possible to equal & parallel forces; these therefore would have no influence on its relative internal state (Note 171).

From Boscovich's Fig. (14) curve we can observe Newton's inverse squares representation flowing into portions of the regions SV and S'V'. It is this region of interest that will be discussed. A very interesting comment concerning Boscovich and the possibility of a unified field interpretation was made by Graves (1971), where he speaks of Boscovich's field of force:

It is this 'substantialization of force', which is one essential requirement for the notion of a field. In field theory a particle interacts directly (i.e. by spatiotemporal contiguity) with the field at the point where it is located, and only indirectly with any possible sources of that field. For Boscovich seems in fact to be creating a trichotomy of space, matter (identified with the point inertial masses), and force. While it is true that mass and force appear to be proportional, they are different sorts of entities, and Boscovich would certainly want to keep inertial and gravitational masses as separate concepts only accidentally related. Inertial mass is localized at the center of force, but gravitational mass really extends throughout space. But most important of all, insofar as Boscovich may be said to have a field of theory, it is a *unified* field theory. There is no multiplicity of forces surrounding the central mass and exerting independent influences on any test particles elsewhere in space, but only one. Although the total force-function may include many terms, they are all functions of r^1, which may be simply added together, i.e., $F = \Sigma_i f_i(r)r$. This force F will then affect all bodies in the same way, depending (presumably only) on their respective inertial masses. Boscovich's vision is certainly admirable. Its main weakness is that he never gave analytic (algebraic) expression for the total force; the most he achieved was a graphical representation of it. A reasonable expression might be a sum of increasing negative powers of r_1 so that $f_i(r) = \alpha_i r^{-i}$, where the first term would be $I = 2, \alpha_2 = -Gm$ (the gravitational term). The terms would alternate in sign, with the last term being opposite to that of the gravitational. (We will see that general relativity introduces correction terms of just this sort into the law for the gravitational field of a single mass-particle.) But there is no indication of what the magnitude of these other terms might be, what physical interpretation could be given to each of them, or whether the α's would require introducing any new parameters which might have to refer to other essential properties of matter (p. 123).

In a footnote, Graves (1971) continues:

Boscovich might in fact have been able to resolve Olbers' paradox that the night sky would be infinitely bright in an infinite universe with uniform average density of matter under Newton's law; he could simply have introduced an additional term (proportional to $\frac{1}{r^1}$ say) effective at great distances (p. 113).

It is this footnote that draws one's interest. Throughout the scientific literature regarding the quantum science of galactic structure, many theorists have commented on the Non-Newtonian aspects of the universe on the large scale.

Following the hint of the $\frac{1}{r}$ term, a search of the literature finds several investigations. One of the many findings is how many cosmologists modify Newton's gravitational law using a Yukawa functional form for the potential given by:

$$V_{Yukawa}(r) = -g^2 \frac{e^{-kmr}}{r} \tag{5}$$

where g is a magnitude scaling constant, m is the mass of the affected particle, r is the radial distance to the particle, and k is another scaling constant. The potential is monotone increasing, implying that the force is always attractive.

There have been many efforts to modify Newton's law. Seeliger (1895, pp. 132-133) felt that Newton's inverse square law was not exact and stated that it was only an empirical formula. In his modification, he made the assumption that an attenuation factor be used to express the gravitational force F between bodies m and m' be given as:

$$F = -Gmm' \frac{e^{-\lambda r}}{r^2} \tag{6}$$

Seeliger's effort was followed by another modification of Newton's law given by C. Neumann (1896) who felt that the problem could be resolved by using a potential of the form

$$\phi(r) = \frac{Aa^{-\alpha r}}{r} \tag{7}$$

which led to a generalized force F given by

$$F = -Gm_1 m_2 \frac{1+\alpha r}{r^2} e^{-\alpha r} \tag{8}$$

These ideas of Seeliger and Neumann in which a slight adjustment of modification of Newton's inverse square dependence has recently been addressed by others. One of these is Fischbach, Talmade, & Krause (1991) whose modification of The Newtonian effects is described using a modified expression for the potential energy V(r) is given by:

$$V(r) = \frac{-G_{\infty} m_1 m_2}{r}\left(1 + \alpha e^{-\frac{r}{\lambda}}\right) = V_N(r) + V'(r) \quad (9)$$

Here, G_{∞} is the Newtonian constant of gravity, and the parameters λ and α, respectively, give the range of the new force and its strength relative to gravity. Also, $V'(r)$ describes the correction to the effective gravitational potential arising from the particular non-Newtonian interaction we are considering (which in this case is a Yukawa).

This in turn leads to a force F(r) given by:

$$F(r) = \nabla V(r) = \frac{-G_{\infty} m_1 m_2 r}{r^2}\left[1 + a\left(1 + \frac{r}{\lambda}\right) e^{-\frac{r}{\lambda}}\right] \quad (10)$$

Fischbach et al. (1991) continue their modification by assuming a model which contains two canceling Yukawa potentials which result in an approximate exponential. It is suggested that the reader consult Fischbach et al. (1991) assumption, since it is too detailed to be presented here.

The fact to be considered is that in all of the efforts regarding the modification of Newton's law with a Yukawa potential, substantiates Graves (1971) comment regarding Boscovich's 1/r dependence for a field of force, in this case gravitation. With this in mind, a modified Yukawa force based on the Boscovich curve in the S, V and S'V' regions was developed in an ad hoc manner is given by:

$$F(x) = \frac{-ne^{px}}{x} \quad (11)$$

The curve using Eq. (11) is shown in Fig. (3)

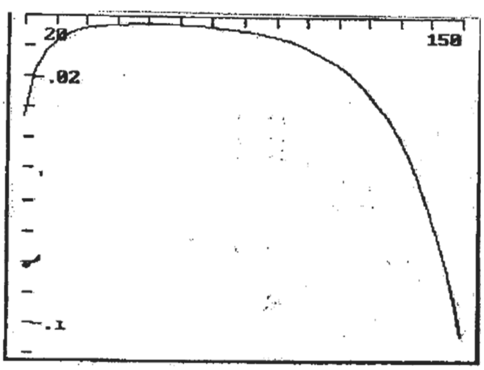

Fig. (3)

$$n = 0.0041214$$
$$p = 0.0549$$

One will note that the exponent is positive instead of the negative exponent that exemplifies the conventional Yukawa force. The reason for this is that in Boscovich's Fig. (14), the curve drops sharply negative in the S'V' and SV region.

As an aside, Bertin & Lin in their book, "Spiral Structure in Galaxies, A Density Wave Theory" (1995, p. 220) produced a curve based on a positive exponent in what appears to be a Yukawa type force, which is depicted in Fig. (4) below:

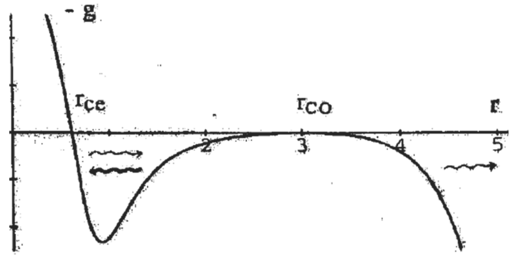

Fig. (4)

They mention that there are two turning points (i.e. two zeros of the function g). One is at $r = r_{ce}$, a simple turning point, and the other at r_{co} a double turning point. The relation between the Newton inverse square law and the $\frac{1}{r}$ dependence seems obvious and compares to Fig. (3) of this work.

This might indicate that Boscovich was ahead of his time again, since an extremely large negative force might imply that this might be due to large masses at extreme distances beyond our observation. Such a situation might explain the so-called "dark matter" that is spoken about in today's cosmology. The concept of dark matter was initiated by Oort (1932), who was studying stellar motions in the galactic region.

This was closely followed by Zwicky (1933) in his study of clusters and galaxies. Then in the 1960 to 1970 interval, Ruben & Ford (1970) established a method using more sensitive instruments to analyze velocity curves of distant galaxies with much more precision.

Based on the findings of Oort (1932), Zwicky (1933) and Rubin & Ford (1970), it can be said that there might be some substantiation of the Boscovich curve. It is also of interest to quote John D. Barrow's "Theory of Everything", who similar to Graves states:

> Newton simplified our apprehension of the world by explaining all gravitational phenomena within a simple scheme, which attributed the observed effects to the action of a single attractive force acting between all massive bodies. Despite the success of this programme, and the other areas of thermodynamics

and optics in which Newton was able to bring logical simplicity to a plethora of confusing observations, he knew that there were areas still shrouded in mystery. He speculated that there must exist other forces of nature – ‘very strong attractions’ – which hold material bodies together, but he could take that intuition no further.

One of the most remarkable and neglected figures in the history of modern European science was Roger Boscovich. Boscovich aimed to extend Newton’s overall picture of Nature in several important ways. In particular, he sought to ‘derive all observed physical phenomena from a single law’. In so doing, he introduced a number of concepts, which still form part of the intuition of scientists. He emphasized the atomistic notion that Nature was composed of identical elementary particles and then aimed to show that the existence in Nature of larger objects with finite sizes was a consequence of the way their elementary constituents interact with one another. The resulting structures were equilibrium states between opposing forces of attraction and repulsion. This was the first serious attempt to understand the existence of solid objects in Nature. He saw that Newton’s inverse-square law of gravitation alone was insufficient to explain the existence of structures with particular sizes because it endowed gravity with no characteristic scale of length over which its effects were especially manifest. The inverse-square law singles out no particular scale of length as special and has an infinite range. To explain objects of particular sizes requires a balance between gravity and some other force.

Boscovich proposed a grand unified force law, which included all known physical effects. This was his ‘Theory’, as he called it. It approached inverse-square law of Newtonian gravitation at large distances as required by observations of the lunar motions. But on smaller length scales, it is alternately attractive and repulsive and so gives rise to equilibrium structures whose sizes are dictated by the characteristic length scales built into the force law. The ‘Law of Forces’ he proposed is shown in Figure 2.1. Boscovich lays great stress upon the fact that this law is not merely a ‘haphazard’ aggregate of forces but needs to be a ‘single continuous curve’, which, he argues, witnesses to the unified all-encompassing nature of the theory. In addition to the pictorial representation of his force law illustrated here, Boscovich also introduced the idea of expressing his law as a convergent series of mathematical terms in powers of inverse distance, each smaller than its predecessor but the longer the sum is extended, the better becomes its approximation to the true force law.

There are many other innovations in Boscovich’s detailed treatise, but we are interested here in drawing attention to just this one point: that he was the first to envisage, seek, and propose a unified mathematical theory of all the forces of Nature. His continuous force law was the first scientific Theory of Everything. Perhaps, in the eighteenth century, only a generalist like Boscovich, who successfully unified intellectual and administrative activities in every area of thought and practice would have the presumption that Nature herself was no less multicultural (pp. 17-18).

With this information, it might be said that this is an adequate description of Boscovich’s famous curve. It’s various regions cover the entire range necessary to describe all phenomena in the microscopic range denoted as Region A (inverse cube), Region B (Newtonian, Coulomb; inverse square) and the so-called “dark matter”, “dark energy” in Region C (inverse). This is depicted in Fig. (5) along with the associated regional equations and respective constants in Eqs. (12) to (14).

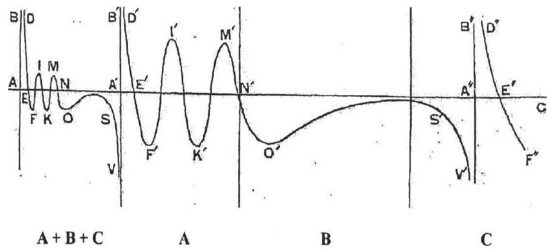

Fig. (5) from *Theoria* Fig. (14)

$$\text{A: } F(x) = \frac{Be^{(-kx)} sin\left\{\frac{1}{5}\pi\left(e^{(kx)} - e^{(-kx)}\cos(\pi x)\right)\sqrt{5}\right\}}{x^3} \quad (12)$$

$$k = 0.481211825$$

$$\text{B: } F(2) = -\frac{gk}{x^2} \quad (13)$$

$$g = 6.668$$

$$\text{C: } F(3) = \frac{-ne^{(px)}}{x} \quad (14)$$

$$n = 0.0041215$$
$$p = 0.0549$$

The numerical values for the constants shown in Fig. (5) have been chosen so as to make the Boscovich curve continuous throughout. One must remember that the curve is a qualitative one and cannot be shown in scale due to the fact that in Region A we measure distances in 10^{-13} cm or less, while in Regions B and C we measure distances in light years, e.g., in Region C we speak of 46-47 million light years. It goes without saying that much needs to be done especially in defining the various constants used in the curve description.

Referring to Seeliger, Kragh (2007) states:

> The modified force law was essentially ad hoc and also arbitrary, since many other modifications might resolve the gravitation paradox in a similar way. The idea of modifying Newton's inverse-square law was not, by itself, very original, as many modifications were proposed in the nineteenth century. The exponential correction factor can be found in 1825 in LaPlace's *Mecanique celeste,* which can hardly have avoided Seeliger's attention. However, what was original in Seeliger's approach was that he used it in a cosmological context and not, as in most other proposals, to solve problems of planetary astronomy (such as Mercury's anomalous revolution around the Sun) (p. 109).

Kragh (2007) also mentions Boscovich and his cosmological ideas:

> Apart from those already mentioned, several other Enlightenment natural philosophers took up cosmological questions. One of them was the Croatian-Italian astronomer and physicist Roger Boscovich, a Jesuit scholar, who in 1758 published his main work *Theoria philosophiae naturalis.* Although best known for its contribution to dynamic atomism and matter theory, the book also included considerations of a cosmological nature. For example, Boscovich imagined that, apart from our space, there might exist other spaces with which we are not causally connected. His conception of the universe was relativistic, such as illustrated by a passage from the end of *Theoria*, which may bring to mind much later cosmological ideas (p. 82).

He continues to quote Boscovich's ideas about space and time:

> If the whole Universe within our sight were moved by a parallel motion in any direction, & at the same time rotated through any angle, we could never be aware of the motion or of the rotation...Moreover, it might be the case that the whole Universe within our sight should daily contract or expand, while the scale of forces contracted or expanded in the same ratio; if such a thing did happen, there would be no change of ideas in our mind, & so we should have no feeling that such a change was taking place.
>
> Boscovich imagined all matter to consist of point-atoms bound together by Newtonian-like attractive and repulsive forces. If no forces were present, a body might pass freely through another without any collision (after all, points have no extension in space). The possibility led him to a daring cosmological speculation:
>
> There might be a large number of material universes existing in the same space, separated one from the other in such a way that one was perfectly independent of the other, & the one could never acquire any indication of the existence of the other (Note 518).
>
> Boscovich did not elaborate. Here we have, in 1758, a new version of the many universe scenarios: not different universes distributed in space and time, but co-existing here and now. It was surely a scenario that harmonized in spirit with ideas that some cosmologists would propose more than two hundred years later. (p. 82)

7. Interpretation of LIGO Observations

First it should be mentioned that according to Newton, gravity propagation is instantaneous, which means that all physical interactions would occur at infinite speed and there would not be any gravitational waves. Usually, astronomical observations do not allow direct measurement. Hulse and Taylor in 1974 examined binary pulsars wherein a radiating neutron star orbited another neutron star (Taylor & Weisberg, 1982). Their work produced a model that used general relativity that produced evidence that gravity waves exist, however this was only an indirect indication of gravity waves. Nonetheless, they did receive the 1993 Nobel Prize in Physics. This is now considered to be the beginning of gravitational astronomy.

On Feb. 11, 2016, the LIGO Scientific Collaboration and VIRGO Collaboration teams announced that they had made the first direct observation of gravitational waves that originated from a pair of merging black holes. This was followed by another announcement on June 15, 2016 who reproduced identical results. This brings us to the analysis of the LIGO results using Boscovich's ideas in cosmological area. Looking at the analytical expression shown in Eq. (12), one sees that there is an inverse cube relationship. This brings us to the question, whether or not this could be employed in the explanation of the LIGO observations.

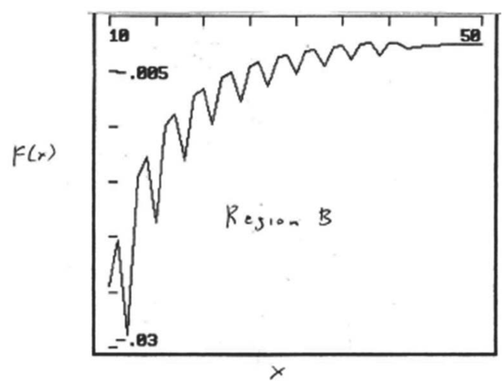

Fig. (6)

Starting with the inverse cube, which is expressed in region A of Fig. (5), which was so instrumental in its application of the microscopic areas, the author decided to increase the magnitude of the constant B in Eq. (12), to see if be applied in the astronomical region.

Figs. (6) and (7) do show that it has structure in the B and C region.

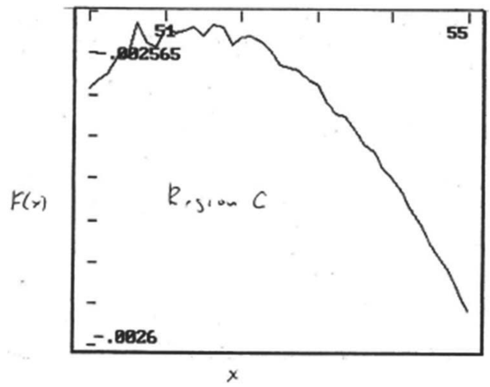

Fig. (7)

Based on this, a decision was made to see if it could be of use in the analysis of the LIGO gravity wave results. Research produced information on the inverse cube concept first investigated by Newton in his *Principia*. Newton showed that in his Propositions 43-45 of the *Principia* (1687) for Prop. 43, an added force might be a central force. In Prop. 44, he derived an equation for a force, which was dependent on the inverse cube of the radius. In Prop. 45, he extended his theorem to arbitrary central forces. It was Chandrasekhar (1995) treatise on "Newton's *Principia* Written for the 'Common Man' ", who studied these theorems and had remained dormant for over 300 years. It is Prop. 44 that specifically mentions the inverse cube idea. Briefly Theorem 44 states

> *The difference of the two forces, by which two bodies may be made to move equally, one in a fixed, the other in the same orbit revolving, varies inversely as the cube of their common altitudes.*

Chandrasekhar (1995) shows that this difference may be expressed as:

$$P'(r) - P(r) = \frac{h^2(\propto^2 - 1)}{r^3} \qquad (15)$$

and when the initial force $P(r) = 0$, then an inverse-cube central force is given as:

$$P'(r) = \frac{h^2(\propto^2 - 1)}{r^3} \qquad (16)$$

Introducing such an inverse-cube force leads to solutions involving equiangular triangles (p. 184).

Chandrasekhar (1995) in his excellent treatment of Newton's *Principia*, expressed surprise that only one book on celestial or dynamic mechanics had treated Newton's Theorem on revolving orbits, namely Whittaker (1997), in his book on *Analytical Dynamics*. This author has found another individual in Lamb (1960). There are many books that treat the inverse cube to show that a spiral results, but only as examples, not as a Newtonian interpretation.

Nonetheless, there is no doubt that an inverse cube does result in a spiral. With this in mind, it was decided to see how the author's analytical form of Boscovich's curve could be used in the investigation of the LIGO observations.

8. Method

Since it has been established that an inverse cube radius produces a logarithmic spiral, it was decided to analyze the Pavia results using the Boscovich inverse cube relation by a set of parametric equations shown below (Eqs. (17) & (18)), which produced Fig. (8).

It should be noted that the x-intercepts occur at both the Fibonacci integer as well as non-integer numbers. The analysis based on Fibonacci numbers is discussed at length in the Pavia paper and are shown to be most important in the development of the Boscovich curve.

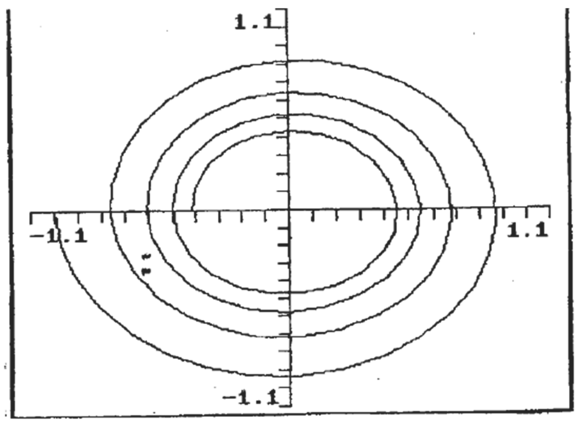

Fig. (8)

$$\frac{Be^{(-kx)}cos\left(\frac{1}{5}\pi\left(e^{(kx)} - e^{(-kx)}\cos(\pi x)\right)\sqrt{5}\right)}{x^3} \qquad (17)$$

$$\frac{Be^{(-kx)}sin\left(\frac{1}{5}\pi\left(e^{(kx)} - e^{(-kx)}\cos(\pi x)\right)\sqrt{5}\right)}{x^3} \qquad (18)$$

$$B = 10^4$$

Having established this parametric relationship, it was decided to see how this translates into a form to be consistent with the LIGO results. We start with the statement given in Andronov, Vitt, & Khaikin (1996, pp. 16-21) using his Eq. (1.28) shown below. This equation produces a family of logarithmic spirals. For small $\frac{h}{\omega}$ produces a family of spirals close to a series of ellipses. (This will be significant in a later discussion on binary stars.)

$$e = Ce^{\frac{h}{\omega}\phi} \tag{19}$$

Andronov et al. (1996, p. 58) continues to show that the motion of the spiral system is also an oscillatory process shown in his Figs. (47) & (48); which are shown here as Fig. (9) and Fig. (10). The maximum deviations are shown to increase with time (Fig. (10)) where the dependence on time is given by:

$$x = Ke^{-ht}\cos(\omega t + \alpha), where\ h < 0 \tag{20}$$

where h is less than zero, and K, ω and α are chosen so as to fit the initial conditions. By changing the instant from where the time is measured, a simpler expression given by:

$$x = Ke^{-ht}\cos(\omega t) \tag{21}$$

where α is set to zero.

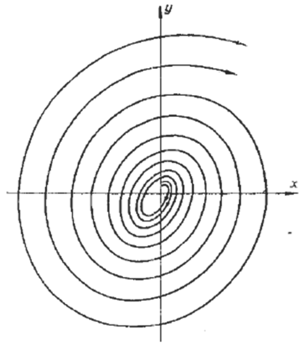

Fig. (9)

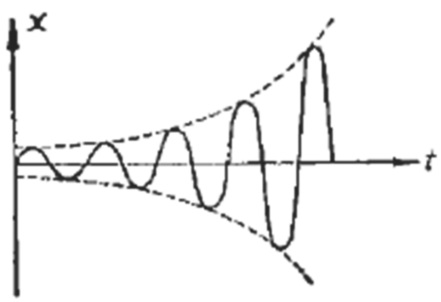

Fig. (10)

Returning to Fig. (48) in Andronov et al. (1996) he states that the oscillations there are not really periodic so one cannot speak of a definite period. However the interval of time between successive maxima, on the same side can be referred to as 'conditional periods' of damped oscillations equal to:

$$T = \frac{2\pi}{\omega} \tag{22}$$

This is shown in Fig. (11). Of course the numerical value of h depends on the choice of the units of time.

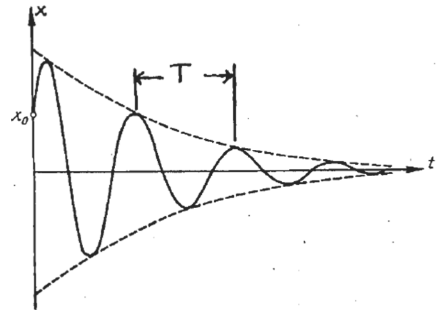

Fig. (11)

If one looks at the LIGO observations, shown below, it is obvious that it displays an oscillatory damped characteristic similar to that shown in Andronov et al. (1996) Fig. (48) (our Fig. (10)), which was determined from a spiral motion of the system.

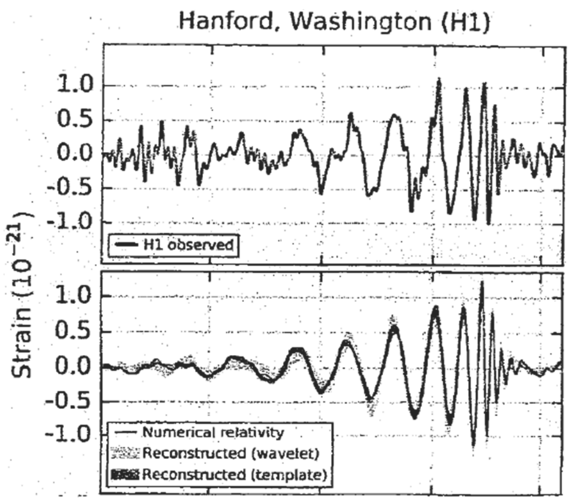

Fig. (12)

Taking the time between successive maxima of the Hanford, WA results yielded an average of $T = 0.02$ sec. Now using this value in:

$$T = \frac{2\pi}{\omega} \tag{23}$$

produces a value for:

$$\omega = \frac{2\pi}{T} \tag{24}$$

Carrying out a non-linear regression calculation yielded the following set of constants:

$$K = 0.0004325$$
$$h = 18.86$$
$$\omega = 314.0$$

Using these constants in:

$$x = Ke^{-ht}\cos(\omega t) \tag{25}$$

resulted in the figure shown below with a comparison of the two LIGO observations. The calculation was for a time interval of 0.175 sec (.425-.25 sec.), which is about the range of the LIGO results where the ordinate is essentially zero. The maxima appear to agree with the LIGO results along with the magnitudes.

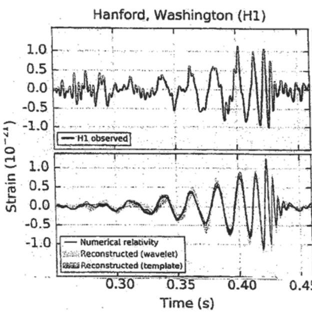

Fig. (13)

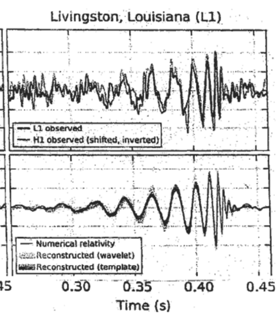

Fig. (14)

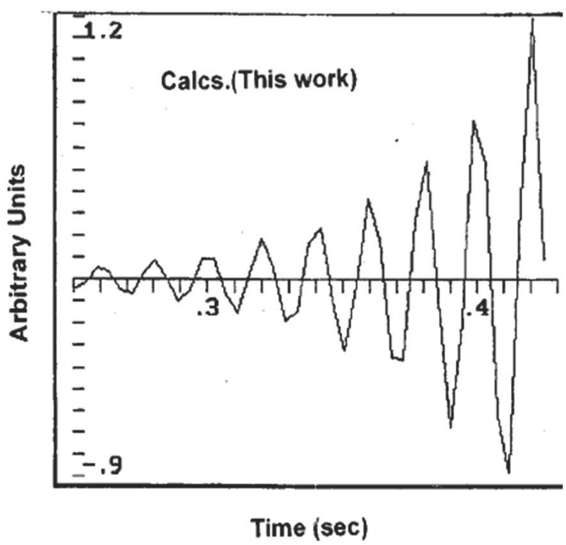

Fig. (15)

9. Conclusion

This analysis seems to confirm the possibility of gravity waves. However, it is somewhat questionable whether or not this is the result of the merging of two black holes. The fact that if one considers the concept of Newton's 'revolving orbits', then this could be applied to a system of binary stars, orbiting around their common barycenter. If we consider the fact that they are actually the result of two stars with an opposite spiral motion, then if certain conditions exist such that they approach an ellipse as mentioned by Andronov, then the possibility of their orbiting in opposite directions could very well produce beats.

To this author's knowledge, this concept has not been analyzed to a depth that might bear scientific scrutiny. However, there has been some work on millisecond pulsars that are said to confirm gravity waves. The timing of these pulsars could well be an indication of beats. With this in mind, this author carried out another calculation using the identical time interval of 0.175 sec. (0.6-0.425). The results are shown in Fig. (16) below.

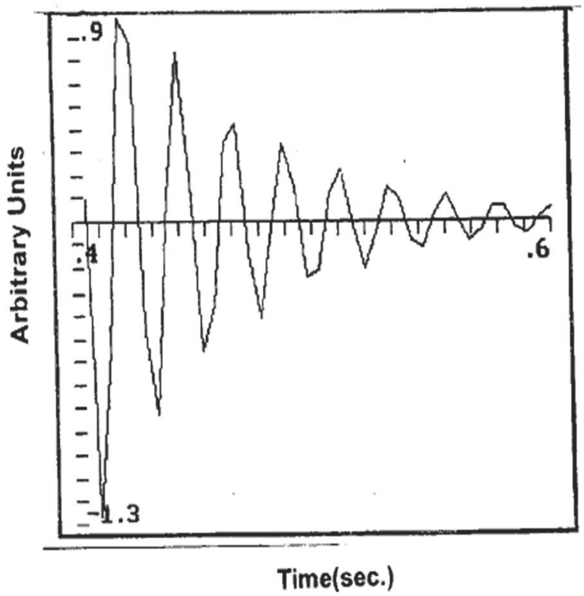

Fig. (16)

This is almost a mirror image of the 0.25 to 0.425 sec. range of the LIGO results. This could very well be an indication of a beat phenomenon. It is hoped that future LIGO type experiments will explore this range. If it turns out to be true then Graves' (p. 113) comment might be correct wherein he states that Boscovich may have had a field theory which might be consistent with a unified field theory. It will be interesting to see if future experiments substantiate this claim.

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Relative Movement of Two Bodies – Hubble's Law, Expanding Universe and Newton's Laws Controversies

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The mathematical expressions for the relative movement of two bodies at parallel and crosswise trajectories are developed. Some controversies concerning the validity of Hubble's law, expanding Universe concept and Newton's laws are discussed.

Keywords: Boscovich, Relative motion, Absolute motion, Hubble's law

1. Hubble's Law and Expanding Universe

Hubble's discovery in 1929 [1] was that recession velocity V_r of a galaxy from our Galaxy is proportional to its distance r:

$$V_r = H_0\, r \tag{1}$$

where H_0 is known as Hubble's constant and the relation (1) as Hubble's law.

In modern studies of velocity–distance relation, accurate velocities of galaxies can be readily derived from their red shifts. It is much more difficult to estimate the distances of galaxies and other extragalactic objects. Thanks to the efforts of many astronomers and a large program of observations using Hubble Space Telescope, there is now general agreement that the value of H_0 is:

$$H_0 = (72\pm8)\text{ km s}^{-1}\text{ Mpc}^{-1} = (2.3\pm0.3)\times10^{-18}\text{ s}^{-1} \tag{2}$$

where the error is at the $\pm1\sigma$ level [1]

The combination of the isotropy of the Universe and Hubble's law shows that the Universe as a whole is expanding uniformly at the present time. In a uniform expansion, the ratio of distances between any two points increases by the same factor in a given time interval $t_2 - t_1$, that is:

$$r_1(t_2)/r_1(t_1) = r_2(t_2)/r_2(t_1) = \cdots$$
$$= r_n(t_2)/r_n(t_1) = \beta = \text{constant.} \tag{3}$$

The recession velocity of galaxy 2 from the galaxy 1, which we can choose as the origin, is therefore:

$$V_{r2} = [r_2(t_2)-r_2(t_1)]/(t_2 - t_1) =$$
$$(\beta-1)r_2(t_1)/(t_2 - t_1) = H_0 r_2(t_1) \tag{4}$$

Similarly, for galaxy n,

$$V_{rn} = (\beta-1)r_n(t_1)/(t_2 - t_1) = H_0 r_n(t_1) \tag{5}$$

Thus, a uniformly expanding distribution of galaxies automatically results in a velocity–distance relation of the form $v = H_0 r$. Note that it does not matter which galaxy we choose as the origin. All observers rightly believe that they are at the center of an expanding Universe with the same Hubble relation, if the observations are made at the same cosmic time.

2. Boscovich's Comprehensions of Absolute and Relative Movements

According to traditional Newtonian comprehensions, [2, 3]. Boscovich (1711-1787) distinguishes between absolute and relative movements. He says: "We do not observe absolute motions, but merely relative motions with respect to the Earth, or at most those with respect to the planetary system or the system of all the fixed stars." [2, p. 386] "Hence it follows that, if the whole Universe within our sight was moved by a parallel motion in any direction, and at the same time rotated through any angle, we could never be aware of the motion or the rotation. Similarly, if the whole region containing the room in which we are, the plains & the hills were simultaneously turned round by some approximately common motion of the Earth, we should not be aware of such a motion; for practically the same ideas would be excited in the mind. Moreover, **it might be the case that the whole Universe within our sight should daily contract or expand** (*bolded by authors DS and RA*), while the scale of forces contracted or expanded in the same ratio; if such a thing did happen, there would be no change of ideas in our mind, & so we should have no feeling that such a change was taking place." [2, Supplement II, & p. 19]

To illustrate the difference between the absolute and relative motions Boscovich applies very simple and comprehensible examples. S. Kutlesha [3] presented three cases considered by Boscovich [4]:

Case I. "Suppose that some point is moving by uniform absolute motion (*vero motu absoluto*) in direction AB and that some other point is moving by the same velocity at the parallel direction CD (Fig. 1). If at the beginning of movement, the distance between points was AC, then at the end of movement it will be BD = AC. Each point has been moved by absolute motion, but the movement of one point toward another did not exist, since neither the distance between them nor the direction has been changed. The points were in rest relatively even if they moved absolutely."

Case II. "If the second point is moved along to some other direction, e.g. CE, which is in some other plane in respect to the plane of movement of the first point along AB, then DE is relative movement of the second point respectively to the first one, and their final distance will be BE."

Case III. "We can realize that DE represents the relative movement of second point against the first point, if we suppose that the first point is at rest in B and that the second point initially is at D. As time elapses, the first point remains at B, while the second point approaches E along the direction DE, where DE represents the relative movement of the second point with respect to the first point resulting in a new distance BE."

It should be quite clear what case I means, but the other two cases might need some clarification. Case II has absolute motion of first point as from A to B the distance AB, and absolute motion of second point as from C to E the distance CE. Now the first point is unaware of its absolute motion AB and unaware of motion of second point from C to D the distance CD, and is only aware of the motion of the second point as from D to E the distance DE, which Boscovich is calling the relative motion of the second point; actually the relative motion of the second point according to the first point. Bearing this sort of analysis in mind, case III should be clear.

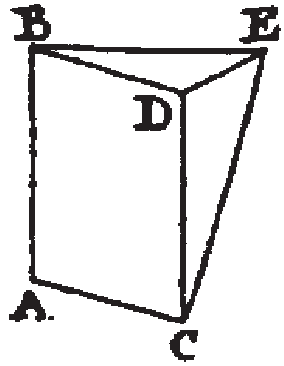

Fig. 1. On Boscovich's comprehension of absolute and relative movement. [3, 4].

If the movements are considered as paths, i.e. distances, that points realized in time interval Δt, then the lines in Fig. 1 are paths of the first and the second points. Their initial distance at $t = 0$ is AC. After some time Δt their distance is BE, and their relative movement is Δr = BE-AC, and their relative velocity is $V_r = \Delta r/\Delta t$. If it is so, it seems that Boscovich made mistake denoting distance DE as their relative movement. If the movements are considered as velocities, however, then lines on Fig. 1 can be treated as vectors: Velocity of the first point is vector AB, and the velocity of second point is vector CE. Then vector AB = vector CD and vector CE = vector CD + vector DE. Hence, vector DE = vector CE – vector CD. Similarly, vector BE = vector BD + vector DE. Hence, vector DE = vector BE – vector BD. If movements are vectors, then vector DE could represent relative motions of the two points, as it was stated by Boscovich.

In order to analyze the relative movements more thoroughly and exactly, we shall derive here the mathematical expressions for the relative movements of two bodies at parallel and crosswise trajectories to explain the cases I and III. (The case II will not be considered here.) We shall propose that **there is no force acting between the bodies that could accelerate or decelerate their movements**.

An issue to remember in this analysis is the Cosine Effect error [5]: "Police microwave and laser radars measure the relative speed a vehicle is approaching, or receding, the radar. If a vehicle is traveling directly at the radar the relative speed is actual speed. If the vehicle is not traveling directly at the radar the relative speed is slightly lower than actual speed. The phenomenon is called the Cosine Effect because the measured speed is directly related to the cosine of the angle between the radar and vehicle direction of travel or speed vector. The greater the angle the greater the speed error and the lower the measured speed. A cosine angle of 90° has 100% error, speed measures zero.” The difference between relative speed and actual speed means there can be apparent acceleration, when there is no actual acceleration involving force.

3. Parallel Movements of Two Bodies

Consider two bodies G and E, having different velocities v_1 and v_2, moving in the same direction at parallel trajectories at distance D (Fig. 2). Initially at t = 0, the distance between them is equal to D. After some time t, G will pass along the path S_1, while E goes along the path S_2, eq. (6), and the distance between the G and E is increased to r, eq. (7).

$$S_1=v_1t; \quad S_2=v_2t; \qquad (6)$$

$$r^2 = D^2 + (S_1-S_2)^2 = D^2 + (v_1-v_2)^2t^2 \qquad (7)$$

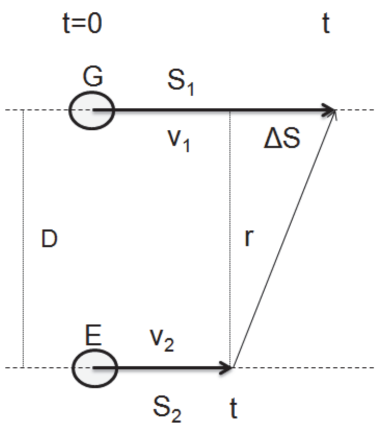

Fig. 2. Two bodies G and E having different velocities v_1 and v_2 are moving in the same direction on parallel trajectories at distance D apart. Here: t is time; S_1 and S_2 are the paths; r is the distance between the bodies, initially $r_0 = D$.

The relative movement of two bodies **at parallel trajectories** can be represented by their relative velocity $V_r = dr/dt$ and distance by (8) and (9).

$$V_r = dr/dt = (v_1-v_2)^2t/[D^2+(v_1-v_2)^2t^2]^{1/2} \qquad (8a)$$

$$V_r = dr/dt = (v_1-v_2)[1-D^2/r^2]^{1/2} \qquad (8b)$$

$$r = [D^2 + (v_1-v_2)^2t^2]^{1/2} \qquad (9)$$

The equations (8 and 9) represent the effects of time (t) and distance between parallel trajectories (D) on relative velocity (V_r) and distance (r) of two bodies moving in same direction.

These equations, with some modifications, can be applied for the following cases of parallel movement of two bodies:

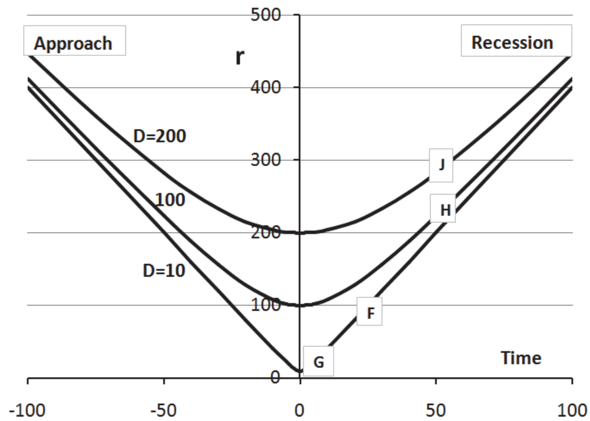

Fig. 3a. Effect of time on distance (r) between a moving body and a motionless body. Where D is a normal distance of motionless body to trajectory of moving one. Calculated by eq. (13) and initial conditions: $v_1 = 4$, $v_2 = 0$ and $r_0 = D$. All values are in arbitrary units.

(a) Case $v_1 = v_2$; movement in the same direction: $V_r = 0$ and r = D (as it is in case I of Boscovich);

(b) Case of recession of bodies, in other words movement of bodies in the opposite directions:

$$V_r = dr/dt = (v_1+v_2)^2t/[D^2+(v_1+v_2)^2t^2]^{1/2} \qquad (10)$$

$$r = [D^2 + (v_1+v_2)^2t^2]^{1/2} \qquad (11)$$

(c) Case $v_2 = 0$; when the first body is moving (by velocity v_1), and the second body is at rest (i.e. $v_2 = 0$ as it is in case III of Boscovich): The corresponding expressions (12) and (13) can be used for both approaching and receding motions of these two bodies (Fig. 3).

$$V_r = dr/dt = v_1^2t/(D^2+v_1^2t^2)^{1/2} = v_1 [1-D^2/r^2]^{1/2} \qquad (12)$$

$$r = (D^2 + v_1^2t^2)^{1/2} \qquad (13)$$

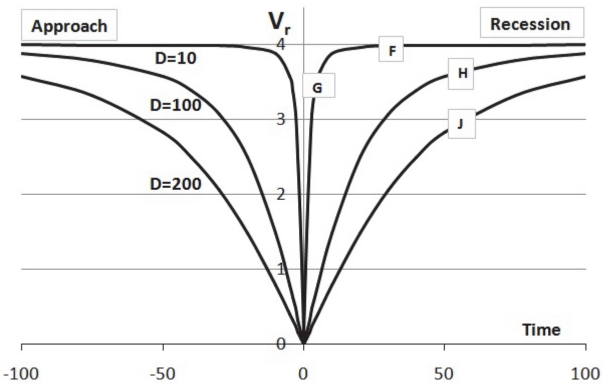

Fig. 3b. Relative velocity V_r plotted against time, and calculated from eq. (12), with conditions the same as in Fig. 3a.

There are several peculiarities that should be noted:
(c1) For D = 0 the second body is on the same trajectory as the first body, with $V_r = v_1$ and $r = v_1 t$.
(c2) For $D \to \infty$ than $r \to \infty$ and $V_r = 0$.
(c3) If $t \to \infty$ than $v_1 t >> D$ and $V_r \to v_1$ whatever the value of D.

Suppose that G and F represent two galaxies moving along the same trajectory, both with absolute velocity v1, but at different distances rG and rF from the Earth E (Fig. 4). An observer on Earth will notice that the recessive velocity of G is lower than that of F, i.e. VrG<VrF (Fig. 3 and eq. (12)). (This is in agreement with Hubble's law.)

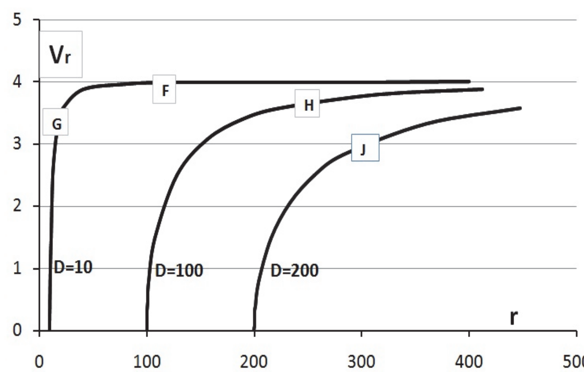

Fig. 3c. Relative velocity V_r plotted against distance r, and calculated from eq. (12), with conditions the same as in Fig. 3a.

Suppose that galaxies H and J both move with absolute velocity v_1 **at parallel trajectories** where $D_H < D_J$) (Fig. 4): hence, an observer on Earth will notice that the recessive velocity of H is higher than that of F, i.e. $V_{rH} > V_{rF}$ (Fig. 3 and eq. (12)). (This is not in agreement with Hubble's law.)

There are many other combinations of v_1, v_2, D and t, resulting in various values of V_r and r that can contribute to very high variations of Hubble's constant.

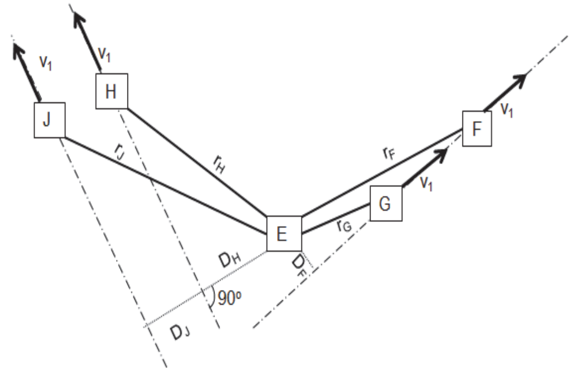

Fig. 4. Even the absolute velocities v_1 of galaxies G, F, H and J could be equal, an observer on Earth (E) will notice different values of their recessive velocities V_r.

4. Crosswise Movements of Two Bodies

We shall derive here the mathematical expressions for the relative movements of two bodies receding from the crossing of their trajectories. We shall propose that **there is no force acting between the bodies that could accelerate or decelerate their movements**.

Consider two bodies G and E, having different velocities v_1 and v_2, moving on crosswise trajectories (Fig. 5). Initially at t = 0, their distances to the crossing are (a) and (b). After some time t, G will pass the path $S_1 = v_1 t$ while E the path $S_2 = v_2 t$, and the distance between the G and E is increased to r, eq. (14).

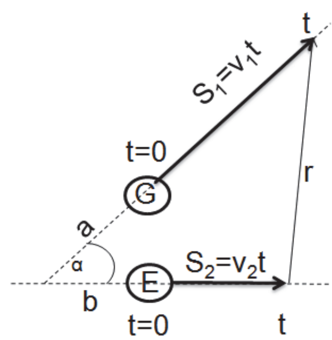

Fig. 5. Two bodies **G** and **E** are moved by different velocities $\mathbf{v_1}$ and $\mathbf{v_2}$ on crosswise trajectories. Initially at t = 0, their distances to the crossing are (a) and (b). Here: t is time, S is path, r is the distance between the bodies, α is angle between trajectories.

$$r^2 = (a+v_1t)^2 + (b+v_2t)^2 - 2(a+v_1t)(b+v_2t)\cos\alpha \quad (14)$$

$$2r(dr/dt) = 2v_1(a+v_1t) + 2v_2(b+v_2t) - 2\cos\alpha$$

$$[(v_1(b+v_2t)+v_2(a+v_1t)] \qquad (15)$$

The relative movement of two bodies **at crosswise trajectories** can be presented by their relative velocity, i.e. $V_r = dr/dt$ by (16) and distance r (17).

$$V_r = dr/dt = [(v_1^2+v_2^2 - 2v_1v_2\cos\alpha)t +$$

$$a(v_1 - v_2\cos\alpha)+b(v_2 - v_1\cos\alpha)]/r \qquad (16)$$

$$r = [(a+v_1t)^2 + (b+v_2t)^2 - 2(a+v_1t)(b+v_2t)\cos\alpha]^{1/2} \qquad (17)$$

An example of what happens over time for angle of trajectories on distance and relative velocity is presented in Fig. 6.

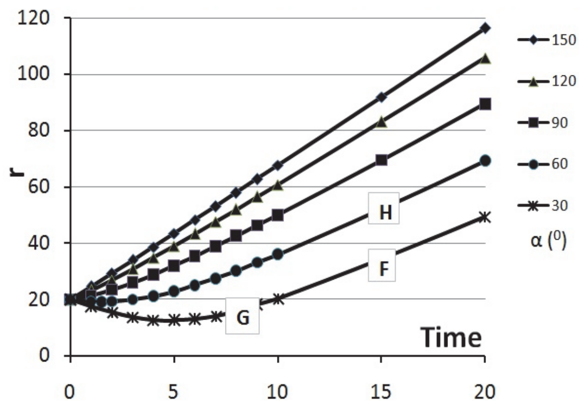

Fig. 6a. Effect over time on distance (r) between two bodies moving at crosswise trajectories under the angle α, calculated from equation (17) with initial conditions: $v_1 = 1$; $v_2 = 4$; $a = 20$ and $b = 0$. All values are in arbitrary units.

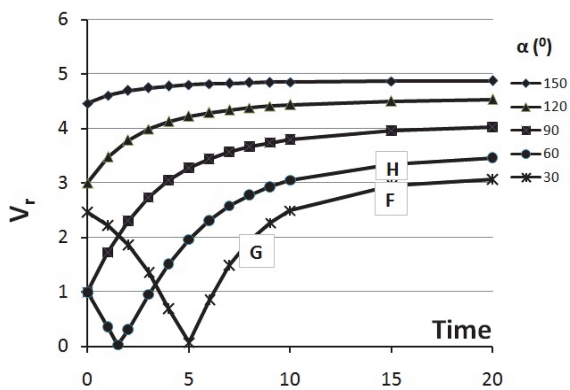

Fig. 6b. Effect over time on relative velocity between two bodies moving at crosswise trajectories, calculated from equation (16). Conditions are the same as in Fig. 6a.

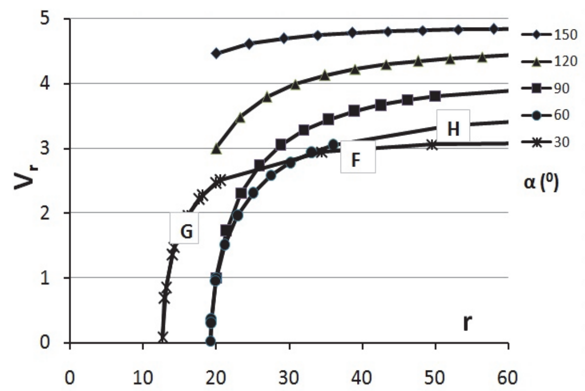

Fig. 6c. Relative velocity V_r versus distance (r) of two bodies moving at crosswise trajectories. Conditions are the same as in Fig. 6a.

Suppose that G and F represent two galaxies moving at the same trajectory both with absolute velocity v_1, but at different distances r_G and r_F from the Earth E (Fig. 6 for $\alpha = 30^0$ and Fig. 7). An observer on Earth will notice that the recessive velocity of G could be lower than that of F, because $t_G < t_F$ and $r_G < r_F$. Now, consider galaxies F and H both with absolute velocity v_1 and $t_H = t_F$ (Figs. 6 and 7). An observer on Earth will notice that the recessive velocity of galaxy F could be lower than that of galaxy H since $\alpha_H > \alpha_F$.

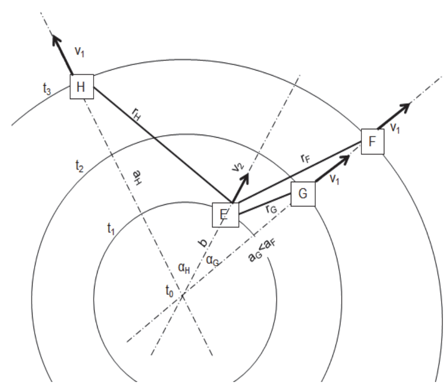

Fig. 7. Even the absolute velocities of galaxies G, F and H could be equal, an observer on Earth (E) will notice different values of their recessive velocities V_r calculated by eq. (16) and presented in Fig. 6 because $\alpha_H \neq \alpha_G = \alpha_F$ and $a_H \neq a_G \neq a_F$.

It should be noted, however, that if a = b = 0, i.e. two bodies (or galaxies) started simultaneously from the same point in various directions, eq. (16) is transformed to eq. (18) and recession velocity between two bodies depends on their absolute velocities and angle of their trajectories, but does not depend on time t nor the distances r between them.

$$V_r = dr/dt = [(v_1^2+v_2^2 - 2v_1v_2\cos\alpha)]^{1/2} \qquad (18)$$

5. Conclusions and Issues

The above analysis of relative movements of two bodies raises several questions:

(a) Hubble's law seems violated by this recession velocities analysis.

(b) The accuracy of the measurements of Hubble's constant is called into question.

(c) Consequently, the concept of expanding Universe is also called into question.

Furthermore, the slope of the curves V_r versus time, i.e. dV_r/dt, is acceleration or deceleration. It is evident (Figs. 3b, 6b and 8) that **an acceleration or deceleration between two bodies exists even when there is no force acting between them**. (Remember the Cosine Effect error, [5] there can be apparent acceleration without a force being involved.)

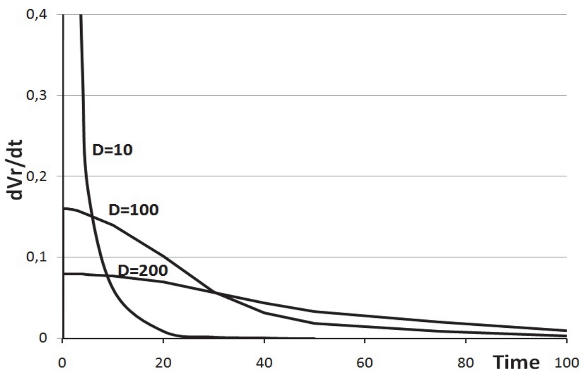

Fig. 8. The change of dV_r/dt (i.e. acceleration by recession of two bodies) plotted against time. The conditions are the same as in Fig. 3. D is a normal distance of motionless body to trajectory of moving one by velocity $v_1 = 4$. Distance of motionless body to moving body when $t = 0$ is $r_0 = D$.

The first and second Newton's laws, however, state [1, p. 138]: "Newton 1: Every body continues in its state of rest, or of uniform motion in a straight line, except in so far as **it is compelled by forces to change that state**. In vector notation:

$$\text{If} \quad \boldsymbol{f} = 0 \quad \text{then} \quad d\boldsymbol{v}/dt = 0 \qquad (19)$$

Newton 2: **Change of motion (that is, momentum) is proportional to the force** and takes place in the direction of the straight line in which force acts. In fact, Newton means that the rate of change of momentum is proportional to the force... In modern notation,

$$f = dp/dt, \quad p = m\,v \qquad (20)$$

(End of citation from [1, p. 138])"

(d) Should these definitions of Newton's laws have to be modified in the case of relative motion of two bodies?

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Why Do Quarks Have Electric Charges?

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The fermionic properties of quarks and leptons are determined by their weak interactions, the ultimate sources of which are their nonzero weak charges, and they are distinguished from each other by the strong interaction, which applies to quarks but not to leptons. There is no obvious reason why electric charge should be relevant to the distinction, but quarks are all required, in the conventional picture, to have electric charges, while leptons are not. It is as though the strong interaction requires electric charge to be present even though it is defined to be electric charge independent. It is proposed that the solution of this mystery lies in subtle aspects of the $SU(3) \times SU(2) \times U(1)$ symmetry breaking of the three interactions and is a consequence of the way in which this symmetry breaking arises.

Keywords: Strong interaction, Quarks, Baryons, Electric, Strong and weak charges, Symmetry-breaking

1. Electric Charges in Quarks and Leptons

The function of the electric charge in quarks is not yet established. The distinctions between the two main classes of particles, bosons and fermions, and between the two subclasses of fermions, quarks and leptons, results from the respective presence and absence of weak and strong, not electric, charges in their composition.

Bosons	Fermions	
no weak charges	weak charges	
	Quarks	Leptons
	strong charge	no strong charge

Quarks and leptons are the only known fundamental particles or fermions. The fermionic properties of these particles are determined by their weak interactions, the ultimate sources of which are their net weak charges. If the presence of nonzero net weak charge separates fermions from the gauge bosons which are the field quanta of their interactions, then the respective presence or absence of nonzero net strong charge separates out the two classes of fundamental particle described as quarks and leptons. The strong interaction is completely independent of the presence or absence of units of electric charge. However, although some leptons (the neutrinos) exist without electric charge units, no quarks appear to be without them.

The weak charge characterizes quarks as fermions. The strong charge separates quarks from leptons, the other kind of fermions. There is no obvious role for the electric charge. Why aren't there quarks without any, just as there are leptons without?

The conventionally accepted electric and strong charge composition of quarks can be represented as follows, with B standing for baryon number (effectively, strong charge):

	Blue	**Green**	**Red**
up	$2e / 3$	$2e / 3$	$2e / 3$
	$B / 3$	$B / 3$	$B / 3$
down	$-e / 3$	$-e / 3$	$-e / 3$
	$B / 3$	$B / 3$	$B / 3$

These may be compared with lepton charge structures, where there is zero strong charge:

	No colour
neutrino	0
	0
electron	$-e$
	0

There seems to be no good reason why we couldn't have:

	Blue	Green	Red
up	0	0	0
	$B/3$	$B/3$	$B/3$
down	0	0	0
	$B/3$	$B/3$	$B/3$

There seems to be no obvious function for the presence of electric charge in quark structures.

2. The Most Fundamental Symmetry in Physics

As we have stated, a key concept of particle physics is that the strong interaction is electric charge independent. The two interactions and their sources do not affect each other in any way. There is a fundamental reason for this. The weak, strong and electric interactions have separate conservation laws, and this is reflected in baryon and lepton conservation. Yet it would appear that the electric charge is essential to the creation of a strongly-interacting *system*. Why?

It seems like a question without an answer. This would be so if we were to take the quark theory as it stands as a given, the starting point. Nevertheless, there is an answer, but it is one that requires going down to the deepest and most foundational level in physics that we can possibly reach. Here, fundamental symmetries determine the physics rather than mathematical equations. The 3-dimensional or colour property of strong charge is impossible without the presence of the electric as well as weak charge, as we will demonstrate.

The most fundamental symmetry that exists in physics is that between the fundamental parameters: space, time, mass and charge. It is a Klein-4 or D_2 group and can be represented in many different ways.[1-5] Ultimately, thee symmetries involved are about 3-dimensionality or anticommutativity, and duality or summation to zero.

mass	conserved	real	continuous (1-D) commutative
time	nonconserved	imaginary	continuous (1-D) commutative
charge	conserved	imaginary	discrete (3-D) anticommutative
space	nonconserved	real	discrete (3-D) anticommutative

Using the symbols, x, y and z, to represent the properties, with $-x$, $-y$ and $-z$ representing the exactly opposite 'antiproperties', indicates that this symmetry incorporates a conceptual zero:

mass	x	y	z
time	$-x$	$-y$	z
charge	x	$-y$	$-z$
space	$-x$	y	$-z$

The set of parameters can also be reduced to algebra, as can all of physics, with the characteristics of the parameters defined entirely through the algebras that represent them. If we want an ontological ordering (universal rewrite system), based on deriving complexity from simplicity, we will have:

mass	scalar	1
time	pseudoscalar (complex)	i
charge	quaternion (pseudovector)	$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$
space	vector	$\mathbf{i}, \mathbf{j}, \mathbf{k}$

An epistemological ordering, based on accessibility to measurement or perception, might be space – time – mass – charge. An algebraic ordering might be based on the most complex algebra and its subalgebras:

space	vector	$\mathbf{i}, \mathbf{j}, \mathbf{k}, i\mathbf{i}, i\mathbf{i}, i\mathbf{k}, i, 1$
		$\mathbf{i}, \mathbf{j}, \mathbf{k}, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, i, 1$
charge	quaternion	$i\boldsymbol{i}, i\boldsymbol{j}, i\boldsymbol{k}, i, 1$
	(pseudovector)	$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, 1$
time	pseudoscalar (complex)	$i, 1$
mass	scalar	1

Here, we see that charge, mass and time have algebras equivalent to the subalgebras of space. But if we put them all together, they create an alternative algebra identical to that of space.

combination	vector	$\mathbf{i}, \mathbf{j}, \mathbf{k}, i\mathbf{i}, i\mathbf{i}, i\mathbf{k}, i, 1$
		$\mathbf{i}, \mathbf{j}, \mathbf{k}, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, i, 1$
charge	quaternion	$i\boldsymbol{i}, i\boldsymbol{j}, i\boldsymbol{k}, i, 1$
	(pseudovector)	$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}, 1$
time	pseudoscalar (complex)	$i, 1$
mass	scalar	1

Since, in the group, they total to zero, we may see the combination as a kind of 'antispace' or alternative space to space itself. Note that its dimensionality comes from that of charge.

If we assume these parameters are the only sources of physical knowledge, then the zero totality looks like a double space (**i**, **j**, **k**, **I**, **J**, **K**), whose two halves mirror each other in some way.

space	vector	$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, $i\mathbf{i}$, $i\mathbf{j}$, $i\mathbf{k}$, i, 1
		$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, i, 1
combination	vector	$\mathbf{I}$, $\mathbf{J}$, $\mathbf{K}$, $i\mathbf{I}$, $i\mathbf{J}$, $i\mathbf{K}$, i, 1
		$\mathbf{I}$, $\mathbf{J}$, $\mathbf{K}$, $\boldsymbol{I}$, $\boldsymbol{J}$, $\boldsymbol{K}$, i, 1

3. The Fundamental Symmetry-Breaking

Let us assume that these parameters are the whole of physics. We can then combine them by taking the tensor product of the algebras.

time	space	mass	charge
i	$\mathbf{i}$ $\mathbf{j}$ $\mathbf{k}$	1	$\boldsymbol{i}$ $\boldsymbol{j}$ $\boldsymbol{k}$
pseudoscalar	vector	scalar	quaternion

Working out every possible combination of the four parameters requires 64 units.

This turns out to be the algebra of the Dirac equation, the relativistic quantum mechanical equation of the fermion, the only true fundamental object that we know must exist. This is another group, this time of order 64, rather than 4:

1	i				-1	$-i$			
$\boldsymbol{i}\mathbf{i}$	$\boldsymbol{i}\mathbf{j}$	$\boldsymbol{i}\mathbf{k}$	$i\boldsymbol{k}$	$\boldsymbol{j}$	$-\boldsymbol{i}\mathbf{i}$	$-\boldsymbol{i}\mathbf{j}$	$-\boldsymbol{i}\mathbf{k}$	$-i\boldsymbol{k}$	$-\boldsymbol{j}$
$\boldsymbol{j}\mathbf{i}$	$\boldsymbol{j}\mathbf{j}$	$\boldsymbol{j}\mathbf{k}$	$i\boldsymbol{i}$	$\boldsymbol{k}$	$-\boldsymbol{j}\mathbf{i}$	$-\boldsymbol{j}\mathbf{j}$	$-\boldsymbol{j}\mathbf{k}$	$-i\boldsymbol{i}$	$-\boldsymbol{k}$
$\boldsymbol{k}\mathbf{i}$	$\boldsymbol{k}\mathbf{j}$	$\boldsymbol{k}\mathbf{k}$	$i\boldsymbol{j}$	$\boldsymbol{i}$	$-\boldsymbol{k}\mathbf{i}$	$-\boldsymbol{k}\mathbf{j}$	$-\boldsymbol{k}\mathbf{k}$	$-i\boldsymbol{j}$	$-\boldsymbol{i}$
$i\mathbf{i}\boldsymbol{i}$	$i\mathbf{i}\boldsymbol{j}$	$i\mathbf{i}\boldsymbol{k}$	$i\mathbf{k}$	$\mathbf{j}$	$-i\mathbf{i}\boldsymbol{i}$	$-i\mathbf{i}\boldsymbol{j}$	$-i\mathbf{i}\boldsymbol{k}$	$-i\mathbf{k}$	$-\mathbf{j}$
$i\mathbf{j}\boldsymbol{i}$	$i\mathbf{j}\boldsymbol{j}$	$i\mathbf{j}\boldsymbol{k}$	$i\mathbf{i}$	$\mathbf{k}$	$-i\mathbf{j}\boldsymbol{i}$	$-i\mathbf{j}\boldsymbol{j}$	$-i\mathbf{j}\boldsymbol{k}$	$-i\mathbf{i}$	$-\mathbf{k}$
$i\mathbf{k}\boldsymbol{i}$	$i\mathbf{k}\boldsymbol{j}$	$i\mathbf{k}\boldsymbol{k}$	$i\mathbf{j}$	$\mathbf{i}$	$-i\mathbf{k}\boldsymbol{i}$	$-i\mathbf{k}\boldsymbol{j}$	$-i\mathbf{k}\boldsymbol{k}$	$-i\mathbf{j}$	$-\mathbf{i}$

We could also use the two 'spaces' as base units:

Space	Time-Mass-Charge
$\mathbf{i}$ $\mathbf{j}$ $\mathbf{k}$	$\boldsymbol{i}$ $\boldsymbol{j}$ $\boldsymbol{k}$
vector	vector

The combined algebra is that of the Dirac equation, equivalent to the gamma matrices.[3-7] That is, it is the algebra of the fermionic state, the most fundamental known state in physics. Nothing is known to be required in physics except fermions and their interactions (which includes the gauge bosons).

Groups do not need to be specified by all their elements. A small number of elements multiplied out can often generate the entire group. Here we only need 5 generators. 5 is a very significant number because it is the point in physics and mathematics at which symmetries become broken and chirality forces itself upon us. Thus, even though we could have generated the group from the six components **i**, **j**, **k**, **I**, **J**, **K**, which would have maintained perfect symmetry between the two 'spaces', the *minimum number of generators*, which is always what nature requires, forces us to break it.

The set of 5 generators is not unique (e.g. any row of the table of 64 can be used), but all sets follow the same pattern, e.g.:

Time	Space	Mass	Charge
i	$\mathbf{i}$ $\mathbf{j}$ $\mathbf{k}$	1	$\boldsymbol{i}$ $\boldsymbol{j}$ $\boldsymbol{k}$

In principle, you have to break the symmetry of one space, **i**, **j**, **k**, or the other, ***i***, ***j***, ***k*** (equivalent to **I**, **J**, **K**). In this instance, we take one of each of ***i***, ***j***, ***k*** on to each of the units of the other three parameters.

$i\boldsymbol{k}$	$\mathbf{i}\boldsymbol{i}$ $\mathbf{j}\boldsymbol{i}$ $\mathbf{k}\boldsymbol{i}$	$\boldsymbol{j}$

Because space is a nonconserved quantity, and its component dimensions **i**, **j**, **k** cannot be uniquely identified or distinguished from each other, the broken symmetry in physics becomes that of charge ***i***, ***j***, ***k***.

And we can show that using 5 generators leads, among many other things, to the broken symmetry between the 3 charges which has troubled physics for more than forty years. The 'compactification' affects the nature of the charges as we observe them, and we can begin to recognise here the particular characteristics of weak, strong and electric charges, governed by the respective *SU*(2), *SU*(3) and *U*(1) symmetries.

The charges adopt properties of the mathematical objects they are connected to. So we find that the charge we have represented by ***k*** becomes associated with a pseudoscalar quantity *i*; that the one represented by ***i*** becomes associated with three vector units **i**, **j** and **k**; and the one represented by ***j*** becomes associated with the scalar unit 1.

$i\boldsymbol{k}$	$\mathbf{i}\boldsymbol{i}$ $\mathbf{j}\boldsymbol{i}$ $\mathbf{k}\boldsymbol{i}$	$\boldsymbol{j}$
weak	strong	electric
pseudoscalar	vector	scalar
SU(2)	*SU*(3)	*U*(1)

We see also that, though physics might require two 'spaces' for its specification, and that, though these two spaces may contain identical information, it presents itself differently within them, through a chirality:

$i\mathbf{K}$	$\mathbf{iI}$ $\mathbf{jI}$ $\mathbf{kI}$	$\mathbf{J}$

and we see that the units of the second 'space' **I**, **J**, **K** now adopt the characteristics of the mathematical objects they are connected to:

pseudoscalar	vector	scalar

4. The Origin of the Dirac Equation

In addition to affecting charge, the combination also affects time, space and mass, for, to create the generators, we distribute the 3 charge units onto the other 5:

i	**i j k**	1
$\boldsymbol{k}$	$\boldsymbol{i}$	$\boldsymbol{j}$

This creates *new* 'compound' (and 'quantized') physical quantities:

$i\boldsymbol{k}$	$\mathbf{i}\boldsymbol{i}$ $\mathbf{j}\boldsymbol{i}$ $\mathbf{k}\boldsymbol{i}$	$\boldsymbol{j}$
energy	momentum	rest mass
E	p_x p_y p_z	m

If we regard E, p_x, p_y, p_z, m simply as scalar values, which are arbitrary in principle, and the algebraic operators $i\boldsymbol{k}$, $\mathbf{i}\boldsymbol{i}$, $\mathbf{j}\boldsymbol{i}$, $\mathbf{k}\boldsymbol{i}$, $\boldsymbol{j}$ as defining the physical meaning, that is, the nature of the physical quantities involved, then we can package the whole information as

$$(i\boldsymbol{k}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m).$$

In physics, this combined object is *nilpotent*, squaring to zero, because

$$(i\boldsymbol{k}E + \mathbf{i}\boldsymbol{i}p_x + \mathbf{j}\boldsymbol{i}p_y + \mathbf{k}\boldsymbol{i}p_y + \boldsymbol{j}m)^2 = 0 \qquad (1)$$

and we can identify this as Einstein's relativistic energy equation

$$E^2 - p^2 - m^2 = 0 \qquad (2)$$

or, in its more usual form,

$$E^2 - p^2c^2 - m^2c^4 = 0.$$

The Dirac equation simply quantizes this nilpotent equation, using differentials in time and space applied to a phase factor for E and p. So (1) or (2) becomes

$$(-\boldsymbol{k}\partial / \partial t - i\boldsymbol{i}\nabla + \boldsymbol{j}m)\,(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)\; e^{-i(Et - \mathbf{p.r})} = 0,$$

the Dirac equation for a free fermion, by simultaneously applying nonconservation and conservation. The most complete possible variation in space and time is defined by a phase factor which associates E with time and $\mathbf{p}$ with space. We then use the differentials $\partial / \partial t$ and ∇ to recover $(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$ from the phase factor.

The operator, $(-\boldsymbol{k}\partial / \partial t - i\boldsymbol{i}\nabla + \boldsymbol{j}m)$, is, in fact, a coding of all the possible space and time variations, which is 'decoded' using a 'phase factor', here $e^{-i(Et - \mathbf{p.r})}$. That is, the most complete set of variations in space and time for a free particle is given by $e^{-i(Et - \mathbf{p.r})}$, and the expression which will recover $(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$ using this as a phase factor is $(-\boldsymbol{k}\partial / \partial t - i\boldsymbol{i}\nabla + \boldsymbol{j}m)$. The phase factor in this case is the minimum expression that the differentials $\partial / \partial t$ and ∇ can be applied to if space and time are to be varied without restriction.

Now, including all possible sign variations of E and $\mathbf{p}$, we obtain

$$(\mp \boldsymbol{k}\partial / \partial t \mp i\boldsymbol{i}\nabla + \boldsymbol{j}m)\,(\pm i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m)\; e^{-i(Et - \mathbf{p.r})} = 0$$

which is equivalent to a nilpotent Dirac equation of the form

$$(\mp \boldsymbol{k}\partial / \partial t \mp i\boldsymbol{i}\nabla + \boldsymbol{j}m)\,\psi = 0.$$

We can also express it in operator form

$$(\pm i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m)\,(\pm i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m)\; e^{-i(Et - \mathbf{p.r})} = 0.$$

where the operators, E and $\mathbf{p}$ become $i\partial / \partial t$ and $-i\nabla$ as in the usual canonical quantization.

The complete Dirac wavefunction in nilpotent form provides the particle state (represented by the first term) and the 3 possible states it could become by P, T and C transformations:

$(i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$	
$(i\boldsymbol{k}E - i\mathbf{p} + \boldsymbol{j}m)$	P
$(-i\boldsymbol{k}E + i\mathbf{p} + \boldsymbol{j}m)$	T
$(-\text{ik}E - i\mathbf{p} + \boldsymbol{j}m)$	C

5. Vacuum

We can interpret the expression

$$(\pm i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m)\,(\pm i\boldsymbol{k}E \pm i\mathbf{p} + \boldsymbol{j}m) \to 0$$

as giving us both relativity and quantum mechanics. In quantum mechanics we take the first bracket as an operator acting on a phase factor. The E and $\mathbf{p}$ terms can include any number of potentials or interactions with other particles. Squaring to 0 gives us the Pauli exclusion principle, because if any 2 particles are the same, their combination is 0.

Fermions appear to be point-like objects with norm 0. In effect, the creation of an object like

$$(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m),$$

which squares to zero, is the only way of defining a point in a space that is rotation and translation symmetric. We define a point as the interaction between two spaces, and the point-like fermion spends its whole existence switching between them (*zitterbewegung*).

In principle, therefore, we cannot define a point, or the space in which it exists, without defining another space with which it interacts to produce a totality zero.[3-5,8-9] One space will appear distorted with respect to the other, but each will contain the same information, or the information in one will be the reverse of the information in the other.

To return to Pauli exclusion, and Nature as a totality of zero, we can imagine creating a particle (with all the potentials representing its interactions) in the form

$$(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$$

and then being forced to structure the rest of the universe *or vacuum*, so that it can be represented by

$$-\,(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$$

So both the superposition and combination states of fermion and vacuum become zero:

$$(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m) - (\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m) = 0$$
$$-\,(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\,(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m) = 0$$

The 'hole' left by creating the particle from nothing is the rest of the universe needed to maintain it in that state. We give it the name vacuum, and the vacuum for one particle cannot be the vacuum for any other. Vacuum tells us 'where' the other 'space', based on ***i***, ***j***, ***k***, or **I**, **J**, **K**, is besides 'real' space, based on **i**, **j**, **k**. Its inaccessibility is demonstrated by the chirality between matter and antimatter, positive and negative energy states, and forward and backward directions in time.

The fermionic structure $(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ posits two states with $+E$ and two with $-E$, which is equivalent to two with + time direction and two with –. And yet only $+E$ and $+t$ states are observed, and the universe is predominantly made of matter rather than antimatter. This is because the second space, the one in which energy and time become negative, is the vacuum space, the one which encompasses the 'rest of the universe', as opposed to the real space defining the point-particle.

The nilpotent structure incorporates both spaces on an equal basis, and it is interesting to recall that it was the seeming inability to do this which led Feynman to develop the path integral method of quantum mechanics as opposed to the previous use of Hamiltonian methods. Perhaps this indicates that, as long as we use the nilpotent formalism, we will be able to reformulate path integral calculations using Hamiltonians.

The nonlocal aspect of the fermionic nilpotent state $(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ is defined by a continuous vacuum – $(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$. The continuous vacuum, with its negative energy, appears to be that associated with gravity. We can consider the fermion itself, with positive energy and $(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$, as being in some sense an inertial term, and discrete fermions as creating their inertial mass through interaction with the continuous vacuum.

Further, we can use the operators ***k***, ***i***, ***j*** to effectively partition the continuous vacuum state, or the inertia which opposes this, into discrete components with a dimensional structure, which can then be identified as the weak, strong and electric components of vacuum, responding respectively to the discrete weak strong and electric charges.

We can postmultiply $(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ by the idempotent $\boldsymbol{k}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ any number of times, without changing its state

$$(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{k}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{k}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m) \ldots \rightarrow (\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$$

because the effect is only to multiply $(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ by a scalar which can be normalized away. The idempotent then acts as a vacuum operator, not changing the fermion's state.

We can show that the same applies if we use the idempotents $\boldsymbol{i}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$ and $\boldsymbol{j}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$:

$$(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{i}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{i}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m) \ldots \rightarrow (\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$$

$$(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{j}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)\, \boldsymbol{j}(\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m) \ldots \rightarrow (\pm\, \boldsymbol{ik}E \pm \mathbf{ip} + \boldsymbol{j}m)$$

In each case, every *alternate* bracket changes the sign of its E, **p** or E and **p** (equivalent to m) terms, leading to the creation of a bosonic state, here defined as a combination of fermion and antifermion, or equivalent.

The only difference between the three is that the *alternate* brackets can be written as

$(-\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$	for	$\boldsymbol{k}$
$(\boldsymbol{ik}E - \mathbf{ip} + \boldsymbol{j}m)$	for	$\boldsymbol{i}$
$(-\boldsymbol{ik}E - \mathbf{ip} + \boldsymbol{j}m)$	for	$\boldsymbol{j}$

This means that they undergo respective T, P and C transformations.

In addition, the combinations with the unchanged bracket $(\boldsymbol{ik}E + \mathbf{ip} + \boldsymbol{j}m)$ mean that the three operators produce different types of bosonic state, respectively:

spin 1
spin 0
paired fermion

We can see how the 3 bosonic states are related to vacua produced by the 3 quaternionic operators:

weak spin 1

$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{k}\, (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{k}\, (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{k}\, (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \ldots$

$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, (-\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \ldots$

electric spin 0

$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{j}\, (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{j}\, (\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{j}\, \boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \ldots$

$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, (-\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, (-\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \ldots$

strong paired fermion state

$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{i}\, (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{i}\, (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, \boldsymbol{i}\, (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \ldots$

$(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, (\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, (\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, (\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \ldots$

6. Vacuum Space and Charge

Now, the vacuum space is the one defined by the units connected with charges, ***i***, ***j***, ***k***. Here we see that the units have multiple roles:

Charges
PCT operators
Dimensions of 'vacuum space'
Generators of the 3 additional terms in the Dirac wavefunction
Creation operators converting fermions to 3 types of boson

The complete vacuum, defined by $(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ or $-(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, can be considered as equivalent to that defined by gravity or inertia, which the gauge forces split into 3 dimensional components. Also, taking $(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ to convey the angular momentum information about a particle state, we can see that the units also say something about the respective conservation laws of handedness, direction and magnitude of this quantity. Ultimately, this introduces the $SU(2)$, $SU(3)$ and $U(1)$ group operators into particle physics, and explains why the symmetry between the 3 charges is broken.

But the key idea is that the total angular momentum information can be obtained either through ***k***, ***i*** and ***j*** of *iE*, *p* and *m*, or through the **i**, **j**, **k** of **p**. The two 'spaces' contain *exactly equivalent information about the whole of physics*, although in one case the symmetry between the units appears to be broken and in the other it is exactly preserved.

To show that the information is equivalent, we will consider how the uniqueness of a fermion wavefunction is determined to maintain Pauli exclusion. So if we know the *iE*, **p** and *m* values in $(\pm \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ for any fermion, we can show that those of any other fermion must be different. But, there is also a completely different way of establishing Pauli exclusion, that is, by realising that fermions have antisymmetric wavefunctions.

The nilpotent structure explains immediately why we have Pauli exclusion between fermions, but the alternative, *conventional*, way of explaining this property leads us to a profound insight on the nature of the information available in quantum systems if we structure it in nilpotent form. We define fermion wavefunctions to be antisymmetric, so that:

$$(\psi_1\psi_2 - \psi_2\psi_1) = -(\psi_2\psi_1 - \psi_1\psi_2)$$

In nilpotent terms, we write $(\psi_1\psi_2 - \psi_2\psi_1)$ as

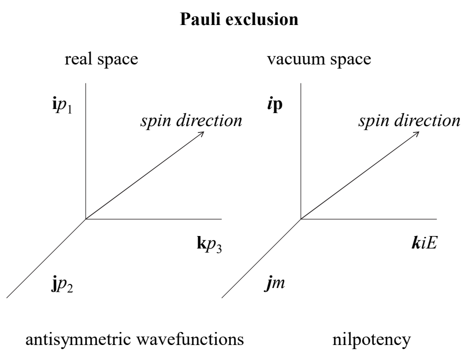
$$\begin{aligned}&(\pm \boldsymbol{ik}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)\,(\pm \boldsymbol{ik}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)\\ &- (\pm \boldsymbol{ik}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)\,(\pm \boldsymbol{ik}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)\\ &= 4\mathbf{p}_1\mathbf{p}_2 - 4\mathbf{p}_2\mathbf{p}_1 = 8i\mathbf{p}_1 \times \mathbf{p}_2 = -8i\mathbf{p}_2 \times \mathbf{p}_1.\end{aligned}$$

This result is clearly antisymmetric, but it also has a quite astonishing consequence, for it requires any nilpotent wavefunction to have a **p** vector, in *real space*, the one defined by the axes **i**, **j**, **k**, at a *different orientation* to any other. The wavefunctions of all nilpotent fermions then instantaneously correlate because the planes of their **p** vector directions must all intersect. At the same time, the nilpotent condition requires the *E*, **p** and *m* combinations to be unique, and we can visualize this as constituting a unique direction in *vacuum* space along a set of axes defined by ***k***, ***i***, ***j*** or **K**, **I**, **J**, with coordinates defined by the values of *E*, **p** and *m*.

The directions of the vectors in each space carry *all the information* available to a fermionic state, and so the information in the two spaces is totally dual, and is equivalent to the instantaneous direction of the spin in the real space. The total information determining the behaviour of a fermion and even of the entire universe is contained in a single spin direction. The information here must be the same as in the combination of *iE*, *p* and *m*. That is, we can represent the unique spin direction in parallel axes in two different spaces.

Though the duality results in fermion and vacuum occupying separate 3-dimensional 'spaces', which are combined in the double Clifford algebra defining the singularity state, these 'spaces', though seemingly different in observational terms, are truly dual, each containing the same information (angular momentum), and the duality manifests itself directly in many physical forms.

Angular momentum, which in some form combines all the information of the phase space of the particle, shows this duality directly, because, as a 3-dimensional pseudovector quantity, it has the rotation symmetric and nonconserved 3-dimensionality of space; but, at the same time, it shows 3 different aspects and 3 different conservation laws connected with them, which are connected by a different and rotation asymmetric 3-dimensionality.[3-4]



If we represent particles in terms of their charge structures, this can be done in two different spaces. If we look at baryons, we can see their behaviour in terms of real space, or that of the momentum operator, and this has rotation symmetry. But we can also see it in terms of vacuum or charge space, and this is rotation asymmetric.

Space and charge show us the dual aspects of angular momentum, nonconserved and symmetric, and conserved and asymmetric. All 3-dimensional quantities are of this kind, one that is ultimately expressed in Noether's theorem. To any 3-dimensionality that gives us nonconservation and rotation symmetry, there is always attached another that gives the same information using conservation and rotation asymmetry.

This is why the ultimate expression of physics requires phase space. To any description involving space or space-time, we require an equivalent description involving vacuum space, manifesting itself through charge, or through the combination of charge and space and time, which is described as energy-momentum. At the quantum (point-particle) level, these cannot be separated. This is how the combination of all 3 charges becomes necessary to describe the behaviour of the particular charges which is associated with spatial 3-dimensionality.

7. Baryons

The behaviour of the strong interaction can be completely determined by the spatial 3-dimensionality of the momentum operator in the nilpotent wavefunction. However, the situation of the strong charge as a 3-dimensional operator within the fermion state requires the application of the other 3-dimensionality associated with charge. Essentially the spatial 3-dimensionality associated with the strong charge can only be expressed through the complementary 3-dimensionality of charge. We can't make a charge dimension rotate through 3 dimensions of a space in which it does not exist unless we make it rotate in one in which it does.

Now, in quantum mechanical terms, the vector aspect of the strong charge requires a source term and corresponding vacuum with three components. Though we clearly cannot combine three components in the form:

$$(\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

as this will automatically reduce to zero, we can imagine a three-component structure in which the vector nature of $\mathbf{p}$ plays an explicit role

$$(\boldsymbol{ik}E \pm \boldsymbol{i}\,\mathbf{i}p_x + \boldsymbol{j}m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\,\mathbf{j}p_y + \boldsymbol{j}m)\,(\boldsymbol{ik}E \pm \boldsymbol{i}\,\mathbf{k}p_z + \boldsymbol{j}m)$$

This has nilpotent solutions when $\mathbf{p} = \pm\, \boldsymbol{i}\,\mathbf{i}p_x$, $\mathbf{p} = \pm\, \boldsymbol{i}\,\mathbf{j}p_y$, or $\mathbf{p} = \pm\, \boldsymbol{i}\,\mathbf{k}p_z$, or when the momentum is directed entirely along the x, y, or z axes, in either direction, though these, of course, are arbitrarily defined, and we can associate the + and – directions of momentum with symmetric and antisymmetric combinations of 3 (arbitrarily-defined) 'colours'. Any other phases can be written as a superposition of these. Using the appropriate normalization, they reduce to

$$\begin{array}{ll}
(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m) & +RGB \\
(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m) & -RBG \\
(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m) & +BRG \\
(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m) & -GRB \\
(\boldsymbol{ik}E + \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m) & +GBR \\
(\boldsymbol{ik}E - \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m) & -BGR
\end{array}$$

with the third and fourth changing, very significantly, the sign of the $\mathbf{p}$ component. Because of this, there has to be a maximal superposition of left- and right-handed components, thus explaining the zero observed chirality in the interaction, and the mass of the baryon.[3,4]

The group structure required to maintain these phases is an $SU(3)$ structure, with eight generators and a wavefunction, exactly as in the conventional model using coloured quarks,

$$\psi \sim (BGR - BRG + GRB - GBR + RBG - RGB).$$

'Colour' transitions in the 3-component structures are produced either by an exchange of the components of **p** between the individual quarks or baryon components, or by a relative switching of the component positions, independently of any real distance between the components. No direction can be privileged, so the transition must be gauge invariant, and the mediators must be massless, exactly as with the eight massless gluons of QCD. Here, we have written only the first term of the 4-component spinors, but we have retained the two spin states, as these will be needed explicitly.

The complete wavefunction will, in effect, contain information from the equivalent of six allowed independent nonlocally gauge invariant phases, all existing simultaneously and subject to continual transitions at a constant rate:

$$\begin{array}{ll}(\boldsymbol{k}E + \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m)\ (\boldsymbol{k}E + \boldsymbol{j}m)\ (\boldsymbol{k}E + \boldsymbol{j}m) & +RGB \\ (\boldsymbol{k}E - \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m)\ (\boldsymbol{k}E + \boldsymbol{j}m)\ (\boldsymbol{k}E + \boldsymbol{j}m) & -RBG \\ (\boldsymbol{k}E + \boldsymbol{j}m)\ (\boldsymbol{k}E + \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m)\ (\boldsymbol{k}E + \boldsymbol{j}m) & +BRG \\ (\boldsymbol{k}E + \boldsymbol{j}m)\ (\boldsymbol{k}E - \boldsymbol{i}\mathbf{j}p_y + \boldsymbol{j}m)\ (\boldsymbol{k}E + \boldsymbol{j}m) & -GRB \\ (\boldsymbol{k}E + \boldsymbol{j}m)\ (\boldsymbol{k}E + \boldsymbol{j}m)\ (\boldsymbol{k}E + \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m) & +GBR \\ (\boldsymbol{k}E + \boldsymbol{j}m)\ (\boldsymbol{k}E + \boldsymbol{j}m)\ (\boldsymbol{k}E - \boldsymbol{i}\mathbf{k}p_z + \boldsymbol{j}m) & -BGR \end{array}$$

Here, six gluons can be constructed from:

$$\begin{array}{c}(\boldsymbol{k}E + \boldsymbol{i}\mathbf{i}p_x)\ (-\boldsymbol{k}E + \boldsymbol{i}\mathbf{j}p_y) \\ (\boldsymbol{k}E + \boldsymbol{i}\mathbf{j}p_y)\ (-\boldsymbol{k}E + \boldsymbol{i}\mathbf{i}p_x) \\ (\boldsymbol{k}E + \boldsymbol{i}\mathbf{j}p_y)\ (-\boldsymbol{k}E + \boldsymbol{i}\mathbf{k}p_z) \\ (\boldsymbol{k}E + \boldsymbol{i}\mathbf{k}p_z)\ (-\boldsymbol{k}E + \boldsymbol{i}\mathbf{j}p_y) \\ (\boldsymbol{k}E + \boldsymbol{i}\mathbf{k}p_z)\ (-\boldsymbol{k}E + \boldsymbol{i}\mathbf{i}p_x) \\ (\boldsymbol{k}E + \boldsymbol{i}\mathbf{i}p_x)\ (-\boldsymbol{k}E + \boldsymbol{i}\mathbf{k}p_z)\end{array}$$

and two from combinations of

$$\begin{array}{c}(\boldsymbol{k}E + \boldsymbol{i}\mathbf{i}p_x)\ (-\boldsymbol{k}E + \boldsymbol{i}\mathbf{i}p_x) \\ (\boldsymbol{k}E + \boldsymbol{i}\mathbf{j}p_y)\ (-\boldsymbol{k}E + \boldsymbol{i}\mathbf{j}p_y) \\ (\boldsymbol{k}E + \boldsymbol{i}\mathbf{k}p_z)\ (-\boldsymbol{k}E + \boldsymbol{i}\mathbf{k}p_z)\end{array}$$

where, as with the baryons, only the lead term is shown for each 4-component spinor.

A representation such as the 3-component baryon above, showing only one 'quark' active at any time in contributing to the angular momentum operator, seems to indicates why only 1/3 of baryon spin has been found to be due to the valence quarks. The rest of the spin then becomes a 'vacuum' contribution, split approximately 3 to 1 in favour of the gluons over the sea quarks, the gluons thus taking half the overall total.

The simultaneous existence of all phases further means that *individual* quarks, and such identifying characteristics as electric charges, are not identifiable by their spatial positions (unlike, say, the proton and electron constituting a hydrogen atom), thus explaining, for example, why the neutron has no electric dipole moment.

Just as $U(1)$ establishes that spherical symmetry of a point source requires the rotation to be performed independently of the length of the radius vector, so $SU(3)$ requires the rotation to be independent of the coordinate system used. In terms of Noether's theorem, while $U(1)$ conserves the magnitude of angular momentum, $SU(3)$ conserves the direction.

8. Strong Interaction Solution

The nilpotent operator has an exact solution for the strong interaction mechanism between quarks in either baryons or mesons, with the quark treated as a point-particle in the spherically symmetric field surrounding a point-centre of force. In this case, the force must have a Coulomb component or inverse linear potential ($\propto 1/r$), just to accommodate spherical symmetry.[3,4]

This has a known physical manifestation in the one-gluon exchange. But there is also at least one other component, which is responsible for quark confinement, for infrared slavery and for asymptotic freedom, and a linear potential ($\propto r$) has long been hypothesized and used in calculations. Here, we see that an exchange of **p** components at a constant rate would, in principle, require a constant rate of change of momentum, which is the signature of a linear potential.

To solve for the interaction with respect to a point-centre of force, we write the operator for the Dirac equation in polar coordinates and add the two specified potentials to the E term.

$$\left(\pm \boldsymbol{k}\left(E + \frac{A}{r} + Br\right) \mp \boldsymbol{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\,\frac{j + ½}{r}\right) + \boldsymbol{i}\boldsymbol{j}m\right)$$

Without needing to write down the full equation we find the phase factor to which this operator must be replied to obtain a nilpotent amplitude. Experience of similar problems tells us that it is of the form:

$$\phi = \exp\left(-ar - br^2\right) r^{\gamma} \sum_{\nu=0} a_{\nu} r^{\nu}$$

Applying the operator to this, and then the nilpotent condition, we obtain:

$$E^2 + 2AB + \frac{A^2}{r^2} + B^2 r^2 + \frac{2AE}{r} + 2BEr =$$

$$m^2 - \left(a^2 + \frac{(\gamma + \nu + ... + 1)^2}{r^2} - \frac{(j + ½)^2}{r^2} + 4b^2 r^2 \right)$$

$$- \left(4abr - 4b(\gamma + \nu + ... + 1) - \frac{2a}{r}(\gamma + \nu + ... + 1) \right)$$

with the positive and negative $i(j + ½)$ terms cancelling out over the four solutions. Then, assuming a termination in the power series (as with the Coulomb solution), we can equate:

coefficients of r^2	to give	$B^2 = -4b^2$
coefficients of r	to give	$2BE = -4ab$
coefficients of $1 / r$	to give	$2AE = 2a(\gamma + \nu + 1)$

We can therefore write ϕ as

$$\phi = \exp\left(\pm iEr \mp iBr^2 / 2\right) r^{\mp iqA-1}$$

where

$$k = \pm iE \mp iBr / 2.$$

The first term in k dominates at high energies, where r is small, approximating to a free fermion solution, which can be interpreted as asymptotic freedom, while the second term, with its confining potential Br, dominates, at low energies, when r is large, and this can be interpreted as infrared slavery. These are the established characteristics of the strong interaction and here we have an explanation, derived on an analytic basis, for a force with these characteristics.

If we now look at how the expression

$$(\boldsymbol{ik}E \pm \boldsymbol{i}\ \mathbf{i}p_x + \boldsymbol{j}m)\ (\boldsymbol{ik}E \pm \boldsymbol{i}\ \mathbf{j}p_y + \boldsymbol{j}m)\ (\boldsymbol{ik}E \pm \boldsymbol{i}\ \mathbf{k}p_z + \boldsymbol{j}m)$$

with its spatial 3-dimensionality, fits into the 3-dimensionality associated with the charge picture, we can recall that the 4 components in the Dirac spinor, can be seen to represent the fermion as seen in terms of gravity / inertia, strong, weak and electric vacua. The 3 momentum components have to be mapped onto the space created by the 3 charges, giving:

$$\begin{matrix} \textit{inertial} \\ \textit{strong} \\ \textit{weak} \\ \textit{electric} \end{matrix} \begin{pmatrix} \boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ \boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \end{pmatrix}$$

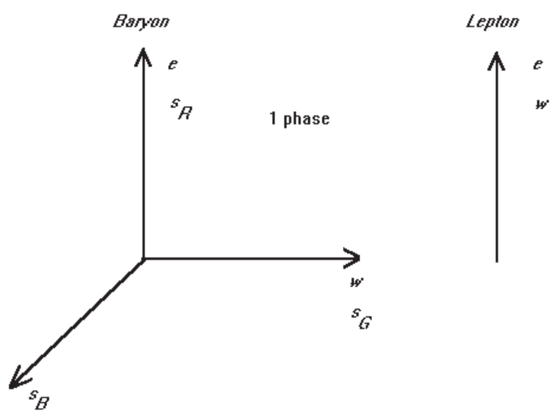
$$\begin{pmatrix} \boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ \boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} \boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ \boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \end{pmatrix}$$

Or

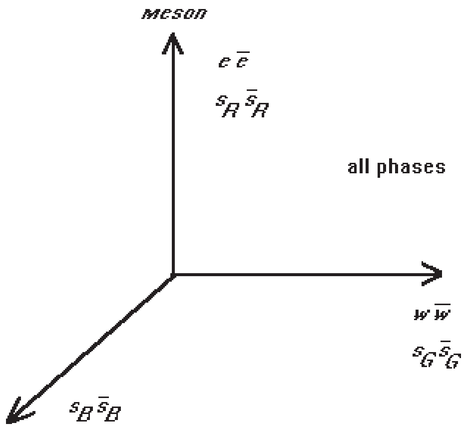
$$\begin{matrix} \textit{inertial} \\ \textit{strong} \\ \textit{weak} \\ \textit{electric} \end{matrix} \begin{pmatrix} \boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ \boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ \boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} \boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ \boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \end{pmatrix}$$

Here, we assume only one dimension of **p** is nonzero, say $\mathbf{p}_3$, so determining the charge structure, with the 3 charges distributed. We can also represent it using a phasor diagrams, with the charges *separated* in baryons, but not in leptons:



For mesons we would have:



If the strong charge is absent, we need only one phase, with all charges aligned, as in:

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} \boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ \boldsymbol{k}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -\boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -\boldsymbol{k}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \end{pmatrix}$$

9. Quarks and Charges

The wavefunctions suggest that the underlying charge structures of quarks are as follows, with the more familiar fractional electric charges appearing in observation because the strong interaction is absolutely gauge invariant:

	Blue	Green	Red
up	*e*	*e*	0
	B	0	0
down	0	0	–*e*
	B	0	0

A similar fractionalization of electric charge occurs in the fractional quantum Hall effect, where the fractional status of the charge on an electron is produced by its *weak* interaction with an odd number of flux lines > 1. In both cases, it is a different electric-charge independent force which ensures that the electric charges become fractional.

The underlying charge structures can represent the Standard Model simply in a set of 4 quark-lepton tables, A-C and L.[3,4,10] They also predict Grand Unification of the 4 forces at the Planck mass, explain *CP* violation and solve a serious anomaly in the Higgs mechanism as applied to finding the masses of fermion states.

A

u	+*e*	1***j***	1***j***	0***i***
	+*s*	1***i***	0***k***	0***j***
	+*w*	1***k***	0***i***	0***k***
d	–*e*	0***j***	0***k***	1***j***
	+*s*	1***i***	0***i***	0***k***
	+*w*	1***k***	0***j***	0***i***
c	+*e*	1***j***	1***j***	0***i***
	+*s*	1***i***	0***k***	0***j***
	–*w*	$z_P\boldsymbol{k}$	0***i***	0***k***
s	+*e*	0***j***	0***k***	1***j***
	+*s*	1***i***	0***i***	0***k***
	–*w*	$z_P\boldsymbol{k}$	0***j***	0***i***
t	+*e*	1***j***	1***j***	0***i***
	+*s*	1***i***	0***k***	0***j***
	–*w*	$Z_T\boldsymbol{k}$	0***i***	0***k***
b	+*e*	0***j***	0***k***	1***j***
	+*s*	1***i***	0***i***	0***k***
	–*w*	$Z_T\boldsymbol{k}$	0***j***	0***i***

B

		B	**G**	**R**
u	+*e*	1***j***	1***j***	0***i***
	+*s*	0***i***	0***k***	1***j***
	+*w*	1***k***	0***i***	0***k***
d	–*e*	0***j***	0***k***	1***j***
	+*s*	0***i***	0***k***	1***j***
	+*w*	1***k***	0***j***	0***i***
c	+*e*	1***j***	1***j***	0***i***
	+*s*	0***i***	0***k***	1***j***
	–*w*	$z_P\boldsymbol{k}$	0***i***	0***k***
s	+*e*	0***j***	0***k***	1***j***
	+*s*	0***i***	0***k***	1***j***
	–*w*	$z_P\boldsymbol{k}$	0***j***	0***i***
t	+*e*	1***j***	1***j***	0***i***
	+*s*	0***i***	0***k***	1***j***
	–*w*	$Z_T\boldsymbol{k}$	0***i***	0***k***
b	+*e*	0***j***	0***k***	1***j***
	+*s*	0***i***	0***k***	1***j***
	–*w*	$Z_T\boldsymbol{k}$	0***j***	0***i***

C

		B	**G**	**R**
u	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{i}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{k}$	$0\boldsymbol{j}$
	$+w$	$1\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{k}$
d	$-e$	$0\boldsymbol{j}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{k}$	$0\boldsymbol{j}$
	$+w$	$1\boldsymbol{k}$	$0\boldsymbol{j}$	$0\boldsymbol{i}$
c	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{i}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{k}$	$0\boldsymbol{j}$
	$-w$	$z_P\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{k}$
s	$+e$	$0\boldsymbol{j}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{k}$	$0\boldsymbol{j}$
	$-w$	$z_P\boldsymbol{k}$	$0\boldsymbol{j}$	$0\boldsymbol{i}$
t	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{i}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{k}$	$0\boldsymbol{j}$
	$-w$	$Z_T\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{k}$
b	$+e$	$0\boldsymbol{j}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$1\boldsymbol{i}$	$0\boldsymbol{i}$	$0\boldsymbol{k}$
	$-w$	$0\boldsymbol{i}$	$1\boldsymbol{k}$	$0\boldsymbol{j}$

L

		$\bar{e}$	$\bar{e}$	ν_e
	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{j}$
	$+s$	$0\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$+w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{k}$
				e
	$-e$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{j}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$+w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{k}$
		$\bar{\mu}$	$\bar{\mu}$	ν_μ
	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{j}$
	$+s$	$0\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{k}$
				μ
	$-e$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{j}$	$1\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$z_P\boldsymbol{k}$
		$\bar{\tau}$	$\bar{\tau}$	ν_τ
	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{j}$
	$+s$	$0\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{k}$
				τ
	$-e$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{j}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$Z_T\boldsymbol{k}$

The tables A-C show the same quark structures as are given in sections 7 and 8, with the strong charge substituting for the 'active' momentum component. Rotation between the A-C representations becomes equivalent to the rotation between p_x, p_y and p_z. The Z_p and Z_T symbols represent the respective violations of parity and time-reversal symmetry which make the unit $-w$ charge behave in the same way as $+w$. The mixtures between $+w$, $-z_Pw$ and $-z_Tw$ indicate the reason for P, T and CP violations in weak interactions, and the reason why there are only 3 generations of fermions. Antiquarks are also identified clearly as those with $-s$, in line with the conservation of baryon number.

With the elimination of the s charge for leptons, and the reduction of $SU(3)$ to $SU(2)$, the first and second columns in L effectively merge. We can also identify these columns as representing the left-handed antileptons and antineutrinos with zero weak charge and zero response to the weak interaction. L is a table, in effect, of all the left-handed states and antistates in the lepton sector. The fact that Z_P and Z_T are indistinguishable in their effects when only weak charges are present suggests maximal mixing for muon and tau neutrinos.

The two $SU(2)$ isospin states in both quark and lepton sectors are determined by the presence or absence of electromagnetic (*zitterbewegung*) mass. In its absence the electric vacuum is filled with $+e$ vacuum charge for all fermion states. In its presence, the electric vacuum is empty, leading to 0 vacuum charge. With $-e$ taken as the basic unit of fermionic charge, the electric charge assignments for the 'up' isospin state (with zero elertromagnetic mass) are respectively $+e$, $+e$ and 0, while for the 'down' isospin state (with nonzero electromagnetic mass) they are 0, 0 and $-e$. It is the fermionic charge that is expressed in the phasor diagrams in section 8.

This model of isospin variation has an interesting consequence in the case of neutrinos, for, though neutrinos have zero electric charge and, according to the Standard Model, have no mass due to isospin, the zero-electric charge, according to the model, is due to two components: $+e$ due to the electric vacuum and $-e$ from the particle itself. We can therefore imagine a small degree of *zitterbewegung* involving the electric charge and so a small amount of mass might be generated by the isospin mechanism.

Significantly, the generators of the Dirac group, which become the coefficients of the nilpotent summation of energy, momentum and rest mass, $\boldsymbol{ik}$, $\mathbf{i}\boldsymbol{i}$, $\mathbf{j}\boldsymbol{i}$, $\mathbf{k}\boldsymbol{i}$, $\boldsymbol{j}$, can be produced by a matrix multiplication of the E-$\mathbf{p}$-m coefficients, and *those of charge in the tables A-C*. Here, we take the product

$$\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & i & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \boldsymbol{i} & 0 & 0 \\ \boldsymbol{i} & \boldsymbol{k} & 0 \\ \boldsymbol{i} & 0 & \boldsymbol{j} \end{pmatrix}$$

and reduce it to the summation of the 5 units by multiplying from the left and right by respective column and row (or bra and ket) unit (or $\boldsymbol{\sigma}$) vectors. It has been claimed previously[3,4,11] that the nilpotent structure ($\boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}p_x + \boldsymbol{j}m$) contains the equivalent of 10 'dimensions', 5 for E-$\mathbf{p}$-m and 5 for charge, and it is evident here that the two sets of 5 contribute equally to the final structure. From the tables in section 2, we can see that the zero totality of the parameter group can be constructed from any single parameter and a combination of the other 3 as an 'anti'-version. Here, the two matrices construct a charge and anti-charge representation, and, as we have shown previously, the Dirac equation can be constructed in terms of charge rather than E-$\mathbf{p}$-m.

10. Conclusion

In principle, the 3 dimensions of space attached to the strong charge in the compactification of the units of the Dirac algebra in the nilpotent representation of the fermion are not a direct expression of charge variation, but rather of variation in momentum direction. Charges, as conserved quantities, are not directly accessible by spatial directions, which are arbitrary. To map the 3-dimensionality required of the strong charge, we need the complementary 3-dimensionality of charge itself. In other words, the dimensional properties of the strong interaction cannot be fully mapped without using the 3-dimensionality of charge, and so electric and weak charges are essential to describing the properties of the strong charge, even though their actions are mutually independent. So, although the strong interaction is not affected by the presence or absence of electric charge, it is not possible to construct strongly-interacting particles without it. The electric and weak interactions, however, are not constructed on dimensional lines, and so particles which are not strongly interacting may have zero electric charges.

This is a very subtle point, but it has profound implications for the way we model the structures of particles obeying the strong interaction. To a large extent, the structures of quarks have been modelled using QED phenomenology. While these structures are valid for QED, they are not necessarily the most fundamental structures which emerge from symmetry considerations. At an early stage in the quark discovery a model of quarks based on integral and zero charges was proposed in the paper which has been most frequently cited as introducing the colour theory.[12] It has never been refuted but gradually faded from view because the main attention from theorists was placed on phenomenology and, if the strong interaction, is gauge invariant and always produces colour singlets, then the electric charges of individual quarks can never be observed. Quarks can be no more separated than individual components of momentum. Exactly the same argument applies to the fractional electric charges observed in the quantum Hall effect,[13] although these are produced by the gauge invariance of the *weak* interaction.

Several considerations suggest that the underlying structure which generates the fractional charges required by QED may actually be built from integral and zero charges. One is that there does not seem to be any sense in having three different units of electric charge, e / 3, 2e / 3 and e. Also, there is no reason to believe that either the electron or the u quark are composite, as these structures would imply. In addition, underlying fractional charges create difficulties with using the Higgs mechanism to derive the masses of fermions, as it means creating hypercharge values for particles which simply do not exist.[3] Finally, using underlying integral and zero electric charges relatively easily produces exact Grand Unification at the Planck mass scale.[3,4,11]

A variety of models and representations for the strong interaction and baryon structure appear to converge on a model in which the underlying electric charges of quarks are either integral or zero. Interestingly, this seems to answer the question posed at the beginning of this paper: quarks with zero electric charges actually exist but strongly interacting systems, such as baryons and mesons, cannot be constructed unless electric charge is part of the system.

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What is Time? What Time is it?

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We live in a reality of time coordinates where time seems absolute. Classical physics, according to Einstein's theories, tells us time may not be absolute and certainly not an isolated quantity. When time is displayed by a chronometer, it is mechanically set, but really time is intimately tied to movements of matter and energy in space. The Minkowski four spacetime is fundamental to modern physics. Yet, we know by relativity theory that time is mutable, whereby objects moving near the velocity of light are time dilated and slowed in the vicinity of a black hole. The velocity of light in a vacuum is determined to be a universal constant *c* and is expressed as a distance moved by the photon in a unit of time. Scientists continue to question both the speed of light and time, itself. The curvature of spacetime near massive objects, as a black hole, moves away the "river of time" from one of the fundamental constants. The relativistic twin paradox and causality can relate to particle interactions at the microdomain which give us a new view of causality, where we encounter cause-effect sequences intimately tied into the nature of time, but in quantum physics it is Quantum Gravity that may return us to the concept of absolute time. Quantum nonlocality may also imply instantaneous "photonic" signal propagation.

Keywords: Gravity, Time, Schrödinger, Quantum gravity

1. Introduction

Fundamental to physics are the concepts of constancy (absolute) and change (relative). Time appears to us as a constant or absolute, but it is mutable, relative to the speed of travel under the force of gravity in spacetime. Newton introduced time as absolute. Einstein transformed the concept in his theory of Special Relativity which stated that the closer an object reaches to the speed of light, the slower time passes [1].

Absolute time proposes a universal time coordinate for all observers, while "relative" time depends on the observer and other quantifiers such as the gravitational field (General Relativity) [2]. In Einstein's universe everything is in motion and constantly changing its state and its position relative to every other thing. There is no absolute "up" and "down"; there is no absolute "here-and-now." As one looks out in space one looks back in time. Thus, the atomic explosion at Hiroshima occurred on earth over 75 years ago, but because light requires time to travel, the event will not be perceived until thirteen years from now by a hypothetical observer on a hypothetical star which is sixty-two light years from earth. What is "past" for us becomes "future" for the observer. In this respect past, present and future are simply a matter of perspective. Time is relative.

Time has quantitative and qualitative attributes. There is the Newtonian clock time, we use to set alarms, in order to get up to go to work or school. Then, there is subjective time. To paraphrase Albert Einstein, "Put your hand on a hot stove for a minute, and it seems like an hour. Sit with a pretty girl for an hour, and it seems like a minute. That's relativity." Dream time is also subjective. Quite a long dream story can occur in a few minutes of REM sleep. In a Lilly sensory deprivation tank, long objective times can seem like a short objective time or vice versa.

Einstein's theories both of special and general relativity lead us to the "twin paradox" where the *inertial observer*, v(t) = 0, who is not "accelerating," experiences time differently when compared to a second moving body. The one traveling faster would age less in comparison to the one standing still because of time dilation. In actuality, the twin paradox is a general relativistic problem because of the acceleration, deceleration and turn around to return to Earth of the outbound twin, and hence, we have a non-inertial frame.

The twin paradox has been confirmed several times by studies using atomic clocks [3]. Here one has an understanding of things like the Lorentz contraction, that if a car passes you at near the speed of light, its length is contracted in the direction of motion, also with the principle of mass dilation, the mass of the car increases towards an infinite mass as the car's velocity approaches that of the speed of light.

The twin paradox describes two twins. One is terrestrial in habit and stays on Earth, the other is into "astronauting" around the cosmos and takes a long and fast space flight (near the velocity of light)! When he does this, his space ship gets shorter (Lorentz contraction), heavier (mass dilation) and his time moves slower (time dilation). He ages less fast, returns to earth and meeting his earth bound twin has effectively moved into his brother's future–now his older brother! Experiments with accelerated particles in "atom smashers" and cosmic ray decay time (which are slowed down) indicate this kind of time machine is very likely.

2. Some Modern Concepts about the Nature of our Universe

Using his own relativistic equations, Einstein worked out a model of the universe. He considered the cosmos as finite, curved in four-dimensions by the matter within it, into a closed, spherical structure. Given the amount of matter density to close the universe, the actual densities of the universe are only 4% of what is needed. To his model, worked out before the "red shift" was fully understood, Einstein added the cosmological constant, Λ, to form static rather than his dynamic equations. A rocket ship, traveling in a "straight line" in this universe, would be like a worm setting out across the earth. It would never reach a boundary but would also not go on indefinitely; it would eventually return to the point from which it started.

In addition to their brain-staggering task of determining whether the universe is limited and closed or endless or open in spacetime, the astronomers are now trying to verify a fantastic and much disputed theory about the behavior of the cosmos, that the whole universe is literally exploding; that each galaxy, except those held together by gravity in local groups, is flying away from every other galaxy at enormous speeds, along with such entities as quasars, pulsars, supernovas and black holes.

The *relative speed* between galaxies depends on the separation linearly according to Hubble's expansion law as:

$$H = \frac{\dot{R}}{R}$$

where $\dot{R} = \frac{dR}{dt} = v$ is the recession velocity, H is the Hubble parameter, and R is the spatial separation. The Absolute Lorentz Transformation (ALT), proposed by Kipreos [4], indicates that increased velocities induce directional "time dilation." This along with increased velocities, seen in the Hubble calculations, reveals time dilation association with our past, where time was faster in the past and slower in the "now."

The first evidence to suggest the exploding universe concept was discovered by V.M. Slipher of the Lowell Observatory [5] and investigated further by Edwin Hubble and Spectroscopist Milton Humason around 1921. Studying the spectra of galaxies, these men were surprised to find that the spectral lines were displaced from their normal positions toward the red end of the spectrum. According to present knowledge about the behavior of light, such a shift should mean that the source of the light is hurtling away from the observer. This so-called "red shift," z, in light waves is like the Doppler Shift or Effect where one can calculate the recession $\dot{R}$ velocity as follows,

$$\dot{R} = c\left[\frac{(1+z)^2 - 1}{(1+z)^2 + 1}\right]$$

The acoustic Doppler Shift Effect is the change in pitch of a sound as the source of the sound approaches (higher frequency) and recedes (lower frequency). Light waves from a tremendously fast-moving source may be compressed so that they shift toward the blue (shorter wavelength) end of the visible spectrum as the source approaches a fixed observer, or stretched relative to the observer toward the red end (in the spectral chart), as the source recedes, as the longer wavelengths appear to the fixed observer.

For decades astronomers have used this shift in the spectrum to measure the movements of local stars and nearby galaxies, but found no consistent pattern in their motion. Then Hubble and Humason checked the spectra of far off stellar systems and found that all of these systems seemed to be rushing away, some at speeds of more than 25,000 miles per second [6]. They also found that the speed of retreat was proportional to the distance of the galaxies from the earth, so that one twice as far away as another, was moving away at twice the speed. A simple two-dimensional analogy to this concept of an expanding universe is given by a balloon being inflated, in which the balloon is covered with equally spaced spots representing galaxies which leads to the same analogy in M-theory's brane cosmology [7]. As the balloon is inflated, each spot moves farther away from every other spot. The greater the distance between any two spots, the more rapidly they recede from each other.

If we take this one step further and accept the premise of Perlmutter [8] and Riess [9, 10] from the Supernovae Ia, pointing out that the universe is not just expanding but accelerating, measured to an accuracy of 5σ, we can see that the "red shift" plays a major role in our understanding of the universe. Although recent evidence by Sarkar and Nielson at the Niels Bohr Institute (University of Copenhagen) in association with Oxford University, from a ten times greater supernova data count, suggests that the measurement of the acceleration may not be as accurate, at least not at a significant rate yielding a statistic instead of "3σ" or even "1σ" [11]. As long as the universe is expanding at the Hubble rate, spacetime interstellar distances are increasing and spacetime is changing. However, for the 3σ to 1σ value, we may not have a need to introduce dark energy. We still have an inconsistency between the density of the universe, for a closed cosmology, and the actual density and the value of the Hubble constant.

How is the relationship between space, time, and the speed of light effected and affected by expansion? Minkowski asserted that it is not space and time, but spacetime. In the continuous creation of more space, then there should be also a change in time. Perhaps in an expanding universe, if space is accelerating then time would be slowing down, described in the formula, velocity $\dot{R} = \frac{dR}{dt}$.

If the speed of light, c, is a constant, and the universe is accelerating or expanding, time could also be changing, but can changes of time ever be measured? We may never perceive that time is changing as Einstein's Principle of Special Relativity indicates that no experimenter or observer could distinguish one inertial frame from another.

We cannot measure a potential change in time if it is changing itself while we are trying to measure it. Mars et al. [12], from the University of Salamanca (Spain) goes so far as to state that sudden singularities may occur where time itself may be replaced by another dimension of space. The theory of Mars et al. [12] is based on string theory models where our universe is confined to the surface of a membrane, or Brane, "floating" in a higher-dimensional space. String theorists use their models also to explain the reason that gravity is weaker in comparison to the other three forces as gravity is considered a unique force that can move between branes.

What about the speed of light? It is well known that the velocity of light varies in a medium. Researchers of quantum physics indicate that all of space, outside basic 3D matter-particles, is filled with a structure created by quanta of matter and sustained by a universal medium [13]. It may be that this universal medium concept contains a mechanism that prevents changes in the speed of light under spacetime influences. However, M. Urban [14], G. Leuchs and L. Sánchez-Soto [15], have also proposed that c may change in a vacuum of space which is actually full of fundamental particles or "virtual" particles such as the Dirac Fermi Sea e+ e-, the Planck density $\rho = c^5/G^2\hbar$ [16, 17, 18].

Leuchs and Sanchez-Soto [15], in their research, suggest virtual charged particle pairs create a state of polarization of the vacuum [19]. They determined that the impedance within the vacuum is key to determining the speed of light which depends, according to their findings, on the sum of the square of the electric charges of particles, not on their masses. If their theory is correct, they have shown an intimate relationship between the properties of the quantum vacuum and the constants in Maxwell's equations and that the speed of light is dependent on variations in the vacuum properties of spacetime. They have also shown how the speed of light plus the impedance give an indication of a finite number of charged elementary particles which correlates to the low-energy properties of the electromagnetic field [15].

With modern technology, we are gaining sufficient evidence to develop various methods to analyze light, as well as space and time and understand it in terms of our models of the universe both in terms of classical science and in quantum physics, where we should also consider that the observer occupies a role [20]. As photons in the medium of spacetime seems to remain a constant, for purposes of this paper, we will maintain the speed of light as a constant and that distance and time are the variables for an expanding and, perhaps, accelerating universe, although we can consider all states of matter and energy as having some variables under certain conditions. We later discuss these issues in terms of quantum gravity. The speed of light may remain constant with only an increase in frequency (wavelength) rather than velocity, $c = \lambda \nu$ for wavelength λ and frequency ν where photons, the velocity, c, remains constant. However, with General Relativity it has been affirmed that when light is associated with a large gravitational field, velocity may change due to the curvature of the path of the photons.

We know that time is associated with frequency that is in its simultaneous relationship of time to a frequency, as $\nu \propto = 1/t$, where t is time. In engineering, the time domain is measured by an oscilloscope and the frequency domain is measured by a spectrum analyzer. As the frequency goes to zero, then time goes to infinity for infinite span. Fourier analysis allows us to formulate a more complex relationship between frequency and time called a dispersion relation [19].

Finally, it is important to cognize that we exist as the observer. "Absolute knowledge" about time or gravity from our spacetime perspective we may never know. As more and more information becomes available, both astronomical and non-astronomical, we certainly can gain a wider view of the "real world"; still we do not know at present how much more information will become available to us about the real world, so we do not know how far we are from the "real truth" of the origin and nature of the universe and it appears as if we shall never be able to know.

3. What is Time Really?

In order to discuss the nature of the universe, we must understand the concept of the quantity we call time. Why does time only seem to run forward at least from our perspective? We know there is a tomorrow and that objects age or get older. Children grow, plants lose their leaves and die, the sun rises and sets again and again, the next day comes. So we have developed devices to "measure the passage" or a clock to determine the quantity called time.

Einstein's theory of general relativity places time in the proper perspective as the "4th dimension of space" or Minkowski space, that is, space consists of three spatial dimensions, the ones we can detect and measure with a yardstick, and one additional time dimension that we measure with a clock or chronometer. Time is, in fact, a quantity measured best by an atomic clock, as at its center, exist electrons and quarks, the same particles that make up matter in our universe.

For time to "change," that is to proceed at a variable rate, rather than a constant rate, could imply that quarks and electrons would have to be moving faster or slower, and likewise the sun, the moon and the stars would also experience a change in rate. Our frame of reference could be part of the change of time viewed, so it may be difficult to perceive a change in time especially if is minute.

We experience reality as a single-valued now in space and time. So in the micro and macro cosmos, time exists symbolically between past and future. Time, unlike space goes only in one direction, that is to the future. We experience life and growth as a uni-directional process of aging and evolution.

Physicists assign symmetry principles to the constant relation between physical variables in their equations describing physical processes, particularly in their study of micro or atomic and nuclear processes. These symmetry principles are forms of conservation laws, such as the conservation of energy, charge and even time in what is termed "time reversal invariance" [21]. The concept of time reversal invariances is that the laws of physics remain unchanged, in form, if time is run forward or in reverse or backwards. A macroscopic analogy is a movie film where information can be run forwards or backwards showing a person diving into a swimming pool or out of it onto the board. As we know from energy conservation, the time reversal dive is unlikely to happen! Essentially, current physical theory describes, in a sense, the "conservation of time" or the linearity and uniformity of time. This is certainly what anyone with a modern chronometer or timepiece would believe.

Time is observed by clocks or sand in an hourglass, which involves the change in matter and energy – yet, matter and energy need space and time to exist in. These four concepts, space, time, matter and energy are intimately interwoven in Einstein's relativity. The famous $E = mc^2$ links matter, m, and energy, E while $s^2 = X^2 - c^2t^2$, the Minkowski line element links space as X = x,y,z as the three dimensions of space and one of time, t. Time is part of a coordinate and Einstein defined the system as having four specific coordinates of *x, y, z, t*. He also considered time as a variable that exists in our universe with a specific one-way direction, at least for us, towards the future.

Like all coordinates, 'time' may be defined throughout the universe or only effecting a local region of the manifold, for example near a black hole. This could mean that the time variable t is defined only for some finite range of values for t or the subset of points t and may not be a complete three-dimensional construction for all of spacetime. Would gravity then be a manifestation of spacetime curvature because the flow of time is not constant throughout the universe but varies according to the distribution of massive gravitational bodies establishing gravity at the heart of the curvature effecting spacetime? Time, seen only as a coordinate on a membrane, has no direct physical significance, but does the property of change of time emerge?

Einstein also pictured time like a river. Stephen Hawking [22] clarified this by stating: "Time flows like a river and it seems as if each of us is carried relentlessly along by time's current. But time is like a river in another way. It 'flows' at different speeds in different places, and that is the key to traveling into the future." Einstein would agree, because special relativity shows that time is elastic and flexible.

A. Two Models of Time

Our first model (Figure 1a.) or definition of time can be stated as follows, time is used as a consequence of the changes in matter and energy, that is to say as matter and energy change, the concept of time also varies. A chronometer changes in matter and energy to measure time.

TIME THEORY A)

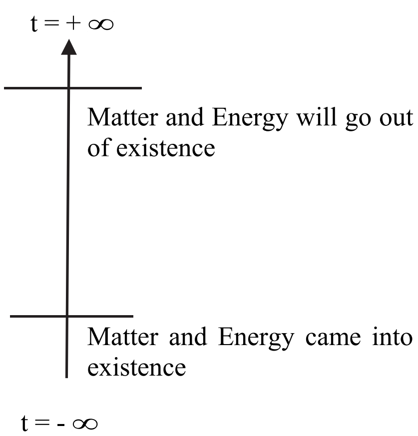


Figure 1a. Time passes along and matter-energy come into existence in the Big Bang and may go out at the Big Crunch.

The second model (Figure 1b.) or second definition of time is that the changes in matter and energy occur within time, that is, time would "go on" whether matter and energy existed or not but how can it be known? And we observe the presence of time by the fact that matter and energy do exist and we see the changes occurring in them and perceive the presence of time. (See Figure 1a. and 1b. for the two models.)

Our theories of time directly imply things about the substance of matter and energy, thus, our models of the universe. So, let us ask "What exists as our universe?" and perhaps later pursue the question "What do we mean by existence?"

TIME THEORY B)

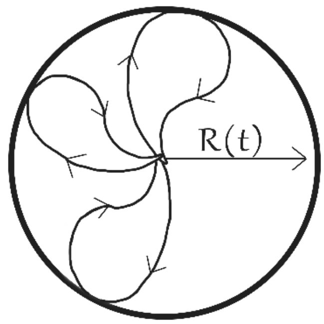


Figure 1b. Matter and Energy always present in the spacetime continuum.

To consider this question about time, we should answer the question "What do we mean by universe and of what does the universe consist?" In Figure 1c., we consider the big bang model in a closed universe to a final big crunch.

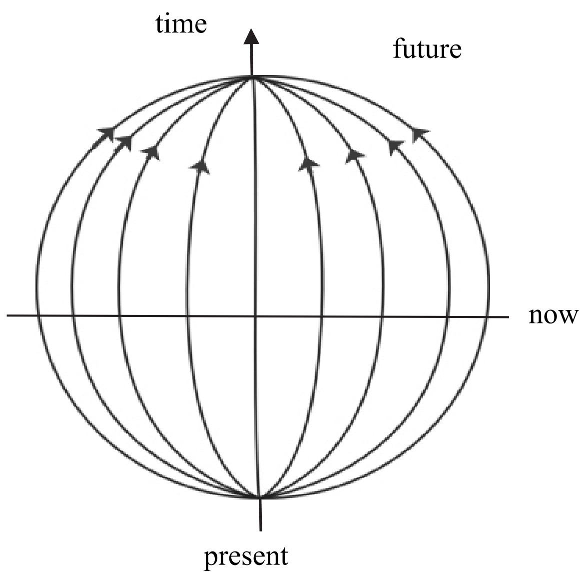
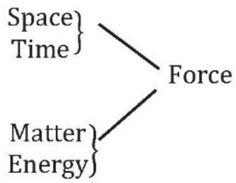


Figure 1c. Cross section of the universe expanding from a "big bang" part to now and to the future.

B. Categories of Existence

All of the existing universe that we are able to determine to exist as of now, can be grouped into four fundamental groups,



Of the four there are only two main divisions which are,

Space + time — $s^2 = X^2 - c^2t^2$
Matter + energy — $E = mc^2$
Force — $F = c^4/G$

Of the two divisions, spacetime is coupled by "matterenergy" and by force, which is the consequence of the geometry of space. By the use of special and general relativity, we see time as the 4th dimension of space, and spacetime occupies a similar coordinate role [16, 17, 18, 23]. As David Bohm [24] said, "Ultimately, the entire universe has to be understood as a single undivided whole, in which analysis into separately and independently existent parts has no fundamental status." In reality, spacetime and "matterenergy" are geometrical representations of the entanglement structure within quantum systems.

It further appears that matterenergy cannot exist without spacetime and force (that produce the changes of matter and energy). This leads to one more important element in the universe and that is gravity. Matter and energy, or matterenergy, and space and time could not exist without gravity. General relativity indicates that what we call spacetime is just another feature of the gravitational field of the universe producing curvature. Space and time would not exist separate from matterenergy that creates the gravitational field. The geometry of space is a consequence of it, whereby we arrive at such properties as anti-deSitter (AdS) space.

Let us prove this latter coupling or equivalence of matter and energy. We begin with the relativistic form of Newton's second law of motion

$$\underline{F} = \frac{d(\underline{P})}{dt} = \frac{d(m\underline{v})}{dt}$$

where F is the constant force connected with gravity and *P* is the relativistic momentum and is a variable with time, *t*. So we have by the chain rule,

$$\underline{F} = m\frac{d(\underline{v})}{dt} - \underline{v}\,\frac{dm}{dt}$$

Let us determine what is the expression for the relativistic kinetic energy, T in E = T + V while E is the total energy and T is the kinetic energy and V is the potential energy. As in classical physics T is the total work done in bringing a particle from rest up to a final velocity v by applying a constant force, F. We have over a path integral, s for T,

$$T = \int_o^s \underline{F} ds = \int_o^s \frac{d}{dt}\,(m\underline{v})\, ds$$

from before, and for ds = $v$dt, we can change to an integration over time. This is derived from space in relation to time, where velocity becomes the time rate of the change of space and where v is the constant

$$T = \int_o^t \frac{d}{dt}\,(mv)\, vdt$$

or

$$T = \int_o^{mv} (v)\, d(mv)$$

for an integration over momentum, m. Now using the relativistic mass formula for momentum

$$\mathrm{P} = mv = \frac{m_0\, v}{\sqrt{1-\beta^2}}$$

for $\beta \equiv v/c$, and we have m_0 is the rest mass

$$T = m_0 \int_o^v vd\left(\frac{m_0 v}{\sqrt{1-\beta^2}}\right)$$

for integration taken over velocity

$$T = m_0 \int_0^v \left\{\frac{v}{\sqrt{1-\beta^2}} + \frac{v\beta^2}{(1-\beta^2)^{3/2}}\right\} dv$$

$$T = m_0 \int_o^v \frac{vdv}{(1-\beta^2)^{3/2}}$$

$$= m_0\, c^2 \left(\frac{1}{\sqrt{1-\beta^2}}\right)\Bigg]_0^v$$

$$= m_0\, c^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right)$$

$$T = m_0\, c^2 - m_0\, c^2 = (m - m_o)\, c^2$$

So we see that the relativistic kinetic energy, T in E = T + V is the increase in mass and/or velocity in P = mv momentum, (due to the motion at velocity v), multiplied by c^2, so that as T the kinetic increases so also does its mass. The change in kinetic energy can be written as

$$T = E - E_0 = mc^2 - m_0c^2$$

and will represent the total change of energy of the particle if there is no change in the potential energy. The energy E_0 is the rest energy of the particle and E is the total energy of the particle, T, if the particle is in motion.

We choose the rest energy $E_0 = m_0c^2$. (A more complex analysis indicates this choice is correct.) We consider the coupling that exists between spacetime. A consideration of relativistic kinematics will demonstrate that space and time are just as inseparably related as matter and energy have been shown to be. This introduces also the concept that matterenergy or gravity may be an important fact in the nature of time.

In Einstein's field equations, the right side is the stress energy tensor as,

$$\frac{1}{8\pi}\frac{G}{c^4}\mathrm{T}_{\mu\nu}$$

where the driving force for expansion of the universe is given as

$$\mathrm{F} = \frac{c^4}{G}$$

where c, is the velocity of light and G is the universal gravitational constant. The mutual causal curvature of spacetime by matterenergy expressed in terms of the matric tensor $g_{\mu\nu}$ as

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R_{\mu\mu} = \frac{8\pi}{F}\, T_{\mu\nu}$$

4. Gravity and Time?

Einstein wrote his General Theory of Relativity to understand a similar effect to the "twin paradox," from gravity waves instead of the speed of travel. Clocks run quicker or slower at different altitudes on Earth. They are faster at higher altitudes and slower at lower ones, so GPS systems in space are continually updated to match time on Earth. This gravitational "time dilation" is too small of an effect to be noticed by us without sophisticated technology, but gravity is what affects time (by warping spacetime). Therefore, in the presence of gravity, the speed of light becomes relative and not absolute.

Gravitational time dilation can be approximated by the Schwarzschild radius of r_s

$$r_s = r\left[1 - \left(\frac{t_r}{t}\right)^2\right]$$

where t_r is the elapsed time for an observer at radial coordinate r within the gravitational field, r is the radial coordinate of the observer. Here t or time relates to the observer who is distant from a massive object.

Gravitational redshifts can also be measured to a precision of one part in 7×10^{-9}, but time dilation may not actually change time, but change the parameters that are used to measure time. Therefore it is relative and a result of different frames of reference in which two observers are moving in some relation to one another. There is a redshift of z which is dilated by $1+ z$. Note that the redshift between $0.1 < z < 0.5$, $s\ (1 + z) - 1.1$ provides evidence for the time-dilation model [25]. General relativity predicted this "slowing" of time.

In 1915, with his theory of general relativity, Einstein predicted that, when matter was accelerated, the moving mass would launch ripples in the invisible web of space and time, tugging momentarily at each point in the universal sea of cosmic space and time as these ripples moved by. These ripples or waves are called gravitational waves or gravitational radiation and, like electromagnetic waves, are thought to travel at the velocity of light. According to Einstein's relativity theory, the presence or acceleration of a large mass in the universe, such as the explosion of a large star, would dramatically change the curvature of space around the star and transmit gravitational radiation.

We are just now beginning to detect gravity waves that were recorded for the first time by scientists at the LIGO (Laser Interferometer Gravitational-Wave Observatory) program. Physicists with the LIGO Scientific Collaboration [26] have concluded that gravitational waves which were detected were likely produced by the dynamic merger of two black holes. A "signal" was recorded at two observatories (one in Livingston, Louisiana and another in Hanford, Washington). The LIGO interferometers that recorded the waves use laser light separated into two beams that travel back and forth down the 4-km long arms of a 1.2 meter diameter tube, kept under a near-perfect vacuum.

According to Einstein's theory of General Relativity, the distance between the mirrors will change by an infinitesimal amount when a gravitational wave passes by the detector. The gravitational waves were detected
on September 14, 2015 by both of the twin Laser Interferometer Gravitational-wave Observatory (LIGO) detectors in the USA [27]. Specifically, there was a 0.007 second or 7 milliseconds delay between the signals being recorded at both Louisiana and Washington. The oscillation or signal sweeps ranged from 35 to 250 Hz and lasted about 0.25 seconds. Four months later, on Dec. 26, 2015, LIGO recorded a second confirmed signal, which was an even smaller signal [28, 29].

It was expected that gravitational waves reaching the Earth would affect our planet. For masses separated by distance *L*, a passing gravitational wave will dynamically stretch and compress matter by an amount *ΔL*, and then compress and stretch the distance by an equal amount in the other perpendicular direction. Therefore, there are two wave cycles, the first comes from the stretching of space along one transverse direction while squeezing occurs along the other, perpendicular direction *ΔL/L*. Then in the next cycle, the system will squeeze along the first direction while stretching along the second [30]. As a gravitational wave passes, the compressing and stretching of spacetime stretches one 4-km arm while compressing the other, changing the distance the light has to travel. Scientists continually measure the light's pattern, and any significant mismatch may reveal the presence of gravitational waves.

Einstein proposed that the speed of gravity would be the same as the speed of light in a vacuum at 2.998×10^{10} cm/sec^{-1}. LIGO reveals that the speed of gravity is between 2.993×10^{10} and 3.003×10^{10} cm/second, which is an amazing confirmation of General Relativity! Earlier measurements were conducted in 2002 by Kopeikin [31] who examined the gravitational bending from Jupiter when Jupiter moved between Earth and a quasar (QSO J0842+1835) using VLBI (very long baseline interferometry). They used the event to measure the speed of gravity to be between 2.55×10^{10} and 3.81×10^{10} cm/second, ruling out infinite speed for gravity and making it consistent with Einstein's predictions.

It is gravity, according to Einstein, that warps the fabric of spacetime primarily due to the mass of objects. Gravitational attraction is where objects follow the warped spacetime path. Near a large mass or object, like a black hole, a gravitational well is formed in curved spacetime. If there is less mass, there is less gravity.

According to Einstein, space and time create a four-dimensional fabric, and any large mass, such as a planet or a star, depresses that fabric. The extreme of this can be seen in or around a black hole where there is not only a curvature of space, but also a swirling of space as rotation is also a factor. Near a black hole in space there is a severe warping of time. Both near the event horizon and inside the horizon of a black hole, spacetime and hence time, become so highly affected and warped. According to Majumdar, for a weak gravitational field we have the formula associated with the simple Schwarzschild metric as [32]:

$$ds^2 = -(1 - 2GM/c^2r)\, c^2dt^2 + (1 - 2GM/c^2r)^{-1}\, dr^2 + r^2d\Omega^2$$

$$r_s = \frac{2GM}{c^2}$$

G is the gravitational constant, M is the mass of the spacetime and $d\Omega^2$ is the Euclidean metric on the 2-sphere, c is the speed of light, t is time and *r* is the radial coordinate associated with the Schwarzschild radius. To analyze the Earth's gravitation "well", NASA's initiated the Gravity Probe-B using four ultra-precise gyroscopes to measure the hypothesized geodetic effect, which is the amount that Earth warps spacetime [33]. The probe confirmed that space is warped by the Earth's gravitational force.

Einstein indicated that a gravitational warp of space introduces a change in time and if we near a black hole time slows down. However, there is the additional possibility that as the universe expands and we move out away from the gravitational field of other objects, time actually speeds up. So an almost empty universe could have time moving faster, or if we as a solar system moved outside the bounds of our galaxy, time might also move faster as it would if we were nearing a white hole. A white hole could be also a time reverse black hole.

5. Black Holes and Other Matters?

A car bumper sticker reads "black holes are out of sight!" They are just that, i.e., "non-visible entities" in which the gravitational field, generated by super-dense matter, becomes so strong as to warp the spacetime around its mass so greatly that light does escape. The notion of black holes is popular with even a movie by that title, but what is the origin of this idea? One of the first solutions to Einstein's field equations was developed by K. Schwarzschild in 1916, a year after the work on general relativity was published for $r_s = \frac{2GM}{c^2}$, where r_s is the Schwarzschild radius, G is the gravitational constant, and *c* is the velocity of light and

M is the mass. This solution relates the radius of a massive body to its mass and gives the condition upon which its mass is condensed or compacted to such a radius, spacetime is extensively bent or warped. Light travels on what "it thinks" are straight lines but this path, in curved space, can be highly curved geodesic paths near "space warps," and may never escape back into space [34].

If stars or galaxies can collapse to the black nothingness of black holes how could we detect such an event? The key to detection of black holes is their activation and excitation of surrounding matter and black hole Hawking radiation. One way might be on a space trip where one sees no object in front of their "spacescape," but their rocket is accelerating with no pressure on the jet accelerators. What to do? Take a quick turn left or right to avoid a black hole! By the signature of the light and X-rays emitted from the visible member one may be able to tell that debris from the visible member is being sucked into and reflected off of a rotating non-visible member. Cygnus X-1 is thought to be such a case.

In fact, some cosmologists, included E.A. Rauscher, suggest that the matter and radius relationship is just right for the universe, as a whole, but would appear to be a black hole to any observer outside it, i.e., we are living in a "Schwarzschild bubble" for $r_s \geq 10^{28}$ cm and m = 10^{56} gm. You may well ask how an observer could exist outside the universe when, by definition, the universe contains all! The black hole universe model would comprise the finite closed cosmological picture [16, 18].

The complete Schwarzschild geometry can be described as comprising a black hole, but a black hole may also lead us to a hypothesized "white hole," and perhaps involve two different universes connected at their event horizons by a wormhole. The Schwarzschild metric has two solutions: a white hole or a black hole, a $+\sqrt{1}$ and $-\sqrt{1}$. If time slows down near a black hole, it would speed up near a white hole.

Suppose we are a Schwarzschild observer staying far away from a black hole. There is also a second observer, stationary or traveling at the same speed of the first observer but nearer the black hole. While their clocks were synchronized at the start, in the Schwarzschild metric, the second observer's clock runs more slowly being closer to the singularity of a black hole. Time passes slower in a stronger gravitational field and faster in a weaker field. For weak fields, the time flow rate becomes proportional to $1+2V/c^2$, where V is the gravitational potential.

In quantum theory, time is not physically observable in the normal sense, instead it is considered a background parameter which applies mainly to non-relativistic quantum theory more so than to relativistic particle dynamics. It is the reason why the meaning assigned to the time-energy uncertainty relation $\Delta t \ \Delta E \geq \frac{1}{2} \hbar$ is quite different from that pertaining to, for example, the position and the momentum of a particle, $\Delta x \ \Delta p \geq \hbar$. Black holes may make their presence known through gravitational radiation, but they are also known through Hawking radiation.

6. Macroscopic Remote Interconnectedness by Mach's Principle

Rotational velocity and Mach's Principle [35, 36] have been confirmed by the change of the OAM (orbital angular momentum). Mach's Principle relates to time and space and gives a frame of reference and orientation in space and time and hence, a measure of time. It is concerned with the relationship of a local phenomenon to cosmic, large-scale phenomena.

If a bucket of fluid is rotated, the meniscus (surface of the fluid) changes shape, from flat to parabolic. Ernst Mach concluded that the configuration of water becomes parabolic rather than flat under rotation, as in a spinning bucket. The faster the rotation, the more parabolic the surface becomes. Mach stated that the rotation of the bucket affects the surface of the liquid because the rotation occurs relative to the fixed star system.

Mach also used pendulums to show the connection of time to force (acceleration). Mach, thus, saw the interconnection of forces and that all physical determinations are relative. He, whose principle and philosophy influenced Einstein [37, 38], appears to imply a nonlocal connection of a remote inertial frame of reference. The inertia of a body or particle is due to the action of forces produced by all other bodies in the Universe!

The logical question is suggested by Mach, "How do we measure the inertial or rest mass or "actual mass" of a body?" From Newton's second law $m_i = F/a$ inertial mass is measured by the ratio of the sum of applied forces F to a mass m_i to accelerate it. The measurement of absolute acceleration requires the measurement of absolute displacement; however, what we can really measure is the displacement of that body relative to other bodies. According to Mach, only by virtue of the presence of other bodies can a given body be said to have inertial mass [37]. Furthermore, both Newton's bucket experiment, as well as the Foucault's pendulum appear to demonstrate that in defining inertial frames of references, large masses at great, nonlocal distances have greater importance than small masses nearby.

Mach's principle relates to the motion of local material bodies and their properties as relative motion to the center of all other masses in the Universe [16]. His explanation like Einstein's involves relative motion whereas Newton considered space to be absolute. The stars and the Earth all have some presence as does the observer. Mach was followed by Hubble, who around 1921, discovered the movement of stars by the analysis of stellar red shift data as the rate of expansion to distance is Hubble's constant, as H =$\dot{R}/R$ where $\dot{R}$ is the velocity of recession of an astrophysical object and R is its distance away [20]. This expansion comprises the before mentioned redshift.

The Hubble expansion yields multiple frames which may not preclude another form of so-termed fixed frame as a type of aether. The relative frames, as in Newton's bucket, may represent a large scale, at least earth size, nonlocal influence. Mach, moreover, never saw time nor space as absolute. We stated Einstein was influenced by Mach's principle in his development of General Relativity. Sciama [39] developed a detailed formulation of Mach's Principle and applied it to develop an accurate model of rotation curves of galaxies without the need for dark matter [40]. Rowlands [38] discusses Sciama's approach and makes a Machian analogy of the so-termed all pervasive Higgs field.

7. Quantum Gravity

Einstein wanted to determine the position and momentum of all particles at any moment. Of course, if one could know the position and momentum of all the particles in the universe, one could perhaps predict the future evolution of the universe. To date this has proved impossible because of the success of the Heisenberg Uncertainty Principle, to the chagrin of Einstein.

A reason for the difficulty of relating classical physics to quantum physics is because quantum physics involves linear superposition. In the Hamilton-Jacobi classical physics, conjugate or paired variables (p, x) and (E, t) occupy a role in moving from classical to quantum physics, where we consider the paired variables and space, x and momentum, p, and also energy and time in the development of a theory of quantum gravity. The many formulations of quantum gravity have mainly come about in an attempt to unify the four fundamental forces where gravity has proved to be the most difficult to quantize. Gravity is nonlinear and so we much reconcile the linear superposition of quantum mechanisms with the properties of a nonlinear gravitational force.

Various attempts to form a picture of quantum gravity or to bring the force of gravity into the quantum picture, have been proposed such as Loop Quantum Gravity, f(R) Gravity, Dilation Gravity, Massive Gravity, Bigravity and Unimodular Gravity have been proposed to name but a few. In our earlier work, Rauscher [16] attempted to unify the four forces fields in a Descartes multidimensional geometry.

By showing that matter is simply "congealed" energy, Einstein changed the concept of matter as made up of cold, hard particles and the quantum de Broglie picture, p = h/λ yields a wave picture of particles for us to observe. Although Einstein also demonstrated that space and time are inseparable and that the universe cannot be understood except in terms of four dimensions, three of space and one of time, we may need also n-dimensional geometries for N > 4 for p as momentum in order to comprehend reality [16].

It is important to further understand from Galileo's rate of objects falling in the Earth gravitational field, the rate of fall in terms of time, t for a distance, s is

$$s = \frac{1}{2} gt^2$$

So s = x ∝ t^2 where g is the gravitational constant of the Earth at sea level, reminding us that gravity is a constant. However, in recent studies [41], gravity varies slightly based on what may be caused by density variations of the Earth, reminding us that gravity is also a wave, which is being correlated with the LOD (Length of Day) on earth, as it shows similar variation correlations.

To further address Einstein's measurement problem of particles, the Wheeler-DeWitt equation (WDE), starts from the Hamiltonian $H\,(x)|\psi\,\rangle$, where $H\,(x)$ is the Hamiltonian operator and $|\psi\rangle$ the wave function of the universe. The Hamiltonian operator can be formulated in terms of momentum dependence, and with or without a time-dependent wave equation. In some quantum formulisms, we can specify time or time is seen as stationary, i.e. frozen in the timeless potential free equation

$$H\psi = E\psi$$

The "Problem of Time" (POT) is based on the now and on the observer where events are happening at a single time. The wave equation ψ (x, t) is a solution to the Schrödinger equation and ψ (p, E) for the Lippmann-Schwinger equation. Here two particles with momenta, p, and the energy of the system, E, can be in a state of superposition instead of the space coordinate x.

A quantum state of a system can be expressed as a linear sum of substrates so that for example ψ (x, t) can be expressed in terms of two other states so that

$$\psi(x, t) = a\,\psi_1(x_1, t_1) + b\,\psi_2(x_2, t_2)$$

where particle positions became hard to determine until observed, since superposition yields an entangled state of matter. Gravity, as before mentioned, is nonlinear and hence, cannot obey linear superpositions.

With quantum physics, we introduce a second premise that time is essentially a change of state of a particle, defined as a carrier of energy, where light and matter are the manifestation of the informational structures that exist throughout the universe called qubits that exist in 3-D, 4-D and, in string theory in 2D or Descartes geometry of N > 4, or in many more dimensions. Time becomes a background parameter to mark the evolution of the system. Quantum mechanics further defines dynamical entities that evolve through Hilbert space (with infinite dimensions) where the parameter *t* is incorporated within the time-dependent Schrödinger equation [42] as,

$$H\psi(\mathrm{x,t}) = 2\hbar \frac{d\psi(\mathrm{x,t})}{dt}$$

The right side of the equation can include time where $\frac{d\psi_t}{dt}$ is a partial derivative with respect to time *t*, hence time dependent.

Page and Wooters (PaW) [43] developed a method to incorporate time in a static entangled state, $|\,\psi\,\rangle$. They considered two observers, one who is external to the universe, who would be expected to see a static universe, and a second observer who is inside the universe who would experience some passage of time and a dynamic universe. They included a clock system, to gauge time evolution that could be viewed by the external observer. Moreva et al. [44], sought to confirm this by considering the state of the "universe" to be $|\,\psi\,\rangle$ identified and enforcing the Wheeler-De Witt equation $H\,|\,\psi\,\rangle = 0$, i.e. by requiring $|\,\psi\,\rangle$ to be an eigenstate of H for the zero eigenvalue. Moreva et al. present their model in which they concluded that by projecting $|\,\psi\,\rangle$ the "universe functions on the states of $|\,\phi\,(\mathrm{t})\rangle_c = e^{-iH_c t/\hbar}\,|\,\phi\,(0)\,\rangle_c$ of the clock, one obtains the vectors,

$$|\,\psi\,(\mathrm{t})\rangle_r := {}_c\langle\phi(\mathrm{t})|\,\psi\,\rangle = e^{-iH_r t/\hbar}\,|\,\psi\,(0)_r$$

which details the evolution of the subsystem r under the action if its local Hamiltonian H_r, the initial state being $|\,\psi\,(0)\rangle_r = {}_c\langle\varphi(0)|\,\psi\,\rangle$" [43].

When Moreva et al. [44] tested the PaW hypothesis, they demonstrated that globally the system appeared static, but the components exhibited dynamical evolution. It was determined because the external observer, known as the "super-observer," outside of their toy model universe did not encounter any photons but only looked at C, the independent clock, noted time as static. However, the internal observer, who is within the universe observing two entangled photons that were sent separately through a birefringent plate which could alter their polarization, was able to observe the photons leaving the plate, saw time as emergent. Moreva et al. results showed that the internal observer could gauge time-evolution when entangled with a photon, but when not engaged, time was static [44]. Verlinde [45], a string theorist, at the University of Amsterdam, considers that gravity may also be an "emergent" phenomena.

We may be living on the surface of a bubble connected by information qubits in 2D. This further allows for a holographic universe, where our 3 + 1 D 'reality' is contained in a 2 +1 surface on its boundaries. Various theories exist, but it is clear that there is a nonlocal environment where everything is entangled and which does not require signals to be sent back and forth, so time becomes "static" in this universe model because of the existence of nonlocality. This approach is similar to that of Wheeler-DeWitt with no external time parameter and operators, such that time appears as stationary in a frozen formalism. In frozen time nothing happens, it is static.

The Frozen Formalism problem as manifested by a frozen equation like the WDE or Einstein's General Relativity can both include Hilbert Space, where the space of solutions of that frozen equation is turned into a Hilbert space. This could occur if behind everything is a network of entangled qubits (or quantum bits of information). Tegmark [46] equates the qubits to a three-dimensional computer game encoded with the classical bits of information. The qubits process time the same way as a computer processes code, in a type of AdS space. The qubits become the information codes that create in de Sitter space a universe with time.

There is some evidence from the study of quantum computers that the bits of information may both correlate from their origin as well as their "future" results [47, 48]. As scientists peer into qubits of information and the more information is coded at different times, time becomes a significant phenomenological event.

Now, the evolution of quantum computers, has led to an understanding of the manner in which Bell's theorem formalism of the EPR paradox and the J. Clauser [49], A. Aspect [50] and Gisen quantum experiments are proving the existence of nonlocality over meter distances [51]. This works for spatial entanglement, but also a similar structure for time which can be labeled "super memory." Quantum information theorist Peres [52] is correct when he says "quantum effects mimic not only instantaneous action-at-a-distance, but also … influence of future actions on past events, even after these events have been irrevocably recorded." Here qubits store quantum information but there are many possibilities which include error correcting codes and quasiparticles that can arise as collective excitations in certain condensed matter systems. We can see this as "super memory" in the universe.

To further solve the "Problem of Time" (POT) we may examine the block universe idea [53, 54] which has a manifold time which comprises a reality where past, present and future are all one within the spacetime continuum that represents all events that have happened and that will happen. This goes back to Minkowski who understood that relativity does not allow for a universal "now", but particular "nows" of a block universe which proposes that reference frames have different "now slices" throughout the universe. However, the block universe does not account for multiple dimensions nor the concept of Eight Space [20]. It does reconsider Newtonian thought where time is absolute and also universal [55]. If we consider the block universe in terms of a time manifold and add that as a central requirement of the scalar product on the Hilbert space, which is the superposition of different universes all coexisting simultaneously [56], then we can determine how the exact nature of the correlation of nonlocal and correlated events take place by information transfer not based on the geometrical structure of the spacetime manifold, but on nonlocality and entanglement [57, 58].

As an extension of Einstein's geometrical model of the spacetime manifold and J.A. Wheeler's *Geometrodynamics,* [59] a "hyperspace" has been developed as a multidimensional geometry and is termed a Descartes's space [16, 23]. The link between the macro and micro domain in terms of the Descartes geometric constraints relates general relativistic descriptions of spacetime and the EPR effect [56, 16]. The dimensions of this space are formulated in terms of Planck-like units which are physical variables uniquely expressed in terms of universal constants and are therefore universal [16]. A generalized metric is found for this new multidimensional space defining a new geometry [17, 23].

Multidimensional reality leads us to the notion of Supergravity in the unreduced Hilbert space. In addition to Loop Quantum Gravity and String Theory, concepts such as Supergravity are based on the particle symmetry, or supersymmetry, which naturally includes gravity along with the other fundamental forces. It is considered that gravity at the quantum level is carried by a particle, or graviton, with an intrinsic angular momentum or spin 2. Its partner, the gravitino, has a spin 3/2. The other forces, and their carriers have a spin of 1 and ½. In this definition, supersymmetry theory would allow gravity to be included in quantum physics and its effects on time as seen in classical physics.

In fact, in a multidimensional model, time can be frozen in one reality and emergent in another as shown by Moreva et al. [43] where the position of the observer becomes paramount, but also where the consciousness of the observer plays a role [57]. Some of the consequences of this geometrical symmetry and multidimensional models are an addition to Einstein's field equations, giving closed cosmological solutions, [17] new Heisenberg-like uncertainty relations, [16] new light cone relations, [23] tachyonic-like "signals", [36] and a unification of micro- and macro-phenomena [16, 17].

In fact, the new Heisenberg uncertainty relations give us a "handle" on a description of possible faster than light information transfer [17, 18, 23]. The exact nature of the propagation of information in the Descartes manifold is not restricted to tachyonic signals, but can be easily formulated in complex Eight Space [36]. Other modes of signal propagation are also accommodated by the Descartes geometry. The geometric constraints of the Descartes space place severe restrictions on the structure of spacetime and the correlation of nonlocal events both macroscopically and microscopically.

For faster than light signaling, considering non-locality, reality is not moving through spacetime but through the superposition or entanglement of qubits. In the domain of quantum mechanics, implications for faster than light information transfer between non-locally connected spacetime points, have been experimentally investigated by J. Clauser et al. [48]. The design and implications of this experiment are presented by Stapp [60] and Herbert [61]. Based on the theorem of J.S. Bell [62] derived from the Einstein, Podolsky and Rosen [63] paradox, one can determine that quantum theory predicts the correlation of events in remote causally connected spacetimes on the level of individual events. The correlation of remote quantum events can be understood in terms of information transfer between non-locally connected quantum events.

Also of interest are quantum coherent collective states in micro and macro quantum systems. Coherent, collective instabilities in plasmas [19], as well as certain nuclear collective vibrational modes of excitation appear to have their basis in information transfer via EPR modalities. Plasma coherent modes are found to be closely related to the superconductivity phenomena, and superconductivity and signal propagation may be particularly amenable to study the detailed mechanism and nature of information transfer at velocities greater than light. They can be facilitated by the work with the new geometrical models of the Descartes space, which allows us to relate the spacetime description in general relativity (light cone relations) to the microscopic Heisenberg relations and non-locally connected quantum events. Utilization of faster than light information transfer may well be made practical by quantum computers and bi-information transfer and remote event correlations by superpositions, but our primary purpose is to understand how time works in the universe and also if the observer plays a role.

8. The Observer Matters

Great minds have grappled with the fundamental conceptual notions of time as A.S. Eddington stated, "*time is a mental construct of our private consciousness…physicists construct the concept of a worldwide time from a string of subjective instances.*" [63].

A concept of time is also considered in a Ziggy cartoon as: "*Time is just nature's way of keeping everything from happening all at once.*"

From the theory of relativity, we learn that the point of view of the observer or frame of reference determines what we perceive, that it is "all relative." With quantum physics, we can say that it matters when one looks, what one sees (observes) as in remote connectedness via Bell's theorem experimental tests. With relativity, one can say that where one looks determines what one thinks he sees (observes) depending on the frame of reference. Remote connectedness does exist in Eight Space [36] which creates an omnipresent observer who can choose his observational perspective. For electromagnetic theory, one can speak of remote connection in and other electromagnetic coherent phenomena such as in MHD plasmas, Young's double slit experiment and super coherence phenomena [20, 36]. The relativity of motion as described by Einstein appears to imply that macroscopically (by large scale observations), one's observational vantage point affects the manner in which one sees (observes). In the quantum picture of the universe, does the act of observation, which utilizes light or particles, affect the system being observed? No observation can be strictly made without affecting the observed!

In Rauscher and Amoroso [36], it is suggested that the particle-wave duality and the results of the Young's double slit experiment can be explained by faster than light information transfer. Note this is not "faster than light" particles, but nonlocal superpositions that apply to consciousness. The nature of faster than light signals relate to that in Clauser experiments in 1969 [49].

In a sense of quantum physics, time only enters when the observer is present. We form an analogy for resolution of the particle-wave duality created by the observer. The specific experiment one conducts makes light or particles appear to act and "look like" a particle or wave. The particular experiment about our perception of reality may determine how we perceive reality to be in terms of the mind-body or mind-brain issue.

John Wheeler took the nonlocal *Gedanken* experiment idea one step farther with his delayed-choice thought experiment in 1978. Although he proposed the theory, extensive experiments were demonstrated and repeated by Jacques et al. [65], where they streamed photons into an apparatus that showed that their states could retroactively change something which had already happened, that is, the polarized states were set after the correlated photons were emitted from the source before their spins were measured. Their experiment used a single-photon source, previously developed for quantum key distribution, based on the pulsed, optically excited photoluminescence of a single nitrogen-vacancy color center in a diamond nano-crystal. Using this device, it was possible to obtain single photons with a well-defined polarization.

According to Jacques et al. [65], "The delayed-choice experiment itself is performed with the fast electro-optical modulator (EOM) randomly switched for each photon sent in the interferometer, corresponding to a random choice between the open and closed configurations." Their final outcome confirmed Wheeler's delayed-choice Gedanken Experiment showing that the behavior of the photon in the interferometer appears to depend on the choice of the observer, even when that choice is made at a position and a time such that the two photons are separated from the entrance of the photon into the interferometer by a space-like interval. In Wheeler's words, since no signal traveling at a velocity less than that of light can connect these two events, "we have a strange inversion of the normal order of time. We, now, by moving the mirror in or out have an unavoidable effect on what we have a right to say about the already past history of that photon" [59].

Just as the double slit experiment illustrates wave-particle duality and the collapse of the quantum wave function into a single reduction of matter with well-defined physical properties as a particle, when all the potential states collapse, the delayed choice experiment is such that it may appear that in the present it can affect what happens in the past or that it already happened before it happens [66].

We contend, based on Ramon and Rauscher [67] and Rauscher and Amoroso, [36] it is in the Eight Space formulation that we have macro and micro nonlocality and not the four space, although by the principle of Lorentz invariance, the laws of physics are invariant or unchanged by the perspective from where one looks, specifically the frame of observation that one observed from. Perhaps, it is moving from the specific to the whole that demonstrates the unity of observation. Another thought is that of the one big problem in the old debate about free will and determinism. Can we change our future with future "information" brought into the now present in the twin paradox? We could explain that Einstein's general theory of Relativity predicts "time travel" into the future, not into the past. Only time can tell!

The dance of reality does not depend on our knowledge as much as on our act of observation and the perspective from which it is made in accord with physical theory and experimental test. This leads us to ask if we have an existence beyond space and time? If we do, does this mean we can survive beyond material death? Maybe beyond death lies a life of pure consciousness. Life, Death and Evolution—behind them are the meaning of time which is fundamental and essential to our comprehension of reality. Time is an essential element in the comprehension of "it all".

9. Conclusions

Max Planck remarked "*I regard consciousness as fundamental. I regard matter as derivative from consciousness. We cannot get behind consciousness. Everything that we talk about, everything that we regard as existing, postulating consciousness*." We agree.

Albert Einstein developed a more complete and accurate model of motion and description of the force of gravity than Newton's, thus demonstrating the non-universality of Newton's description of time. Both men dealt with the nature of mechanical force, motion and acceleration as well as the formulation of the properties of the non-linear gravitational force in general relativity.

Minkowski stated, "*You can't have place without a time or time without a place*." He and others laid the ground work and Einstein put these concepts together, spacetime and relative motion! He discussed time as the fourth dimension and now we are also discovering additional dimensions which we affirm contain states of past, present and future events as well. We would conclude that in the world of quantum physics and relativity, one needs to involve $N > 4$ to especially include Eight Space to understand time and the role of the observer.

We can conclude that time is not constructed from geometric data alone, but from matter fields ϕ coupled to gravity, or from a combination of matter and gravity fields. The universe described by Einstein was a dynamic one and this view has been experimentally verified, but one unknown is whether this dynamic process "Universe" will expand forever, limitless in space and time, where eventually space and time will disappear or whether the universe is finite, closed and is with beginning and end. The general relativistic equations of Einstein do not specify the curvature of space as closed or open, only that space and time are inexorably linked and curved by the presence of gravity created by matter and energy.

In the conventional view, when most of us think of the universe, we think of things that we can see, stars, planets of matter floating in space. Space means that which is up and down, left or right, front and back, time may be that which travels through our consciousness like a river moving in one direction. But if we think of the universe only in this way we are accepting appearance as being the ultimate reality.

What do we mean by universe and of what does the universe consist? Over the millennia, the concepts about the nature of the universe have changed and evolved as our knowledge becomes more and more refined. But when do we say we "know it all"? We are never right. What we can do is do the best we can to discover and understand "reality" and the nature of the "real world" and strive towards this purpose and not be deluded into, too soon, proclaiming to have the "absolute truth" or a theory of everything (TOE). It is about time, we figure out time!

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The Observer Omission in Einstein's Rail-Car Experiment

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Einstein special relativity theory provides an ontological interpretation of the Lorentz transformations and Maxwell's Eqs. that predicts the physical retardation of clock rates and shrinkage of measuring rods. A logical explanation for these effects is demonstrated by comparing the measurements of common phenomena made by an observer in moving train versus those obtained by a stationary observer at rest on the embankment. Though the mathematical treatment properly accounts for observations made by coordinate systems built upon the constant speed of light assumption, I will show that these coordinate frames analysis cannot be transferred to actual physical situations without encountering contradictions with the original experiment assumptions. The ontological view of physical reality implied by these assumptions, which underlying special relativity and much of conventional physics, is therefore in question. By eliminating these assumptions an alternative interpretation of experimental evidence is provided. Specifically, the careful construction of clock rates, rod lengths, and time offsets required to explain the constant speed of light reported by all observers is not necessary if the speed of light is observer dependent. I will argue that light propagates in the space attached to the material from which observer's are built. The speed of light in this material is a property of the gravitational potential the material is subject to. The space supplied by the material of each observer is more accurately described as possessing a constant speed-of-now, which determines the rate of activity of all processes the observer carries out. This ontological interpretation of special relativity is compatible with quantum theory since it treats observables, including the observation of empty space, as the internal observer dependent phenomena resulting from measurements rather than a ubiquitous a priori container in which all things happen.

Keywords: Special Relativity, Rail Car Example, Universal Background Space, Cognitive Action Theory

1. The Train-Embankment Thought Experiment

Einstein's thought experiment [1] utilizing a moving railroad car and a stationary railroad embankment as a stationary is shown in Fig. 1. An observer M outfitted with an un-primed coordinate frame is stationary on an embankment. A second observer M' outfitted with a primed coordinate frame is located on a passing rail car. Exactly when the midpoint of the rail car is lined up with the stationary observer simultaneous lightning strikes hit the coordinate clocks located at the front and back end of the car. These also illuminate the stationary clocks located on the embankment.

As the train moves with velocity "**v**" light travels with constant velocity "*c*" in the medium of Einstein's imagination toward the two observers. The images of the primed clock pointers travel to M' and those from the un-primed frame travel to M. The clock pointers in each frame were synchronized so that the speed of light travel time multiplied by the speed of light equals the distance between clocks. For example, the equation,

$$(t_M - t_B)\cdot c = x_B, \qquad (1)$$

provides the synchronization criteria for the right clock shown on the embankment. From these readings and the assumption that the speed of light is constant, both observers set the clocks so that they define a single now time.

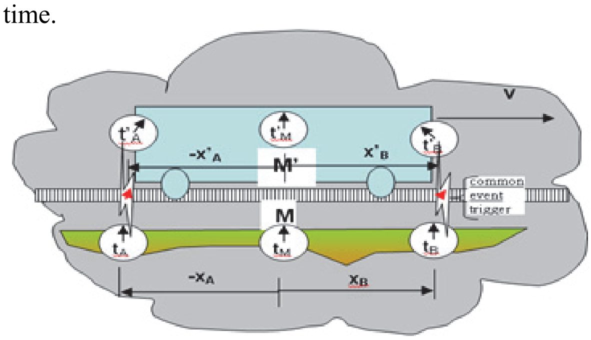

Figure 1. Einstein's rail car visualization explaining Special Relativity.

In Fig. 2 we now introduce a 3D observer above the scene who can look down and take objective snapshots, as Einstein envisioned of the two observers coordinate frames, when deriving the Lorenz transformations [2]. This observer is stationary with respect to the embankment but using a lens the path length to the ground is constant over a wide range of the x-axis. This means the god's-eye-view observer would see the entire length of the experiment as a synchronized now and we readers can envision the results of such snapshots because we could see both sets of clocks in the gods-eye-view records shown on the pages of this article.

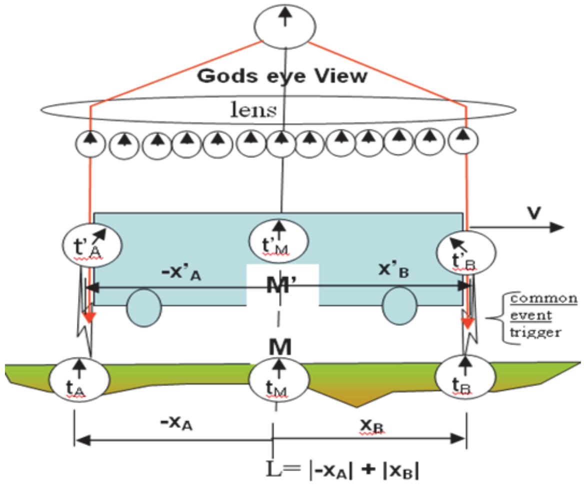

Figure 2. Einstein's rail car visualization with an explicit 3D person.

In order to conform to the requirement that the speed of light is constant both coordinate frames are synchronized using the speed of light delay technique described above (Eq. (1)). This assures the clock and position numbers can be transformed from the stationary to the moving frame by the Lorentz transformations. These give the clock time and length for the left clock A' as,

(2) $$t'_A = [t_A + x_A \cdot (v/c^2)]/(1-v^2/c^2)^{1/2}$$

$$x'_A = [-x_A - t_A \cdot v]/(1-v^2/c^2)^{1/2}$$

and for the right clock B' gives.

(3) $$t'_B = [t_B - x_B \cdot (v/c^2)]/(1-v^2/c^2)^{1/2}$$

$$x'_B = [x_B - t_B \cdot v]/(1-v^2/c^2)^{1/2}$$

in terms of the stationary clocks on the embankment. It is important to remember that these parameters t', x', t, x for any clock express the number of time or space units the labeled point is distant from the origin. The larger the number of space units between two fixed points the smaller the rods used to define those units so the physical rods shrink. However, the larger the number of time units between two fixed events the smaller the time units. But the smaller the units the faster the clock pointer moves.

Let's assume lightning strikes the clocks at the front and back of the train exactly at the same time. The A and B clock tell the same time i.e. $t_A = t_B$ since signals from these clocks are the same distance from the midpoint $|x_A| = |x_B|$, travel at constant speed "c", and they are synchronized.

Subtracting the distances measured between clocks A and B gives,

(4) $$\Delta x' = x'_B + x'_A = (x'_B + x'_A)/(1-v^2/c^2)^{1/2}$$

so that the number of units between the two clocks in the primed frame is larger, which means the physical units and hence the physical material from which those units are made, i.e. the rods, in the primed frame have shrunk as expected.

If we let the distance between the clocks go to zero L = > 0, then both end clocks being now co-located in the middle will tell the same time. This equals the time told by the midpoint clock so that all stationary clocks tell the same time $t_A = t_B = t_M$. But with the midpoint clock at the origin of the stationary frame moving clocks would according to Eqs. (2) and (3) would be,

(5) $$x'_A = x'_B = -t_M \cdot v/(1-v^2/c^2)^{1/2},$$

This means the moving clocks will not be located at the center unless the midpoint clock time is also set to zero. The experiment will only work out correctly according to the Lorentz transformations if both the midpoint clocks in both coordinate frames are set to zero exactly at the time the lightning flash hits the front and back of the train.

Because the readings, $t_A = t_B = 0$, the clock A' offset from Eq. (2) is,

(6) $$t'_A = x_A \cdot (v/c^2)/(1-v^2/c^2)^{1/2}$$

and the clock B offset from Eq. (3) is,

(7) $$t'_B = -x_B \cdot (v/c^2)/(1-v^2/c^2)^{1/2}.$$

Now let's take a snapshot of the clock settings in both coordinate frames at precisely the instant the two lightning strikes hit. Instead of using vectors for position we construct the coordinate systems so that the clock position label is also written on the clock that holds that position and therefore the clock's address on the x axis is also its name. In Fig. 3 the snapshots of both coordinate frames as seen by the third observer or the reader are shown. The two coincident clocks in the primed frame are the same length but the number of prime coordinate units required to measure this distance has increased.

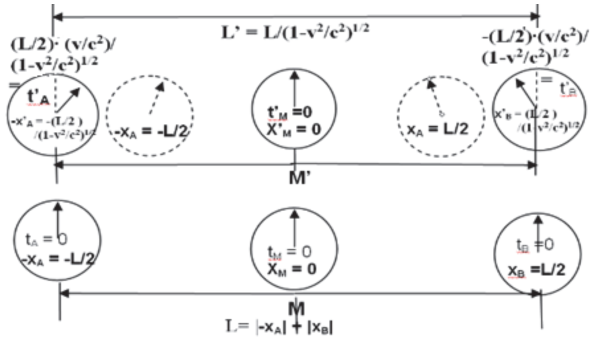

Figure 3. The rail car experiment when only coordinate frames are considered.

As mentioned above at this instant the distant dependent offset calculated by Eq. (6) advances the clocks on the left and the offset calculated by Eq. (7) retards the clock pointers on the right. The dependence on distance assures that the farther we go in both directions the greater this offset.

The clock position name is larger in the negative direction on the left side and larger in the positive direction on the right side. This means that the clocks in the moving frame which are labeled L/2 and –L/2 must be closer to the center as shown by dashed lined clocks in Fig. 3. The distance between these two clocks has shrunk by the usual Lorenz contraction. This is shown by comparison with the larger distance L' at the top of Fig. 3.

2. Actualization Analysis

So far, the experiment shows exactly what Einstein claimed. Rods shrink and clocks are offset exactly as required by application of the Lorentz transformations, and both observers would claim their clocks are synchronized because light traveling from either end clock would arrive at the center showing pointers delayed by L/2c and L'/2c respectively. However, when we apply these results to actual physical situations there are inconsistencies we must now point out. Einstein's entire experiment is based upon reading synchronized coordinate clock pointer positions locally where the events, in this case two simultaneous lightning strikes happen. To give these local pointer positions meaning they must be synchronized. The synchronization procedure, however, assumed light propagates in a background media attached either to Einstein's imagination or a god-eye-view 3D person frame we have introduced. It is certainly possible to synchronize coordinate frames based upon such assumptions however tying such constructions to actual hardware is problematic.

The top portion of Fig. 4 shows how we might construct such coordinate frames both on the embankment and on a rail car exactly at rest. The embankment clocks at both ends of the car are labeled x_A and x_B to indicate they are collocated with the exact duplicate. The clock labels exactly line up clock for clock with the same labels given to the same position and all the clock pointers show the same time state and are moving are the same rate. Simultaneous signals from both ends of reach the other observers coordinate frame and are judged to be simultaneous in both frames.

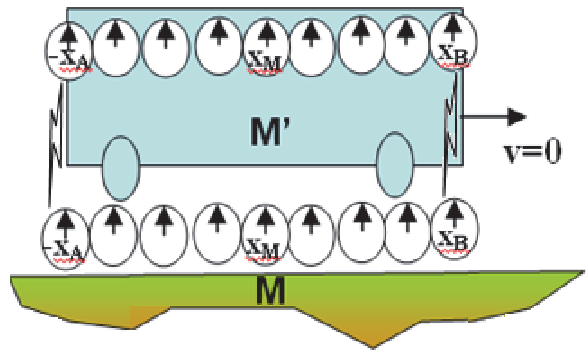

Figure 4. Construction of coordinate frame with a stationary car.

Then the rail car is brought into motion on the track somewhere far to the left of the embankment. Acceleration will send the car moving back to the position of observer M. According to general relativity this acceleration will compress the car and its attached coordinate frame. When the rail car has reached the desired velocity "v" the synchronization procedure (Eq. (1)) is carried out. When all the offsets are properly dialed in and the moving car has reached the exact position relative to the embankment the coordinate frames, then according to special relativity theory the clocks will line up next to each other exactly as specified by the Lorentz transformations. The god's-eye-view snapshot of that instant is shown in Fig. 5.

There is nothing wrong with Einstein's argument when applied to the coordinate frames placed on a background space. If the theory is applied the snapshot prediction it would show the moving clocks offset and shrunk exactly as required to make the speed of light constant. However, Fig. 5 clearly shows that the physical car and the clocks labeled $-x_A$ and x_A which are securely fixed to the front and back of the car are no longer coincident with the clocks on the embankment. In other words, the symmetry of the experiment fails. If only the physical car is considered there would not be any clocks x_A' and x_B' for light to bounce off. This does not mean the Lorentz transformations or Maxwell's Eqs. are wrong. If we extended the moving frame beyond the end of the moving car (not shown in Fig. 5) the signals would bounce off the extended frame clocks x_A' and x_B' as calculated. However, applying these calculations to the real world physical experiment uncovers a contradiction. If the SRT application of the Lorentz transformations applies then each observer would say that the other observer's moving lengths shrink and clocks slow down and neither could tell who is moving relative to an absolute background. But the real thought experiment is not symmetric because the moving observers can determine that he is moving relative to the absolute space introduced in Figs. 1 and 2.

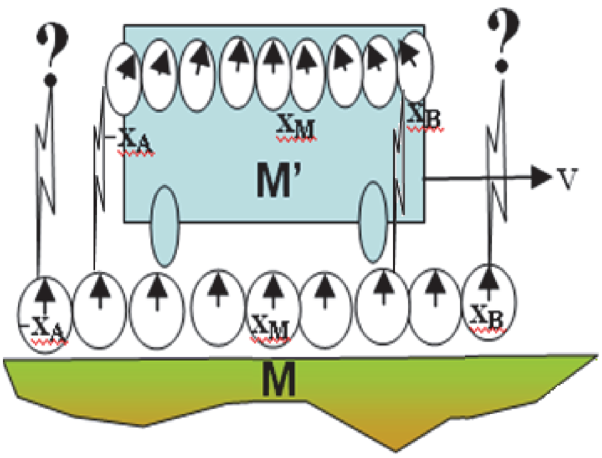

Figure 5. Construction of coordinate frame with a moving car.

Of course, the asymmetry in this thought experiment is also evident in the twin paradox, which has been well documented when its principles are applied to real situations. And, as A. Wheeler [17] pointed out it is absolutely absurd to believe that a clock on a table would run slower just because one walks past it. Aware of these difficulties Einstein attempted to address them by including gravity in general relativity. However, all these symptoms of inconsistency can be traced to the basic assumptions adopted by Einstein [2]. The very derivation of the Lorentz transformations assume light propagates relative the god's-eye-view perspective. If we switch perspectives and assume we are moving with the coordinate frame of the moving rail car then the Special Relativity logic will only work if we assume the god's-eye-view is now also moving with the rail car and the embankment and the coordinate frame constructed on it is moving in the opposite direction.

Typical statements such as "*the observer in the moving frame will see*", and then applying the space shrinking or time slowing interpretations never take into account the additional requirement that the god's-eye-view, which is in fact the space attached to background space in which the experiment is conceived, must also be transformed. Neglecting the role of the observer leads to false conclusions regarding the nature of physical reality.

Of course, light speed is constant when its speed is determined by an implied background media in which it propagates. And yes, if moving coordinate frames are envisioned to move in that background space then to achieve constant speed of light measurement results the coordinate frames have to be physically modified to produce such a consistent result. However, the logic and world view proposed to explain the electromagnetic phenomena in moving coordinate systems is suspect. Specifically, Einstein, and many who follow, tacitly assume the back ground space and the speed of light in it can be eliminated once relationships between observers have been established. Therefore, they conclude it is the rods and clocks themselves that are shrinking and slowing not the imagined or observed phenomena.

2.1. *Interpretation of Clock Rates*

A second issue that should be addressed is to clarify the interpretation of the numerical values produced by the Lorentz transformations on clock rates. If two identical clocks are constructed at the same location, their clock pointers once synchronized will move at the same rate and therefore point to identical locations as shown on the left two stationary clocks in Fig. 6 below.

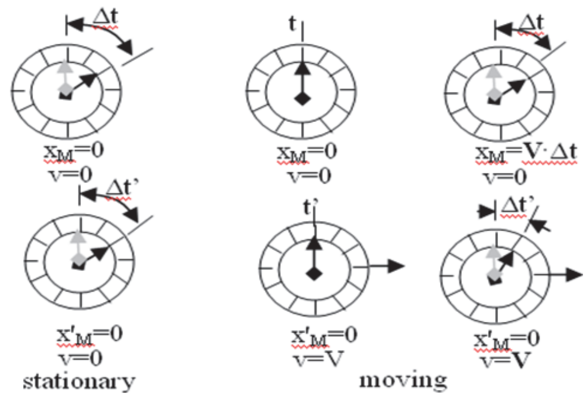

Figure 6. God's eye view comparison of clock rates.

Next, we accelerate the lower primed clock to a velocity **V** and arrange the offsets so that when it passes the upper clock both pointers point up to the same high noon position. After some time, interval "Δt" the upper clock pointer moves to 2PM while the lower clock has moved to a new position and its pointer to a new time. The new position as seen by the stationary observer is $x_M = V \cdot \Delta t$ and the pointer on the stationary clock at that position is also 2PM This is because our god-eye-view is stationary with respect to the non moving frame and all the clock pointers move the same distance on the clock dials irrespective of their position along the x-axis. Substituting x_M into the Lorenz transformation Eq. (2) we calculate the time interval in the moving clock as

$$\Delta t'_M = \Delta t_M \cdot (1-v^2/c^2)^{1/2} \quad (8)$$

As in the spatial interval comparison in Eq. (4) this tells us that there are a fewer number of time units in the moving clock or the numbers $\Delta t'_M < \Delta t_m$. In the spatial situation we noted that if the number of spatial units fit into the same length (L vs L' in Fig. 3) has increased the units must have shrunk and hence the unit rod material, called the meter, must have shrunk. Here the number of unit clock intervals became less in the traveling clock, so applying the same spatial logic one would say the unit of time, the second, must have lengthened and whatever one might call the material of time must have expanded. But *time is not space* so this interpretation is not applicable.

The units of time marked on the clock dials of the moving clocks are not changed just because the clock is moving. If the dial face is perpendicular to the direction of motion there is no effect predicted by general relativity. If it is turned so that the 3 O'clock to 9 O'clock line coincides with the velocity direction then the face becomes an oval but in no case do the number of units marked on the clock face increase or decrease. Therefore, if $\Delta t'_M < \Delta t_M$ the pointer on clock must have move more slowly and only gotten to 1PM as shown on the lower left clock. But this means the clock pointer moves more slowly as commonly taught.

The difference in interpretation between space interval readings and time is that if time units get smaller, the distance between units shrinks, when time is mapped onto a spatial axis, but this spatial display should not be confused with the rate of time. If a second, as defined by a physical mechanism, gets smaller then the physical mechanism is executing more quickly. *Real clocks run slower after energy has been added by acceleration not faster.*

2.2. *What Problem was Einstein Solving?*

Einstein quite properly made the assumption that the speed of light must be constant in all directions for all observers because all observers, specifically the Michelson-Morely experiments, reported the same value no matter what velocity the Earth was going. He then produced a thought experiment which demonstrated how two observers moving at constant velocity with respect to each other would report this result. The only logical conclusion was that the coordinate frames used by such observers must be adjusted so that light sending the time information between the two coordinate clocks separated by a distance L would be delayed by the travel time interval L/c. He failed to acknowledge that in such a thought experiment light would travel in the media of his imagination so that the speed of light transmitting the information between clocks was actually determined by that media.

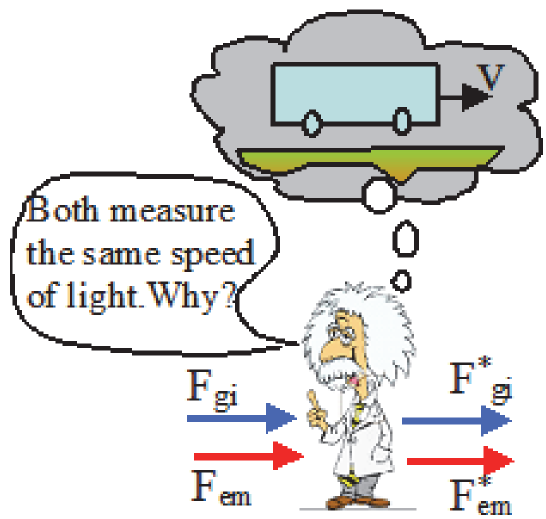

Figure 7. Einstein's actual situation.

His actual situation, shown in Fig. 7 above, depicts the material, from which he is constructed, subjected to incoming and outgoing gravito-inertial and electro-magnetic fields. These influences are interpreted as two observers in a rail car and embankment. These interpretations are then displayed in his perceptual space. The speed of light in that space is whatever his material supports when under the external force influences. Since both observers report the same light speed and the only difference is their velocity Einstein naturally concluded their rods and clocks must automatically adjust.

The error in his thinking was to assume that the images of the observers and their coordinate frames he imagined were equivalent to real entities in Nature. This error is forgivable because such an assumption is the foundational underpinning of classic physics and our secular western society as a whole. It is called the "naïve reality" assumption by philosophers. It states that reality is essentially identical to what we see. In other words, we believe the objects we see and the space we see them in are representations of reality. In this regard, Einstein was a realist. He never came to grips with the fundamental assumption proposed by quantum theories that reality is completely different from what we see. In quantum theory reality is described by a wave function that is only converted into the classic world of objects by the measuring instruments which surround us. Given the Zeitgeist of the early 20th century Einstein's conclusions were the only logical outcome consistent with what was believed at the time.

Our analysis and many others have pointed out inconsistencies with the interpretation of special relativity and the logic used to apply its equations to physical situations [3, 4]. Of course, it may be that some of the predictions are valid for other reasons than those supplied by Einstein. To explore this possibility some alternative explanations will now be investigated.

3. The Alternative Interpretation

If we acknowledge that Einstein's imagined observers are incorrect representations of physical reality and give up the "naïve reality" assumption what alternative explanations to the observed facts might be available?

If the background space supplied by Einstein and his imaginings in that space were withdrawn then some reality of these observers would still exist, only they would not exist in any ones background space. Fig. 8 shows the icons of two observers no longer imbedded in a space. In this diagram, we no longer place the stationary observer on the embankment because this image strongly suggests he is firmly planted in a real objective world background. To avoid this suggestion the stationary observer has been given his own railroad car, which now defines the material owned and controlled by this observer.

As before he constructs a coordinate axis using the usual time equals distance divided by speed formula only this time light signals used to synchronize the clocks no longer propagate in an external background space but in the material specific to the car in which he is embedded. The speed of light is now an observer dependent number designated with a prime like the other coordinate values in the primed frame. The time distance relations in the two systems are now,

$$x_A = c \cdot t_A, \quad x'_A = c' \cdot t'_A. \tag{9}$$

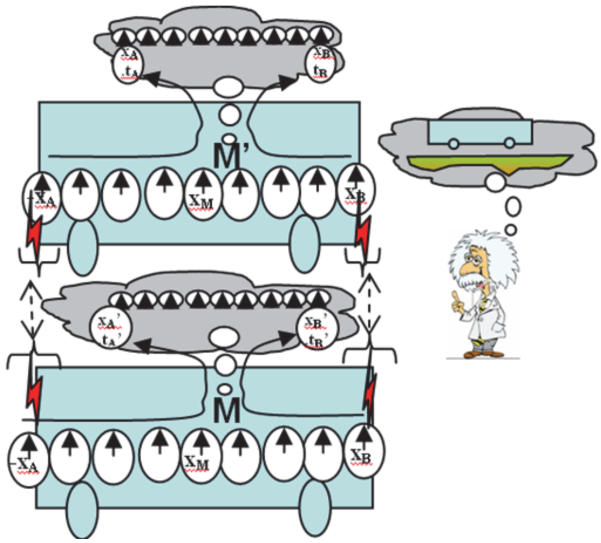

Figure 8. Two observers without a 3D person background space.

The rail car experiment is again defined as both observers sending out signals that are simultaneous in their own coordinate frames and are judged to be simultaneous in the receiving observers frame. Fig. 8 now shows the predicted result for any velocity under the assumption that the speed of light propagates inside each rail car at a speed determined by the material from which the car and coordinate frames are built. Einstein is still in the picture and his imagined rail car and embankment are properly placed where they belong i.e. in his own perceptive space shown as a thought bubble. The two observers are also outfitted with their own display space. Einstein may be imagining the other two, and his space still provides the background with a constant speed of light "*c*" for his imagined representations of the other two observers. But his space is attached to his material and no longer provides a physical or objectively representational background for the real observers which exist outside of his imagination. All three observers contain their own space, which are their respective thought bubbles. All information regarding the external world is derived from interactions reported by the detectors whose measurement data is tagged with local coordinate clocks address and time state labels. This information is then transmitted in the material making up observer to the display and there all equal time state data is displayed in their respective thought bubbles at once. This each observer experiences his own defined Now.

3.1. *The Event Oriented World View*

To understand the significance of Fig. 8 and the alternative world view it describes we must take an aside to quickly summarize the principles of Cognitive Action Theory (CAT) [5-8]. CAT is a physical theory supporting an event oriented world view originally proposed by Whitehead [9] and further developed by the ideas of Bohm [10] among many others. The reason for pursuing the development of the CAT model is to create a physical theory which could explain subjective experiences. The critical assumption was made by Everet [11] who assumed that all systems are observers. This meant all systems execute an activity which includes 1) a measurement 2) a change in its internal structure to accommodate the measurement, and 3) a transmission informing the external force field of its changes associated with the internal accommodation. In other words, a simple input, think, output process. The "think" comes in when the system contains feed forward processing cycles to augment primitive reflex reactions. The CAT model associates force fields internal to material with perceptive space so that all systems are described by a flow of action through the material [12]. An internal Now plane provides a kind of perceptive space between the input and output to the external fields. The content of this internal space is depicted as a thought bubble above the three observers in Fig. 8. It is shown as above the material for clarity, but should be considered the perceptive Now plane inside the material through which action flows.

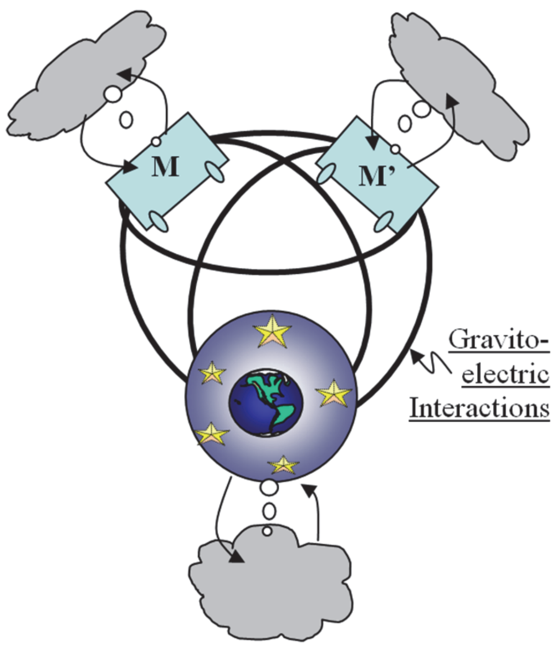

Figure 9. CAT model of Reality with 3 interacting parts.

When such observers are completely isolated they are independent existences which form closed cycles of change. They are not seen by other such systems unless there are interactions. *It is of utmost importance to realize that we scientists are observers who do not see objects as they are, but rather interpret interactions and display those interpretations in our models of reality.* A useful model showing 2 observers and the rest of the universe is included in Fig. 9 below.

The material composing Einstein in Fig. 8 has now been incorporated into the rest of the universe. Besides explicitly introducing subjective experiences into physics the critical difference between the CAT model and our classic or quantum physics concepts is that there is no independent space-time continuum containing all material, instead space is explicitly shown as a thought bubble that represents a connected and internal property of each material part. The content of each personal space are interpretations of interactions and these interpretations make up each individual's experiences. In this diagram two observers, M and M' have explicitly been, taken out of the rest of reality along with all their interaction lines. Since the whole is equal to the sum of its parts plus all their mutual interactions, Fig. 9 is a convenient depiction of the whole of reality when we are interested in calculating the behavior of these two observers.

To emphasis the special relativity problem the observers in Fig. 8 are shown without interactions to the rest of the Universe. Only the two interactions required to implement the Einstein's thought experiment were included. These are used by each observer to determine the location and size of external systems by processing interaction signals. Of all the interaction lines shown in Fig. 9 only the two carrying signals from the back and front of the rail cars used in the special relativity thought experiment are needed in Fig. 8.

It is important to emphasize that space is represented by the thought bubbles attached to the material of each observer and not the page upon which that bubble is shown to the reader. The position of independent activities nor their size and motions, as they appear to the reader on the page, have any significance in CAT. The symbols of CAT theory only relate to each other not to the medium in which they are presented on the page. The space provided by the background page is not part of the CAT model. A rigid page is necessary to keep the shape of the symbols fixed. Like the poles holding a spinning globe modeling the earth, the page is only a support tool for the model. Only items drawn on the pages, including thought bubble icons of internal space, represent actualities. Until the page is formatted with explicitly drawn feature no metric for physical reality exists in the model. The fact that you, the reader is looking down on the page providing you with a kind of god's-eye-view of the entire model does not mean you are god and does not give you the right to introduce or extract illegal changes. Such intervention would be miracles, which by definition, would violate the rules of the theory. By sticking to the rules, you can manipulate the symbols of the model to calculate results. However observable results only appear in each system's internal space. You only experience the content of your own thought bubble. If you take on the role of one of the observers then all you can actually observe are appearances in the selected observers thought bubble. Given this limited information the fundamental question pursued by all physical theories is, "What model of reality explains my observed display?

3.2. *The Special Relativity Experiment in the New World View*

The special relativity experiment then consists of observer M instructing transmitters, attached to the clocks A and B at the front and back of his rail car, to send simultaneous space-time signals containing t_A, x_A and t_B, x_B to the respective clocks in the M' observer. As in the original special relativity experiment design this transmission must take place when the respective clocks are in coincidence so that there is no time delay in the transmission that would otherwise have to be taken into account. Though the front and back of the rail-cars are shown separated on the page they are actually connected by distances as close as coincidence allows, when two clocks are located on parallel x-axis. We assume the transmission delays are negligible.

Upon receiving a signal the clocks in the primed observer coordinate system echo their own signal and time-space stamp the received signals so that records can be sent to a comparison location in relative leisure. The record in each observer will then have eight values. These are, t_A, x_A, t_B, x_B and t'_A, x'_A, t'_B, x'_B, which represent the transmission and reception space-time values for the M and M' observer respectively. The M observer clocks are internally synchronized $tA = t_B = t_M$, which defines the un-primed observers Now. And because no time delay was introduced during the transmission the signals arrive at the same time as measured by the primed observer clocks.

The un-primed observer will conclude he has sent a simultaneous signal from the ends of his car while the primed observer will conclude he has received a simultaneous signal from the ends of his car. The reverse signals sent use the same protocol so the same eight values recorded in M' are available in M. Only the roles of receiver and sender are reversed. Because both comparators would have exactly the same set of eight numbers, each would conclude they have measured the linear size of the other car as $\Delta x = x_A - x_B$ in M and $\Delta x' = x'_A - x'_B$ in M'.

The space length of the thought bubble is directly mapped into linear array of clocks. Normally the positions are labeled implicitly so that images of one's own arrays do not clutter one's own display surface. In Fig. 8 we explicitly included images of each observer's arrays so that his perception of his own extension and Now time is easily identified.

The curved arrow processing paths, from the reception of the signals along the distance of the physical arrays and into the thought bubbles, show how images of the other rail car's clocks are displayed as observations. From this information, each observer effectively sees the end points of the other rail-car next to the end point of his own rail car. For example the observer M' will see x_B, t_B next to his own clock reading x_B', t_B'. Taken together each observer will believe he sees the end points of the other car at the end points of his own. Exactly the way we human observers believe to be seeing objects in each of our own perceptive spaces. In fact, if the entire array of clocks along the length of each car is sent and received signals to their coincident partner the display in each observer's thought bubble would contain the entire length of the others car.

There is now no need for the coordinate frame adjustments to explain what is observed. The second observer's car is the same size as the first, because the speed of light in the CAT-model is propagated in the media of the material which defines each observer's simultaneous events. Clock rates can not be compared in this one-time interaction experiment. This requires a second set of interactions.

Assume for example the M and M' clocks have exchanged their time and position information as before so that t_M, x_M, $t'_{M'}$, $x'_{M'}$ are recorded in each observer. Let's further assume that the coincident position relative to each other has moved. Of course, our habit is to look down on the page as god's-eye-view observers and identify one car as moving and the other as stationary. This view point would make the two rail cars observable images in the reader's thought bubble not independent events in our theoretical construct. All we need is to record a second interaction hit at different coincident locations. We do not need, and if quantum mechanics is correct, perhaps cannot know what happened between the interactions. Thus, lets assume in a second interaction clock A sends a coincident signal at time t''_A to clock M' which returns its signal immediately. Now each observer has the information t''_A, x_A, $t''_{M'}$, $x_{M'}$. Each observer calculates the velocity of the interaction. If the un-primed frame the M' clock has moved from M to A with a velocity between the interactions

(10) $$v = x_A - x_M / t''_A - t_M$$

and in the primed frame M' is stationary but M has moved and been replaced by A. The velocity is calculated as,

(11) $$v' = x_A - x_M / t''_{M'} - t'_{M'}.$$

The distance interval is the same since it was only measured by the number of internal clocks between the front and back of each car. If the clock intervals are identical the velocities are also the same. Both observers would conclude the other is moving past at the same velocity. From this exchanged data, the times could be compared and if they are the same then

(12) $$1 = t''_A - t_M / t''_{M'} - t'_{M'},$$

the ratio would be one. If the clocks are identical they should run at the same rate and as long as they have not experienced additional interactions there is no reason to assume they would be different.

We have thereby provided an alternative explanation to Einstein's problem. By acknowledging the difference between one's observations, no matter how realistic they appear, and the reality which caused them, we have proposed a new model based upon interacting events not objects in a space time continuum. This model allows us to conclude, along with common sense that clock and rod material does not dilate or shrink simply because they are in relative motion. However, we have also not included gravitational interactions and this deficiency will be addressed in the next section.

4. Clock Rates in a Gravitational Field

Einstein himself acknowledged the incompleteness of special relativity and went on to propose his general theory in order to include gravitational effects. Fig. 8 only showed two electromagnetic interactions required to demonstrate the special relativity thought experiment. This would be adequate if the two observers and Einstein were all that existed. Clearly there is more to reality and our two observing systems will interact with it. Fig. 9 shows interaction lines between our two observers and the rest of the universe. The influence of electromagnetic interactions can to some extent be shielded, however, gravitation permeates all matter and its influence on the clocks and rods of the coordinate frames must be considered.

The largest gravitational influence comes from the distribution of distant masses in the rest of the universe. Mach's principle suggests these distant masses produces inertial effects [14]. The clock rates however may also be effected. From cosmological considerations, the speed of light is related to the gravitational scalar potential φ_g by the formula

(13) $$c^2 = \varphi_g =\sim G \cdot M_U / R_U$$

Where M_U is the mass of the Universe shell surrounding us and R_U is its radius. This strongly suggests that the speed of light depends upon the gravitational influences the system is under. The speed of light is a surrogate for all happenings so that if it changes rates one would expect any electromagnetic clock mechanism to change as well. Because of its complete domination of all activity the constant "c" should be called the *speed-of-Now* specific to any observer. The speed of light was introduced as a rail-car dependent variable in Eq. (9).

4.1. *Internal Observer Effects*

If the two rail cars are in different gravitational fields the clocks may run at different rates. If this actually happens the numerical distances between the rail car end points, L and L', as measured by counting clocks will remain the same. The clock pointer synchronization implemented by the $L = c \cdot \Delta t$ protocol will remain the same. This is because a lower gravitational potential will slow the speed of everything including the oscillators driving the clocks so the Δt numbers get smaller and the "c" number gets larger and the two effects cancel. Therefore, all clock pointers will be set to the same angle to define a Now plane for both M and M' observers. This reciprocal relationship holds in both observers so if the rail car lengths are identical and the number of clocks fitting into the length will be identical as shown in Fig. 4. In each car, the speed of light can be measured by dividing the length by the transit time as measured by local clocks. Since the internal distances are identical the following holds.

(14) $$c' \cdot \Delta t' = L' = L = c \cdot \Delta t.$$

The question we are not allowed to ask in relativity is, "What is the velocity of light in the each of the observers?" To address this question in CAT consider the general relativistic formula relating clock rates in two systems when spherically symmetric mass distributions are involved. This is, [12]

(15) $$\Delta t' = \Delta t_0 \cdot (1 + 2 \cdot \varphi_g / c^2 - v^2/c^2)^{1/2}.$$

The gravitational term can also be interpreted as the escape velocity when starting from any point where the potential is φ_g. Therefore, Eq. (13) can be written as

$$(16) \qquad \Delta t' = \Delta t_0 \cdot (1 - v_\infty^2/c^2 - v^2/c^2)^{1/2},$$

where v_∞ is the velocity required to get a test mass to a region of zero gravitational potential energy which in principle is at infinity and Δt_0 is the rate of the clock at rest at infinity.

For two stationary systems in interstellar space with no local masses the local gravitational potential φ_g and the velocity v are both zero. However, the potential energy from the distant masses are not eliminated. If we calculate the escape velocity to get outside the distant masses we would need to add enough kinetic energy to overcome the negative potential energy hole we are in. This can be calculated by,

$$(17) \qquad m \cdot G \cdot M_U/R_U = 1/2 \cdot m \cdot v_\infty^2$$

If we identify $m \cdot c^2$ as the gravitational potential energy of a mass inside the ring of distant masses then the velocity required to escape our universe equals the speed of light energy. We must now distinguish between the group velocity of light "c_g", which determines the rate at which energy is carried from one place to another, and the phase velocity "c_p", which determines the amount of energy carried in the group. The wave mechanics multiplying the group and phase velocity for any localized wave gives the constant "c", usually interpreted as the speed of light.

$$(18) \qquad c^2 = c_g \cdot c_p$$

Using these definitions of the speed of light we can now return to our two observers in inter galactic space. Their two clock rates are equal and determined by their gravitational potential as

$$(19) \qquad \Delta t' = \Delta t = \Delta t_0 \cdot (1 - v_\infty^2/c^2)^{1/2}$$

Since both are initially at rest the speed of light is the same for both observers and Eq. (14) holds. If the primed system is accelerated to a relative velocity "v" then the escape velocity in the prime system would be modified. During the acceleration period the mass of the primed system would be lifted to a higher energy so the residual escape velocity will decrease. The kinetic energy increase is calculated by subtracting the moving from the stationary system energy.

$$(20) \qquad m \cdot c^2/(1 - v^2/c^2)^{1/2} - m \cdot c^2 =\sim ½ \cdot m \cdot v^2$$

This must now be subtracted from the kinetic escape energy of the remaining stationary observer and equated to the new kinetic escape energy of the moving observer.

$$(21) \qquad ½ \cdot m \cdot v^2_{esc} - ½ \cdot m \cdot v^2 = ½ \cdot m \cdot v'^2_{esc}$$

Solving for the new escape velocity gives

$$(22) \qquad v'^2esc = v^2_{esc} \cdot (1 - v^2/v^2_{esc})$$

Since the escape velocity is the speed of light group velocity the moving observer the speed of light carrying energy required to escape is

$$(23) \qquad c'_g = c_g \cdot (1 - v^2/c^2)^{1/2}$$

This implies that as energy is added, the system is raised from a lower negative value to a higher negative value and therefore a slower speed is required to escape.

If we now compare this to the calculation of the speed made inside each observer using the standard retardation formula and the equality of distance measured in number of clocks as expressed by Eq. (14) we get,

$$(24) \qquad c' \cdot \Delta t' = c' \cdot \Delta t\, (1 - v^2/c^2)^{1/2} = c \cdot \Delta t.$$

Dividing by Δt gives the phase velocity of light inside the moving system as

$$(25) \qquad c_p' = c/(1 - v^2/c^2)^{1/2}$$

Multiplying Eq. (25) by 23 we get Eq. (18). Hence c^2 emerges as a constant but its meaning is no longer the speed of light since it is the group times the phase velocity that remains constant. The individual speeds c_g and c_p differ depending how light is used. If phase is used in measurements such as the definition of length by counting krypton wave fringes in an interferometer the phase velocity is dominant. This is the speed used when rail car lengths are compared in Eqs. (24) and (25). When the speed of energy propagation is involved such as the transport of energy out of a potential well the group velocity is required.

4.2. *Experimental Support for Varying Speed of Light*

The constant speed of light assumption is a basic tenet of modern physics, which, as we have seen, leads to adjustments in coordinate rods and clocks. Suggesting an alternative location dependent variation would require showing all experimental verifications of general relativity can be explained. The bending of light around massive objects can be explained with equal accuracy by assuming the gravitational field introduces an effective index of refraction [14]. In this case, the phase velocity is involved since by Eq. (25) this velocity increases with altitude.

In a similar vein, the Pound-Rebka experiment showing a frequency shift in light traveling between the top and bottom of a 22.5 meter tower is equally well explained by assuming light gains speed when falling in a gravitational field [15, 16]. This effect utilized the group velocity since particle detectors i.e. scintillation counters, are used to measure the effect.

5. Conclusion

We have shown that the special relativistic thought experiment interpretation of the Lorentz transformations predicting a slow down of clocks and shrinkage of rods as suggested by special relativity does not produce reasonable predictions and inconsistencies when applied to realistic situations. This interpretation is dependent on the following assumptions. First that a physical system is imbedded in a background space in which light travels at constant velocity and therefore clock and rods of coordinate frames must adjust to explain the result of Michelson-Morley type experiments. Second that a single observer independent objective space time continuum exists in which the speed of light is constant. By including gravitational and accelerated coordinate frames and expanding special to general relativity some of the false predictions and inconsistencies can be addressed by introducing curved space time background space. This not only leads to complex mathematics and visualization difficulties but also conflicts with the principles of quantum theory.

An alternative Cognitive Action Theory approach, which identifies space as an internal phenomenon of matter rather than a container of matter, and time as a measure of change in the state of that matter, eliminates the inconsistencies of special relativity while more fully explaining the experimental results of phenomena in a gravitational field. The cost for these simplifications requires us to give up the Aristotelian view that we experience reality directly and adopt a Platonic view that what we see directly are, like the shadows in a cave, only indirect evidence of a true reality.

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Genus Two Prime Form Formula for Vertex Operator Characters

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We find an expression for the self-sewn genus two Riemann surface counterpart of the torus formula for for an n-vertex operator character for the Heisenberg vertex operator algebra an n-point correlation function for a vertex operator algebra module with complex parameterization of corresponding states.

Keywords: Prime forms, Vertex operator algebras, Correlation functions, Prime form identities

1. Introduction

From the algebraic point of view, vertex operator algebras represent a generalization of Lie algebras constructed in terms of series with non-commutative Fourier modes, and further relations applied on endomorphisms called vertex operators. Both from pure algebraic and mathematical physics points of view, it is important to compute and understand the structure of the spaces of characters for modules over vertex operator alegbras.[1,3] In addition to algebraic computation methods based on vertex operator algebra properties, algebro-geometric ways to calculate characters are also very important. By comparison of these two approaches, one is then able to deduce new identities for modular objects opening up deep number theory meaning of vertex operator algebras.

In[4] (see also[8]) a formula relating a lattice vertex operator algebra module n–point function and a product of genus one prime forms[2] was introduced. Later, in[7] and[10] we have generalized this formula for the case of a vertex operator algebra module with $\mathbb{C}$–parametrized states.

In this paper we introduce the genus two analogue of the above mentioned formula by using the gluing procedure for lower genus n–point functions[5,10] to form higher genus partition or n–point functions associated to Yamada self-sewing of Riemann surfaces scheme.

For a module $\mathcal{V}$ of a vertex operator algebra V, we find closed formulas for characters of vertex operators $\mathcal{Y}$ on the torus with $q = e^{2\pi i\tau}$, $z_i \in \Sigma^{(g)}$, $v_i \in V$:[7]

$$Z^{(1)}_{\mathcal{V}}(v_1, z_1, \ldots, v_n, z_n; q) = \mathrm{STr}_{\mathcal{V}} \left(\mathcal{Y}(v_1, z_1) \ldots \mathcal{Y}(v_n, z_n)\, q^{L(0)-C/24} \right),$$

where $L(0)$ is the Virasoro algebra generator, and C is central charge. Final expressions are given by determinants of matrices with elements being coefficients in the expansions of the regular parts of corresponding differentials (Bergman (bosons) A_a, $a = 1$, 2 or Szegő (fermions) Q kernels).[7] It is important to know n-point functions on all genua, in particular on genus two Riemann surfaces.

2. Genus one prime form formula and supporting results

2.1. *General parameter formula*

One defines[10] the genus one intertwining n–point correlation function on $M \otimes e^{\alpha}$ for n vectors $u_1 \otimes e^{\beta_1}, \ldots, v_n \otimes e^{\beta_n} \in \mathcal{M}$ by

$$\begin{aligned} &Z^{(1)}_{\alpha} \left(u_1 \otimes e^{\beta_1}, z_1; \ldots; u_n \otimes e^{\beta_n}, z_n; q\right) \\ &= Tr_{M\otimes e^{\alpha}} \left(\mathcal{Y}\left(q_1^{L(0)}(u_1 \otimes e^{\beta_1}), q_1\right) \right. \\ &\qquad \left. \ldots \mathcal{Y}\left(q_n^{L(0)}(u_n \otimes e^{\beta_n}), q_n\right) q^{L(0)-1/24} \right), \end{aligned}$$

for formal $q_i = e^{z_i}$ with $i = 1, \ldots, n$, and intertwined vertex operators $\mathcal{Y}$.[10] First, recall a natural generalization of previous results[4,7] is obtained in.[10] Consider the n–point functions for n highest weight vectors $\mathbf{1} \otimes e^{\beta_i}$, that we abbreviate to e^{β_i}, for $i = 1, \ldots, n$.

Proposition 2.1. *For* $\sum\limits_{i=1}^{n} \beta_i = 0$ *then*

$$\begin{aligned} &Z^{(1)}_{\alpha} \left(e^{\beta_1}, z_1; \ldots; e^{\beta_n}, z_n; q\right) \\ &= \frac{q^{\frac{1}{2}\alpha^2}}{\eta(\tau)} \exp\left(\alpha \sum_{i=1}^{n} \beta_i z_i\right) \prod_{1\le r<s\le n} K^{(1)}(z_{rs}, \tau)^{\beta_r\beta_s}, \end{aligned} \tag{1}$$

where $z_{rs} = z_r - z_s$ *and* $K^{(1)}(z, \tau)$ *is the genus one prime form* (16).

Then in[10] we proved the following proposition:

Proposition 2.2. *For $\sum_{i=1}^{n}\beta_i=0$ then*

$$Z_\alpha^{(1)}\left(u_1\otimes e^{\beta_1},z_1;\ldots;u_n\otimes e^{\beta_n},z_n;q\right)$$
$$=Q_\alpha^{\beta_1,\ldots,\beta_n}(u_1,z_1;\ldots u_n,z_n;q)$$
$$\cdot Z_\alpha^{(1)}\left(e^{\beta_1},z_1;\ldots;e^{\beta_n},z_n;q\right), \qquad (2)$$

where $Q_\alpha^{\beta_1,\ldots,\beta_n}(u_1,z_1;\ldots u_n,z_n;q)$ was explicitly given in.[4] It is a sum of elliptic and quasi-modular forms (see[4] for details).

2.2. *The character formula in the self-sewn formalism*

In[6] (see also[10]) they introduced the formula for the genus two n-point correlation function on the genus two Riemann surface formed as a result of self-sewing of the torus. Following[6] we defined in[10] the genus two n–point function for $u_i\otimes e^{m_i}\in V_{\mathbb{Z}}$ inserted at z_i on the sewn genus two Riemann surface in the ρ sewing scheme as the following infinite sum of genus one intertwined $n+2$ point functions:

$$Z_{V_{\mathbb{Z}}}^{(2)}(u_1\otimes e^{m_1},z_1;\ldots,u_n\otimes e^{m_n},z_n;\tau,w,\rho)$$
$$=\sum_{\Psi_\kappa}Z_\alpha^{(1)}(u_1\otimes e^{m_1},z_1;\ldots,u_n\otimes e^{m_n},z_n;$$
$$\sigma f_2\Psi_\kappa,w;\overline{\Psi}_\kappa,0;q). \qquad (3)$$

Here $Z_\alpha^{(1)}\left(u_1\otimes e^{m_1},z_1;\ldots,u_n\otimes e^{m_n},z_n;\sigma f_2\Psi_\kappa,w;\overline{\Psi}_\kappa,0;q\right)$ is the twisted genus one n-point function defined in,[10] and $q=e^{2\pi i\tau}$. The sum is taken over the square bracket Fock basis $\Psi_\kappa\in V_{\mathbb{Z}+\kappa}$ with square bracket λ–dual $\overline{\Psi}_\kappa$ for $\lambda=(e^{i\pi B}\rho)^{\frac{1}{2}}$ of (23) and with σf_2 acting on Ψ_κ (see appendix).

In[9] we have also derived the formula relating the genus g+1 and genus g prime forms, i.e., $K^{(g+1)}\left(x,y,\Omega^{(g+1)}\right)=K^{(g)}\left(x,y,\Omega^{(g)}\right)e^{-\frac{1}{2}b_aX_{\bar{a}\bar{a}}(\rho)b_a^T}$, and in particular, $K^{(2)}\big(x,y,\Omega^{(2)}\big)e^{\frac{1}{2}b_a(x,y;k)X_{\bar{a}\bar{a}}(\rho)b_a^T(x,y;k)}=K^{(1)}(x-y,\tau)$. Here $b_a(x,y;k)=\int_y^x a_a(\cdot,k)$ where $a_a(x)X_{\bar{a}\bar{a}}(\rho)a_a^T(y)=\sum_{k,l\geq 1}a_a(x,k)\ X_{\bar{a}\bar{a}}(k,l,\rho)\ a_a(y,l)$, with $a_a(x,k)$ a certain one-form[6] on the initial Riemann surface $\widehat{\Sigma}^{(g_a)}$ of genus g. Here $X_{\bar{a}\bar{a}}(k,l,\rho)$ an infinite matrix determined from genus g data (see[4] for details).

3. Self-sewn genus two prime for formula for characters

Let us formulate the main result of this paper, the genus two version of proposition 2.2.

Proposition 3.1. *With $z_{n+1}=w$, $z_{n+2}=0$, $u_{n+1}=\sigma f_2\Psi_\kappa$, and $u_{n+2}=\overline{\Psi}_\kappa$,*

$$Z_\alpha^{(2)}(u_1\otimes e^{\beta_1},z_1;\ldots,u_n\otimes e^{\beta_n},z_n;\tau,w,\rho)$$
$$=\frac{q^{\frac{1}{2}\alpha^2}}{\eta(\tau)}\sum_{\Psi_\kappa}Q_\alpha^{\beta_1,\ldots,\beta_{n+2}}(u_1,z_1;\ldots;u_{n+2},z_{n+2};q)$$
$$\cdot\exp\left(\alpha\left(\sum_{i=1}^{n+2}\beta_iz_i\right)\right)\prod_{1\leq r<s\leq n+2}\left(K^{(2)}(z_r,z_s)\right)^{\beta_r\beta_s}$$
$$\cdot e^{\frac{1}{2}\beta_r\beta_s\ b_a(z_r,z_s;k)\ X_{\bar{a}\bar{a}}\ b_a^T(z_r,z_s;k)}. \qquad (4)$$

This formula generalizes (1) for the genus two case.

Proof. Let us take $u_{n+1}\otimes e^{\beta_{n+1}}=\sigma f_2\Psi_\kappa$, and $u_{n+2}\otimes e^{\beta_{n+2}}=\overline{\Psi}_\kappa$. Then we obtain

$$Z_\alpha^{(2)}(u_1\otimes e^{\beta_1},z_1;\ldots,u_n\otimes e^{\beta_n},z_n;\tau,w,\rho)$$
$$=\sum_{\Psi_\kappa}Z_\alpha^{(1)}\left(u_1\otimes e^{\beta_1},z_1;\ldots,u_n\otimes e^{\beta_n},z_n;\right.$$
$$\left.\sigma f_2\Psi_\kappa,w;\overline{\Psi}_\kappa,0;q\right)$$
$$=\sum_{\Psi_\kappa}Z_\alpha^{(1)}\left(u_1\otimes e^{\beta_1},z_1;\ldots,u_n\otimes e^{\beta_n},z_n;\right.$$
$$\left.u_{n+1}\otimes e^{\beta_{n+1}}w;u_{n+2}\otimes e^{\beta_{n+2}},0;q\right)$$
$$=\sum_{\Psi_\kappa}Q_\alpha^{\beta_1,\ldots,\beta_n}(u_1,z_1;\ldots u_n,z_n;q)$$
$$\cdot Z_\alpha^{(1)}\left(e^{\beta_1},z_1;\ldots,e^{m_n},z_n;e^{\beta_{n+1}}w;e^{\beta_{n+2}},0;q\right)$$
$$=\sum_{\Psi_\kappa}\frac{q^{\frac{1}{2}\alpha^2}}{\eta(\tau)}Q_\alpha^{\beta_1,\ldots,\beta_n}(u_1,z_1;\ldots u_n,z_n;q)$$
$$\cdot\exp\left(\alpha\left(\sum_{i=1}^{n+2}\beta_iz_i\right)\right)\prod_{1\leq r<s\leq n+2}K^{(1)}(z_{rs},\tau)^{\beta_r\beta_s}$$
$$=\sum_{\Psi_\kappa}\frac{q^{\frac{1}{2}\alpha^2}}{\eta(\tau)}Q_\alpha^{\beta_1,\ldots,\beta_n}(u_1,z_1;\ldots u_n,z_n;q)$$
$$\cdot\exp\left(\alpha\left(\sum_{i=1}^{n+2}\beta_iz_i\right)\right)$$
$$\cdot\prod_{1\leq r<s\leq n+2}\left(K^{(2)}(z_r,z_s)\right)^{\beta_r\beta_s}e^{\frac{1}{2}\beta_r\beta_sb_aX_{\bar{a}\bar{a}}b_a^T},$$

Thus we obtain (4). □

Note that corresponding generalizations are possible at higher genua.

4. Appendix

4.1. *The prime form*

Let us recall the definition and properties of the prime form on a genus g Riemann surfaces.[2] Consider a compact Riemann surface $\Sigma^{(g)}$ of genus g with canonical homology cycle basis $a_1, \ldots, a_g, b_1, \ldots, b_g$. In general there exists g holomorphic 1-forms ν_i, $i = 1, \ldots, g$ which we may normalize by

$$\oint_{a_i} \nu_j^{(g)} = 2\pi i \delta_{ij}. \quad (5)$$

The genus g period matrix $\Omega^{(g)}$ is defined by

$$\Omega_{ij}^{(g)} = \frac{1}{2\pi i} \oint_{b_i} \nu_j^{(g)}, \quad (6)$$

for $i, j = 1, \ldots, g$. $\Omega^{(g)}$ is symmetric with positive imaginary part, i.e., $\Omega^{(g)} \in \mathbb{H}_g$, the Siegel upper half plane. The canonical intersection form on cycles is preserved under the action of the symplectic group $Sp(2g, \mathbb{Z})$ where

$$\begin{pmatrix} b \\ a \end{pmatrix} \to \begin{pmatrix} \tilde{b} \\ \tilde{a} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} b \\ a \end{pmatrix}, \quad (7)$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(2g, \mathbb{Z}).$$

This induces the modular action on $\mathbb{H}_g$

$$\Omega^{(g)} \to \tilde{\Omega}^{(g)} = \left(A\Omega^{(g)} + B\right)\left(C\Omega^{(g)} + D\right)^{-1}. \quad (8)$$

One introduces the normalized differential of the second kind defined by

$$\omega^{(g)}(x, y) \sim \frac{dxdy}{(x-y)^2}, \quad (9)$$

for $x \sim y$, for local coordinates x, y, with normalization $\int_{a_i} \omega^{(g)}(x, \cdot) = 0$ for $i = 1, \ldots, g$. Using the Riemann bilinear relations, one finds that $\nu_i^{(g)}(x) = \oint_{b_i} \omega^{(g)}(x, \cdot)$. We recall also the definition of the theta function with real characteristics

$$\vartheta \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \left(z|\Omega^{(g)}\right) = \sum_{m \in \mathbb{Z}^g} e^{i\pi(m+\alpha).\Omega^{(g)}.(m+\alpha)+(m+\alpha).(z+2\pi i\beta)}, \quad (10)$$

for $\alpha = (\alpha_i)$, $\beta = (\beta_i) \in \mathbb{R}^g$, $z = (z_i) \in \mathbb{C}^g$ and $i = 1, \ldots, g$ with

$$\vartheta \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \left(z + 2\pi i(\Omega^{(g)}.r + s)|\Omega^{(g)}\right) = e^{2\pi i\alpha.s} e^{-2\pi i\beta.r} e^{-i\pi r.\Omega^{(g)}.r - r.z} \vartheta \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \left(z|\Omega^{(g)}\right), \quad (11)$$

$$\vartheta \begin{bmatrix} \alpha + r \\ \beta + s \end{bmatrix} \left(z|\Omega^{(g)}\right) = e^{2\pi i\alpha.s} \vartheta \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \left(z|\Omega^{(g)}\right), \quad (12)$$

for $r, s \in \mathbb{Z}^g$.

There exists a (nonsingular and odd) character $\left[\begin{smallmatrix} \gamma \\ \delta \end{smallmatrix}\right]$ such that[2]

$$\vartheta \begin{bmatrix} \gamma \\ \delta \end{bmatrix} (0|\Omega^{(g)}) = 0, \quad (13)$$

$$\partial_{z_i} \vartheta \begin{bmatrix} \gamma \\ \delta \end{bmatrix} (0|\Omega^{(g)}) \neq 0. \quad (14)$$

Let

$$\zeta^{(g)}(x) = \sum_{i=1}^{g} \partial_{z_i} \vartheta \begin{bmatrix} \gamma \\ \delta \end{bmatrix} (0|\Omega^{(g)}) \nu_i^{(g)}(x), \quad (15)$$

a holomorphic one-form, and let $\zeta^{(g)}(x)^{\frac{1}{2}}$ denote the form of weight $\frac{1}{2}$ on the double cover $\widetilde{\Sigma^{(g)}}$ of $\Sigma^{(g)}$. We also refer to $\zeta(x)^{\frac{1}{2}}$ as a (double-valued) $\frac{1}{2}$-form on Σ. We define the prime form $E(x, y)$ by

$$E^{(g)}(x, y) = \frac{\vartheta \left[\begin{smallmatrix} \gamma \\ \delta \end{smallmatrix}\right] \left(\int_y^x \nu^{(g)} |\Omega^{(g)}\right)}{\zeta^{(g)}(x)^{\frac{1}{2}} \zeta^{(g)}(y)^{\frac{1}{2}}} \sim (x-y) dx^{-\frac{1}{2}} dy^{-\frac{1}{2}}, \quad (16)$$

for $x \sim y$, where $\int_y^x \nu^{(g)} = \left(\int_y^x \nu_i^{(g)}\right) \in \mathbb{C}^g$. $E^{(g)}(x, y) = -E^{(g)}(y, x)$ is a holomorphic differential form of weight $(-\frac{1}{2}, -\frac{1}{2})$ on $\widetilde{\Sigma^{(g)}} \times \widetilde{\Sigma^{(g)}}$. We denote by $K^{(g)}$ the functional part of $E^{(g)}$.

4.2. *Self-sewing of a Riemann surface*

Consider the construction a Riemann surface $\Sigma^{(g+1)}$ formed by self-sewing a handle to a Riemann surface $\Sigma^{(g)}$ of genus g. We review the Yamada formalism[11] in this scheme which we call the ρ-formalism. Consider a Riemann surface $\Sigma^{(g)}$ of genus g and let z_1, z_2 be local coordinates in the neighbourhood of two separated points p_1 and p_2. Consider two disks $|z_a| \leq r_a$, for $r_a > 0$ and $a = 1, 2$. Note that r_1, r_2 must be sufficiently small to ensure that the disks do not intersect. Introduce a complex parameter ρ where $|\rho| \leq r_1 r_2$ and excise the disks

$$\{z_a : |z_a| < |\rho| r_{\bar{a}}^{-1}\} \subset \Sigma^{(g)},$$

to form a twice-punctured surface

$$\widehat{\Sigma}^{(g)} = \Sigma^{(g)} \backslash \bigcup_{a=1,2} \{z_a : |z_a| < |\rho| r_{\bar{a}}^{-1}\}.$$

As before, we use the convention $\bar{1} = 2$, $\bar{2} = 1$. We define annular regions $\mathcal{A}_a \subset \widehat{\Sigma}^{(g)}$ with $\mathcal{A}_a = \{z_a :$

$|\rho|r_{\bar{a}}^{-1} \leq |z_a| \leq r_a\}$ and identify them as a single region $\mathcal{A} = \mathcal{A}_1 \simeq \mathcal{A}_2$ via the sewing relation

$$z_1 z_2 = \rho, \tag{17}$$

to form a compact Riemann surface $\Sigma^{(g+1)} = \widehat{\Sigma}^{(g)} \backslash \{\mathcal{A}_1 \cup \mathcal{A}_2\} \cup \mathcal{A}$ of genus $g+1$. The sewing relation (17) can be considered to be a parameterization of a cylinder connecting the punctured Riemann surface to itself. In the ρ-formalism we define a standard basis of cycles $\{a_1, b_1, \ldots, a_{g+1}, b_{g+1}\}$ on $\Sigma^{(g+1)}$ where the set $\{a_1, b_1, \ldots, a_g, b_g\}$ is the original basis on $\Sigma^{(g)}$. Let $\mathcal{C}_a(z_a) \subset \mathcal{A}_a$ denote a closed anticlockwise contour parameterized by z_a surrounding the puncture at $z_a = 0$. Clearly $\mathcal{C}_2(z_2) \sim -\mathcal{C}_1(z_1)$ on applying the sewing relation (17). We then define the cycle a_{g+1} to be $\mathcal{C}_2(z_2)$ and define the cycle b_{g+1} to be a path chosen in $\widehat{\Sigma}^{(g)}$ between identified points $z_1 = z_0$ and $z_2 = \rho/z_0$ on the sewn surface.

4.3. *The free fermion vertex operator superalgebra (VOSA)*

Consider the rank two free fermion VOSA $V(H, \mathbb{Z} + \frac{1}{2})^{\otimes 2}$ of central charge 1.[3] The weight $\frac{1}{2}$ space $V_{\frac{1}{2}}$ is spanned by ψ^+, ψ^- with vertex operators

$$Y(\psi^{\pm}, z) = \sum_{n \in \mathbb{Z}} \psi^{\pm}(n) z^{-n-1},$$

with modes satisfying the anti-commutation relations

$$[\psi^+(m), \psi^-(n)] = \delta_{m,-n-1}, \tag{18}$$
$$[\psi^+(m), \psi^+(n)] = [\psi^-(m), \psi^-(n)] = 0.$$

The VOSA is generated by $\psi^{\pm}$ with V spanned by Fock vectors of the form

$$\Psi(\mathbf{k}, \mathbf{l}) \equiv \psi^+(-k_1) \ldots \psi^+(-k_s)\, \psi^-(-l_1) \ldots \psi^-(-l_t)\mathbf{1}, \tag{19}$$

for distinct $0 < k_1 < \ldots < k_s$ and $0 < l_1 < \ldots < l_t$. The Virasoro vector

$$\omega^{(g)} = \frac{1}{2}(\psi^+(-2)\psi^-(-1) + \psi^-(-2)\psi^+(-1))\mathbf{1}.$$

generates a Virasoro algebra with central charge $c = 1$ for which the Fock vectors have weight

$$(\Psi(\mathbf{k}, \mathbf{l})) = \sum_{i=1}^{s}\left(k_i - \frac{1}{2}\right) + \sum_{j=1}^{t}\left(l_j - \frac{1}{2}\right). \tag{20}$$

The mode $a(0)$ generates continuous $V_{\mathbb{Z}}$-automorphisms $g = e^{-2\pi i \alpha a(0)}$ for all $\alpha \in \mathbb{C}$. In particular, we define the fermion number involution

$$\sigma = e^{\pi i a(0)},$$

where $\sigma u = (-1)^{p(u)} u$ for u of parity $p(u)$.

4.4. *The invariant form on $\mathcal{M}$*

It is convenient to define[10] for formal parameter z and $\chi \in \mathbb{C}$

$$(-z)^{\chi} = e^{i\pi B \chi} z^{\chi}, \tag{21}$$

where B is an *odd integer* parametrizing the formal branch cut. In[10] we introduced an invariant bilinear form $\langle \cdot, \cdot \rangle$ on $\mathcal{M}$ associated with the Möbius map

$$\begin{pmatrix} 0 & \lambda \\ -e^{i\pi B}\lambda^{-1} & 0 \end{pmatrix} : z \mapsto -\frac{\lambda^2}{e^{i\pi B} z}, \tag{22}$$

for $\lambda \neq 0$. We are particularly interested in the Möbius map $z \mapsto \rho/z$ associated with the sewing condition (17) so that we will choose

$$\lambda = e^{\frac{1}{2} i\pi B} \rho^{\frac{1}{2}}, \tag{23}$$

for the odd integer B of (21). Thus we reformulate the sewing relationship (17) as $z_1 = -\frac{\lambda^2}{z_2}$ so that $dz_1^{\frac{1}{2}} = \xi \rho^{\frac{1}{2}} / z_2\, dz_2^{\frac{1}{2}}$ for $\xi = e^{\frac{1}{2} i\pi B}$.

Define the adjoint vertex operator

$$\mathcal{Y}^{\dagger}(u \otimes e^{\alpha}, z) \tag{24}$$
$$= \mathcal{Y}\left(e^{-z\lambda^{-2}L(1)}\left(\frac{\lambda}{e^{i\pi B} z}\right)^{2L(0)} (u \otimes e^{\alpha}), \frac{\lambda^2}{e^{i\pi B} z}\right).$$

A bilinear form $\langle \cdot, \cdot \rangle$ on $\mathcal{M}$ is said to be invariant if for all $u \otimes e^{\alpha}$, $v \otimes e^{\beta}$, $w \otimes e^{\gamma} \in \mathcal{M}$ we have

$$\langle \mathcal{Y}(u \otimes e^{\alpha}, z)\ (v \otimes e^{\beta}),\ w \otimes e^{\gamma} \rangle$$
$$= e^{-i\pi B \alpha\beta} \langle v \otimes e^{\beta},\ \mathcal{Y}^{\dagger}(u \otimes e^{\alpha}, z)\ w \otimes e^{\gamma} \rangle. \tag{25}$$

(25) reduces to the usual definition for a VOSA when $\alpha, \beta, \gamma \in \mathbb{Z}$.[10] Choosing the normalization $\langle \mathbf{1}, \mathbf{1} \rangle = 1$ then $\langle \cdot, \cdot \rangle$ on $\mathcal{M}$ is symmetric, unique and invertible with[10]

$$\langle u \otimes e^{\alpha}, v \otimes e^{\beta} \rangle = \lambda^{-\alpha^2} \delta_{\alpha,-\beta} \langle u \otimes e^0, v \otimes e^0 \rangle. \tag{26}$$

Thus the dual of the Fock vector $\Psi = \Psi(\mathbf{k}, \mathbf{l})$ is

$$\overline{\Psi}(\mathbf{k}, \mathbf{l}) = (-1)^{st + \lfloor (\Psi) \rfloor} \lambda^{2(\Psi)} \Psi(\mathbf{l}, \mathbf{k}),$$

where $\lfloor x \rfloor$ denotes the integer part of x.[10] Applying (23) and (26) it follows that $\Psi_{\alpha} = \Psi_{\alpha}(\mathbf{k}, \mathbf{l})$ has dual

$$\overline{\Psi}_{\alpha}(\mathbf{k}, \mathbf{l}) = (-1)^{st + \lfloor (\Psi) \rfloor} \lambda^{2(\Psi_{\alpha})} \Psi_{-\alpha}(\mathbf{l}, \mathbf{k}) \tag{27}$$
$$= (-1)^{st + \lfloor (\Psi) \rfloor} e^{i\pi B (\Psi_{\alpha})} \rho^{(\Psi_{\alpha})} \Psi_{-\alpha}(\mathbf{l}, \mathbf{k}).$$

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On Neutrino Oscillations and Flavour Persistence

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Using the quantum-mechanical amplitude analysis of an unstable system in motion, we investigate the persistence of flavours for the neutrino propagation and the emergence of the relativistic effect.

Keywords: Neutrino, quantum mechanics and special relativity

1. Introduction

The notion of particle masses, especially for neutrino, remains an actual problem of particle physics where neutrinos have been a field of wide studies.[1–3] In the SM, neutrinos were presented as verily massless fermions, in contrast with the recent evidences of their flavour oscillations bringing the first sign of the non-zero tiny neutrino masses.[4–6] This makes the propagation behaviour of neutrinos an important toul in investigating various subjects beyond the standard model of particle physics.[7–9]

In relativistic motion, it is well known that the lifetime of an unstable system is increased with respect to the same system at rest by the relativistic factor[a].[10–13] This standard relativistic increase of the lifetime of an unstable system in motion follows from elementary considerations based on the relativistic time dilation and on the physical requirement that the fact that a system has evolved or not does not depend on its velocity with respect to the observer. In case of neutrino propagation, the evolution of the flavour states is subject to the relativistic effect and hence the persistence of flavours.

In what follows we start with a brief review of the quantum mechanical treatment of an unstable flavour neutrino in the rest frame, then we discuss its extension to the relativistic motion and finally we investigate the emergence of the relativistic effect.

2. Neutrino Oscillations, Flavor Persistence

In non-relativistic quantum mechanics, the persisted amplitude that an unstable neutrino flavour $f = e, \mu, \tau$ in motion with a relativistic velocity $\overrightarrow{v}$ described by the state $\left|\upsilon^f_{\overrightarrow{v}}\right\rangle$ is not evolved at the time t is,[10]

$$A^f_{\overrightarrow{v}}(t) = \left\langle \upsilon^f_{\overrightarrow{v}} \right| e^{-iHt} \left| \upsilon^f_{\overrightarrow{v}} \right\rangle, \tag{1}$$

where H is the Hamiltonian operator. Since it is always possible to expand the state $\left|\upsilon^f_{\overrightarrow{v}}\right\rangle$ over the eigenstates of the Hamiltonian, we write,

$$\left|\upsilon^f_{\overrightarrow{v}}\right\rangle = \sum_f C_{p_f,m_f} \left|p_f, m_f\right\rangle, \tag{2}$$

with C_{p_f,m_f} are complex coefficients. The state $\left|\upsilon^f_{\overrightarrow{v}}\right\rangle$ is defined by,

$$\begin{aligned} H\left|p_f, m_f\right\rangle &= E_f \left|p_f, m_f\right\rangle \\ &= \sqrt{p_f^2 + m_f^2}\left|p_f, m_f\right\rangle, \end{aligned} \tag{3}$$

with the normalization,

$$\left\langle p_f, m_f \right| p'_{f'}, m'_{f'} \rangle = \delta_{mm',p_f p_{f'}}. \tag{4}$$

In the rest frame $\overrightarrow{p}_f = \overrightarrow{0}$, the unstable flavour neutrino is described by,

$$\left|\upsilon^f_{\overrightarrow{0}}\right\rangle = \sum_f C_{m_f} \left|m_f\right\rangle, \tag{5}$$

Here $\left|C_{m_f}\right|^2 = \left|C_{p_f=0,m_f}\right|^2 = |\langle m_f \left|\upsilon^f_{\overrightarrow{0}}\right\rangle|^2$ is the distribution of mass (energy in the rest frame) of the flavour neutrino, which determines the persisted amplitude of the flavour f 1,

$$A^{\overrightarrow{0}}_f(t) = |\langle m_f \left|\upsilon^f_{\overrightarrow{0}}\right\rangle|^2 e^{-im_f t}, \tag{6}$$

In a reference frame in which the neutrino is in motion with velocity $\overrightarrow{v}$, we have $\overrightarrow{p}_f = \gamma m \overrightarrow{v}$ and the

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[a] With v is the velocity of the relativistic particle and the velocity of light taken $c = 1$, the corresponding relativistic factor is $\gamma = (1 - v^2)^{1/2}$.

neutrino state is transformed by the unitary $U_{\overrightarrow{v}}$ operator,

$$U_{\overrightarrow{v}} \left| \upsilon^{f}_{\overrightarrow{0}} \right\rangle = \left| \upsilon^{f}_{\overrightarrow{v}} \right\rangle, \qquad (7)$$

such that,

$$U_{\overrightarrow{v}} \left| p_f = 0, m_f \right\rangle = \left| p_f = \gamma m v, m_f \right\rangle. \qquad (8)$$

Therefore, in this reference frame, the system is now described by the state 2 as,

$$\left| \upsilon^{f}_{\overrightarrow{v}} \right\rangle = \sum_{f} \langle p_f, m_f \left| \upsilon^{f}_{\overrightarrow{v}} \right\rangle \left| p_f, m_f \right\rangle. \qquad (9)$$

now, to obtain the correct persisted amplitude of the falvour f, we must take into account both the time and space evolutions of the system within the evolution operator; the relation 1 becomes then,

$$A^{f}_{\overrightarrow{v}}(t) = \left\langle \upsilon^{f}_{\overrightarrow{v}} \right| e^{-i(Ht-Px)} \left| \upsilon^{f}_{\overrightarrow{v}} \right\rangle, \qquad (10)$$

where P is the momentum operator whose the action is,

$$P \left| p_f, m_f \right\rangle = p_f \left| p_f, m_f \right\rangle, \qquad (11)$$

at this level, if we accept that the neutrino coordinate reads $\overrightarrow{x} = \overrightarrow{v} t$ since it is moving with velocity $\overrightarrow{v}$, we finally obtain from the relation 10 the desired persisted amplitude of the flavour f[b],

$$A^{\overrightarrow{v}}_{f}(t) = |\langle m_f \left| \upsilon^{f}_{\overrightarrow{0}} \right\rangle|^2 e^{-i m_f t/\gamma}, \qquad (12)$$

involving the relativistic factor γ. Indeed, confronting with the survival amplitude 6 of the system at rest, we end up with,

$$A^{\overrightarrow{v}}_{f}(t) = A^{\overrightarrow{0}}_{f}(t/\gamma). \qquad (13)$$

where one can see that the relativistic effect influences the neutrino flavour changing amplitude during its propagation.

3. Conclusion

In this work, we have shown that the correct measure of the persisted amplitude of an oscillating flavour during neutrino propagation is subject to relativistic effect. In particular, we have shown that elementary considerations based on relativistic time dilation lead to the standard relativistic relation 13 between the flavour survival amplitudes of neutrino in motion and at rest. This relativistic effect has been obtained with a relativistic approach taking into account, in addition to the time evolution as done usually, the space evolution of the unstable neutrino flavours.

The phenomenon of neutrino oscillations still deserve deep investigations. The connection with other interpretations such as extra-dimensions, quantum gravity, ... could bring more insights in the future.

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[b]Note that using $x = \overrightarrow{v} t$ we obtain the desired quantity as a function of time only as in the case of the rest frame.

BiEntropy of Knots on the Simple Cubic Lattice

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Binary representations of the trefoil and other knots of up to ten crossings in the simple cubic lattice were created. The BiEntropy of each knot was computed using a variety of binary encodings and compared against controls. This showed that binary encoded knots are highly disordered information objects. The BiEntropy of knots on the simple cubic lattice increases slightly as the number of crossings and length of encoding increases. The non-alternating knots of nine and ten crossings are more disordered than equivalent alternating knots.

Keywords: Entropy Knots Cubic Lattice

1. Introduction

We developed our BiEntropy function [4] as a simple method for determining the order and disorder of finite binary strings of arbitrary length. We successfully tested our BiEntropy function in a variety of domains including number theory, cryptography, quantitative finance and random number generation. We have started further work enumerating the algebras of BiEntropy [5] for possible application in bit string physics [16]. Other teams have been able to successfully apply BiEntropy in cryptographic, internet information processing, mobile computing and random number generation domains [3, 10, 11, 14, 20].

We have become aware of Kauffman's work on knots [12] and physics [13] over a period of ten years or more. It was inevitable that we should at some point want to consider whether we could measure the BiEntropy of knots expressed in binary form.

Although the entropy of knots has been addressed from a variety of mathematical and statistical perspectives in the past [2, 7, 17] we believe our work is the first to quantify the entropy of a knot from its precise geometrical configuration using a generically applicable methodology.

This paper is based upon on the work of Scharein et al. [18] who have computed and tabulated the minimal or near minimal forms of knots of up to ten crossings on the simple cubic lattice. Using Scharein et al. as a data source, we convert their NEWSUD knot encodings into various binary equivalents and compute the BiEntropy of the resultant binary encoded knots. In this paper we provide a brief introduction to Shannon's Entropy [19], Binary Derivatives and define the several forms of BiEntropy we have previously used. Since knots are cyclical structures we describe a new variation of BiEntropy, Knot BiEntropy, which is effectively adapted to their geometric form. We describe the NEWSUD and subsequent binary encodings and then present our results in graphical and tabular form.

2. Shannon Entropy, Binary Derivatives & Weighting Methods

2.1. *Shannon Entropy*

Shannon's Entropy of a binary string $s = s_1, \ldots, s_n$ where $P(s_i=1) = p$ (and $0 \log_2 0$ is defined to be 0) is:

$$H(p) = -p \log_2 p - (1-p) \log_2 (1-p) \qquad (1)$$

For perfectly ordered strings which are all 1's or all 0's i.e. $p = 0$ or $p = 1$, $H(p)$ returns 0. Where $p = 0.5$, $H(p)$ returns 1, reflecting maximum variety. However, for a string such as 01010101, where $p = 0.5$, $H(p)$ also returns 1, ignoring completely the periodic nature of the string. BiEntropy seeks to compensate for this omission of consideration of the periodicity (i.e. the order and disorder) of a string by using the binary derivatives of s.

2.2. *Binary Derivatives, Binary Knot Derivatives & Periodicity*

In our previous work on the BiEntropy of linear strings, the first binary derivative of s, $d_1(s)$, is the binary string of length n - 1 formed by XORing adjacent pairs of digits. In the present work on the BiEntropy of knots, which are cyclical structures, the first binary knot derivative of s, $d_1(s)$, is the binary string of length n formed by XORing adjacent pairs of digits *including the last and the first*.

In either case, we refer to the kth derivative of s $d_k(s)$ as the binary derivative of $d_{k-1}(s)$. There are n-1 binary derivatives of s in the linear string version of BiEntropy and infinitely many in the knot version of which we use the first *n-1*. In either case let *p(k)* denote the observed fraction of 1's in $d_k(s)$ where $p(0)$ denotes the fraction of 1's in s.

By calculating the (first) *n-1* binary derivatives of s we can discover the existence of repetitive patterns in binary knots and strings of arbitrary (even) length. If a binary string is *periodic* then $d_{n-1}(s) = 0$. A binary string is *aperiodic* if $d_{n-1}(s) = 1$ or all 1's. A binary string is *nperiodic* if $d_{n-1}(s) = 0$, but it is not periodic.

For example, the first binary knot derivative of 01010101 (with *period*, $P = 2$) is 11111111 ($P = 1$), following which all the higher derivatives are all 0's indicating periodicity. The third derivative of 00010001 ($P = 4$) is 11111111, following which again all the higher derivatives are 0, again indicating periodicity. The seventh binary knot derivative of 10100011 is 11111111 indicating aperiodicity.

The properties of infinite binary strings, their derivatives and the notions of periodicity and eventual periodicity are given more fully elsewhere [8, 15]. We rely here solely upon the binary derivatives of a *finite* string or *finite cyclical* knot to resolve the issue of the degree of periodicity (and hence the degree of order and disorder) within the string or knot.

2.3. *Weighting Methods*

In order to obtain a single value that represents the order and disorder of a binary string or knot we need to combine the Shannon Entropies of the derivatives in one or more ways. One obvious method is to combine them using the powers of two such that the last used binary derivative has the most weight (as the last derivative gives a final determination of periodicity and aperiodicity) and also in order that the weights of each derivative are separated from each other. We abbreviate this method "BiEn" noting its meaning in the French language. Another method is to use a logarithmic weighting such that the order and disorder discovered in a binary string of *any* length through the enumeration of *all* its binary derivatives is fully taken into account, though in an exponentially diminishing fashion. We abbreviate this method Tres Bien or TBiEn for short. Linear weights would of course be LBiEn ("Les BiEn") and zero weights would be PBiEn ("Pas BiEn").

3. BiEntropy

BiEntropy, or BiEn for short, is a weighted average of the Shannon binary entropies of the string and the first n-2 binary derivatives of the string using a simple power law. This version of BiEntropy is suitable for shorter binary strings where $n \leq 32$ approximately as the weights of the first derivatives tend rapidly to zero.

$\mathrm{BiEn}(s) = W * D$ where $W = \left(1/\left(2^{n-1}-1\right)\right)$ and

$$D = \left(\sum_{k=0}^{n-2}\left(\left(-p(k)\log_2 p(k) - \left(1-p(k)\right)\log_2\left(1-p(k)\right)\right)\right)2^k\right).$$

The derivative d_{n-1} is not used as it makes no contribution to the total entropy. The highest weight is assigned to the highest (last used) derivative d_{n-2}.

If the higher derivatives of an arbitrarily long binary string are periodic, then the whole sequence exhibits periodicity. For strings where the latter derivatives are not periodic, or for all strings in any case, we can use a second version of BiEntropy, which uses a Logarithmic weighting, to evaluate the complete set of a long series of binary derivatives.

$\mathrm{Tres\ BiEn}(s) = W * D$ where $W = \left(1/\sum_{k=0}^{n-2}\log 2(k+2)\right)$ and $D = \left(\sum_{k=0}^{n-2}\left(-p(k)\log_2 p(k) - \left(1-p(k)\log_2\left(1-p(k)\right)\right)\right)\log_2(k+2)\right).$

The logarithmic weighting (TBiEn for short) again gives greater weight to the higher derivatives.

					KTBiEn4	0	0	0.22	0.22	0.56	0.56	0.56	0.56	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
					N	0	15	5	10	3	6	9	12	1	2	4	7	8	11	13	14
Knot TBiEn of an 8-bit sequence					8	0	1	0	1	0	0	1	1	0	0	0	0	1	1	1	1
					4	0	1	1	0	0	1	0	1	0	0	1	1	0	0	1	1
					2	0	1	0	1	1	1	0	0	0	1	0	1	0	1	0	1
					1	0	1	1	0	1	0	1	0	1	0	0	1	0	1	1	0
KTBiEn4		8	4	2	1																
0	0	0	0	0	0	0.00	0.43	0.58	0.58	0.74	0.74	0.76	0.74	0.89	0.89	0.89	0.95	0.89	0.97	0.97	0.95
0	15	1	1	1	1	0.43	0.00	0.58	0.58	0.74	0.76	0.74	0.74	0.95	0.97	0.97	0.89	0.95	0.89	0.89	0.89
0.22	5	0	1	0	1	0.58	0.58	0.07	0.43	0.76	0.76	0.76	0.78	0.92	0.95	0.92	0.92	0.97	0.95	0.92	0.97
0.22	10	1	0	1	0	0.58	0.58	0.43	0.07	0.78	0.76	0.76	0.76	0.97	0.92	0.95	0.97	0.92	0.92	0.95	0.92
0.56	3	0	0	1	1	0.74	0.74	0.76	0.78	0.17	0.60	0.60	0.43	0.94	0.94	0.97	0.94	0.95	0.94	0.97	0.95
0.56	6	0	1	1	0	0.74	0.76	0.76	0.76	0.60	0.17	0.43	0.60	0.97	0.94	0.94	0.94	0.97	0.95	0.95	0.94
0.56	9	1	0	0	1	0.76	0.74	0.76	0.76	0.60	0.43	0.17	0.60	0.94	0.95	0.95	0.97	0.94	0.94	0.94	0.97
0.56	12	1	1	0	0	0.74	0.74	0.78	0.76	0.43	0.60	0.60	0.17	0.95	0.97	0.94	0.95	0.94	0.97	0.94	0.94
0.96	1	0	0	0	1	0.89	0.95	0.92	0.97	0.94	0.97	0.94	0.95	0.29	0.76	0.58	0.78	0.74	0.60	0.78	0.43
0.96	2	0	0	1	0	0.89	0.97	0.95	0.92	0.94	0.94	0.95	0.97	0.76	0.29	0.76	0.60	0.58	0.76	0.43	0.78
0.96	4	0	1	0	0	0.89	0.97	0.92	0.95	0.97	0.94	0.95	0.94	0.58	0.76	0.29	0.78	0.76	0.43	0.76	0.60
0.96	7	0	1	1	1	0.95	0.89	0.92	0.97	0.94	0.94	0.97	0.95	0.78	0.60	0.78	0.29	0.43	0.76	0.58	0.74
0.96	8	1	0	0	0	0.89	0.95	0.97	0.92	0.95	0.97	0.94	0.94	0.74	0.58	0.76	0.43	0.29	0.78	0.60	0.78
0.96	11	1	0	1	1	0.97	0.89	0.95	0.92	0.94	0.95	0.94	0.97	0.60	0.76	0.43	0.76	0.78	0.29	0.76	0.58
0.96	13	1	1	0	1	0.97	0.89	0.92	0.95	0.97	0.95	0.94	0.94	0.78	0.43	0.76	0.58	0.60	0.76	0.29	0.76
0.96	14	1	1	1	0	0.95	0.89	0.97	0.92	0.95	0.94	0.97	0.94	0.43	0.78	0.60	0.74	0.78	0.58	0.76	0.29

Table 1. Logarithmic Knot BiEntropy (KTBiEn) of the 4 and 8-bit strings.

We use the Logarithmic Knot BiEntropy (KTBiEn) exclusively in the analytics of this paper as the minimum length of a knot encoded in binary is 24 * 3 = 72 bits. We show in Table 1 KTBiEn for the 4 and 8 Bit strings.

The BiEntropy of the perfectly ordered strings 00 and 11 is zero. The BiEntropy of the perfectly disordered strings 10 and 01 is one. For finite strings of arbitrary length greater than two, the BiEntropy is greater than or equal to zero and less than one.

4. Minimal Knots in the Simple Cubic Lattice

Diao [6] proved some time ago that the minimum number of steps needed to construct a knot on the simple cubic lattice was 24, and that the only knot available at this length is 3_1, the trefoil knot. Minimum lengths for 4_1 and 5_1 of 30 and 34 steps respectively have been recently proven as outlined in Scharein et al. Except for these three proofs, determination of the minimum number of steps for knots on the cubic lattice is by heuristic methods and is therefore incomplete and somewhat uncertain.

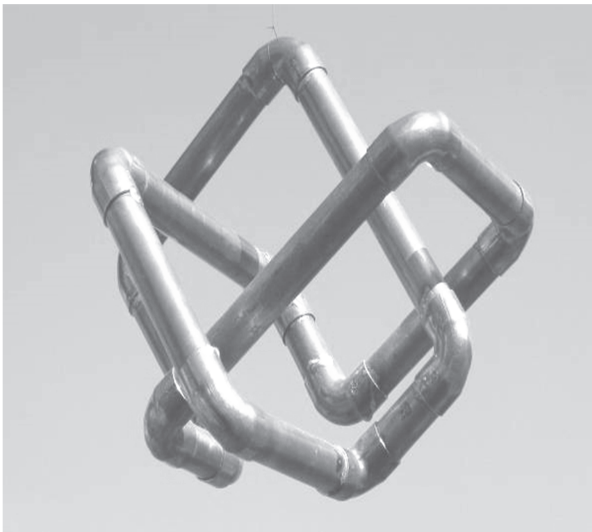

Figure 1. The Trefoil knot 3_1 of canonical form 2-50 in the simple cubic lattice. The start point is at the bottom left hand corner of the photograph, with DDD running upwards and to the right.

Scharein et al. correct and complete earlier work to show by an exhaustive method that there are 75 minimum canonical forms of the trefoil knot (in three distinct groups) and tabulate each form in their Table 6 using the NEWSUD encoding. A NEWSUD string is a sequence of letters from the set {N, E, W, S, U, D} representing edge directions in the cubic lattice of North, East, West, South, Up, and Down. For example: DDDEEUUSWWWNNEEDSSSUUNNW is the trefoil 2-50 depicted in Figure 1 below. Scharein et al. document the geometric realisation of each of the 75 canonical forms of the trefoil knot using stick and ball diagrams with the start point denoted by a small dot at the top.

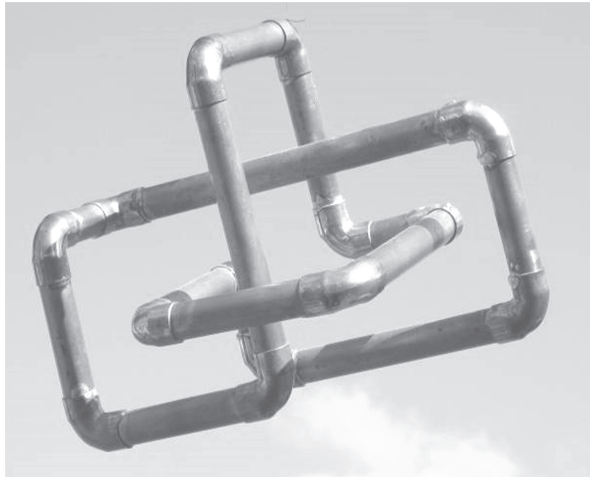

Figure 2. A Figure of Eight knot (4_1) in the simple cubic lattice.

Scharein et al. then give in their Table 7 a single example of a definite, potential or probable minimum encoding of each knot from 3_1 up to 10_{165}. Including the Unknot, the Granny and the Square knot by way of comparison. Other tables give the numbers of canonical forms of these more complex knots. Tables 6 & 7 of Scharein et al. is the raw data for the rest of this paper.

We show in Figures 1 & 2 our first physical implementation of two knots in the simple cubic lattice.

5. Binary Encodings of Trefoils, Knots & Controls

The simplest and most obvious binary encoding for the six member {N, E, W, S, U, D} set is a three bit encoding such as {000, 001, 010, 011, 100, 101}. An encoding this simple would be naïve as a number of issues need to be taken into consideration which we list in the following sections.

5.1. *Selection of Encoding Bits*

There are 8! / 2! = 20,160 ways of allocating a three-bit binary number to each member of the NEWSUD set. Each encoding would result in a differing BiEntropy for each knot as the different sub patterns created in the resultant binary string representing the knot would be different. For example, the Unknot DEUW could be encoded in a three-bit encoding as 000111010101 or 101010110011 which have differing BiEntropies. It is necessary therefore to compute the BiEntropy on a large sample of encodings using a monte carlo method in order to obtain statistically repeatable and reliable results.

5.2. *Length of Encoding*

The existence and influence of sub patterns in the binary string representing a knot will vary according to the length of the encoding. For example, in an 8 bit encoding the Unknot DEUW could be represented as 00000000111111110000000111111110 which is relatively well ordered with a low BiEntropy or as 01110101110001101101001001001001 which is relatively disordered with a high BiEntropy. With a longer binary encoding, the three bits corresponding to the direction instructions implied in the NEWSUD encoding become lost in the noise. Note that there are 256! / 250! = 2.65E14 ways of selecting an 8 bit NEWSUD encoding. After some experimentation with 3 and 4 bit encodings we settled on an 8 bit encoding.

In our analysis, we used two different encodings ENCODING_A and ENCODING_B. Each encoding consisted of 256 different randomly selected 8 bit encodings from the myriad available. For example, the first four encodings (out of 256) of ENCODING_A were:

N	E	W	S	U	D
44	82	201	21	245	214
32	231	121	87	252	179
6	28	80	33	137	77
111	52	163	120	238	147

Table 2. First four encodings (out of 256) of the NEWSUD encodings of ENCODING_A.

N	E	W	S	U	D
84	41	102	101	67	222
43	107	20	118	66	113
227	111	65	189	142	99
145	108	159	3	248	240

Table 3. First four encodings (out of 256) of the NEWSUD encodings of ENCODING_B.

Where the 44 of the N in the first row of ENCODING_A translates to 00101100 and the 3 of the S in the fourth row of ENCODING_B translates to 00000011.

5.3. *Start Points*

We initially ran this analysis using the linear string version of BiEntropy which required us to randomly select a start point within the NEWSUD encoding. Following initial peer review, we changed the design of BiEntropy to accommodate the cyclical or hoop like nature of knots [1]. The Knot version of BiEntropy detects the identity of the eight strings 00000001, 00000010, … 10000000 (and others similar) and computes identical BiEntropies. Thus, we were able to use the NEWSUD strings exactly as given in Scharein et al for the trefoils and knots without needing to rotate them around an arbitrarily selected start point. This second analysis was a decimal order of magnitude faster. The results were similar, providing useful confirmation that our implementation and computations were secure.

5.4. *Trefoils and Knots*

To repeat for clarity, we worked with two data sets. Table 6 of Scharein et al documents the NEWSUD encodings for the 75 Canonical forms of the Trefoil knot. These results are denoted TREF_N. Table 7 of Scharein et al gives the NEWSUD encoding for a single suspected (or actual in three cases) minimal form of all the knots from 3_1 through to 10_{165}, which we denote SOME_N. Note that there was a mistake in 3_1 of Table 7 as this was only 23 characters long. The initial "D" had been omitted due to a typographical error, which we corrected. We excluded the unknot, granny and square knots of Table 7 from our analysis giving 249 knots in total.

5.5. *Randomised Trefoil and Knot Controls*

We computed the distribution of NEWSUD instructions within TREF_N and created two randomised controls TREF_A and TREF_B with similar NEWSUD character distributions. Each string in TREF_A and TREF_B is 24 characters long.

We computed the distribution of NEWSUD instructions within SOME_N and created two randomised controls SOME_A and SOME_B with similar NEWSUD character distributions. Each string in SOME_A and SOME_B varied from 24 to 64 characters in length depending upon the knot length in the equivalent row of SOME_N.

The resulting four NEWSUD controls were pure random and were not self avoiding polygons. There is a very small probability that a control might contain a knot.

6. Computations

After a little experimentation to determine the sensitivities of various encoding lengths and so on we performed two similar computations, firstly on the

Trefoils and secondly on the knots of up to 10 crossings. These took ten minutes and one hour respectively on a 4GHz single core Pentium.

We examined every trefoil (TREF_N) and every knot (SOME_N) using each of the 256 encodings in ENCODING_A and ENCODING_B. For each trefoil, knot and encoding we created a binary string *s* and computed the logarithmic knot BiEntropy (KTBiEn(*s*)). We then repeated the exercise with the two pairs of control data TREF_A, TREF_B, SOME_A and SOME_B using both ENCODING_A and ENCODING_B. Note that the 8-bit binary encoding of a trefoil knot encoded in NEWSUD is 24 * 8 = 192 bits and that the 8-bit encoding of a more complex knot of up to 10 crossings is 64 * 8 = 512 bits.

7. Results

The first and most important result is that ENCODING_A and ENCODING_B, despite being completely different from each other having been sampled randomly from a large space (***O***E14), gave almost identical results.

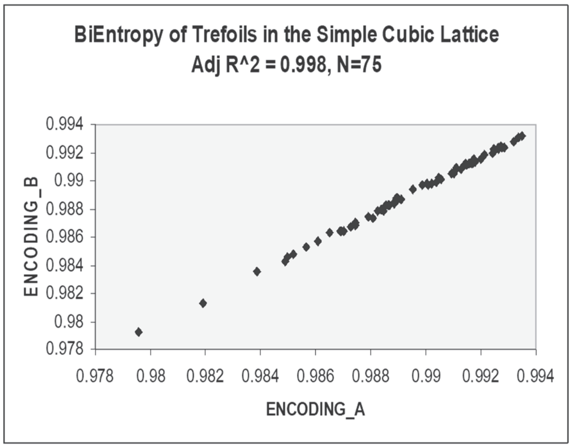

Figure 3. BiEntropy of Trefoil Knots (TREF_N) in the Simple Cubic Lattice.

These results were almost exactly repeated in the controls TREF_A, TREF_B, SOME_A, SOME_B using the two encodings. This suggests that we can reliably use sampled encodings in this and other related experimental domains in order to obtain reliable estimates of BiEntropy. We have combined the results from ENCODING_A and ENCODING_B in the presentation of our remaining results.

As expected, the BiEntropy of knots on the simple cubic lattice was high (> 0.979) indicating that these knots are relatively disordered binary objects. The knot with the lowest BiEntropy is the 2-50 variation of the trefoil which has a clear 3 way symmetry about a central axis, which might be determined from the photograph in Figure 1. We show the small variance in BiEntropy among the Trefoils in Figure 5.

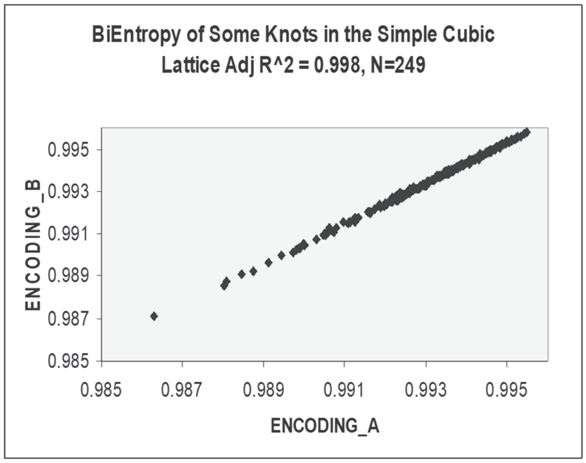

Figure 4. BiEntropy of Knots up to 10 Crossings (SOME_N) in the Simple Cubic Lattice.

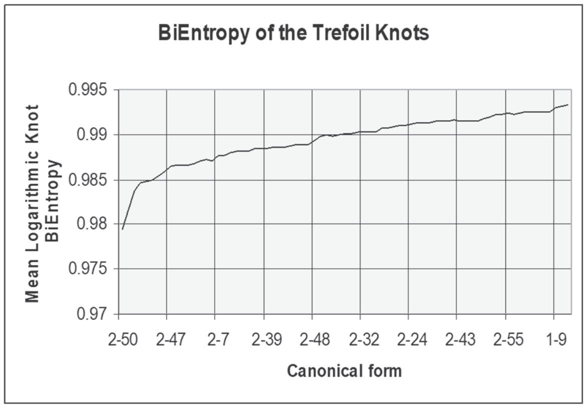

Figure 5. BiEntropy of the Trefoil Knots.

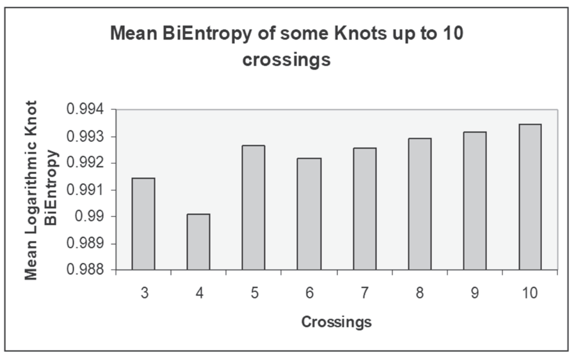

Figure 6. Mean BiEntropy of some knots up to 10 crossings.

Figures 3 and 4 show that whereas there is reasonable certainty as to which trefoils and knots have the lowest BiEntropy, there is sample size based uncertainty for the more complex knots. With larger sample sizes, it may be possible to exactly sequence the knots in terms of their BiEntropy. The 8-bit encoding space is probably large enough to permit refined estimates of BiEntropy for the more complex knots, but at a significant computational cost.

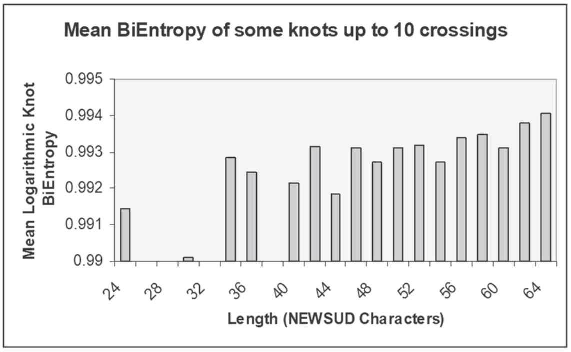

Figure 7. Mean BiEntropy of some knots up to 10 crossings by NEWSUD encoding length.

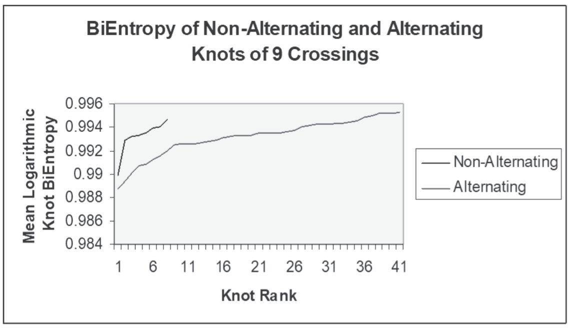

Figure 8. The BiEntropy of Non-Alternating and Alternating knots of 9 crossings.

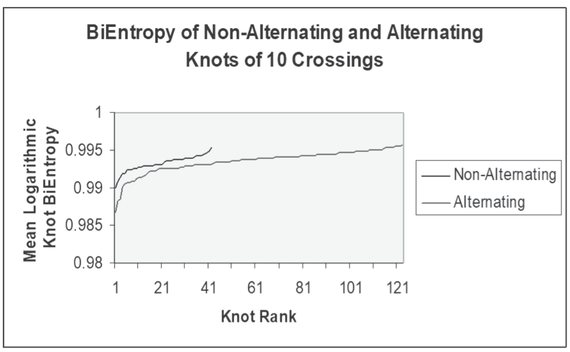

Figure 9. The BiEntropy of Non-Alternating and Alternating knots of 10 crossings

As expected, the mean BiEntropy of more complex knots with greater numbers of crossings slightly increased which we show in Figure 6, noting that there are single or few data points for the knots with lower crossings. As also expected, the BiEntropy of knots increased as the length of their NEWSUD encoding increased (Figure 7) noting again the low number of data points for the lower knots.

There were some small but significant ($p < 0.01$) differences between the BiEntropies of the non-alternating (or quasi-alternating [Jablan, 2014]) versus alternating knots of differing lengths and crossings, which we show in Figures 8 and 9.

8. Summary

We have adapted our BiEntropy measure for the cyclical world of knots encoded in binary on the simple cubic lattice. We have measured the Logarithmic Knot BiEntropy of the 75 canonical forms of the Trefoil knot and of a single instance of each other knot up to and including ten crossings. It is clear that binary encoded knots are highly disordered binary strings. Despite the high level of disorder, we were able to discriminate between the average entropy of knots of increasing crossings and length of encoding, between some alternating and non-alternating knots and between the trefoils of various canonical forms.

We have shown that we can use a monte carlo method to encode knots in binary and produce statistically consistent results with relatively low computational effort.

This study is constrained by our use of the limited output from a single prior experiment, but hopefully should point the way for wider studies into the entropy – the order and disorder – of binary encoded knots and other information structures.

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Interactions, Symmetry, Lorentz Invariance: The Struggle of Physicists About These Concepts

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As well known, a still large number of physicists understands the whole corpus of theoretical physics as a complex system which, after all, is based on three primeval and fundamental pillars, constituted by the concepts of Interaction, Symmetry, and (excluding Cosmology) Lorentz Invariance. In the last century some new ideas, like the ones arising from quantum theory or disordered systems, seemed to introduce deep changes into this sound conceptual picture. In this contribution, I will try to evidence that this state of affairs is actually changing, on the basis of new suggestions introduced by researchers such as Jean-Pierre Vigier, who put forward strong arguments supporting the need for a radical change of the conceptual structure of physics. These arguments came mainly from the applications of physics to highly complex systems. This situation gave rise to a sort of struggle between the holders of different conceptions and it is becoming clearer that the old concepts of interaction, symmetry, and Lorentz invariance will survive only when applied to enough simple systems. Among the many evidences of the aforementioned change I will limit myself to discuss two cases: 1) the introduction, within classical field theories, of new theoretical proposals which generalize in a somewhat 'heretical' way the traditional constructs of interactions and symmetry used in mathematical physics; 2) the need for using, within mathematical models of complex system behaviours (like, for instance biological neural networks), methods for assessing the validity of the introduced models based not only on general mathematical arguments, but also on computer simulations of model dynamics.

Keywords: Loss of Lorentz invariance, Inadequacy of particle concept in QFT, QM application to cognitive phenomena

1. The Problem of Complexity

As well known, the accrual Theoretical Physics is similar to a solid building grounded on three main pillars: the concepts of *interaction*, of *symmetry*, and of *Lorentz invariance*. They are arisen from a long history, lasting from the Newton times and reaching its apex in the Fifties (some historical references can be [1, 2]). In the last years, however, the situation appears to change, owing mainly to the need for coping with *complexity*, characterizing many actual researches in Theoretical Physics. In this regard we remark that the effects of complexity come from many different sources: the complexity of *phenomena* under study, the complexity of *measurement procedures*, the complexity of *research goals*, and the complexity of the introduced *theories*.

Starting from the complexity of phenomena, even a superficial observation shows that the actual Physics deals with a number of phenomena far wider than in the Fifties. To make an example, the huge development of Astroparticle physics, related to the enormous growth of available energies (like in LHC), discloses in an endless way an entire world of new (and even more complex) phenomena, which were impossible to study by resorting to the first accelerators of the Fifties.

But the field of Astroparticle physics is only the most popular tip of the iceberg including all new achievements of Physics in a number of different domains. Another example is given by Condensed Matter Physics, ranging from Bose-Einstein condensation to Nanotechnologies, as well as from Fractional Quantum Hall effect to high temperature superconductors, opening the door to a look on a world of phenomena evidencing an extreme complexity. Besides, we cannot forget the contributions of Econophysics, dealing with the problem of emergence within the domain complex social phenomena, searching for a generalization of the traditional theory of phase transitions. And, last but not least, the recent progress of Biophysics widened in an unexpected way the traditional physical research domain, investigating the relationships between microscopic and macroscopic features of biological matter, from DNA molecular level to brain neurons, leading then to take into consideration mental processes and consciousness phenomena.

As regards the complexity of measurement procedures we observed, mainly in the last years, a growth of "unusual" situations, due to the fact that the increased complexity of phenomena and of technological tools began to cast some doubts on the traditional methodological rules applied by physicists to measurement procedures. A first example is given by the fact that actually most measure operations (related to particles or signals) are charged to a computer rather than to a human being. While in most cases the researchers introduce suitable criteria granting the reliability of the obtained results, the latter remains a not completely solved problem. In any case the experimental output becomes the outcome of a complex mental construction made by the scientists, far more complex than the one used by their predecessors of a century ago. But the complexity of this problem increases if we deal with measurement procedures operating on biological matter. Namely, in these cases the measure operation itself produces, in principle, an unavoidable perturbation of the measured entity. As it is well known, this problem is ubiquitous when we deal with quantum systems. However, also in this context, despite the efforts of quantum information theorists, the problem cannot be considered as fully solved. Finally, when dealing with social or psychological domains, we have a far serious problem, consisting in the fact that many concepts, used to formulate the research hypotheses and to design the experimental procedures, are themselves badly defined. Of course, notwithstanding this fact, we have in these domains a lot of data (collected, for instance, through tests) which are often used to support some conclusions. But how to assess their significance?

Focusing now our attention on the complexity of the research goals, we remark that the actual research activities – at least of theoretical physicists – are characterized by a plurality of different goals. This plurality, however, implies a plurality of conceptual tools, and therefore of different competences. Such a circumstance entails that the main pillars on which is grounded the building of theoretical physics are less solid than suspected, mainly because the goals of a number of theoretical physicists do not depend on this solidity. A situation of this kind implies, in an obvious way, a high complexity of the world of possible theories and theoretical frameworks. Even if we still lack a precise definition of complexity (see, for instance, [3, 4]), it is evident, by looking at this world, that, whatever can be the adopted definition of complexity, not only this world is highly complex, but it is also impossible that a single model belonging to this world can capture all features of a complex system existing within our environment (ourselves included).

Now, once quoted the possible causes for the actual complexity of theoretical physics, in the next sections we will spend some words about specific problems whose presence can undermine the stability of the pillars mentioned above.

2. The Possible Loss of Lorentz Invariance

Since many times, the Lorentz invariance (LI) constitutes a sort of "dogma" of the actual Theoretical Physics, both classical and quantum. In fact, LI acts as the fundamental hypothesis to be used in most theorems and concrete applications. There are, of course, many domains of theoretical physics where LI is not so important or even neglected, typically when one deals with low velocities or low energies, such as in nonrelativistic Quantum Mechanics.

Besides these well known cases, it is to be remembered that a fundamental pillar of theoretical physics – the General Theory of Relativity (GTR) – yet allows in principle a *Global* loss of LI. Unfortunately, the latter is taken into consideration only by physicists interested in cosmology or, at most, in large-scale phenomena. In this regard, the main problem for GTR stems from the fact that it has been originally designed to account for the large-scale gravitational phenomena. Thus, the main field equations of GTR, that is:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{1}$$

include, in the left-hand side, only terms describing the large-scale geometrical structure of the Universe, while the right-hand side is a tensor describing matter behaviour whose form is, necessarily, derived only from our *Local* knowledge about Special Relativistic dynamics. This conceptual anomaly prevents, so far, from the use of GTR as the main tool for building a unified theory of all physical fields.

Anyhow, the majority of physicist is interested in the loss of *Local* rather than *Global* LI. A number of models endowed with such a feature have been built by resorting to effective field theories, in turn derived from more general approaches. An example is given by the celebrated non-relativistic model of superfluid Helium-4 film [5], described by the Lagrangian:

$$\mathcal{L}_{4Me} = i\varphi^{\dagger}\partial_i\varphi - \left(1/2m\right)\partial_i\varphi^{\dagger}\partial_i\varphi +$$

$$\mu\varphi^{\dagger}\varphi-\kappa\left(\varphi^{\dagger}\varphi\right), \tag{2}$$

or the explicitly violating LI electrodynamics model of Bailey and Kostelecky [6] based on the Lagrangian:

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}\left(k_F\right)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu}$$
$$+\frac{1}{2}\left(k_{AF}\right)^{\kappa}\varepsilon_{\kappa\lambda\mu\nu}A^{\lambda}F^{\mu\nu}-j^{\varpi}A_{\mu}. \tag{3}$$

where

A_{μ} = electromagnetic potential

$F_{\mu\nu}\equiv\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}$ = field strength

j^{μ} = current source

$\left(k_F\right)_{\kappa\lambda\mu\nu}$, $\left(k_{AF}\right)^{k}$ = coefficients controlling Lorentz violation.

Among the theories predicting a violation of Lorentz invariance one of most interesting is the *Deformed Special Relativity* (DSR), introduced by Cardone & Mignani (see [7-8]), which is based on a deformation of the usual metric of Minkowski space-time having the form:

$$ds^2=b_0^2(E)c^2(dt)^2-b_1^2(E)(dx)^2+$$
$$-b_2^2(E)(dy)^2-b_3^2(E)(dz)^2 \tag{4}$$

where the symbol E denotes the energy of the process under consideration.

Without entering into a discussion about the experimental validation of DSR two important remarks are in order:

1) the metric coefficients depend not only on the energy scale but also on the kind of interaction taken into consideration;

2) the geometrical structure of space-time in DSR is very rich (see also [9-10]), including not only a curvature, but also a torsion; these features depend only partially on the energy but are also partly due to a sort of inner gauge field generated by the structure itself.

The previous circumstances could be exploited to build geometrical models of unified field theories, pursuing a line of thought initiated by the well-known Kaluza-Klein model.

3. The Inadequacy of Particle Concept in Quantum Field Theory

Whatever can be the opinion about the relevance of quantum theory, the Quantum Field Theory (QFT) still represents the most important implementation of the 'Maxwellian dream' of primacy of the field concept in physics. Born with the initial aim of relating quantum theory to Special Relativity, QFT undergo a tumultuous development, giving rise to a large number of different models. As a consequence, actually QFT offers the best theoretical tools for dealing with problems in particle physics, cosmology, and condensed matter physics.

As widely known, since its beginning QFT has been tailored to fulfil the needs of growing particle physics. Namely its formalism can be expressed in terms of suitable *creation* and *annihilation* operators satisfying canonical commutation (or anticommutation) relations. This entails that, once introduced a suitable *vacuum* state, the action on it of a creation operator gives rise to a new state containing a 'field quantum', normally identified with a 'particle'. However, starting from the Fifties, physicists proved some mathematical theorems about QFT which make this identification problematic. Here, without entering into useless mathematical details, we will limit ourselves to mention the intuitive content of the most important ones among these theorems.

We start with the *Reeh-Schlieder*theorem: it is impossible to count the number of field quanta within a specific bounded space-time region. We continue with a theorem related to the so-called *Unruh effect*: it is impossible to define in a unique way the total number of field quanta, as this number depends on the adopted representation of canonical commutation relations. We end our list by quoting the *Haag's* theorem: the concept of total number of field quanta cannot be defined in presence of interacting fields.

These theorems clearly evidence that the features of field quanta are very different from the ones traditionally attributed to the concept of 'particle'. What to do in such a situation? A possible approach would be the one considering the concept of 'particle' as a construct endowed only with a statistical meaning. But, how to use statistics if the count operations are devoid of any meaning? Another possibility could consist in renouncing to the concept of 'particle' and working only with the concept of 'field'. But it is very difficult to implement even this strategy (see, e.g., [11]). Namely in QFT the fields are not numbers or observables but are rather treated as *potentialities* distributed on the whole space-time. And it is very easy to acknowledge that most methods used in QFT are essentially devoted to evidence the correlations between these potentialities. In order to transform these distributions into numbers we must necessarily resort to some sort of averaging operations, but these latter just prevent from understanding all localized aspects. As evident, many conceptual aspects of QFT are still unsolved and this situation calls for a deep reformulation of the old QFT.

4. The ‘Unreasonable’ Effectiveness of Quantum Mechanics in Accounting for Macroscopic Cognitive Phenomena

In recent times, many psychologists and psychophysiologists evidenced a number of macroscopic empirical phenomena concerning human subjects mainly consisting in non-local correlations between distant (in space or in time or in both) events which could not be explained by resorting to models based on classical physics (for a recent review of the available data see, e.g., [12]). This circumstance suggested the need for the introduction of a suitable generalization of traditional quantum mechanics, useful to account for these new phenomena. Such a generalization has been classed ‘Generalized Quantum Theory’ (see, among the others, [13-16]) or ‘Quantum Probability’ (see, for instance, [17]). So far such a generalization is somewhat limited, lacking some aspects of traditional quantum mechanics, such as the existence of a commutator or – what is equivalent – of a suitable value of Planck’s constant. It is, however, possible to describe the general features of the evolution of a stochastic quantum-like system in which the time evolution of probabilities is computed not by resorting to Brownian motion formulae but through the quantum law of state evolution based on the wave function. More precisely, if we denote by $\psi(t)$ the value of the wave function at instant t, the associated probability of state will be given by the square of the modulus of the wave function $|\psi|^2$. The value of the wave function at the next instant $t+\Delta t$ is given by the quantum law from Schrödinger equation):

$$\psi(t+\Delta t)=\psi(t)\cdot e^{-\frac{i}{\hbar}\cdot H\cdot\Delta t} \tag{5}$$

Here the symbol H denotes the Hamiltonian operator of the system under consideration while $\hbar$ is the (hypothetical) value of Planck’s constant.

While remarking that these rules allow, in principle, a direct discrete simulation on a computer of the evolution of a quantum-like system, we must take into account that this is possible only if we know in advance the form of H and the value of $\hbar$. In order to gather this knowledge many different strategies can be used, depending on the kind of system we are considering. For instance, in many cases in which people is interested only in the general features of the evolution under study, the most convenient choice is to put $\hbar=1$. Such a choice is, however, not so popular among the people assuming from the start that the system under consideration can be represented as a stochastic system performing some sort of Brownian motion driven by the presence of a suitable noise. This assumption first allows to hold that the quantum-like nature of the evolution depends not so much on the stochastic nature of system dynamics, but rather on the rule adopted in computing the probabilities. In the second place, this assumption itself allows to find the correct choices to be adopted for $\hbar$ and H. More precisely, the contributions made in [18, 19] show that the appropriate value for $\hbar$ is given by the amplitude of the two-point function characterizing, the noise, provided the latter is Gaussian with zero mean.

As concerns the form of the Hamiltonian, the methods of the theory of Markov processes allow to derive it starting from the initial hypotheses about the probabilities involved in the process of stochastic evolution of the system under consideration (see, e.g., the references [20, 21]). It is, however, possible to shorten this deductive process in the case of a small noise amplitude, by resorting to the so-called ‘Fokker-Planck Hamiltonian’, obtained as a particular case of the more general Fokker-Planck equation ruling the evolution of probabilities in Markov processes (for further technical details see, e.g., [22]).

These tools allow to build software programs designed to perform numerical simulations of the time evolution of quantum-like systems within the framework of Generalized Quantum Theory. Among these simulations, the most interesting are the ones concerning the *quantum neural networks*. In the simplest cases these latter consist in networks of excitable elements (represented by computational neurons), located within a suitable discrete lattice and interconnected by *links* vehiculating the output activation of each neuron to the inputs of other neurons. Often, in order to simplify the study of the network, the links as well as the signals they transmit are treated as *classical* objects while the only aspect which is of quantum nature is the law of inner evolution of each single neuron from the time t to the time $t+\Delta t$, once its input has been fixed by the signals coming from outside. In this way, we do not deal properly with a quantum system but rather with a *classical-quantum* system. Of course, the classical counterpart of such a system is constituted by a system having the same structure and the same links, but in which the inner evolution of each neuron is ruled by a traditional Markovian-like stochastic law.

Without discussing the lot of results obtained from computer simulations of these or similar systems (see, e.g., [23-24]) we limit ourselves to remark that, in general, they evidence a consistent difference between the cases in which a quantum component is present and the ones in which it is absent. Moreover, in most simulations the presence of a quantum component gives rise to behaviours which are stable against the occurrence of external influences (producing, on the contrary, strong effects on the classical counterparts). It could be conjectured that these observations can be explained by hypothesizing that the presence of a quantum component could give rise to non-local effects, absent in the classical case. Even if this hypothesis would be incorrect, it is indubitable that the picture of a physics based on the loss of locality principle and on the study of wizard-like phenomena would have been intolerable for a theoretical physicist of the post-second world war age, devoted to the search for the sacred pillars of elementary interactions theory!

However, happily or unfortunately, this picture is becoming more and more concrete.

5. The Existence of Both Bottom-Up and Top-Down Types of Causation

This circumstance concerns all systems described by using (at least) two different levels (or scales): *microscopic* and *macroscopic*. In this regard a long-lasting philosophical tradition, almost universally adopted in science, tells us that the macroscopic description is (when possible) derived from the microscopic one through a bottom-up form of causation (the only one allowable). The large majority of scientific explanations is based on this strategy and, while it sounds as obvious that the observed behaviours of chemical substances is explained by the laws ruling the elementary particles of matter, nobody would explain the behaviours of elementary particles by starting from the laws of macroscopic chemistry.

In more recent times, however, the concept of top-down causation has been reconsidered, owing to the work of statistical physicists, which introduced suitable tools to detect its occurrence (see, for reviews, [25, 26]). These tools have been designed mostly to work on time series of experimental data. They helped to measure the amount of top-down causation in a number of phenomena concerning the relationships between microscopic and macroscopic aspects (as, for instance, the interaction between a whole neuronal cell and the microtubules present within it). Such a circumstance is bound to change the actual structure of science.

6. The Intrinsic Unpredictability of Complex Systems Behaviours

Actually, the study of complex systems constitutes a new frontier for physics. Without introducing a formal definition of complexity (see, e.g., [27]) we can roughly say that a complex system is characterized by the fact that it shows *emergent* behaviours. Their occurrence (whose detection is a subjective and observer-dependent fact) typically takes place in self-adaptive systems living within a dynamic environment. The latter circumstance implies the need for different levels of description, required also by the fact that emergent behaviours are most often characterized by the presence of *coherent* entities or coherent sets of events (like in biological processes). In turn, the existence of different levels of description entails, on one hand, the lack of reducibility of higher levels to lower ones, and, on the other side, the fact that the properties holding at a given level cannot be owned by lower-level components. Moreover, as the presence of some sort of coherence depends on the structure of relationships within the system, the whole set of features so far listed implies the presence also of downwards causation effects in all emergent phenomena. Other corollaries stemming from the above features are that the behaviour at the higher levels is unpredictable in traditional mechanistic terms and the behaviour at the higher level is a genuine novelty.

Can the set of these characteristics be accounted for by the traditional tools of theoretical physics? The answer is negative, for a number or reasons. First of all, the mathematical tools useful to *talk about* these characteristics cannot predict or control the occurrence of the characteristics themselves, as these tools have been designed only to deal with very simple situations. In the second place any form of control on an adaptive system coming from outside produces automatically a reaction of the system on the outside environment, modifying the latter in such a way as to make impossible to describe what the system itself was *before* the control intervention (a circumstance reminding some aspects of quantum physics). In the third place the occurrence (or the disappearing) of new levels of description is not allowed by theories in which the number and the type of interactions is fixed and stated in advance.

7. Conclusions

All previous arguments evidenced that the actual edifice of theoretical physics is very far from being based on solid pillars. Then, what could be the future destiny of this discipline? According to our opinion, instead of assuming a nihilist attitude, we should take in the right consideration all tools available to theoretical physics in order to gather information about the phenomena. In the actual stage of the science we have essentially three tools: the resort to *experiments*, the use of *theories*, and the resort to *computer simulations*. Unfortunately, all these tools are scarcely reliable, for a number of different reasons.

Concerning the experiments, it is widely known that in general they are difficult to perform, full of conceptual and practical errors, often not repeatable, giving results which can be unreliable and prone to wrong interpretations, due to incorrect data analyses. However, the theories are not better tools, as often the traditional mathematical methods are unsuited and the risk of obtaining wrong conclusions lies in ambush. Besides, most models used in theoretical physics are nothing but toy models, unsuited to deal with real cases. The things are not going better if we resort to computer simulations. Namely they often produce unreliable results because the existence of bugs in the used software, of unsuited algorithms for discretisation, and of hardware problems.

Within this context, however, it is more convenient to describe the research in physics as a sort of *risky game*, played by resorting to all available tools (see, e.g., [28]). This game stops just in presence of a large discrepancy between the results gathered by two different tools. But when three different tools give simultaneously similar results, then all is OK: the game *continues*! In this situation, we are authorized to say that our tools added a new piece of knowledge to our previous reservoir. This representation, rather than describing theoretical physics as a sort of static edifice, depicts it as moving dynamical system, which travels in a risky way within the infinite sea of all possible phenomena, of all possible theories, and of all possible computations. This picture, in syntony with the conceptual approach held by Jean-Pierre Vigier within his life, supports the validity of the attribution to theoretical physicists of the role of *explorers* of the unlimited world of knowledge.

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What Physics Needs Today: A Few Good Anachronisms

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Because of the way academic subjects are organized for formal education, we often get the impression that real-world Science occurs in an ordered sequence, without false starts, or blind alleys, or re-works. This is just not so. Sometimes we have to go back, recognize an old problem that was not addressed, and in response to that old problem reconsider some aspects of current theory. This paper explores one such old problem, and brings together from different historical eras the bits and pieces needed to solve that problem in an orderly way. The result is a revised version of Special Relativity Theory (SRT) that is more compatible with Quantum Mechanics (QM), more convenient for Quantum Chemistry (QC), and likely better for extending to Gravity Theory (GT).

Keywords: Electromagnetic Theory (EMT), Special Relativity Theory (SRT), Quantum Mechanics (QM), Gravity Theory (GT)

1. Introduction

An 'anachronism' is an idea from one historical era that gets injected into a narrative rooted in another historical era. In literature, history, and other subjects in the humanities, anachronisms are usually *not* appropriate. Science fiction might be the one exception to this rule. But in the actual sciences, the opposite can be the case. An idea from another historical era can be just the right tool to expose, and maybe resolve, a current big Problem.

The big Problem discussed here is as follows: Something appears to have gone astray in Physics in the early 20th century, but the demonstration of a definite Problem *could not have even been attempted until the* ***mid*** 20th century, and, in fact, the demonstration discussed here *was not started until the late* 20th century, and has been in continuing development up to the present day, in the early 21st century.

In the early 20th century, the world of science was mired in confusion over the so-called 'aether', and the problems of observing and characterizing the propagation of light in this aether. The best response at the time was Einstein's development of Special Relativity Theory (SRT) [1], which did not invoke any kind of aether, and so avoided those problems.

Besides avoiding the problem of aether, SRT has another very positive feature. Einstein identified assumptions that everyone else had also been making, but only implicitly, without recognizing them as Assumptions. Einstein made them explicit, as his First Postulate and Second Postulate. The First Postulate says the Laws of Physics look the same to all inertial observers, and the Second Postulate says the speed of light is the number c relative to any inertial observer.

Any time a hidden assumption gets recognized and formalized as an Explicit Assumption, that formalization is a good thing. We need to do more of that. It is to be hoped that even more discovery and examination of our hidden assumptions can help us even better address all our known Big Problems.

Today, in the early 21st century, most physicists believe that SRT was well founded, completely developed, and physically true. But a few physicists, as well as many engineers, chemists, and other technical professionals, still wonder some about SRT. This author is one of those who still wonder about it. I see trouble; indeed, I see Big Trouble!

Here are three symptoms of Big Trouble in Physics: **1)** At the observational level: we see obvious contradiction within every scale of phenomena. On the smallest scale, we have a staggering proliferation of different particles, all of which we deem 'elementary'. On the largest scale, we see mysterious behaviors that we do not understand, and so we presently attribute them to so-called 'dark matter' and 'dark energy', and invent entirely new branches of Physics for them.

2) At the conceptual level, we have obvious conflict between two major branches of Physics: Quantum Mechanics (QM), and Relativity Theory (RT) [including Special (SRT), and General, (GRT)]. QM seemingly features instantaneous global connection, whereas RT is based on c-speed-only communication.

3) At the mathematical level, we have a rather obscure problem: physicists seem to be more casual than engineers, or mathematicians, or anybody else, with regard to so-called 'generalized functions' (GF's), such as the Dirac delta and the Heaviside step. GF's lack the property that mathematicians call 'uniform convergence'. As a result, they don't respond well to exchanges of the operations of differentiation, integration, and going to the GF limit. End results can depend on operation order. [2] So how can we tell which, if any, result is actually correct?

But looking to past historical eras, I see some ideas that seem potentially useful for removing these symptoms of Big Trouble in Physics. Coming from different historical eras, such ideas are Anachronisms with regard to early 20th century Physics. Here are three ideas:

1) The first idea goes back millennia, to ancient Greece. Some time around 300 BC, Euclid of Alexandria articulated a disciplined mathematical approach to the subject we now know as Geometry. He set out a few founding statements that he called Axioms. He never introduced any *additional* Axioms, and he developed a great collection of Theorems from just his few founding Axioms. The message is: Don't be making up more than the necessary number of foundational Axioms. Why? Because too many Axioms can lead to redundancies, or worse: Paradoxes.

Today, we speak less about Axioms, and more about Postulates. This wording change mainly reflects military history: around 150 BC, Rome conquered Greece, and Latin joined Greek as a language of civilized learning. So SRT has founding Postulates, rather than founding Axioms. And just like too many Axioms, too many Postulates can lead to Paradoxes. The rule is: if you add a *new* Axiom/Postulate, then *also delete an old one*! Einstein did delete Newton's universal time, but that did *not* eliminate Paradoxes from SRT. And though generally considered entertaining, SRT Paradoxes should *really* be viewed as signs of Big Trouble!

2) The second idea comes from 19th century Europe. It is the Standard Math Routine (SMR) for dealing with problems in continuum math/physics; *i.e.*, Classical Field Theory: **1)** Write down the governing laws as differential equations; **2)** Identify the family of solutions that the differential equations allow; **3)** Apply the boundary conditions that your particular problem requires. The message is: Don't be looking for any shortcut; just do the SMR.

Indeed, it is *not* doing the SMR that really ought to require justification. But in the case of SRT, none was offered, or even requested. The SMR simply was not considered. The message is: we should try the SMR. Indeed, normal human curiosity *requires* that we try it!

3) The third idea comes from mid 20th century America. Claude Shannon developed a mathematical theory for practical use in the communication industry. He borrowed from Thermodynamics the foundational concept of Entropy, put a minus sign to it, and used negative entropy, *neg*-entropy, to quantify Information. Thus, was born modern Information Theory (IT). [3, 4].

Shannon discovered some very important facts. For example, Information cannot be conveyed with an everlasting constant, or an everlasting sinusoid, or any such repetitive continuing function. In order to convey any Information at all, a signal has to be detectable, so it has to *stand out*: it has to have *finite duration*, so it has to have *finite energy*. That means finally that the SRT signal has to be rather pulse-like. The message is: the founding concept for SRT is its 'Signal', and this Signal concept needs elaboration based on IT.

With these three ideas, it is possible to revisit SRT and update it. We can initialize the Signal with finite-energy pulses in **E** and **B**. We can track the travel and evolution of this Signal using Maxwell's equations. We can visualize the effects of boundary conditions. So we can fully characterize the SRT Signal without invoking Einstein's Second Postulate.

2. The Differential Equations

Recall first the situation in the 19th century. Scientific development of all kinds dominated the times. What is relevant here is that James Clerk Maxwell gave us four coupled field equations expressed in terms of space and time derivatives. These equations are the basis for modern Electromagnetic Theory (EMT).

Maxwell wrote his equations in quaternion notation, which is not so widely familiar today. In modern vector notation, and in Gaussian units [5], Maxwell's first-order coupled field equations go:

$$\nabla \times \mathbf{E} = -\frac{1}{c}\partial \mathbf{B} / \partial t \, , \;\; \nabla \cdot \mathbf{B} = 0 \, , \qquad \text{(1a,b)}$$

$$\nabla \cdot \mathbf{D} = 4\pi\rho \, , \; \nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\partial \mathbf{D} / \partial t \, . \quad \text{(1c,d)}$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, and $\mathbf{D} = \varepsilon \mathbf{E}$, and where ε is 'electric permittivity', and $\mathbf{H} = \mathbf{B} / \mu$, where μ is 'magnetic permeability', and where ρ is charge per unit volume, and $\mathbf{j}$ is ρ times local charge velocity. In empty space, $\rho = 0$ and, $\mathbf{j} = 0$. Also in empty space, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$, and

$$c = 1 \Big/ \sqrt{\varepsilon_0 \mu_0} \, . \quad (2)$$

This constant c is commonly called 'the speed of light' – an interpretation worthy of the further discussion given below.

Note that the four field equations are 'first-order', meaning that only first-order derivatives occur in them. Note too that the equations are 'coupled', meaning that each equation involves *two* of the fields. Note finally that the space derivatives of one field drive the time derivative of the other field. In this situation, oscillations can result. For that reason, people naturally looked first at oscillating wave solutions.

Typically, one sees wave equations expressed in terms of potentials, rather than fields. There is a scalar potential Φ and a vector potential $\mathbf{A}$ such that:

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\partial \mathbf{A} / \partial t \, , \; \mathbf{B} = \nabla \times \mathbf{A} \quad \text{(3a,b)}$$

The scalar potential Φ is driven by the source charge per unit volume ρ, and the vector potential $\mathbf{A}$ is driven by the source current per unit volume $\mathbf{j}$:

$$\frac{1}{c^2}\partial^2\Phi / \partial t^2 - \nabla^2\Phi = 4\pi\rho \, , \quad \text{(4a)}$$

$$\frac{1}{c^2}\partial^2\mathbf{A} / \partial t^2 - \nabla^2\mathbf{A} = \frac{4\pi}{c}\mathbf{j} \, . \quad \text{(4b)}$$

The main feature of this presentation format is that the coupling that was so clearly evident in Maxwell's equations is seemingly removed, making the situation look simpler. This simplicity is, however, an illusion. Somewhere out of view there has to be a continuity equation about the source charge density ρ and current density $\mathbf{j}$:

$$\partial\rho / \partial t + \nabla \cdot \mathbf{j} = 0 \, . \quad \text{(4c)}$$

This is where the coupling is hiding out.

If one ignored the coupling hidden in the continuity equation, or if one avoided it by limiting attention to regions of space where the sources vanish, one would have homogeneous wave equations for the potentials:

$$\frac{1}{c^2}\partial^2\Phi / \partial t^2 - \nabla^2\Phi = 0 \, , \quad \text{(5a)}$$

$$\frac{1}{c^2}\partial^2\mathbf{A} / \partial t^2 - \nabla^2\mathbf{A} = 0 \, . \quad \text{(5b)}$$

One could then be tempted to hope for uncoupled homogeneous wave equations for the field amplitudes $\mathbf{E}$ and $\mathbf{B}$. That is, one could be tempted to hope for:

$$\frac{1}{c^2}\partial^2\mathbf{E} / \partial t^2 - \nabla^2\mathbf{E} = 0 \, , \quad \text{(6a)}$$

$$\frac{1}{c^2}\partial^2\mathbf{B} / \partial t^2 - \nabla^2\mathbf{B} = 0 \, . \quad \text{(6b)}$$

Here it looks as if the fields are uncoupled. For example, it looks as if there is no mandatory relationship between the magnitudes of $\mathbf{E}$ and $\mathbf{B}$. We could have all $\mathbf{E}$ and no $\mathbf{B}$, or *vice versa*. This would indeed be possible if the non-zero field(s) would be constant in time, but that is not a very interesting solution.

So, consider oscillatory fields instead. Can we have a sinusoidal $\mathbf{E}$ without a sinusoidal $\mathbf{B}$? Maybe, but it would not transfer energy, so that, too, is not a very interesting solution. We need both $\mathbf{E}$ and $\mathbf{B}$, and so we *have to* accept coupling between $\mathbf{E}$ and $\mathbf{B}$.

And in fact, built into Maxwell's field theory there is even *more* coupling. The fact is that while the sources ρ and $\mathbf{j}$ drive the fields, the fields in turn drive the ρ and $\mathbf{j}$. That aspect of EMT is captured in the Lorentz force law [5]. The Lorentz force $\mathbf{F}_\mathrm{L}$ acts on a single source charge q according to:

$$\mathbf{F}_\mathrm{L} = q(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \, , \quad (7)$$

where $\mathbf{V}$ is the velocity of the charge. The existence of the Lorentz force law means there is a 'feedback' connection between sources and fields.

Feedback, and system instability due to unwanted feedback, and system control via deliberately applied feedback - all are important topics in Engineering Science in general, and in Plasma Physics in particular.

The issue of stability is central. Timing can make the difference between stability and instability, and timing is determined by signal speed.

3. The Possible Solutions

Now recall the situation that existed in the early 20th century. Technology development dominated the times. Railroads were allowing high-speed travel. Electricity was starting to light up the dark night. Telegraph lines were providing long-distance communication. Notice that the trains were like 'coordinate frames' in relative motion, the lights were like 'sources', and the telegraph dots and dashes were like 'signals'. This is all exactly the stuff of SRT.

Here, what we are especially interested in is the 'signal' at the foundation of SRT. For that, an EM signal has to travel, and so it has to have *both* $\mathbf{E}$ and $\mathbf{B}$, perpendicular to each other, and with equal energy content in each. But in what shape shall the functional descriptions of $\mathbf{E}$ and $\mathbf{B}$ be cast?

In the early 20th century, people were familiar with traveling waves, particularly pure sinusoidal traveling plane waves, in the context of Fourier analysis and Fourier synthesis. Einstein thought about such sinusoidal waves, even in his boyhood, and his thought experiments led him to focus on light speed as a central concept for SRT. [1a]

But there is a problem: the pure sinusoid is literally *monotonous*. And metaphorically, there can be no *music* in that monotone. That is to say, without some kind of modulation, a sinusoid cannot convey any message, and so cannot serve as a signal for SRT. For the SRT signal, we need an EM *pulse*. That means finite energy. That makes the EM pulse similar to the *photon*, Einstein's even bigger idea of 1905. [1b]

Note: the one difference between photon and signal is that the photon requires circular polarization, whereas the signal *allows* it, but does not *require* it.

If there could be *un*coupled wave equations for $\mathbf{E}$ and $\mathbf{B}$, we could use Fourier synthesis to create a pair of pulses in $\mathbf{E}$ and $\mathbf{B}$, forming a finite-energy EM signal that could travel perpendicular to both $\mathbf{E}$ and $\mathbf{B}$, from a source to a receiver. It would look a lot like a bug with two wings, flying from source to receiver.

But this EM bug image does not just fly, like one might expect it to do. Because of the *coupling* in Maxwell's equations, this EM bug will not only *travel*, but also *evolve in shape* while it travels.

For example, suppose we model an initial input pulse in $\mathbf{E}$ as a simple Gaussian [6]. The first of Maxwell's four coupled field equations, $\nabla \times \mathbf{E} = -\frac{1}{c}\partial \mathbf{B} / \partial t$, Eq. (1a), will then generate a $\mathbf{B}$ field that has *two* peaks, one positive and one negative, in the direction perpendicular to the original $\mathbf{E}$ pulse. In fact, the generated $\mathbf{B}$ will have the shape of the initial input Gaussian pulse multiplied by a Hermite polynomial [6] of first order, which is a straight line through the coordinate origin.

The same thing will happen with an initial input Gaussian pulse in $\mathbf{B}$: it will lead to two pulses in $\mathbf{E}$, one positive and one negative, in the direction perpendicular to the original $\mathbf{B}$ pulse.

And then in turn, the first-order Hermite polynomials will generate second-order Hermite polynomials, and two opposing pulses will become three pulses alternating in sign. This escalation of Hermite polynomial order and proliferation of opposing pulses will go on and on. So our initial input pulses in $\mathbf{E}$ and $\mathbf{B}$ will inevitably morph into extended wavelets. So in flight, this EM bug will undergo a total metamorphosis!

Note: this wavelet development is in the *longitudinal* direction. Compare it to diffraction, which makes fringes in the *transverse* plane. Coupling makes peaks along the wave path; interference makes fringes in the wave face.

The problem is this: with the information-bearing wavelet not only traveling, but also elongating in its propagation direction, what can the phrase '*the* speed of light' actually mean? Clearly, the speed of light *cannot* be just a constant *number*; it has to be a *function*, depending on the spatial coordinate along the propagation direction, and on time into the propagation process. This is a feature entirely absent from SRT.

4. The Boundary Conditions

In any problem involving differential equations, the purpose of boundary conditions is to limit attention to the spatial volume that is actually of interest. In the signal-modeling problem, we are really only interested in modeling the signal in the space between the source and the receiver. So the natural boundaries are the source and the receiver.

What we want to accomplish with boundary conditions is:

1) No backflow of energy behind the source;

2) No overflow of energy beyond the receiver.

So, we need $\mathbf{E} \times \mathbf{B} = 0$ at the boundaries. One way is to demand $\mathbf{E} = 0$ at the boundaries. Alternatively, we could demand $\mathbf{B} = 0$ at the boundaries. But the first way is more familiar today because it is how we normally model the mirrors in a laser cavity. So, let us use the specification $\mathbf{E} = 0$ at the source and receiver.

What does it take to meet the specifications? We have to imagine some phantom signal sources, *i.e.* charges and currents, outside the problem domain. First, we need a phantom source just behind the real source, sending a signal the opposite direction. This can definitely make $\mathbf{E}=0$ at the source. Then we need a phantom beyond the receiver, at a distance always matching that of the source to the receiver, sending a signal in the opposite direction. This will make $\mathbf{E}=0$ at the receiver. But it will slightly spoil $\mathbf{E}=0$ at the source. So then we need another round of corrective phantoms. This correction process doesn't ever conclude, but it does converge rapidly, and we do *not* have to actually compute it; we only have to imagine it. The point is: the finite-energy EM signal is very complicated, so it is no trivial matter to formulate a short and usable description of its 'speed'.

5. A Reasonable Model for Signal Speed

Recall Eq. (2): the speed of light c is presently understood as just a number, determined by the other numbers, ε_0 and μ_0 that occur in EM wave equations: $c = 1\big/\sqrt{\varepsilon_0\mu_0}$. That number c would describe the speed of light as an infinite plane wave. But with a situation so complicated as an actual signal is, the one thing that is clear is that the little article word 'the' cannot be applied to actual signal speed. It implies way too much.

One readily available way to bridge the complexity gap between the light-speed number c and the actual speed of a realistic signal is to look more into the so-called 'reference' for light speed c.

Observe that there is nothing in any of the wave equations themselves to specify the reference for light speed. There can be no *single* signal speed because there *is* no *single* reference for light speed c.

From experiment, we do know what to expect. If we were to measure light speed using a lab at rest with respect to the source, we would surely get c with respect to the source. If we were to measure light speed using a lab at rest with respect to the receiver, we would surely get c with respect to the receiver. If we could somehow contrive to measure light speed using a lab permanently located halfway between the source and the receiver, we would surely get c with respect to that halfway location.

This elusive behavior of light speed recalls a situation familiar from QM: the so-called 'observer effect'. In QM, the act of making a measurement forces the system observed into just one from a set of possible 'eigenstates', and thereby forces the measurement result to assume one particular value from a countable set of possible values. That is, when a measurement is attempted, a wave function that may have been a superposition of many allowed states gets 'collapsed' into just one of them.

So, what alternative characterization for signal speed might be more acceptable? We need a definition that allows for evolution of the reference as a propagation scenario progresses. Here are three plausible ideas to consider:

1. The center peak of the developing wavelet could serve as the reference. This is simple to say, but complicated to implement, since the location of the center peak at a given time depends on the history up to that time.
2. A point that moves uniformly between the source and the receiver could serve as the reference. This, too, is simple to say but hard to implement, since the motion of such a point depends on the signal speed we are trying to model.
3. The point defined as always instantaneously equidistant between the source and the receiver could serve as the reference. This would be a 'scenario-averaged' reference.

And at least for a simple scenario without significant accelerations, the scenario-averaged reference appears a very reasonable choice to try first.

6. The Difference the Signal Model Makes

In the late 19th and early 20th century, there was a prototypical problem addressed by many researchers: formulating the potentials and fields of a rapidly moving source. Everyone made the same assumption and everyone got the same results, although possibly in different formats. The universal assumption was the same one that Einstein later formalized as his Second Postulate: the speed of light is *always* c relative to its ultimate receiver, *even* at its initial emission.

Those results today carry the names of Liénard and Wiechert (LW) [7, 8]. The LW potentials were

$$\Phi(\mathbf{r},t) = e\big[1/\kappa R\big]_{\text{retarded}}\,, \tag{8a}$$

and

$$\mathbf{A}(\mathbf{r},t) = e\big[\boldsymbol{\beta}/\kappa R\big]_{\text{retarded}}\,. \tag{8b}$$

where $\kappa = 1 - \mathbf{n}\cdot\boldsymbol{\beta}$, $\mathbf{n}$ is propagation direction, $\boldsymbol{\beta} = \mathbf{v}/c$, where is $\mathbf{v}$ source velocity, with magnitude v, limited to the number c, and 'retarded' means evaluated at the earlier time that allows for propagation to the receiver at speed c relative to the receiver.

The LW potentials gave the fields:

$$\mathbf{E}(\mathbf{r},t) = e\left\{\frac{(\mathbf{n}-\beta)(1-\beta^2)}{\kappa^3 R^2} + \mathbf{n}\times\frac{(\mathbf{n}-\beta)\times(d\beta/dt)}{c\kappa^3 R}\right\}_{\text{retarded}} \quad (9a)$$

$$\mathbf{B}(\mathbf{r},t) = \mathbf{n}_{\text{retarded}} \times \mathbf{E}(\mathbf{r},t)\,. \quad (9b)$$

The $1/R^2$ fields were called 'Coulomb-Ampere fields', or 'near fields', and the $1/R$ fields were called 'radiation fields', or 'far fields'. But the distinctions are meaningless, because radiation is better described with the Poynting vector $\mathbf{P} \propto \mathbf{E}\times\mathbf{B}$, and in the far field $\mathbf{P}$ becomes $\propto 1/R^2$ just like the near fields are.

Observe that the LW results say that the $1/R^2$ Coulomb part of the $\mathbf{E}$ vector lies in the direction of

$$(\mathbf{n}-\beta)_{\text{retarded}} \approx \mathbf{n}_{\text{retarded}} - \beta \approx \mathbf{n}_{\text{present}}, \quad (10a)$$

whereas the $1/R^2$ part of the $\mathbf{P}$ vector lies in the direction of $\mathbf{n}_{\text{retarded}}$. That is, the directions of the Coulomb 'tug' and the radiation 'torch' *disagree*. This amounts to Conflicting Information. It makes no sense.

But suppose we change to using the spatial midpoint of the scenario as the appropriate scenario-averaged reference. Then, in effect, 'retarded' changes to '*half* retarded'. The Coulomb field lies along

$$(\mathbf{n}-\beta)_{\text{half retarded}} \approx \mathbf{n}_{\text{present}} - \beta/2 \approx \mathbf{n}_{\text{half retarded}} \quad (10b)$$

and the Poynting vector *also* lies along $\mathbf{n}_{\text{half retarded}}$. That is, the Coulomb tug and the radiation torch come into alignment. This makes physical sense.

This adjustment may look small, but it is in fact very significant. It allows a different approach to Quantum Mechanics (QM), so that one need not start with yet another new founding Postulate, or put up with a sense of mystery, or enlist big computers to accomplish meaningful computations in application areas such as Chemistry [9]. It also provides a different point of departure for Gravity Theory [10]

7. Conclusion

This paper has been about the founding concept of SRT, the Signal, and how to think about the Signal from the viewpoint of the much later developed Information Theory, IT. The paper has identified requirements that would not have been obvious in the early 20th century.

One can think about the problem of the Signal: in terms of modern Set Theory. Recall that the coupling that was plainly evident in Maxwell's four coupled field equations became obscure in the two un-coupled wave equations. So take caution: second order uncoupled wave equations admit more solutions than first order coupled field equations admit. The Signal needs to be described in terms of solutions to the four first-order coupled field equations. These solutions make a *subset* of the set of solutions to the two un-coupled wave equations. We just need to stay within this special subset.

This paper has showed how to stay within the required special subset, and how at least one problem gets resolved when we do that. With the new point of departure for QM, many scenarios in Chemistry have been re-analyzed in a new and simpler way. With a new point of departure for Gravity Theory, we can start to hope that the now mysterious 'dark matter' and 'dark energy' can eventually be seen as kinds of 'phlogiston' peculiar to our present age.

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Projective Geometrical Space, Duality, Harmonicity and the Inverse Square Law

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Having adopted the fundamental distinction between Perceptible space and Geometrical space, we select the Projective space as Geometrical space of choice for the description of the correlations between the natural world's elements. This description is attempted by the Theory of the harmonicity of the field of light, which always introduces a localized observer or a scientific instrument. As the speed of light is finite, the results of our observations and measurements of a linearly moving element of matter always refer not to its present position, but rather to a previous one, which we call Conjugate position, (the "shadow" in the Plato's Allegory of the Cave). In the Projective Geometrical space the Conjugate positions, in a linear motion, are two which are harmonically connected through the observer. As a consequence of the axioms of the Projective geometry these Conjugate positions are both accepted. Moreover, those axioms introduce automatically the Principle of Duality of the Geometrical space's fundamental elements, which leads to the well-known Duality in Physics. A result of the Geometrical Principle of Duality is the Inverse square law in the Gravitational field, as well in the Electric field. Thus, this work produces the Inverse square law from First Principles.

Keywords: Projective space, Localized observer, Conjugate position, Harmonic cross-ratio, Theory of the harmonicity of the field of light, Duality, Inverse square law, Briefest Computation Principle (BCP)

1. Introduction

After the fundamental distinction between Perceptible Space and Geometrical Space given by my late professor Panagiotis Ladopoulos [1, 2], and since all the logically consistent geometries are equally valid, we select the Projective Space as the Geometrical Space of choice to formulate the Theory of the Harmonicity of the Field of Light [3, 4].

Projective Geometry was established at the beginning of the 19th century mostly by French mechanical engineer and mathematician Jean-Victor Poncelet [5]. This Geometry was dynamically developed during the 19th and 20th centuries, through the efforts of many illustrious mathematicians. Although Poncelet and his predecessors (Pappus of Alexandria, Johannes Keppler, Gerard Desargues) applied the synthetic method, today Projective Geometry is mostly developed by following an analytical one [6, 7]. In this paper, however, we shall adhere to the synthetic geometric spirit.

2. The Axioms of Projective Space

Projective Space is established with eight (8) axioms and its Geometry is established with nine (9). I quote them from Prof. Ladopoulos' book [1], noting their decisive contribution in the structure of Theory of the harmonicity of the field of light. These axioms are separated in three independent and distinct groups:

2.1. *Positional Axioms*

I. Two points define a straight line, on which they lie.
II. Three points, not on the same straight line, define a plane on which they lie.
III. A point and a straight line, not passing through it, define a plane on which they lie.
IV. Two planes define a straight line, which lies on them.
V. Three planes, not intersecting at the same straight line, define a point, which lies on them.
VI. A plane and a straight line not lying on it, define a point, which lies on them.

Before we proceed to the second group of axioms, we have to define the notion of the "first degree geometric formation", whose elements, in Analytic Geometry, are defined by a single parameter. The fundamental first degree geometric formations are:
a. The straight-line point series comprised of the set of points lying on a straight line. Any element of this point series (its point) is determined by its Cartesian abscissa in relation to another point of the same point series, arbitrarily designated as the origin of measurements.
b. The pencil of ray-lines, i.e. the set of straight lines of a plane intersecting at a certain point. Here the parameter is the angle in relation to another randomly selected ray-line used as the origin of measurements.
c. The sheaf of planes, i.e. the set of planes in space intersecting at one straight line. Here too, the parameter is the angle. An example of a sheaf of planes is the set of possible places of a door pane in relation to the axis defined by the hinges.

2.2. *The Axiom of Order and the Axiom of the Projective Character of the Direction of Movement*

VII. If an element O is defined on a first-degree geometric formation, the remaining of its elements can be ordered in such a way so that element O precedes all the other elements. In this order, there is always an element preceding every other element and between two elements A and B of the formation, if A precedes B, there is always an element that comes after A and before B.

VIII. In a first-degree geometric formation there are two specific directions of movement opposite each other. If a first-degree formation results from another via a finite number of projections and sections, a specific direction of movement on the first corresponds to a specific direction of movement on the second.

2.3. *Dedekind's Continuity Axiom*

IX. If AB is a first-degree geometric formation segment on which there has been defined a direction of movement, and if this segment is divided into two parts so as that:

a. Every element of the segment AB belongs to one of the two parts.
b. A, belongs to the first part and B to the second.
c. A random element of the first part precedes a random element of the second. Then, there is an element C of the segment AB (which may belong to one of the parts), so that every element of AB preceding C belongs to the first part and every element after C belongs to the second part.

3. Remarks on the Axioms of Projective Geometry

3.1. *The Six Positional Axioms Were Named so to Place Emphasis on the Importance of "Position"*

3.2. *The Axioms of Projective Geometry Introduce Automatically to the Projective Space*

Also its elements "at infinity", which are considered totally equivalent to those "not at infinity" and thus undistinguishable. As a result of this introduction and equivalence, the fifth Euclidean postulate, (parallel straight lines do not intersect), has absolutely no meaning in Projective Space.

3.3. *The VII Axiom of Order "Closes" the Straight Line*

The projective straight line is a closed line via its point at infinity, as opposed to the Euclidean which is an open one. It follows that the projective plane is also a closed surface via its straight line at infinity (ideal line). In order to divide a Euclidean straight line into two parts, it is necessary and sufficient to define one point on it. This does not apply in a projective one. To divide a projective straight line into two parts, two points are necessary and sufficient. In addition, to define the direction of movement in the Euclidean straight line, it is necessary and sufficient to name in a specific order two of its points. This does not apply in the case of the Projective Geometry. That is because when in the Euclidean Geometry we say A, B, the direction has been defined, whilst in Projective Geometry, it has not been properly defined, because you can go from point A to B either in the classical way (Euclidean direction) or in the opposite direction via its "point at infinity".

3.4. *The Six Positional Axioms Were Formulated Utilizing Three Concepts: Point, Straight Line and Plane*

These concepts, however, were not defined. How could they be defined? The answer is obvious: These concepts are so rudimentary in nature that no simpler ones exist to define them with. These three fundamental concepts are automatically defined simply by stating the six Positional Axioms. It is not the "mathematical entities" that we define and investigate, but their relationships.

So, what is a point? A point is what we understand it to be simply by reading the six Positional Axioms. The same is true for the straight line and the plane. There is one thing for sure: All of us have understood the same thing and that is due to the strictness of the axioms.

3.5. *Projective Space was Established with the First Eight Axioms*

Dedekind's Continuity Axiom does not contribute to the establishment of Projective Space. It was introduced, however, to Projective Geometry by F. Enriques to offer Projective Geometry autonomy and to help it overcome a certain weakness present in the proof of its fundamental theorem. Dedekind's Continuity Axiom, as formulated, constitutes the geometrical expression of the continuity principle of real numbers in Analysis.

4. The Principle of Duality in Projective Geometrical Space

If we classify the Positional Axioms in two groups of three; the 1st group comprises of the first three, and the 2nd of the last three, then by comparing the axioms of two groups 1st to 1st etc., we observe that the last three result from the first three, respectively one to one if, in the wording of each axiom, we respectively replace the word "point" with the word "plane" and vice versa and keep the word "straight line" unchanged. Certain changes in the syntax would of course be required.

It seems therefore, that the concept of an infinitesimally small point is equivalent to the concept of an infinitely extended plane (non-scientific comparisons notwithstanding), that is to say, a valid statement containing the concepts of points and planes, retains its validity (it is invariant) if the two concepts swap their roles.

This property, introduced automatically in Geometrical Space by the formulation of the six Positional Axioms, is called: Principle of Duality in Geometrical Projective Space.

Moreover, since the formulation of axioms VII and VIII (establishing Projective Space) does not discriminate between "point" and "plane", it follows that the Duality Principle, introduced by the first six axioms, remains valid also for those two.

Based on this Principle, from a true geometrical statement defining relations between points, straight lines and planes, we can pass on to an equally true geometrical statement if we interchange the roles of "point" and "plane", while maintaining on both the "straight line".

Taking it a step further, the Duality Principle holds its validity also at the planar level, where it is the "point" ↔ "straight line" that now interchange their roles. Furthermore, the Duality Principle is also valid for any "Central Beam" (the set of all the straight lines and planes of space passing through a common point), where the concepts now interchanging roles are "straight line" and "plane".

I consider the Duality Principle, in combination with Axioms VII and VIII, to be of fundamental importance to the Theory of the harmonicity of the field of light and of great assistance not only in the interpretation of certain «Mysteries» of modern Quantum Mechanics as, for example, the Superposition of Quantum States (Mystery of Schrödinger's cat) [8], but also for the extraction of fundamental laws of Physics from First Principles.

There exist exemplary works by other researchers, that point out the significance of the Duality Principle in modern Physics, based on the analytic method [9-11]. In this paper, we shall follow the geometrical (synthetic) method, that results from the aforementioned Axioms establishing the Projective Space, a method we first introduced many years ago [3, 4, 12].

5. The Theory of the Harmonicity of the Field of Light

The foundation of this theory is briefly presented below.

5.1. *The Philosophy of the Theory*

The Science of Physics, contrary to Mathematical Science, is basically empirical, practiced by human beings, who, being of a material nature, are restricted by their locality. We adhere to the philosophical thought of Weizsacker and Heisenberg, as it is presented by the latter in his well-known book, "Physics and Philosophy" [13], and can be summarized in the following statement by the former: "Nature is earlier than man, but man is earlier than the natural science". We therefore believe that every natural science ought definitely not to ignore the existence of the Observer who observes and measures the events. Of course, the "Observer" concept is rather elastic and able to refer even to the instrument through which the registration and measurement of events takes place.

5.2. *The First Fundamental Hypothesis of the Theory*

The second of the two Hypotheses of the Special Relativity Theory (SRT) refers to the independence of the speed of light from the speed of its source [14]. We shall extend it to cover all interactions of matter but refer it to the Geometrical Space. Thus, our first hypothesis is as follows: "Matter interactions occur in the Geometrical Space at a speed that is essentially constant and independent of the relative speed of the interacting elements of matter. More specifically, mater interactions conducted through light, occur in Geometrical Space at a speed essentially constant in magnitude, independent of the relative speed of interacting elements of matter and equal to the speed of light which, however, I measure in my Perceptible Space at the place where I am located". I note that when I write: "…the speed of light which, however, I measure in my Perceptible Space at the place where I am located".

I mean the measurement performed via the method described by Einstein in his original paper [14], using one local clock, to avoid the problems caused by the use of two clocks.

5.3. *The Concept of the Linear Array of Synchronized Clocks (LASC)*

We adopt the definition of the synchronization of two distant clocks that Einstein gave in his original article [14] and we consider that any logical being performing physical experiments, would use synchronized clocks in the various positions of his frame of reference. It is via the readings of those clocks, and only those, that two events can be characterized as synchronous in experimental physics [15, 16]. Specifically, when rectilinear motion is observed and measured, said synchronized clocks would need to be arranged along the length of the path of the object in motion.

Thus, is created the concept of the Linear Array of Synchronized Clocks (LASC), via which the notion of speed is determined in Classical Mechanics, a notion that is valid in all of Natural Science.

5.4. *Kinematics of the Material Point Moving with Subluminal Speed (υ < c) Measured by the LASC*

According to our first fundamental hypothesis and since the speed of light is finite, a localized real Observer does not see nor measure the present position of a linearly moving material point, but a previous one, which we call Conjugate Position [2-4, 8]. This previous position has been called the "retarded position" by Richard Feynman [17]. Thus, in reality, the objects of study by real Observers, and not by omni-present spirits, are not the moving bodies' positions themselves, but rather their conjugates. This conclusion, evocative of the "shadows in the cave" in the Plato's Allegory, carries critical implications for the whole of modern Physics [18, 19].

Let a material point moving with a constant speed υ, measured by the LASC, on a straight-line E furnished with the LASC, and let a real Observer, furnished with a local clock, which is synchronized with the LASC clocks. The Observer, been posed at a random position O outside the straight-line E, is studying the Kinematics of the material point (Fig. 1).

Let the material point now be at position A. However, in the Observer's Perceptible Space, it is not located at A, but rather at its conjugate position A′. Thus, we have:

$$\frac{A'A}{\upsilon} = \frac{A'O}{c} \Rightarrow \frac{A'A}{A'O} = \frac{\upsilon}{c} \tag{1}$$

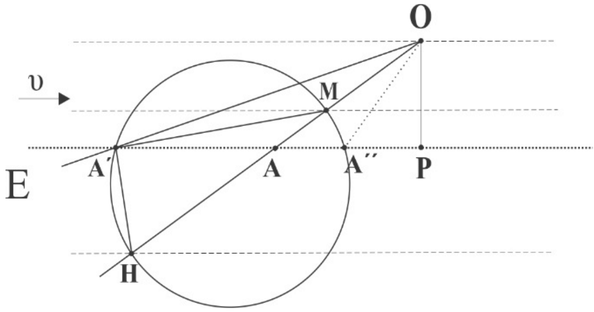

Figure 1. The two conjugate positions for a given position.

If the position A, the direction of movement and the speed of the material point measured by the LASC are given, we are asked to locate the conjugate position A′.

Let us for a moment assume that A′ has been located. In triangle OA′A I draw the internal bisector of the angle A′. Suppose that said bisector crosses OA at point M. I also draw the external bisector of angle A′, which crosses the extension of OA at H. MA′ is perpendicular to A′H. From the angle bisector theorem, it holds that:

$$\frac{MA}{MO} = \frac{HA}{HO} = \frac{A'A}{A'O} = \frac{\upsilon}{c} \tag{2}$$

Thus, given the position A, I draw OA and extend it. I divide OA internally in a υ/c ratio. There is one and only one point M so that: $\frac{MA}{MO} = \frac{\upsilon}{c}$. I divide OA externally in a υ/c ratio. There is one and only one point H so that: $\frac{HA}{HO} = \frac{\upsilon}{c}$. With diameter MH, I draw the Apollonian Circumference (Apollonian Circle), which crosses line E at points A′ and A′′. A′ is the conjugate position of A for a measure of speed υ and direction of movement as per figure 1, whereas A′′ is the conjugate position of A for the same measure of speed υ and opposite direction of movement. This is so because the so defined Apollonian Circumference, i.e. with MH as its diameter, is the geometrical locus of points (on a plane) whose ratio of the distances from the given points A and O is the given one (υ/c not equal to 1).

We observe that the two conjugate positions A′ and A′′ of position A, for the same speed measure υ and for the two opposite directions of movement, are not symmetrical with regards to it. Here we have discovered Harmony. This is because the four points H, M, A, O, constitute a Harmonic Tetrad. We observe that the double ratio or cross-ratio (with signs):

$$(\mathrm{HMAO}) = \frac{(\mathrm{HA})}{(\mathrm{AM})} : \frac{(\mathrm{HO})}{(\mathrm{OM})} = \frac{(\mathrm{HA})}{(\mathrm{HO})} : \frac{(\mathrm{AM})}{(\mathrm{OM})} = \frac{\upsilon}{c} : \left(-\frac{\upsilon}{c}\right) = -1$$

equals -1, which constitutes the necessary and sufficient condition for a harmonic tetrad.

This property of Harmonicity is fundamental in Nature and is a direct result of the stability of the speed of light, i.e. its independence from the speed of its source. The consequences of Harmonicity are very important to modern Physics [20].

5.5. *Kinematics of the Material Point Moving with Superluminal Speed (υ > c) Measured by the LASC*

We have shown [21] that the relationship $\upsilon < c$ is not a conclusion of the Lorentz Transformation (LT), but rather a hidden pre-condition of its analytical production. In other words, the LT can only be valid for $\upsilon < c$.

Consequently, there does not exist a definite and rigid law of Physics forbidding matter to travel with superluminal speed in vacuum. I examine (Fig. 2) the case where the material point moves away from the foot of the perpendicular P and Observer O with a speed $\upsilon > c$ and is now located at position A. What is the location of its conjugate positions, i.e. where is it now seen by the Observer? The answer is provided by the Apollonian Circumference.

I divide the segment OA with point M such as: $MA/MO = \upsilon/c$. There exists one and only one such point M satisfying the aforementioned relation. Similarly, on the extension of AO I locate point H such as: $HA/HO = \upsilon/c$. There exists one and only one such point H satisfying the aforementioned relation.

With diameter MH, I create the Apollonian Circumference that intersects straight line E generally in two points A′ & A′′, which are the conjugates of A that we seek. This is so because the Apollonian Circumference thus created, is the geometrical locus, on the plane, of those points whose ratio of their distance from A and O is the given one, i.e. $\upsilon/c > 1$. In other words, it holds that:

$$\frac{\mathrm{A''A}}{\mathrm{A''O}} = \frac{\mathrm{A'A}}{\mathrm{A'O}} = \frac{\upsilon}{c} > 1 \qquad (3)$$

The tetrad of points H, M, A, O is harmonic, because the double ratio, with signs, (HMAO) equals -1.

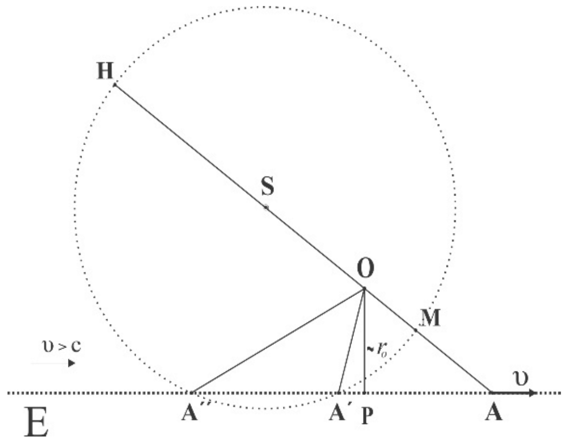

Figure 2. The two conjugate positions for superluminal speed.

All the above mean that the signal (image) transmitted by the material point when it was located at A′′, reaches the Observer O at the time said point has already reached A, and also that the signal (image) transmitted from A′, reaches the Observer O also by the time it has already reached A. In other words, the images from A′′ and A′, both reach the Observer O simultaneously.

6. From the Projective Space to the Duality in Physics

From Fig. 2 it becomes obvious that at the Observer's position O, the signals from the moving point's positions A′′ and A′, although not transmitted at the same time, do indeed arrive simultaneously. This is exactly where the concept (matter-wave) is born: Matter travelling ultra-fast demonstrates wave-like behavior. This phenomenon is not apparent for subluminal ($\upsilon < c$) speeds of matter (Fig. 1). This is exactly where the introduction of Projective Geometrical Space is necessary. In other words, while in the Euclidean Geometrical Space to each direction of movement corresponds only one Conjugate Position, in the Projective Geometrical Space both Conjugate Positions are accepted, because the Projective straight line is a closed line via its point at infinity. Thus, in the Geometrical Space, which exists only in our minds, we consider that, in as much time it took the material point to travel the noetic segment A′′∞ A, light travelled the noetic segment A′′∞ O. (Fig. 1). However, in Perceptible Space, we can only measure finite segments via which the Apollonian Circumference is constructed.

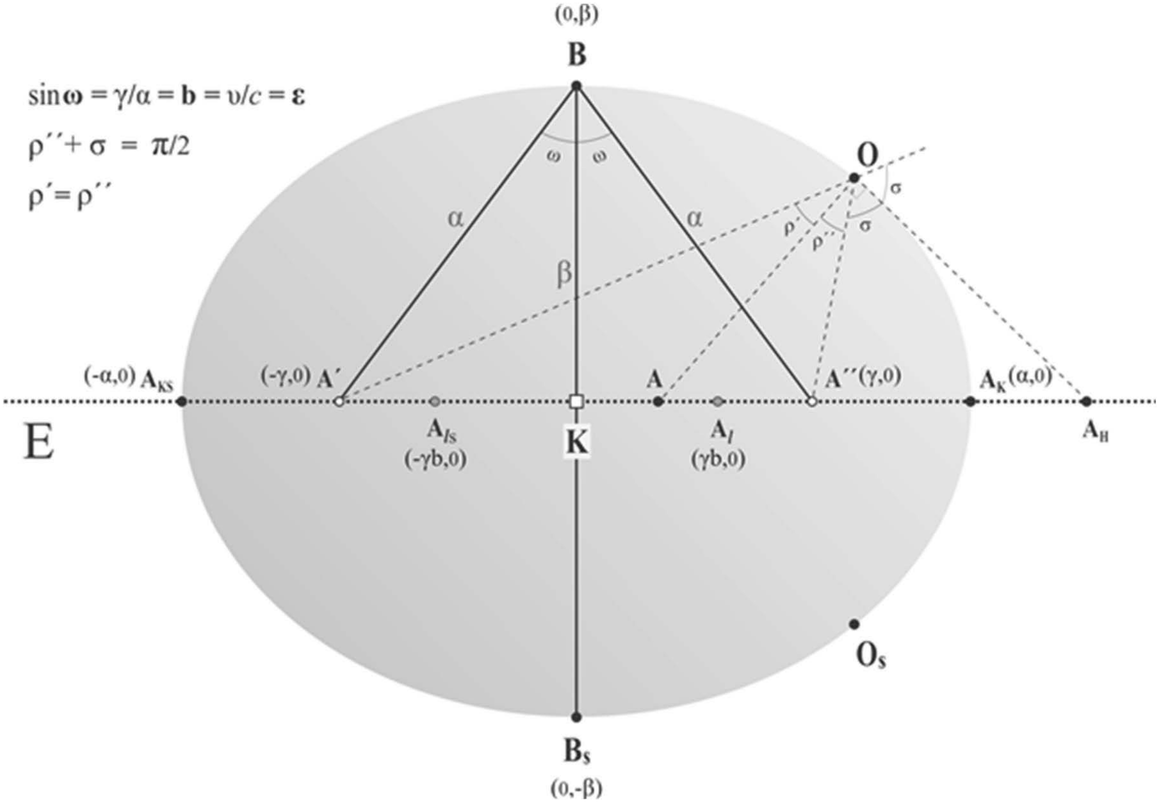

Figure 3. The Geometrical Locus of Observation Position for given Conjugate Positions (υ < c).

In [2] we have shown that the Superposition of Quantum States is a consequence of the Projective Space and we presented two fundamental theorems relative to this phenomenon as well as some of the many corollaries [4]. Therefore, in the Projective Space, both solutions provided by the Apollonian Circumference are accepted for the complete spectrum of speeds of moving matter.

If we now consider both conjugate positions as well as the speed of the moving material point as given, then the geometrical locus of the observing positions O, on the plane, is a conic section. For example, when υ < c, it is an ellipse (Fig. 3), with eccentricity ε= υ/c and its minor (secondary) semi-axis β equals its major (primary) semi-axis α, contracted by the Lorentz contraction factor.

That is:

$$\sin\omega = \frac{\gamma}{\alpha} = b = \frac{\upsilon}{c} = \varepsilon \tag{4}$$

and:

$$(\mathrm{KB}) = \beta = \sqrt{\alpha^2 - \gamma^2} = \alpha \cdot \sqrt{1 - \frac{\gamma^2}{\alpha^2}} = \alpha \cdot \sqrt{1 - \frac{\upsilon^2}{c^2}} \tag{5}$$

We observe that in both Figure 1 and Figure 3, line OA connecting Observer O with the observed moving material point A, is the bisector of the angle formed by the two conjugate positions A′ and A′′ and the Observer O (angle A′OA′′). This observation of ours is of cardinal importance when it comes to connecting the Principle of Duality with the Inverse Square Law in Physics.

7. From the Projective Principle of Duality to the Inverse Square Law

It is well known that the Inverse Square Law, in both the Gravitational and Electric Fields, is empirical resulting from the observation and experiment. In other words, this law does not stem from First Principles. Newton's famous dictum "*Hypotheses non fingo*", written in the final "General Scholium" on his "*Philosophiae Naturalis Principia Mathematica*", is contained in the relevant passage which, translated in English, is as follows:

"Hitherto we have explained the phenomena of the heavens and of our sea by the power of gravity, but have not yet assigned the cause of this power … I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses [*hypotheses non fingo*]; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy … To us it is enough that gravity does really exist, and acts according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea." [22].

Furthermore, Feynman's position, regarding the fundamentality of the Inverse Square Law in Gravitation that there doesn't exist a more fundamental machinery to explain said law, is also well known: "…But up to today, from the time of Newton, no one has invented another theoretical description of the machinery behind this law which does not either say the same thing over again, or make the mathematics harder, or predict some wrong phenomena. So there is no model of the theory of gravitation today, other than this mathematical form." [23]

We, further on, shall demonstrate that there is indeed a "machinery" behind this law and that is none other than the Principle of Duality in the Projective Geometrical Space. That is to say, we shall produce theoretically the Inverse Square Law solely from First Principles. Thus, perhaps, Galileo's position who proclaimed that "…the book of nature is written in the language of geometry, without which we cannot understand a single word of it." [22], might be vindicated.

In the Projective Plane the dual notions are the point and the straight line. So, from a theorem that is true in the Projective Plane we can get another true theorem, simply by interchanging the points for straight lines and straight lines for points. A good example of dual theorems is that of Pascal's and Brianchon's. This duality we shall now explore in Gravitation Law.

Let us assume that in Nature, there stands a Law of Gravitation between two point-like masses m1 and m2.

Let also the Gravitational Force be analogous to the two masses and dependent on the distance R between them, raised to an unknown exponent n, which means that the measure of the Gravitational Force is of the form:

$$F = G.m_1.m_2.R^n \tag{6}$$

where G is a constant coefficient.

By applying the Principle of Duality in the Projective Plane, we try to detect the Gravitational Law between a point-like mass at position O and a linearly expanded mass with a constant linear density λ kg/m. In other words, in accordance to law (6), we try to determine the impact on a random point in space O, of a uniformly distributed mass in the linear section A′A′′. Point O and linear section A′A′′ define a plane (Axiom III) upon which we operate (Fig. 4).

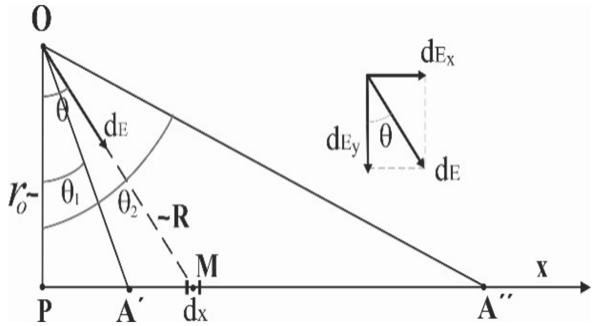

Figure 4. The Gravitational Interaction between the point-mass O and the linearly expanded mass A′A′′.

The measure of the intensity of the gravitational field at position O, stemming from the linearly distributed mass m = λ.IA′A′′I, we shall call E. The elemental intensity of the field of an elemental mass of length dx, based on law (6), will have a measure at point O:

$$dE = G.(\lambda.dx).R^n \tag{7}$$

Which is split in two components an horizontal and a vertical one.

$$dE_x = dE.\sin\theta \tag{8}$$

and

$$dE_y = dE.\cos\theta \tag{9}$$

Therefore, the total impact at O of the distributed mass will result from synthesizing the two components, after they have been first computed by integration.

From Fig. 4 we get:

$$x = r_0.\tan\theta \tag{10}$$

(x=0 at P) thus:

$$dx = \frac{r_0}{\cos^2\theta}.d\theta \tag{11}$$

also

$$R = \frac{r_0}{\cos\theta} \tag{12}$$

Thus:

$$dE_x = dE.\sin\theta = G.\lambda.r_0^{n+1}.\frac{\sin\theta.d\theta}{\cos^{n+2}\theta} = -G.\lambda.r_0^{n+1}.\cos^{-(n+2)}\theta.d\cos\theta \quad (13)$$

and the total horizontal component is:

$$E_x = -G.\lambda.r_0^{n+1}\int_{\theta_1}^{\theta_2}\cos^{-(n+2)}\theta.d\cos\theta \quad (14)$$

Finally:

$$E_x = \frac{G.\lambda.r_0^{n+1}}{n+1}\left[\cos^{-(n+1)}\theta_2 - \cos^{-(n+1)}\theta_1\right] \quad (15)$$

From (9) the elemental vertical component is:

$$dE_y = dE.\cos\theta = G.\lambda.r_0^{n+1}.\frac{\cos\theta.d\theta}{\cos^{n+2}\theta} = G.\lambda.r_0^{n+1}.\cos^{-(n+1)}\theta.d\theta \quad (16)$$

And the total vertical component is:

$$E_y = G.\lambda.r_0^{n+1}\int_{\theta_1}^{\theta_2}\cos^{-(n+1)}\theta.d\theta \quad (17)$$

So, what needs to be calculated is the integral:

$I = \int\cos^{-(n+1)}\theta.d\theta = \int\cos^{-(n+3)}\theta.\cos^2\theta.d\theta = \int\cos^{-(n+3)}\theta.(1-\sin^2\theta).d\theta$ Thus:

$$I = \int\cos^{-(n+1)}\theta.d\theta = \int\cos^{-(n+3)}\theta.d\theta - \int\cos^{-(n+3)}\theta.\sin^2\theta.d\theta$$

But

$$\cos^{-(n+3)}\theta.\sin^2\theta.d\theta = \frac{1}{n+2}d\left[\cos^{-(n+2)}\theta.\sin\theta\right] - \frac{1}{n+2}\cos^{-(n+1)}\theta.d\theta \quad (18)$$

Using (18), the integral we seek becomes:

$$I = \int\cos^{-(n+1)}\theta.d\theta = \int\cos^{-(n+3)}\theta.d\theta - \frac{1}{n+2}\int d\left[\cos^{-(n+2)}\theta.\sin\theta\right] + \frac{1}{n+2}\int\cos^{-(n+1)}\theta.d\theta \quad (19)$$

And by doing the calculations in (19), we get:

$$I = \int\cos^{-(n+1)}\theta.d\theta = -\frac{\cos^{-(n+2)}\theta.\sin\theta}{n+1} + \frac{n+2}{n+1}\int\cos^{-(n+3)}\theta.d\theta \quad (20)$$

Thus, we observe that the calculation of the integral I that we seek falls into an infinite series of calculations of similar integrals, with an exponent that each time is diminished by 2. In other words, to calculate $\int\cos^{-(n+1)}\theta.d\theta$, we first need to calculate $\int\cos^{-(n+3)}\theta.d\theta$. And before that, $\int\cos^{-(n+5)}\theta.d\theta$ and so on & on…, without a determined end since n is an unknown. Thus, in order that the calculation is swiftly completed, only one capability exists: The factor before the resulting integral of (20) to become zero, i.e.

$$\frac{n+2}{n+1} = 0 \quad (21)$$

Which can be achieved only if $n = -2$. This way, the Gravitation Law becomes an Inverse Square Law and (6) is written:

$$F = \frac{G.m_1.m_2}{R^2} \quad (6a)$$

This conditionality via which we arrived at the Inverse Square Law I name: Briefest Computation Principle (BCP).

Thus, for n = -2 the integral *I* that we seek becomes:

$$I = -\frac{\sin\theta}{(-1)} = \sin\theta$$

and by replacing in (17) we get:

$$E_y = G.\lambda.r_0^{-1}\left[\sin\theta\right]_{\theta_1}^{\theta_2} = \frac{G.\lambda}{r_0}\left(\sin\theta_2 - \sin\theta_1\right) \quad (22)$$

and the resulting Ex from (15) becomes:

$$E_x = -\frac{G.\lambda}{r_0}\left[\cos\theta_2 - \cos\theta_1\right] = \frac{G.\lambda}{r_0}\left(\cos\theta_1 - \cos\theta_2\right) \quad (23)$$

The total intensity of the Gravitation field in O is Eo:

$$E_o^2 = E_x^2 + E_y^2 = \left(\frac{G.\lambda}{r_0}\right)^2\left[\left(\cos\theta_1 - \cos\theta_2\right)^2 + \left(\sin\theta_2 - \sin\theta_1\right)^2\right]$$

and following the necessary calculations we arrive:

$$E_o = \frac{2G.\lambda}{r_0}.\sin\frac{\theta_2 - \theta_1}{2} \quad (24)$$

I replace $\theta_2 - \theta_1 = 2\rho$, thus:

$$E_o = \frac{2G.\lambda}{r_0}.\sin\rho \tag{25}$$

Finally, angle φ is calculated as follows:

$$\tan\varphi = \frac{E_x}{E_y} = \frac{\cos\theta_1 - \cos\theta_2}{\sin\theta_2 - \sin\theta_1} = \frac{2\sin\frac{\theta_1+\theta_2}{2}.\sin\frac{\theta_2-\theta_1}{2}}{2\sin\frac{\theta_2-\theta_1}{2}.\cos\frac{\theta_1+\theta_2}{2}} = \tan\frac{\theta_1+\theta_2}{2} \tag{26}$$

Thus

$$\varphi = \frac{\theta_1+\theta_2}{2} = \theta_1 + \rho \tag{27}$$

Therefore OA, on which lies the total intensity of the Gravitation Field, is the bisector of angle A′O A′′. If we observe Fig. 1 & Fig. 3 of the Theory of the Harmonicity of the Field of Light, we notice that also there OA that connects the Observer with the moving material point, is also the bisector of the conjugate rays.

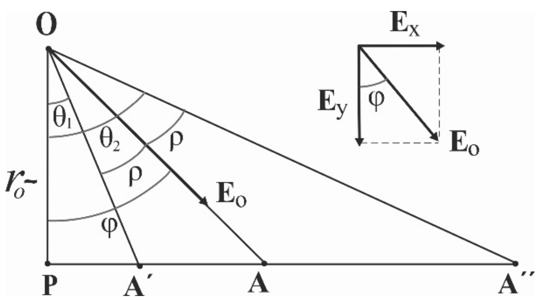

Figure 5. The Total Gravitational Interaction Eo lies on the Bisector of the angle A′OA′′where A lies (see Harmonicity).

Fig. 2 that refers to superluminal speeds, OA is the bisector of the external angle of the conjugate rays. Is this a coincidence? Of course not.

The Briefest Computation Principle, that leads to the Inverse Square Law, is deeply connected with the behavior of Light in the Projective Space.

Nature is self-consistent. Projective Space, Duality, Harmonicity & the Inverse Square Law are all causally and integrally interconnected.

8. The Inverse Square Law in the Electric Field

By applying the exact same reasoning resulting from the Principle of Duality in the Projective Plane, as well the Briefest Computation Principle, we produce the Inverse Square Law in the Electric Field of a linear, uniformly distributed electric charge.

In this case also, the resultant Field intensity lies on the bisector OA of angle A′O A′′. Let us however combine the above ascertainment with the uniform and rectilinear motion of an electric charge, considering that a point-like electric charge is equivalent with its distribution between its conjugate positions A′ and A′′ in Projective space (Principle of Duality). Then Fig. 1 (or Fig. 3) and Fig. 5 combine, that is to say the kinematics of the material point in the Projective Space, is combined with the dynamics. Thus the resultant intensity of the Electric Field located on the bisector of angle A′O A′′, explains the strange phenomenon [24] that the intensity of Coulomb's Field stems from the present position, and not from the retarded position, as if the Electric Field was moving with infinite speed.

8.1. *The Relationship between the Electrostatics and the Kinematics of the Material Point in the Projective Space*

Let us consider one material point moving with constant relativistic speed υ, measured by the LASC, on a Projective straight line E. Now it is at the foot of the perpendicular A, driven from the observer O onto the straight line E. The two conjugate positions A′and A′′ in the Projective Space are found via the Apollonian Circumference and they are symmetrical with regards to A. (Fig. 6). The known [4] equations of the Theory of the Harmonicity of the Field of Light apply:

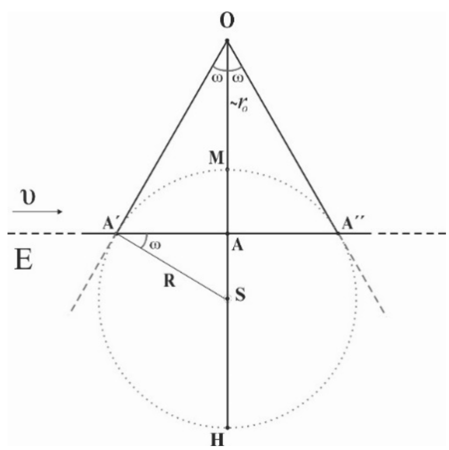

Figure 6. The Apollonian Circumference at the Foot of the Perpendicular (υ < c).

$$\frac{AM}{MO}=\frac{HA}{HO}=\frac{\upsilon}{c}=b,\ \sin\omega=b,\ \cos\omega=\sqrt{1-b^2}$$

Naming OA = r0, we get $HO=\frac{r_0}{1-b},\ MO=\frac{r_0}{1+b}$

As OA´ & OA´´ are tangential of the Apollonian Circumference at the points A´ and A´´ respectively [4], it follows that:

$$(OA')^2=(OA'')^2=(HO)(MO)=\frac{r_0^2}{1-b^2},$$

thus, $OA'=OA''=\frac{r_0}{\sqrt{1-b^2}}=\frac{r_0}{\sqrt{1-\upsilon^2/c^2}}=\frac{r_0}{\cos\omega}$

Setting s = OS, where S the center of the Apollonian Circumference we get: OM= s - R & OH = s + R thus: $s=\frac{OM+OH}{2}=\frac{r_0}{1-b^2}$. Also $R=\frac{r_0.b}{1-b^2}$

Thereby, in Fig. 6 appear all the known mathematical means:

a. OA = r0 = the Harmonic Mean of HO & MO.
b. $OA'=\frac{r_0}{\sqrt{1-b^2}}$ = the Geometrical Mean of HO & MO.
c. $OS=s=\frac{r_0}{1-b^2}$ = the Arithmetic Mean of HO & MO.

Those three means constitute elements of a Geometric progression with first term r0 and ratio $\frac{1}{\sqrt{1-\upsilon^2/c^2}}$

I note that the aforementioned mathematical means have been observed also in the kinematic elements of the moving material point [20]. Furthermore, because R2 = AS.OS it follows that radius R is the Geometric Mean of the distances AS and OS.

Let us now move to the Electrostatics. Let, at position O, a positive electric charge q. Then an Electric Field, that follows the Inverse Square Law, is introduced in the Space. Let us now assume a grounded metallic sphere with center S and radius R (Fig. 7). We seek the Electric Potential V in said space. Inside the sphere, the electric potential is zero, as is also on the surface of the grounded sphere where, however, electric charges are created by induction.

The calculation of the electric potential in Space, excluding the sphere, will be performed utilizing the method of image (or mirror) charges [25].

According to this method, one negative mirror charge q´ is introduced inside the sphere and on the symmetry axis SO, such as $|q'|=\mu.q$ Its position on SO is on P, at an unknown distance a = SP from the center of the sphere.

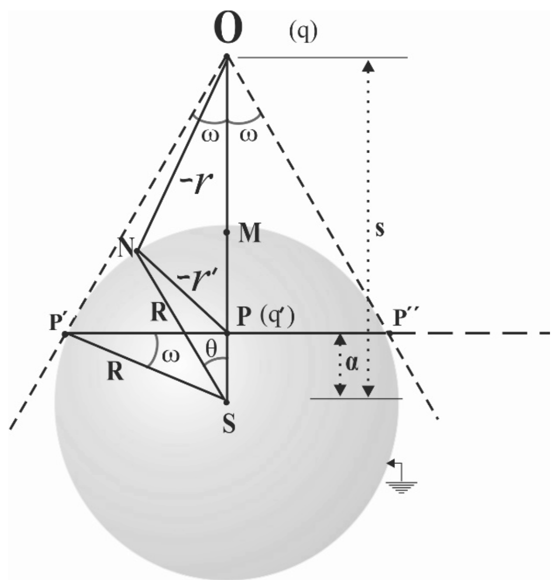

Figure 7. The Electrostatic Field outside a grounded conductive sphere with a charge q at O.

This problem can be solved once factor μ and distance a are determined. And that is because then, the Electrostatic Field that we seek, would be the field of two known electric charges q and q´ at a known distance PO = SO – a = s– a with the metallic sphere ignored.

At a random point, N of the sphere the Electric Potential would be:

$$V=K\left[\frac{q}{r}-\frac{\mu q}{r'}\right]=0 \tag{28}$$

where K a constant factor. This leads to equation:

$$R^2+s^2-2Rs\cos\theta=\frac{R^2}{\mu^2}+\frac{a^2}{\mu^2}-\frac{2Ra\cos\theta}{\mu^2} \quad \text{or}$$

$$R^2(\mu^2-1)+\mu^2s^2-a^2+2R(a-s\mu^2)\cos\theta=0 \tag{29}$$

And because (29) has to be valid for every angle θ, it follows that:

$$a-s\mu^2=0 \tag{30}$$

By replacing $a=s\mu^2$ In (29) we get:

$$R^2(\mu^2-1)-\mu^2 s^2(\mu^2-1)=0 \tag{31}$$

and since $\mu^2 \neq 1$ and $\mu > 0$ it follows:

$$\mu=\frac{R}{s} \tag{32}$$

and
$$a=\frac{R^2}{s} \tag{33}$$

From (33) it arises that radius R is the Geometric Mean of distances SP & SO, thus point P in Fig. 7 coincides with point A in Fig. 6. Thus, straight line P΄PP΄΄, vertical on OS at P, is the Polar Line of Point O with respect to the circle with center S and radius R, just like in the Apollonian Circumference in Fig. 6 [4].

Finally, from the rectangular triangle SP P΄ it follows: $\sin\omega=\frac{a}{R}=\frac{R}{s}=\mu$ thus:

$$\frac{|q'|}{q}=\mu=b=\frac{\upsilon}{c} \tag{34}$$

In other words, the ratio of the absolute value of the mirror charge over the initial electric charge, equals the ratio of speed υ of the material point over the speed of Light!! The grounded metallic sphere is tangential to the cone with vertex O and half-angle ω where:

$$\sin\omega=\frac{\upsilon}{c} \quad \left(\cos\omega=\sqrt{1-\upsilon^2/c^2}\right)$$

and the locus of tangent points is the circumference of circle with diameter P΄P΄΄.

Conclusion.

The kinematics of the material point in the Projective Space coincides with the Principles of Electrostatics!

The above conclusion leads us to ask the fundamental questions:
What is the Electric Charge?
What is electrically charged matter?
What is the Electric Field?

8.2. *What is the Electric Field?*

We usually ask: Why is the Gravitational Field so week? And we have yet to receive an answer accepted by all. We shall reverse the question here: Why the Electric Field is so much stronger than the Gravitational? Our previous deliberations might provide an answer here.

Let us consider a mass of linear density λ distributed along the Projective Straight Line E between points A΄ and A΄΄. Let us examine the Gravitational Field, resulting from this distribution, at position O which is located on the perpendicular on straight line E at point A΄, which position is at a distance r0 from it (Fig. 8).

According to the analysis in unit 7, the resultant total intensity of the Gravitational Field of this distribution is given by equation (25), which can be written:

$$E_0=\frac{2G\lambda\sin\rho}{r_0}=\frac{G\lambda\sin 2\rho}{r_0\cos\rho} \tag{35}$$

But, $\sin 2\rho=\frac{L}{\sqrt{r_0^2+L^2}}$, where L is the length of A΄A΄΄. Thus:

$$E_0=\frac{G\lambda L}{r_0\cos\rho\sqrt{r_0^2+L^2}}=\frac{G\lambda}{r_0\cos\rho\sqrt{r_0^2/L^2+1}} \tag{36}$$

Let us now consider that point A΄΄ moves away infinitely, mass increases infinitely and when L tends to infinity, the intensity of the Gravitational Field tends to:

$$E_0=\frac{G.\lambda}{r_0\cos\rho} \tag{37}$$

where: $\rho\to\frac{\pi}{4}\left(\cos\rho\to\frac{\sqrt{2}}{2}\right)$

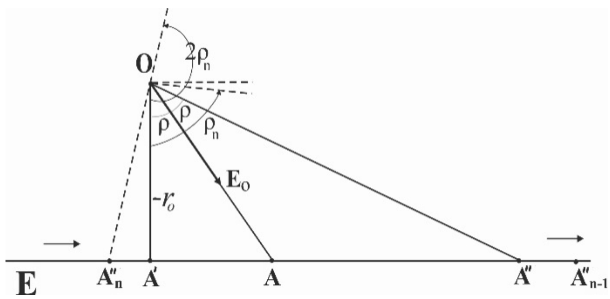

Figure 8. The Gravitational Field when the Conjugate position A΄ is at the Foot of the Perpendicular.

Thus, the infinitely distributed mass creates a finite field at position O. As straight line E is projective, point A΄΄ that moves away infinitely will re-appear from the “other side”, as An΄΄. Let us now consider that An΄΄ tends to A΄.

Then angle $2\rho_n\to\pi$, angle $\rho_n\to\frac{\pi}{2}$ and $\cos\rho_n\to 0$.

Therefore, the Field at O increases infinitely, as the denominator in (37) tends towards zero.

However, according the Principle of Duality in the Projective Plane, and according to our previous analyses, the linearly distributed mass A΄A΄΄ is equivalent to the Kinematics of the Material Point on the Projective Straight Line E. Thus, the question posed in the title translates to this: Could it be that the Electric Field is the Gravitational Field of a material point, which is moving very fast and whose one conjugate position (A΄΄) has already passed the point at infinity of the Projective Straight Line E, has appeared from the "other side" and is on its way towards the other conjugate position A΄?

This kinematics has been thoroughly analyzed in the Theory of the Harmonicity of the Field of Light [4] in all its sub-categories. We also observe that points A and the point at infinity of E, separate the conjugate points A΄ και A΄΄. However, when A΄΄n appears from "the other side" then points A and the point at infinity do not separate the conjugate positions A΄ και A΄΄n. Then the ellipsis of Fig. 3 tends to transform to a hyperbola that appears in superluminal speeds of matter measured by the LASC [2]. It might just prove to be that the unification of the four Fields, is a direct consequence of the unified structure of the four conic sections, which in its turn is an achievement of Projective Geometry. Thus, Galileo's aforementioned proclamation in unit 7, might after all be confirmed.

9. Summary

The Theory of the Harmonicity of the Field of Light is based on two fundamental acceptances:

1. The adoption of the natural philosophy of Werner Heisenberg and the school of Copenhagen, according to which a consistent natural description of the Cosmos shouldn't ignore the existence of the Observer or at least the instrument of observation and measurement and
2. The choice of the Projective Space as the Geometrical Space of its natural description. This choice is validated following the fundamental separation of the Perceptible Space, which is objective, and the Geometrical Space, that exists only in our minds. As all logically consistent Geometries are accepted in Mathematical Science, the adoption of a Geometrical Space by a Theory of Physics is free.

Further on, this theory adopts as its first fundamental hypothesis the second hypothesis of the Special Relativity Theory, properly modified. Then, during the study of the kinematics of the material point, the harmonic cross-ratio emerges practically automatically. However, as the Principle of Duality is a fundamental property of the Projective Space, this principle governs the development of the whole theory and leads to some very important conclusions in both the Relativistic and the Quantum Mechanics.

One application of the Principle of Duality in the research of the Gravitational Field, guides to the creation of the Inverse Square Law, during which the Briefest Computation Principle (BCP) also emerges practically automatically, a principle that is probably related to the Principle of Least Action, with its well-known important consequences on Classical as well as Quantum Mechanics.

Moreover, this work establishes that there is an internal relation between the Electrostatics and the Relativistic Kinematics of the Material Point in the Projective Space, which might offer future researchers, alternative routes of approach leading towards the final unification of the four known dynamic Fields.

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Λ- Units and Λ- Quantum of Action Have They Any Physical Sense?

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The list of Λ- Units determined by three Einstein's constants *c, κ* and Λ will be introduced. Some of them are already used since long time in relativistic cosmology e.g. lambda density of mass $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G} = \frac{\Lambda}{\kappa c^2}$ and the pressure of the physical vacuum $P_\Lambda = \frac{\Lambda c^4}{8\pi G} = \frac{\Lambda}{\kappa}$. Others like e.g. lambda quantum of action $H_\Lambda = \frac{c^3}{8\pi G\Lambda} = \frac{1}{c\kappa\Lambda}$ and lambda charge $Q_\Lambda = (H_\Lambda c)^{1/2} = \left(\frac{c^4}{8\pi G\Lambda}\right)^{1/2} = \left(\frac{1}{\kappa\Lambda}\right)^{1/2}$ are not yet used. Their physical meaning will be looked for. Have they any part to play in the theory of dark energy? Which part do they play in Hubble spheres? Can the mega quantum of action $H_\Lambda = \frac{1}{c\kappa\Lambda}$ be interpreted as mega physical event? The physical meaning of the Kittel's gravitational mega quantum of action will be also examined.

Keywords: Planck's Units, Stoney's Units, Kittel's Units, Λ- Units, Quanta of Action as physical events

1. Introductory Historical Data

As it is well known, at the turn of XIX and XX century, Max Planck (1858–1947) has not only introduced his very important constant $h = 6.626 \times 10^{-34}$ J · *s* called by himself "the elementary quantum of action", but he has also, at the same time, introduced his Natural Units determined by three universal constants *c* – velocity of light, *G* – Newton's gravitational constant and his constant *h* [1]. The scientific community had soon recognized the importance of Planck's constant that became the quantization parameter of the new born Quantum Mechanics. As regards Planck's Units the scientific community has longtime ignored them. So, Planck during 12 years has added to all his papers his Units believing that the community will finally recognize also the importance of his Units. Later especially the cosmologists have recognized their importance and nowadays in cosmology we speak even about the Planck's era existing at the beginning of the cosmic evolution.

Something similar happed earlier, when in 1874 the Irish physicist George Johnston Stoney (1826–1911), who is famous for his introduction of the term "electron" to describe the elementary unit of electricity and for his calculation of its value from Faraday's law of electrolysis, introduced his "physical units of nature" determined by *c, G* and the elementary unit of electric charge *e*.

$$l_s = \left(\frac{Ge^2}{c^4}\right)^{\frac{1}{2}} \quad t_s = \left(\frac{Ge^2}{c^6}\right)^{\frac{1}{2}} \quad m_s = \left(\frac{e^2}{G}\right)^{\frac{1}{2}} \tag{1}$$

The scientific community has recognized his discovery of the "electron" the existence of which was proved experimentally in 1897 by Joseph J. Thomson (1856–1940). But Stoney personally was convinced that the discovery of his natural units is more important and therefore he published his paper with the title *Physical Units of Nature* [2].

Stoney and Planck, believed that their natural units of physical quantities are more important than the used in mechanics conventional units cgs and now SI. Their belief was based on the fact that their units are not conventional but natural because they are determined by universal constants.

However, note that there are as many sets of so-called natural units as many there are charges of interactions because they are determined not only by *c* and *G* but also by the respective charges. However in Planck's, Stoney's and other units there are some units determined only by *c* and *G* e.g. the fore $\frac{c^4}{G}$, the power $\frac{c^5}{G}$. They are common to all sets of the natural units. Note that these units exist as coefficients in equations of General Relativity and relativistic cosmology [14].

Note that among Stoney's units there is a quantum of action $h_s = \frac{e^2}{c}$ smaller than Planck's one. A. Einstein [3], M. Planck [4], A.S. Eddington [5] and E.

Schrödinger [6] have taken seriously Stoney's quantum of action and have stressed its importance. I've presented more details in my paper published in *Physics Essays* [7]. Let's mention here only that all of them indicated that the Sommerfeld's fine structure constant $\alpha=\frac{e^2}{hc}=\frac{h_s}{h}$ is a ratio of the two quanta.

Note that using $h_s=\frac{e^2}{c}$ the Stoney's units can be written in a Planck like form as follows

$$l_s=\left(\frac{Gh_s}{c^3}\right)^{\frac{1}{2}} \quad t_s=\left(\frac{Gh_s}{c^5}\right)^{\frac{1}{2}} \quad m_s=\left(\frac{ch_s}{G}\right)^{\frac{1}{2}} \tag{2}$$

2. The Physical Meaning of the Physical Quantity Called Action

In physical processes, we are dealing with a transfer of momentum and of energy along certain path and during certain time and with the transfer of angular momentum when the angle of rotation increases. The physical quantity called action expresses just these physical processes.

Let's give a deeper explanation of the physical meaning of the used in physics quantity called action. In the uniform motion with constant velocity the action manifests itself as the transfer of constant momentum *mv* along certain path Δl.

$$mv \cdot \Delta l = action \tag{3}$$

and as the transfer of constant energy *E* during certain time interval Δt.

$$E \cdot \Delta t = action \tag{4}$$

In a rotational motion action manifests itself in the transfer of angular momentum *mvr* when the angle of rotation Θ increases.

$$mvr \cdot \Theta = action \tag{5}$$

In the accelerated motion the action is expressed as the product of the acting force *F* and the path Δl and time interval Δt of its acting.

$$F \cdot \Delta l \cdot \Delta t = action \tag{6}$$

Since $F\cdot \Delta l$ is the work *W* and $F\cdot \Delta t$ is the impulse of the force *I* we obtain

$$W \cdot \Delta t = action \tag{7a}$$

and

$$I \cdot \Delta l = action \tag{7b}$$

The known Polish physicist Czeslaw Bialobrzeski has often pointed out that action is the richest in meaning physical quantity because it expresses a physical dynamical process in which dynamical quantities are connected with space-time quantities [8]:

Dynamical quantities Space-time quantities

momentum · path
energy · time
angular momentum · angle of translation
force · path · time
work · time
impulse of the force · path

3. Some Historical Data Concerning the Physical Quantity Called Action

The physical quantity called *action,* (taking into account all kinds of motion), has been introduced into physics by Pierre Louis Moreau. de Maupertuis (1698–1759).

He formulated also the "Least Action Principle". The principle of the least action states that the trajectory of the transfer of momentum and energy between two point in the space and time is always the least.

However, when the Variational Calculus had been introduced into the examination of the mentioned principle it was recognized that action is submitted to a larger Variational principle because it is not only a minimum but, in certain cases, it can be also a maximum (see e.g. [9]). Therefore, it is now more correctly to call it principle of extremal action. It is often called also principle of stationary action. The principle was central in the classical physics and remains central in modern physics being applied in the theory of relativity (special and general), in quantum mechanics and quantum field theory.

In General Relativity applied to the cosmos i.e. with Einstein's cosmological constant Λ the Lagrangian of action has the following form.

$$S=\int\left[\frac{1}{2\kappa}(R-2\Lambda)+\mathcal{L}_{\mathrm{M}}\right]\sqrt{-g}\,\mathrm{d}^4x \tag{8}$$

where $g=\det(g_{\mu\nu})$ is the determinant of the metric tensor matrix, R is the Ricci curvature scalar, and $\kappa=8\pi Gc^{-4}$, where G is the Newton's gravitational constant and c is the speed of light in vacuum. The term $\mathcal{L}_{\mathrm{M}}$ is describing any matter fields appearing in the theory and the term Λ is the mentioned above Einstein's cosmological constant. The integral is taken over the whole space-time if it converges [10].

4. Planck's and Stoney's Quanta of Action are Very Small and Constant

In the SI system of units Planck's constant is given by the following numerical value $h = 6.626 \cdot 10^{-34}\, J \cdot s$.

There are also other very small quanta of action connected with the four fundamental interactions [11]. For instance, the quantum connected with the electromagnetic interactions h_{em} called also Stoney's quantum of action h_s, $h_{em} = h_s = \frac{e^2}{c}$ in cgs units and $h_{em} = h_s = K\frac{e^2}{c} = 7{,}69 \cdot 10^{-37}\, J \cdot s$ in SI system of units, where e is the elementary electrical charge and $K = \frac{1}{4\pi\varepsilon_0}$.

5. Micro Quanta of Action Interpreted as Elementary Extended Physical Events

In Special and General Relativity events are considered as space-time points. However, in the world of quanta the physical events are considered as spatially and temporally extended.

Thanks to Planck, Einstein and other known physicist we know that in the micro-world action is always spatio-temporally extended and quantized. Such quanta of action are constant and can be interpreted as elementary real indivisible physical events. The real transfer of energy, momentum and angular momentum is an real physical event that happens along certain real path, during certain real time interval or with the real increase of the angle of rotation. So a quantum of action has a nature of an indivisible real elementary spatio-temporal physical event and therefore in the micro world we are dealing with real indivisible elementary atoms of action. For example, A. S. Eddington called the Planck's and Stoney's indivisible portions of action, atoms of action" [12].

Note that Planck's Stoney's etc. quanta of action are the limitary smallest ones. The others are the multiple of them hn, $h_s n$ etc. (where $n = 1, 2, 3\ldots$).

Although in Heisenberg's uncertainty relations the pairs of quantities energy-time, momentum-space location, angular momentum-rotation angle are marked by uncertainties, their products show the elementary physical events expressed by the quanta h, $\frac{h}{2\pi}$.

The quanta of action h and $h_S = h_{em}$ are the basic parameters of quantization of other physical quantities.

Planck's quantum h is the source of energy quantization $E = h\nu$, of momentum quantization $mv = \frac{h}{\lambda}$ and angular momentum quantization $mvr = \frac{h}{2\pi}$ in real microphysical phenomena like emission, absorption and scattering of real photons. These phenomena are examples in which we are dealing with the real elementary Planck's h events [13].

Stoney's quantum of action $h_s = K\frac{e^2}{c}$ is the source of energy ($E = h_s\nu$) and momentum ($p = \frac{h_s}{\lambda}$) quantization in virtual (i.e. near to real) phenomena e.g. in the interaction between two elementary electrical charges in which we are dealing with an exchange of virtual photons. These phenomena are examples of elementary virtual (i.e. near to real) Stoney's h_s events [13].

Are there also mega quanta of action connected with Λ- and gravitational interactions? To answer this question let's introduce Λ- Units and Kittel's gravitational units.

6. Λ- Units and Λ- Mega Quantum of Action

Let's first introduce the Λ – units using the three Einstein's General Relativity constants: c, κ and Λ. The Λ-mega quantum of action is among these units. Since the exact numerical value of the so-called cosmological constant Λ is not yet known we shall use its approximate value: $\Lambda \sim 1.28 \cdot 10^{-52} m^{-2}$. (The whole list of Λ – units expressed using c, G and Λ and also with c, κ and Λ can be find in the Appendix of this paper)

$$l_\Lambda = \frac{1}{\Lambda^{½}} \sim 8{,}84 \cdot 10^{25}\, m \tag{9a}$$

$$t_\Lambda = \frac{1}{c\Lambda^{½}} \sim 2{,}95 \cdot 10^{17} s \tag{9b}$$

$$m_\Lambda = = \frac{1}{c^2\kappa\Lambda^{½}} \sim 4.75 \cdot 10^{51}\, kg \tag{9c}$$

$$E_\Lambda = \frac{1}{\kappa\Lambda^{½}} \sim 4{,}27 \cdot 10^{86}\, J \tag{9d}$$

Now we can introduce the Λ- mega quantum of action. Note its esthetical form

$$H_\Lambda = \frac{1}{c\kappa\Lambda} \sim 1.26 \cdot 10^{86}\, J \cdot s \tag{10}$$

As we can see the Λ- mega quantum of action is the inverse of the product of the three fundamental Einstein's constants of General Relativity c $\varkappa$ and Λ.

Let's add that several Λ – units are used, since long time, in General Relativity and Cosmology. For instance, the Λ - density ρ_Λ is known from Friedmann equation

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G} = \frac{\Lambda}{\kappa c^2} \tag{11}$$

Here are other examples. Let's begin with the Λ – pressure of the physical vacuum or Λ – density of energy

$$P_\Lambda = \rho_{E\Lambda} = \frac{\Lambda c^4}{8\pi G} = \frac{\Lambda}{\kappa} \qquad (12)$$

In the Lagrangian (1) we are dealing with the very great lambda force placed at the first place

$$\Lambda - \text{force } F_\Lambda = \frac{1}{\kappa} = 4{,}82 \cdot 10^{42}\ N \qquad (13)$$

Note that we can transform the Lagrangian (1) in such a way that the Λ – mega quantum becomes explicit and placed first

$$S = \int [H_\Lambda (R\frac{\Lambda c}{2} - \Lambda^2 c) + \mathcal{L}_M]\sqrt{-gd^4x} \qquad (14)$$

In such a way we can see how the action in General Relativity with Λ depends in a natural way on the Λ-mega quantum of action. This aspect is here explicitly shown. We can conclude that in General Relativity with Λ applied to the universe as a whole the action is quantized in certain sense. Is this fact connected with the existence of finite causally bounded zones because of the finite speed of transmission of interactions? For us observers on Earth our Hubble sphere constitutes our causally bounded zone. Every Hubble sphere connected with a chosen observational point constitutes such a causally bounded zone. There are Hubble spheres partialy superposed and other totally separated. It depends upon the place where the observer is. The Λ-mega quantum of action concerns every Hubble sphere Note that the aproximative numerical values of the lambda length $l_\Lambda \sim 10^{25}\ m$, lambda time $t_\Lambda \sim 10^{17} s$ and lambda volume $V_\Lambda \sim 10^{77}\ m^3$ are close to the approximate numerical values of the Hubble length $t_H \sim 10^{25}\ m$, time $t_H \sim 10^{17} s$ and volume $V_H \sim 10^{77}\ m^3$. They are of the same numerical order.

Note that the product of lambda energy (that is the source of the Hubble expansion) and lambda time gives the Λ- mega quantum of action. $E_\Lambda t_\Lambda = H_\Lambda \sim 1.26 \cdot 10^{86}\ J x s$.

This mega quantum is contained in the bounds of a Hubble sphere internally causally linked.

7. Charge of Λ- Interactions in a Causally Linked Hubble Sphere

In Quantum Mechanics, we use the notion of Planck charge $q = (hc)^{1/2}$. If we introduce in similar way the notion of lambda charge $Q_\Lambda = (H_\Lambda c)^{1/2}$ then the action Lagrangian can be written

$$S = \int [Hc\,(R\frac{\Lambda}{2} - \Lambda^2) + \mathcal{L}_M]\sqrt{-gd^4x} \qquad (15a)$$

or

$$S = \int [\,Q_\Lambda^2\,(R\frac{\Lambda}{2} - \Lambda^2) + \mathcal{L}_M]\sqrt{-gd^4x} \qquad (15b)$$

In such a way another aspect of the physical process called action i.e. the dependece on $Q_\Lambda = (H_\Lambda\ c)^{1/2}$ is shown in the Lagrangian.

Let's add still the Langrangian in which the lambda pressure P_Λ is shown and put at the first place

$$S = \int [P_\Lambda\,(\frac{R}{2\Lambda} - 1) + \mathcal{L}_M]\sqrt{-gd^4x} \qquad (16)$$

8. An Introductory Trial of Interpretation of the Lambda Action as Mega Physical Event in Hubble Sphere

Let's try to interprate the mega lambda action as a real Lambda Mega Event that happens in every Hubble sphere.

In a Hubble sphere the lambda force, $F_\Lambda = \frac{1}{\kappa}$ causes the lambda pressure $P_\Lambda = \frac{\Lambda}{\kappa}$ and therefore the sphere expands with the lambda acceleration $a_\Lambda = c^2 \Lambda^{½}$ and the dark energy performs a work equal to the lambda energy $E_\Lambda = \frac{1}{\kappa\Lambda^{½}}$ during the lambda time $t_\Lambda = \frac{1}{c\Lambda^{½}}$ The resulting momentum along the lambda radius equal to lambda length gives the lambda action. Since action consists in the transport of energy and momentum during a certain time and along certain path therefore the product of lambda energy and lambda time and the product of lambda momentum and lambda length give the lambda action $H_\Lambda = \frac{1}{c\kappa\Lambda}$ taking place during the lambda time and along the lambda radius. So the mega lambda action $H_\Lambda = \frac{1}{c\kappa\Lambda}$ is a mega event that happens in a Hubble sphere.

However, is such an interpretation well done or is it simply faulse? Let's enter into details, because, as we will see, some of the enumerate lambda units are only purely theoretical limitary paramiters.

Note still that the Λ- Units can be written in a Planck's like form as follows:

$$l_\Lambda = \left(\frac{8\pi G H_\Lambda}{c^3}\right)^{\frac{1}{2}}\ t_\Lambda = \left(\frac{8\pi G H_\Lambda}{c^5}\right)^{\frac{1}{2}}\ m_\Lambda = \left(\frac{c H_\Lambda}{8\pi G}\right)^{\frac{1}{2}} \qquad (17)$$

9. Some Clarifications of Lambda Units

To do a good physical interpretion of the lambda action we need first some clarifications of others lambda units. A good cognitive transparence of the lambda units is needed.

The lambda mass $m_\Lambda = \frac{1}{c^2\kappa\Lambda^{½}}$ is the lambda mass contained in every single lambda volume that is every singular Euclidian cube with the side equal to lambda length.

In a similar way the lambda energy $E_\Lambda = \frac{1}{\kappa\Lambda^{½}}$ is the lambda energy contained in every single lambda Euclidian volume.

The lambda density of mass $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G} = \frac{\Lambda}{\kappa c^2} \sim 6{,}86 \cdot 10^{-27}\ kg\ x\ m^{-3}$ is the lambda mass $m_\Lambda = \frac{1}{c^2\kappa\Lambda^{½}}$ contained in a lambda Euclidian volume divided by this volume.

In a similar way the density of lambda energy $\rho_\Lambda\ c^2 = \frac{\Lambda c^4}{8\pi G} = \frac{\Lambda}{\kappa} \sim 6{,}17 \cdot 10^{-10}\ J \cdot m^{-3}$ is a lambda energy $E_\Lambda = \frac{1}{\kappa\Lambda^{½}}$ contained in a lambda Euclidian volume that is divided by this volume.

Note that both of them used in the relativisic cosmology have a larger meaning. The mass and energy density concern the whole universe and not only the every single lambda volume.. Also the lambda force and presure (both used in the realtivistic cosmology) have such a general sense. We are dealing with them not only in single lambda volumes but in the whole universe. However for an observer only his Hubble sphere is causally bounded because of the finite velocity of the propagation of the interactions.

Let's add that we must be careful when interpreting the lambda units, because, as we could see, there is an underling Euclidian geometry. But in the relativistic cosmology we use not only geometry with the curvature parameter *k = 0,* but also with the parameters *k= + 1* and *k = -1.*

10. Interpretative Difficulties with Lambda Units

According to several authors (especially those who formulated the great inflation theory) the so-called cosmological constant *Λ* is timedependent and then all lambda units are also timedependent because they depend on lambda.

Multiplying the lambda mass with lamba acceleration we receive the lambda force. But doing so we are dealing with a Newtonian force. Can a Newtonian force play a part in the relativistic cosmology? However, the lambda force $F_\Lambda = \frac{1}{\kappa}$ is used in relativistic cosmology e.g. in the Lagrangian (1). Does this create a difficulty? How to interpretate the lambda accelaration? We know that the real acceleration of the universe expansion is not constant and it increases. This problem has to be resolved. We shall try to do it in this paper later.

The product of the used in cosmology lambda force, length and time gives us the lambda action. Has such great quantum of action any physical sence? Can it serve as a parameter of quantisation in the mega scale? If the universe is flat, as it seems to be, then the Hubble spheres causaly bounded are very small and in number infinite. Perhaps we have to introduce a Quantum Mechanics of mega scale. There is another mega quantum of action connected with gravitation. It must be also taken into account.

11. Kittel's Units for Hubble Spheres and Kittel – Mega Quantum of Action

In a Hubble sphere, we are dealing not only with dark energy (lambda energy) that is the source of its expansion but also with gravitational interactions that causes because of the curvature of space-time the gravitational attraction. Therefore, we have to look also for the mega gravitational units which concern the Hubble spheres as zones that are causally bounded also by gravitation. Ch. Kittel has indicated gravitational units for every concrete mass *m* [14]

$$l_G = \frac{Gm}{c^2};\ t_G = \frac{Gm}{c^3}\ ;\ m = m_G \tag{18}$$

If we introduce in Kittel's units the approximate gravitational mass $M_G \sim 2.107 \cdot 10^{52}\ kg$ (of ordinary matter plus dark matter) gravitationally bounded in a Hubble sphere then we can introduce a *gravitational* mega quantum of action

$$H_G = \frac{GM_G{}^2}{c} = \frac{\kappa c^3}{8\pi} M_G{}^2 \sim 9{,}88 \cdot 10^{85} J \cdot s \tag{19}$$

Using H_G we can write Kittel's Units in Planck like form as follows

$$l_G = \left(\frac{GH_G}{c^3}\right)^{\frac{1}{2}}\ t_\Lambda = \left(\frac{GH_G}{c^5}\right)^{\frac{1}{2}}\ m_\Lambda = \left(\frac{cH_G}{G}\right)^{\frac{1}{2}} \tag{20}$$

Note that $\frac{H_\Lambda}{H_G} = 1{,}27$. As we can see lambda mega quantum of action is greater than the gravitational one.

The Kittel's acceleration is given by

$$a_G = \frac{c^4}{8\pi GM_G} = \frac{1}{\kappa M_G} = 2{,}29 \cdot 10^{-10}\ \text{m s}^{-2} \tag{21}$$

The relation $\frac{a_\Lambda}{a_G}$ = 4,45 shows that lambda acceleration is 4,45 times greater than the gravitational acceleration and therefore the universe is expanding. But, this conclusion must be still examined in more detail way.

12. A Very Important Distinction Which Must be Made in Every Set of Units Determined by Universal Constants and Parameters

In order to make a cognitively transparent interpretation of units determined by universal constants and parameters we must make in every set of them a clear distinction between units which are purely theoretical limitary quantities and those which are permanently real constant or real time dependent parameters.

Let's give some examples. The Planck's, Stoney's etc. force $\frac{c^4}{G}$ and power $\frac{c^5}{G}$ are purely theoretical limitary parameters which indicate the greatest possible force and power in Nature [15]. They are not real parameters but purely theoretical quantities which show us that it is e.g. impossible to construct an accelerator in which the increase of the momentum of a particle pro second could be greater than $\frac{c^4}{G}$ and it is impossible to construct a collider the power of which could be greater than $\frac{c^5}{G}$. Both of them are limitary quantities [15]. But Planck's constant is a real portion of action, it is a real event which happens in immense cases in the processes in the micro-world. In micro-world the real processes of transfer of energy and momentum during certain time and along certain time are real and equal to Planck's constant *h* or to its multiple *hn* (i.e. where *n* =1, 2, 3…).

Let's add that some artificially introduced lambda units have not any physical sense. For example, when we combine the lambda energy with Boltzmann constant *k* we obtain lambda temperature

$$\Lambda - \text{temp. } T_\Lambda = \frac{c^4}{8\pi G k \Lambda^{1/2}} = \frac{1}{\kappa k \Lambda^{1/2}} = 3{,}093 \cdot 10^{91}\ K^o \quad (22)$$

This unit has not any physical meaning because the notions of heat and temperature cannot be applied to the so-called physical vacuum.

13. Can the Acceleration Units Determined by Universal Constants and Parameters Resolve the Problem of the Increasing Acceleration of the Expansion of the Universe?

If Λ is time dependent and decreasing than lambda acceleration is also time dependent and decreasing because the latest depends in a way directly proportional upon the square root of lambda

$$a_\Lambda = c^2 \Lambda^{1/2} \quad (23)$$

The gravitational acceleration a_G depends in a inversely proportional way upon the gravitational mass M_G contained in a Hubble sphere. M_G increases though its density decreases and it is in a random way distributed in the Hubble sphere. When galaxies are sufficiently close then they can collide because locally the gravitational attraction is greater than the expansive activity of dark energy which is uniformly distributed. Which is the resulting global acceleration in a Hubble sphere? We must be aware that the solution of this problem is much more sophisticated and cannot be resolved by simple comparison of units determined by universal constants and parameters. The acceleration notion of the expansion of space-time is in no way an acceleration of a body.

14. Conclusions

In the micro-world, the gravitation and lambda interactions are very feeble and therefore totally negligible e.g. gravitational interactions between two neutrons. However, both interactions have to be taken into account in the mega scale in which they are respectively very strong and can in no way be neglected. Perhaps the two mega quanta of action play a real general part as mega events in the development of our observational world.

Has the beautiful, from the estetical point of view, lambda quantum of action $H_\Lambda = \frac{1}{c\kappa\Lambda}$ to say anything in the relativistic cosmology or nothing? The future will show if it will be recognized by the scientific community.

Perhaps General Relativity is able to be quantized in the mega scale and there will be constructed a mega scale quantum mechanics for it.

All considerations presented in this paper are only hypothetic statements that must be further investigated. We must be aware, however, that in all systems of units determined by universal constants and cosmological parameters (called often also *constants*) there is a quantum of action determined by these constants and parameters. Such quanta of action can have some physical meaning which must be examined. H_Λ and H_G belong to them.

Final concluding hypothesis. Perhaps the visible part of our universe is a result of two action processes: lambda and gravitational. In such a case the mega lambda event and mega gravitational event are time dependent and increasing. They are also referential because every observer in our universe has its own observational world performed by two time depended portions of action H_Λ and H_G. In these two action processes, we are dealing with energy and momentum transfer during the time equal to the age of our universe and along the radius of our observable world.

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Appendix: The List of *Λ*- Units Determined by *c, G (or κ = 8πGc⁻⁴)* and *Λ* ~ 1.28 · 10⁻⁵²m⁻²

Λ - length $l_\Lambda = \frac{1}{\Lambda^{½}} \sim 8{,}84 \cdot 10^{25}\, m$

Λ - surface $A_\Lambda = \frac{1}{\Lambda} \sim 7{,}81 \cdot 10^{51} m^2$

Λ - volume $V_\Lambda = \frac{1}{(\Lambda^3)^{\frac{1}{2}}} \sim 6{,}9 \cdot 10^{77} m^3$

Λ - time $t_\Lambda = \frac{1}{c\Lambda^{½}} \sim 2{,}95 \cdot 10^{17}$s

Λ - mass $m_\Lambda = \frac{c^2}{8\pi G\Lambda^{½}} = \frac{1}{c^2\kappa\Lambda^{½}} \sim 4.75 \cdot 10^{51}\, kg$

Λ - momentum $p_\Lambda = \frac{c^3}{8\pi G\Lambda^{½}} = \frac{1}{c\kappa\Lambda^{½}} \sim 1.42 \cdot 10^{60}\, kg \cdot m \cdot s^{-2}$

Λ - energy $E_\Lambda = \frac{c^4}{8\pi G\Lambda^{½}} = \frac{1}{\kappa\Lambda^{½}} \sim 4{,}27 \cdot 10^{68}\, J$

Λ - mass density $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G} = \frac{\Lambda}{\kappa c^2} \sim 6{,}86 \cdot 10^{-27}\, kg \cdot m^{-3}$

Λ – pressure of physical vacuum $P_\Lambda = \frac{\Lambda c^4}{8\pi G} = \frac{\Lambda}{\kappa} \sim 6{,}17 \cdot 10^{-10}\, N \cdot m^{-2}$

Λ - energy density $\rho_\Lambda\, c^2 = \frac{\Lambda c^4}{8\pi G} = \frac{\Lambda}{\kappa} \sim 6{,}17 \cdot 10^{-10}\, J \cdot m^{-3}$

Λ – acceleration $a_\Lambda = c^2 \Lambda^{½} \sim 1{,}02 \cdot 10^{-9}\, m \cdot s^{-2}$

Λ – force $F_\Lambda = \frac{1}{c^2\kappa\Lambda^{½}} c^2\, \Lambda^{½} = \frac{1}{\kappa} = 4{,}82 \cdot 10^{42}\, N$

Λ – action $H_\Lambda = \frac{c^3}{8\pi G\Lambda} = \frac{1}{c\kappa\Lambda} \sim 1.26 \cdot 10^{86}\, J \cdot s$

Λ – charge $Q_\Lambda = (H_\Lambda\, c)^{½} = \left(\frac{c^4}{8\pi G\Lambda}\right)^{½} = \left(\frac{1}{\kappa\Lambda}\right)^{½} \sim 1{,}94 \cdot 10^{47}$

Λ – power $W_\Lambda = \frac{c^5}{8\pi G} = \frac{c}{\kappa} = 1.445 \cdot 10^{51}\, J \cdot s^{-1}$

Λ – scalar (anty-gravitational) potential $V_\Lambda = \frac{c^2}{8\pi} = 3.58 \cdot 10^{15} m^2\, s^{-2}$

Λ – temperature $T_\Lambda = \frac{c^4}{8\pi G k\Lambda^{½}} = \frac{1}{\kappa k\Lambda^{½}} = 3{,}093 \cdot 10^{91}\, K^o$

The Stueckelberg Off-Mass Shell Model for Particle Interaction as Template for New Insight into Hidden Dimensions of Time and Mass

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Time, space, energy and mass form the four-fold conceptual basis for gauging physical reality. Through a reappraisal of the manifest covariant Stueckelberg off-mass shell model and its recent key extension by Horwitz and Piron, this paper will seek to reveal fresh important perspectives on the four yardsticks of physical reality alluded to above, as they are applied in both classical and quantum mechanics. In particular, it will be demonstrated how the conceptual foundations of these branches of *causal linear* physics can be formulated under the off-mass shell ansatz to disclose the common structural edifice of a time-symmetric (*acausal*) mechanics that is fueled by the primordial dynamic *non-linear* multidimensional quantum engine which underpins the extant *causal* wheelwork of nature. In this regard, it will be shown how the recently discovered phenomenon designated as "entanglement in time" can only be adequately explicated via the off-mass shell model, and is beyond the scope of current standard on-shell models of non-relativistic quantum field theory. The off-mass shell model will also be shown to possibly provide satisfying conceptual reasons for documented empirical phenomena currently beyond the reach of orthodox paradigms, such as low-energy nuclear reactions (LENR), and reveals new perspectives on gravitation as well, supporting the revolutionary worldview of the possible non-singular nature of the "black hole" entity, and the associated revelation of general relativity as being an approximation of a more over-arching eikonal gravitational equation of quantum nature. Experimental tests will be advanced to test the off-mass hypothesis with the unprecedented aim of placing various poorly understood phenomena in particle physics squarely in the arena of the manifest covariant Stueckelberg theory. These include neutrino flavor oscillations, K-meson interactions, new insight into tachyon/bradyon dynamics, and a new high-temperature Bose-Einstein condensate (BEC) which is predicted to exist. This last is most important since in a more direct practical sense, it might herald the development of future technology that incorporates the BEC as an integral operating component in the first hybrid matter/non-matter energy-efficient mechanical, optical and electrical systems, implying novel engineering protocols amenable to all the tools of non-linear and quantum optics.

Keywords: Hidden dimensions, Particle ineractions, Stueckelberg model

1. Introduction

One of the deepest and most difficult problems in theoretical physics in the past century has been the construction of a simple, well-defined one-particle theory which unites the ideas of quantum mechanics and relativity. In fact, such a problem has existed in the consistent formulation of classical relativistic dynamics as well. A central problem in formulating such a theory is posed by the description of a state of a system that has spatial properties. Early attempts, such as the construction of the Klein-Gordon equation and the Dirac equation were inadequate to provide such a theory since, as shown by Newton and Wigner [1], they are intrinsically non-local, in the sense that the solutions of these equations cannot provide a well-defined probability distribution. Relativistic quantum field theories, such as quantum electrodynamics, provides a manifestly covariant framework for important questions such as the Lamb shift and other level shifts, the anomalous moment of the electron and scattering theory, but the discussion of quantum mechanical interference phenomena and associated local manifestations of the quantum theory are not within their scope; the one-particle sector of such theories display the same problem pointed out by Newton and Wigner since they satisfy the same field equations. On the other hand, the non-relativistic quantum theory, making the explicit use of the Newtonian notion of a universal, absolute time, provides a description of such a state in terms of a square-integrable function over spatial variables at a given moment in time. This function is supposed to develop dynamically, from one moment of time to another, according to Schrodinger's equation, with some model Hamiltonian for the system. Also, the non-relativistic quantum theory carries a completely local description of probability density; it can be used as a rigorous basis for the development of nonrelativistic quantum field theory, starting with the construction of tensor product spaces to build the Fock space, and on that space to define annihilation and creation operators.

Now, the essential properties of the quantum theory, such as the notions of probability, transition amplitudes, linear superposition, observables and their expectation values, are realized in terms of the structure of a Hilbert space. Hence, a central problem in formulating such a theory is posed by the requirement of constructing a description of the quantum state of an elementary system as a manifestly covariant function of a manifold of observable coordinates which belongs to a Hilbert space.

Let us examine the mathematical reasons why the standard nonrelativistic quantum theory cannot be used to predict interference in time. For example, Ludwig [2] has pointed out that the time variable cannot be a quantum observable, since there is no imprimitivity system involving this variable. (i.e., no observable exists that does not commute with coordinate time (t) in the nonrelativistic theory). Note that the Hamiltonian of the standard theory evolves quantum states in time, but does not act as a shift operator since it commutes with t. Dirac [3] has argued that if t was an operator, then the resulting t, E commutation relation would imply that the energy is not bounded from below (with no gaps), from which he concluded that the time cannot be an observable in the nonrelativistic quantum theory. Basically, in the general practice of the use of the nonrelativistic quantum theory and as the axiomatic treatment of Piron [4] shows, the Hilbert space of the quantum theory is constructed of a set of wave functions satisfying a normalization condition based upon integration over all space, for a single particle $\int|\psi_t(x)|^2 d^3x \leq \infty$, for each value of the parameter t. Since t is not integrated over, ψ_t does not carry a probability distribution for values of t. Consequently, since the Hilbert spaces associated with different times are distinct, it therefore loses its interpretation as a description of a state.

Moreover, as pointed out from Wick, Wightman and Wigner [5], a Hilbert space decomposes into incoherent sectors if there is no observable that connects these sectors; hence, if there were a larger Hilbert space containing a representation for t, the absence of any observable that connects different values of t in the standard nonrelativistic physics would induce a decomposition of the Hilbert space into a (continuous) direct sum of superselection sectors [4]. Therefore, no superposition of vectors for different values of t would be admissible.

This description of a state is inconsistent with special relativity from both mathematical and physical points of view. The wave function, as a function of spatial variables, and parameterized by the Newtonian time, described in the frame of inertial motion with respect to another and related to it by a Lorentz transformation, undergoes a transformation which makes its interpretation in the new frame very difficult. In particular, if an event is predicted by this function with a certain probability to take place at the point $\mathbf{x}$ at the time t in the original frame, that event should occur with the same probability, as seen in the new frame, at the point $\mathbf{x}'$ at the time t'. According to the structure of the Lorentz transformation, the time t' depends on the location of the point $\mathbf{x}$ in the original frame as well as t so that it is inconsistent to label the wave functions in the new frame according to t', now no longer a parameter, but partly dependent on the variable $\mathbf{x}$, with a value associated with the probability distribution defined by the original wave function. The wave function, described in a frame in motion with respect to the frame in which the state is originally defined takes on the meaning of a probability amplitude at a set of different times, depending on the value of the spatial variables. The variables of the phase space, under the transformations of special relativity, are mapped into a new set in which the time parameter for each of them depends on the spatial location of the points; in addition there is a structural lack of covariance of the phase space variables themselves. The restoration of the original description by extrapolating the dynamical evolution through this family of times is not satisfactory since, in addition to being highly impractical, the specification of a state should be independent of the model for the dynamical evolution of the system.

On the other hand, observed interference phenomena, such as the Davisson-Germer experiment, showing the interference pattern due to the coherence of the wave function over the spatial variables at a given time, clearly should remain so when observed from a moving frame (detectors in motion relative to the original experiment). It is clear, from the (spatial) double slit interference of light, which travels at a fixed velocity, that the sections of the wave front passing through the two slits must pass at different times if they are to arrive simultaneously at the detection plate off-center. The arrival of pieces of a *particle* wave packet which have passed through the two spatially separated slits simultaneously on a plate off-center is made possible by the dispersion of momenta in the wave packet, permitting a range of velocities. If the two contributions to the linear superposition on the screen were not taken to be simultaneous at the two slits, they would not interfere, since they would have originated on wave packets at different values of time. In this case, the parts of the wave function that interfere appear to pass the scattering centers (or slits, in a double slit experiment) at different times, and would not be coherent in the framework of nonrelativistic theory.

Hence from all the above considerations, one would expect that there is a more general, covariant, description of the state of a system, which would predict such an interference pattern, modified only by the laws of special relativity when observed from a moving frame. The resolution of the problem of localization posed above lies in the formulation of a quantum theory for particles which are not precisely constrained to a pointwise mass-shell; or in different words, the relativistic particle is not restricted, in its definition, to an irreducible representation of the Poincare group as advocated by Wigner in his fundamental paper of 1939 [6]. The kinematic definition

$$E^2 = \mathbf{p}^2 + m^2$$

is, of course maintained but the quantity m^2 is to be considered as a *dynamical variable*, with values determined by the interactions in the physical system [7]. To construct a theory of particles on a phenomenological level, which does not make direct use of the energy stored in the fields that surround them, it is necessary to admit this degree of freedom. For example, the electron is seen to have an effective mass, in the context of Lamb shift calculations [8] differing from its free value when it is bound to an atom, and the effective masses of nucleons bound in the nucleus differ from their free values.

At any rate, considering the mass as a dynamical variable in the above equation, $\mathbf{p}$ and E must be considered as independent dynamical variables. The complementary variables dual to these are $\mathbf{x}$ and t, and we must conclude that these are also independent dynamical variables.

In the succeeding sections of this paper, we shall explore such an off-mass shell theory based on the original work of Stueckelberg [9] and reinitiated by Horwitz and Piron starting in 1973 [7], which we will concisely designate as the SHP theory, and describe some important results that have been achieved in this framework.

In section 2 we begin by demonstrating how the SHP model, unlike other theories/orthodox paradigms, can possibly explain all the phenomena that have been continually observed over the past few decades in connection with the unconventional nuclear reaction dynamics associated with the so-called "cold fusion" scenario. This has been renamed more appropriately low energy nuclear reactions (LENR) or lattice assisted nuclear reactions (LANR). We remark that the cold fusion phenomenon, originally spurned and summarily discredited by the mainstream nuclear physics community, may garner greater understanding and achieve higher level of respectability by viewing it through the lens provided by the SHP off-mass shell theory. Correspondingly, this might lead to greater respectability for the off-mass shell model itself in both quantum and classical mechanics, provided the dynamics of cold fusion could be viewed as the so-called "poster child" for off-mass shell sub-atomic interactions.

Section 3 argues that the new quantum phenomenon termed "entanglement in time" [11] can only be properly understood through the off-mass shell models of QED [8]. This then would be the first experimental evidence of a phenomenon that cannot be explained through current orthodox models of either relativistic quantum field theory or nonrelativistic quantum mechanics, and as such deserves the prompt attention of all in the physics community. Along the same lines, section 4 demonstrates how the currently ascertained sacrosanct theory of Einsteinean general relativity might have an Achilles heel as well, which comes to light by the treatment of so-called "gravitational collapse" through the quantum off-mass shell Stueckelberg-Schrodinger equation [10]. Indeed, general relativity might actually be shown to be an *emergent* phenomenon – specifically an approximation of a more precise 5-dimensional non-linear eikonal equation.

In section 5 it is demonstrated that probability of neutrino mass oscillation (flavor mixing), calculated using the SHP ansatz, provides an expression for the probability of transition from one neutrino flavor to another that has similarities with the result of the conventional theory, but differs in details that are experimentally testable.

Section 6 continues this theme by focusing on the SHP formalism in analyzing the stability (decay dynamics) of the neutral K-meson (K^0). The mass differences, between the long-lived and short-lived states, predicted in the conventional theory of two-state particle decay is shown to differ from those calculated using the SHP theory. This discrepancy, which can be experimentally verified, arises due to the key markedly different treatment of time with the SHP model.

The existence of the hypothetical tachyon particle receives a boost of support in section 7, since in the SHP theory all values of energy and momentum must be considered, and not just those values that yield on-shell masses. Accordingly, utilizing potentials dependent on the unique concept of time alluded to above, SHP theory, which allows for transitions between mass states, describes a system that allows for quantum transitions across the light cone. In other words, tachyon creation within the context of the off-mass theory, does not require continuous acceleration of a classical time-like entity through the light cone. Instead tachyons are created or annihilated by quantum transitions.

Finally, in section 7 it is demonstrated that, by incorporating particles and antiparticles in statistical mechanical models, utilizing the so-called scalar universal evolution parameter τ (t, the time coordinate is treated as an operator and as a quantum observable), in off-mass shell models, that a high temperature phase transition is predicted for Bose-Einstein condensation (BEC). Similar to the technology that could be possibly developed by applying the off-mass shell dynamics attendant with LENR, such new high temperature BEC dynamics might pave the way for future technology that incorporates the BEC in an integral fashion as an operating component for development of the first hybrid matter/non-matter energy efficient mechanical, electrical and optical systems.

2. Low Energy Nuclear Reactions and Off-Mass Shell Dynamics

Off-mass shell dynamics may possibly play a key role in room temperature energetic phenomena, particularly those exhibiting documented persistent unexplained anomalies that have been unable to be produced on-demand. One of the most controversial of these is the low energy nuclear reaction (LENR) process originally termed "cold fusion". This potential paradigm revolution was inaugurated in 1989 through the corresponding work of Pons and Fleischmann [12]. It immediately ignited a firestorm of controversy and instant notoriety for these researchers, whose work was not consistently reproducible. This research was summarily discredited by mainstream physics, which continues to this day to stringently uphold the thesis that nuclear fusion can only occur at high energies. However, over the two decades since this phenomenon surfaced, continued research in this frontier field of energy research has shown unquestionably that low energy nuclear reactions are genuine, and that accompanying excess energy, excess power gain, commensurate helium-4 (alpha particle) production, and minimal neutron and gamma ray emissions are undeniable, heralding an important new clean form of energy production enabled by highly loaded metal hydrides. Also, since these particular types of LENR have been observed to occur exclusively on the metal surfaces, afforded by the loading of its lattice sites by deuterium or hydrogen, as opposed to its bulk, the process has been recently been more appropriately termed: lattice assisted nuclear reactions (LANR). These surfaces have been called by Storms [14], Nuclear Active Environments (NAE).

As an important byproduct of the eventual demonstration and certification of the viable nature of LANR and its relatives, physicists have accordingly determined that the original difficulties associated with sustained energy production in the early studies of LENR can be attributed to several factors: (1) the complexities of the experimental set-ups involving many materials (including impurities), (2) numerous variation of experimental and input parameters, (3) poor loadings and a poor appreciation of the requisite metallurgy and engineering, (4) the reported results were small effects. This situation has prevented the development of a coherent theoretical understanding or working theoretical model of the phenomenon, which can be used as a guide in carrying out the new experimental tests to sort out essential parameters and controls needed to achieve reproducibility on-demand.

The branching ratios for the three channels for standard plasma fusion are given below [13]:

$$d{+}d \rightarrow t(1.01\,MeV) + p(3.02\,MeV), (Q = 4.04\,MeV), 51\%\ (a)$$
$$d{+}d {\rightarrow}^{3}\,He(.82\,MeV) + n(2.45\,MeV), (Q = 3.27\,MeV), 49\%\ (b)$$
$$d{+}d {\rightarrow}^{4}\,He(.076MeV) + \gamma(23.7MeV), (Q = 23.8MeV), .08\%\ (c)$$

where Q is the kinetic energy released by the reaction in the center of mass Lorentz frame. There are also extensive experimental claims of a variety of elemental transformations taking place inside or on deuterated palladium and also in other metal-hydrogen alloys [14, 15]. The transmutation data are much harder to ignore than ^4He production which might be due to environmental contamination, and they represent a strong evidence for nuclear reactions, although not necessarily fusion.

Attributing the orthodox thermonuclear reaction channels shown above in (a), (b), or (c) as the only access to nuclear fusion, considering the possibility d + d fusion in a d_2 molecule, the standard calculation for the penetration factor through the Coulomb barrier at room temperature gives reaction rates that are 50 orders of magnitude less than the claimed empirical result of Pons and Fleischmann [12]. This, along with the difficulty of reproducibility on-demand, are the primary reasons the claims of nuclear fusion via LENR protocols were summarily discounted as wrong or due to errors in measurement.

Accordingly, it was calculated that the required electron mass would have to be $10m_e$ in order for the tunneling rate in d_2 to explain the Pons-Fleischmann results for excess heat. There is beam-scattering evidence of enhanced screening in deuterated palladium. This enhancement has been attributed to the higher density of surrounding electrons without resorting to heavy electrons.

Nevertheless, although the mass of elementary particles in a bound (stationary) state are, by current paradigms assumed to be constant, the required increase in electron mass for the viability of the LENR scenario alluded to above, caused physicist Mark Davidson to consider such off-mass shell dynamics as a possible actual occurrence which would then explain, unlike any previous ideas advanced, the effectiveness for all the anomalous empirical effects documented in association with LENR [16].

Here is the scenario Davidson speculates for off-mass shell enabled nuclear reactions. Consider two neighboring deuterons in a palladium lattice. The masses of the deuterons and possibly that of the nearby electrons too are moving slowly off the mass shell due to interaction with the condensed matter system according to an off-shell SHP theory. We further assume that the final state masses in the fusion process are the usual rest masses of the particles, that special conditions inside the solid (such as the formation of NAE), are required for this process to occur, and that these are roughly equivalent to the conditions required for anomalous LENR effects to occur. Finally, we assume that after a period of time, the system returns to normal and all masses return to their standard values, except for those that have experienced a nuclear reaction. It is proposed that an active d + d pair reduces its mass slowly until it is approximately equal to the mass of the ^{4}He (alpha particle), about a .63% reduction, or 11.9 MeV per deuteron. Although this is a radical assumption which has never been observed in nature, it could be possible if an off-mass shell effective Lagrangian were describing a small volume of the lattice. Resonant tunneling would occur if the sum of the two deuterons equaled the mass of ^{4}He, regardless of the type of screening. In this case, any photon produced would have low energy. The increase in electron mass enhances tunneling, and the decrease of the deuterium mass allows resonant tunneling directly into an alpha particle and the suppression of neutrons and tritium. Therefore, off-mass shell-enabled fusion depends on two tuning parameters – the mean deuteron mass and the mean electron mass. Figure 1 illustrates these ideas qualitatively.

We see in Figure 1 that the main fusion is occurring at the instant that $\overline{m}_{2d} = m_a$, but this is not when most of the heat is added to the solid, because the Q value for fusion is essentially zero then as the masses of the two deuterons sum to very nearly the mass of an alpha particle at resonance. The energy has been given up to the condensed matter prior to fusion due to the continuously varying masses, and transients continue until all masses eventually return to their on-shell values.

Assuming that this theory is correct, and that masses of deuterons are decreasing slowly as the loading factor *d/pd* increases, then we can make a very simple experimental prediction that is testable. It is expected that the Q values for the reactions in the channels (a), (b), and (c) will be observed to decrease with time as the loading progresses, in those systems that exhibit excess heat production. Perhaps observable effects will be seen in systems where no excess heat is produced, as excess heat requires that the deuteron mass must decrease until it is resonant with the ^{4}He channel, but less mass change could still reduce the Q values for the reactions. As the phase space for all three channels will change as the Q values diminish, then the relative branching ratios for the three channels will change as well. The first channel that would zero out would be the $^{3}He + n$ channel. Then once the masses of the deuterons decrease below the threshold for producing the $t + p$, the only channel open would be the $^{4}He + \gamma$. Such a reduction in Q values for a fundamental nuclear reaction has never been observed before. It would be a clear and undeniable proof that the deuteron rest mass was changing in these settings. It is also expected that the energy of the gamma rays will be reduced from the expected 23.77 Mev continuously down to zero at the resonant point.

Also, the fact that resonance in this off-mass shell model is required for significant fusion, serves as a safeguard which prevents harmful radiation from being produced because at resonance there is no energy (or Q) left over to produce it. This would be an enormous benefit to this form of nuclear energy if it can be verified experimentally. The energy given to the palladium lattice by the fusion event is actually given up prior to the event as reflected in the reduced masses of the two deuterons which subsequently fuse into ^{4}He with zero Q. The d_2 molecule has been treated as a closed system, but the slow mass variation would require continuous soft electromagnetic interaction with the lattice. As the deuteron mass decreases, the $n + {}^{3}He$ phase space reaches zero before the $t + {}^{3}He$ channel does. Moreover, as the electron mass is supposedly increasing during this time, the $t + {}^{3}He$ channel benefits from a relatively lower Coulomb barrier. This would explain why significantly more tritium is produced than neutrons. The energies of the charged particles produced (tritons, Helium-3 nuclei, protons and alpha particles) are functions of the deuteron rest mass. If the deuteron rest masses are changing, then this would show up as a broadening of the energy spectrum for a given type of charged particle. Moreover, after all reactions have ceased we expect all particles to return to their normal rest masses and so the energy stored in the electrons' higher masses would be returned to the solid too. This relaxation process may take some time, and might result in apparent heat production after all driving factors such as electrolysis have after-death phenomenon which has been observed in LENR reactions [15].

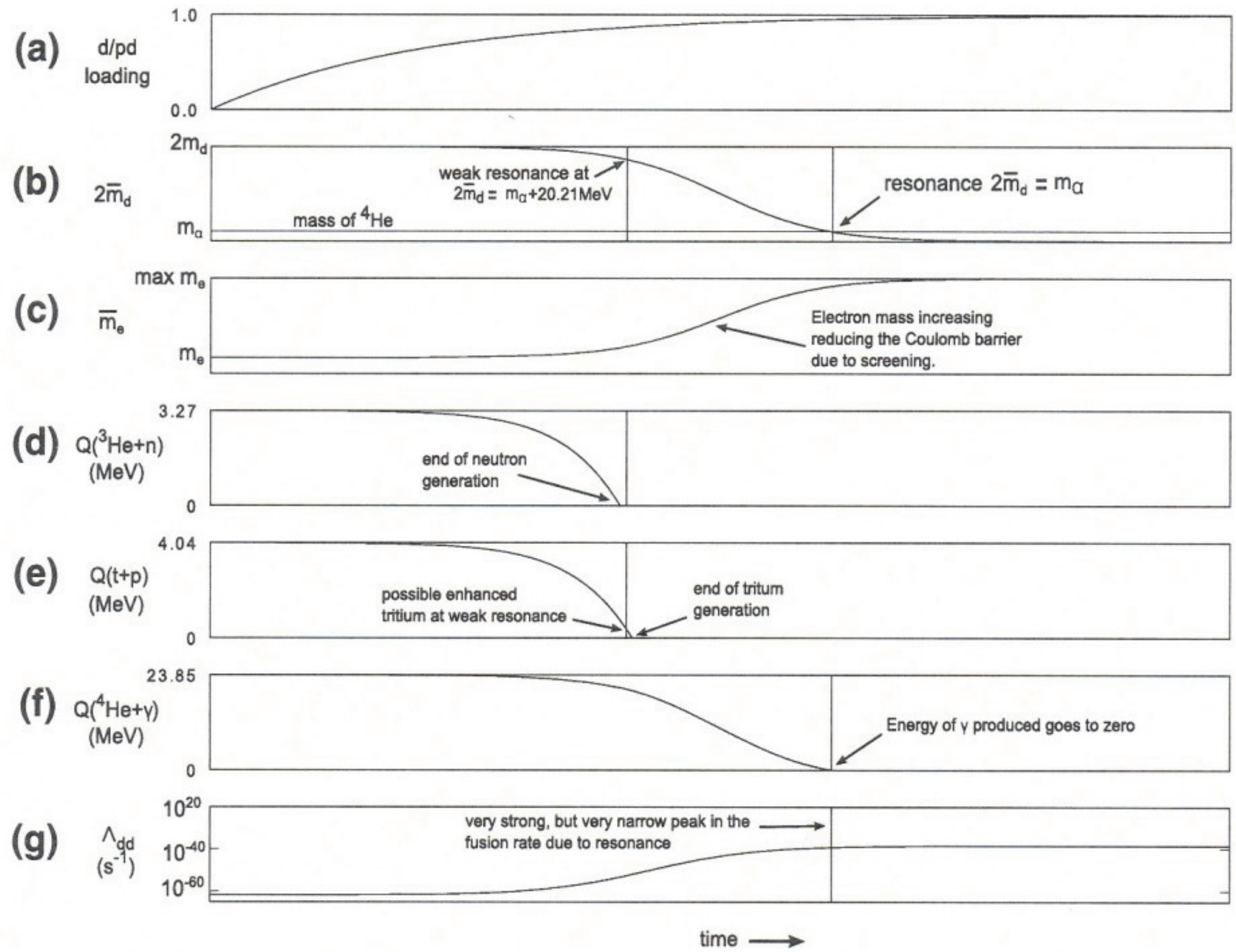

Fig. 1. Qualitative time evolution of $d + d$ mass variation to resonant tunneling and fusion **a** shows a plausible but fictional loading process; **b** shows presumed decrease in deutron mass; **c** shows presumed increase in electron mass; **d** shows the resulting reduction in the energy release from d + d $\rightarrow^3$ He + n; **e** the energy release from d + d $\rightarrow^3$ He + t; **f** the energy release from d + d $\rightarrow^4$ He + γ; **g** the non-resonant fustion rate of Koonin and Nauenberg with the very sharp resonant peak superimposed.

Another effect that has been noticed in these experiments is micro-crater damage to the palladium surface after LENR activity. Although it has commonly thought that these craters were evidence of micro-explosion, the model expounded upon here could give a different explanation. If all (or a substantial fraction) of the electron masses were to increase in a small local region, the lattice spacing would be reduced approximately inversely proportional to their average mass, and this would cause a severe mechanical deformation of the surface of the palladium which could leave a crater caused by shrinkage. This would explain how transmutations are often observed to have occurred in or near these craters, as these are locations where the mass variation would be expected to have been the largest. Later, after all reactions were over and the electrons returned to their original mass, the volume would re-expand and possibly look like excess material on the rim of a crater.

Admittedly, Davidson was circumspect in his development of these speculative ideas [16] and was quick to stop short of endorsing this hypothetical physical scenario in 2013. However, in his more recent papers [17, 18] he takes a considerably more positive stance towards his thesis, by expanding the examination to incorporate many other protocols dealing with special circumstances in condensed matter that could represent instances of LENR dynamics. In fact, in these recent articles, he remarks that he sees no other option for explaining all LENR phenomena alluded to previously, than the variation of mass of the various interacting nucleons. Also, in [17] he remarks that many other poorly understood phenomena - some dealing with everyday macro-objects at room temperature, and even nuclear processes within the Earth's mantle or atmosphere might have actually represented unheralded signatures of LENR effects and/or nuclear scale energies in common objects. Some of these interesting examples include:

1) The 'Reifenschweiler effect', originally discovered in early 1960's, showing that the beta-decay of tritium (half-life 12.5 years) is reduced visibly by about 25-30% when the isotope is adsorbed into 15 nm. titanium clusters in a temperature window between 160-275°C. The reported decay rate reduction can be explained if the tritium mass and consequently the phase space for the decay were reduced.

2) Oppenheimer-Phillips processes whereby a deuteron gives up a neutron to a metal nucleus through quantum tunneling. Enhanced electron screening is required for the cross sections of these reactions to be large enough to account for the observed effects. Increased electron mass would provide such a mechanism.

3) Radioactive isotopes implanted into metals at low temperature $\sim 12^oK$ show variations of decay rates in some cases, which could be due to electron screening, which in turn could be due to mass variation. These results have not been completely reproducible so far, and the theory proposed so far, and the theory proposed based on conventional electron screening has been criticized.

4) Time varying decay rates may indicate mass variation. Experiments show decay rates varying with time for a number of isotopes. Frequency analysis has shown annual, diurnal, and approximate monthly variations. Again these results are controversial, and may involve solar neutrinos as the reason for the decay-rate variation.

5) The unexplained disparity between the atmospheric ratio of ^3He to ^4He compared to the much larger ratio of these isotopes within thermally active locations on the Earth's surface; such as volcanic emissions, geothermal hot springs etc. It has been suggested that part of the sources of high ratio of ^3He over ^4He in volcanoes or deep lakes are LENR reactions occurring in the Earth's mantle or crust.

6) X-ray production from peeling of transparent tape in a vacuum (10^{-3} Torr). The X-ray photons range in energy from zero to 100 KeV with a mean of about 20 KeV. An avalanche discharge effect is suggested due to many photons emitted during nanosecond bursts. The standard explanation of triboluminescence producing photons via bremsstrahlung is discounted because the angular distribution of photon energies is more sharply peaked than in simple bremsstrahlung. Thus, unexpected high energy radiation can be produced by mild conditions at room temperatures in everyday objects. Although this phenomenon is probably not an effect of LENR, nevertheless it is a dramatic example of how nuclear scale energies can be produced in everyday objects. It turns out, however, that tape speed and atmospheric pressure are critical in determining the X-ray output, which is similar to the LENR effect in that with knowledge and experience more things that can affect the performance are being found.

However, in all papers, he does aver that if off-mass shell quantum mechanics is needed in order to understand LENR results, then our understanding of quantum mechanics will be affected at a fundamental level. The standard on-shell wave equations would have to be considered as approximations to a more general, possibly higher dimensional, off-shell theory. Support from LENR for off-mass shell dynamics would be of great relevance for this field. As we have mentioned in the introduction, the standard relativistic wave equations (Klein-Gordon, Dirac, Proca, etc.) all have problems with a localized position operator, or negative energies/negative probabilities. Thus, modern physics regards *quantum fields* as preeminent over *n-particle* wave mechanics. But a resurgence of the Stueckelberg theories could change this, and make n-particle wave mechanics prominent once again.

3. Quantum Interference in Time Due to Off-Mass Shell Influence

In this and section 4 we will explore some important ideas that might lead to key paradigm shifts in various areas of physics. As we have learned in the introduction, the nonrelativistic Schrodinger equation cannot be used to predict quantum interference in time. Yet, it has been demonstrated recently (Lindner *et al.*) [11], that an electron wave packet undergoing sequential ionizing perturbation in time (from Argon gas) undergoes quantum interference phenomena. It is here, that the Stueckelberg theory, in its full relativistic form does account for these striking results in a simple and consistent way, when time intervals involved lie within the spread of time of the wave packets. Although the Stueckelberg theory is essentially relativistic, and the energies of the macroscopic motions of the particles involved in the Lindner experiment are low, the very high frequencies used to establish excitations and pulse rates involve high energy components of the wave packets and thus the use of a relativistically covariant theory is appropriate.

In 1976, Horwitz and Rabin [19] pointed out that the relativistic quantum theory predicts interference in time. In this theory t is necessarily treated as a quantum observable, since the Einstein variables x,t are considered in relativity as the nontrivial outcome of experiments measuring the place and time of the occurrence of events. Since this description of dynamics of events rests on the identification of observables, the set of observables assigned to each particle, often called an *event* is comprised of all four Minkowski coordinates $x^\mu = (t, x_1, x_2, x_3)$ as well as $p^\mu = (E, p_1, p_2, p_3)$.

We begin with the discussion of the Lindner experiment and its implications for the nature of the observed time. In this experiment, laser light of about 850 nm wavelength is radiated onto a sample of Argon gas in a short pulse of one and a half wavelengths, constituting two peaks in the electric field in one direction, and one in between, in the opposite direction. An electron may be emitted as a result of the interaction with the first peak or the third, separated by about one femtosecond in time. At the detector, one sees an interference pattern between the two possibilities corresponding to an ejection at the first or third maximum in the wave, much like the double slit experiment in space. The second peak in the opposite direction, which exhibits no perceptible interference effect, was used to confirm that just a single electron was involved in the process.

The interference observed in the spatial double slit experiment is accounted for by the coherence of the wave function in space, and was one of the earliest experimental confirmations of the structure of the quantum theory as it emerged from its formulation in Hilbert space. Consequently, it was only natural for the Lindner team to wonder whether interference in *time* could be observed in their experiment. This question was answered in the positive due to the remarkable success of this experiment.

However, the results are discussed in the Lindner *et al.* paper [11] in terms of a very precise solution of the time-dependent nonrelativistic Schrodinger equation, which, as stated above, by the basic principles of quantum theory, cannot be used to predict interference phenomena in time. Consequently, the very striking results of this remarkable experiment, beyond the obvious technical advances which they represent, possibly point to a more fundamental paradigmatical importance, raising significant questions on the role of time in quantum theory. These results imply that the time variable t, as in the Horwitz/Rabin paper [19], must be adjoined to the set of standard quantum variables so that the standard ket $| x,t >$ for the representation of the quantum state, in Dirac's terminology [20], can be constructed. It is this structure for the wave function $\psi(x,t) = < x,t|\psi)$, where x and t are the spectra of self-adjoint operators, that provides the possibility of coherence in t, and therefore, interference in time.

Essentially, the significance of the Lindner experiment is that it demonstrates at least one class of phenomena actually seen to occur in nature at low energies (but high frequency), for which the standard nonrelativistic quantum theory, or even relativistic quantum field theory does not provide an adequate description, and therefore requires the development of some theoretical tools which are a proper generalization of the standard theory.

In this regard, the related proposed experiment of Palacios *et al.* [21] is equally significant. This protocol involves the supposition that the spin of two particles at different times retain the entanglement characteristics of two particles at equal times. In their proposed experiment, they consider a situation in which two short ultraviolet pulses with different central frequencies doubly ionize a Helium atom to produce electrons with slightly different energies; the short subfemtosecond duration of each pulse gives it an appreciably wide energy bandwidth, so that the pulses overlap. Therefore an electron with a given energy in the overlap region could have been emitted by either pulse. It is assumed that the electrons are indistinguishable after emission at two different times, and that the spin correlations between them remain as determined by their coupling in the initial state (singlet in their case). They argue that the resulting probabilities for ejecting two electrons restricted in their total energy by the ionization energy of the *He* and the photon energies, should show interference oscillations that depend on the time delay between the pulses as well as their durations. However, the detailed calculations of Palacios *et al.* are based on the assumption of nonrelativistic entanglement and the use of the nonrelativistic Schrodinger equation for the evolution of the two body system, as done by the Lindner *et al.* group. The addition of angular momentum for a two or more body system, according to the usual Clebsch-Gordon coefficients, however, is valid in the nonrelativistic theory only for systems at equal times. In nonrelativistic theory, the correlation is destroyed by the successive emission of the electrons. Also, the tensor product for the two body system is constructed for equal times, corresponding to the parameter labeling the Hilbert space under unitary evolution. This structure is not covariant.

4. Possible Abrogation of Gravitational Singularities Due to Quantum Interference Inside Event Horizon

It is generally regarded that all aspects of gravitational field effects can be codified by Einsteinean general relativity. In relativity, the time of an event as measured in the laboratory is subject to variation according to the velocity of the apparatus related to the transmitting system and in addition may be affected as well by forces(such as gravity). A relativistic quantum theory should therefore incorporate *time* in a manifestly covariant manner and also permit definition of a global *causal* parameter to generate evolution of the 4D states. For this dynamics, Stueckelberg first considered the spacetime diagram of the orbit, called a world line of a free particle, expected to be simple straight line. He then supposed that there is some force acting on the particle that makes the world line bend during the interaction. Thus, in this model the interaction may be strong enough to make the world line turn back and run in a direction opposite to that of the t axis. This means that the system admits quantum superposition for the states in the total 4D picture where coordinate time acts as another spatial dimension and events can move backward or forward in time t. It is clear that Stueckelberg was thinking of this process as reflecting the fact of some dynamical laws on the evolution of a sequence of events constituting a world line rather than a global manifestation of the world line. In contrast to the view of Weyl [22] who suggested that the particles we see are the intersection of the observer's plane of time with pre-existing world lines, comprising a static universe, with apparent motion generated by the effect of this plane cutting the world lines at a succession of points in t, the world line is envisaged here as generated by a motion of a single event evolving according to dynamical laws, in a similar way to the formation of an orbit of a particle in nonrelativistic mechanics, generated as a function of the Newtonian time. Stueckelberg observed that in the extreme case of the reversal in the sense of time of this motion, the physical processes of pair annihilation could be represented in the framework of *classical* mechanics of the path running backward in time could be considered as an antiparticle.

To describe the dynamical evolution of such a system, Stueckelberg and Horwitz and Piron (SHP) [23, 24] noted that the standard use of *t* as a parameter would be inadequate to describe this curve, but that an invariant parameter τ, along the curve – a "world time" coinciding with the Newtonian world time, accounting for classical as well as quantum relativistic evolution, had to be introduced to construct a consistent description. This ("historical") time provides a parameter that labels the dynamic evolution of the covariant system. Horwitz and Piron further assumed, in order to treat many-body systems, that this historical time parameter, τ, is universal, playing the role of the universal time postulated by Newton in his Principia. For free on-shell motion of a single particle, the Einstein proper time can be taken equal to the world time. The evolution parameter, τ, is an invariant parameter that can be identified with Newton's time. In Newton's classical mechanics and in quantum mechanics one makes use of a global time that has causal meaning. In standard nonrelativistic quantum mechanics, coordinate *time* (*t*) is interpreted as a causal parameter, where, for each value of the parameter, the quantum states are coherent. However, the manifestly covariant quantum Stueckelberg formalism is based on the idea that there is an invariant parameter τ of evolution of the system; wave functions, as covariant functions of space and the Einstein time *t* form a Hilbert space (over R^4) for each value of τ. Thus, there are two types of time or temporal parameters, one transforming covariantly (*t*), and the second, a parameter of evolution (τ). Stueckelberg basically proposed and formulated a manifestly covariant form of classical and quantum mechanics in which space *and* time (*t*) become dynamical observables. They are therefore represented in quantum theory by operators on the Hilbert space on square integrable functions in space and time. The dynamical development of the state is controlled by the invariant parameter τ, coinciding with the time on the (on-mass) shell of freely falling clocks in general relativity.

One of the features of this dual-time formalism, is the significance of both the retarded and advanced parts of the particle propagator for the correct description of direct particle interaction (in the 5D electrodynamics associated with the Stueckelberg-Schrodinger equation, the τ- retarded propagator contains both *t*-retarded and *t*-advanced components). The symmetry between past and future in the prescription of the fields as a consequence of the QED theory, leads to the understanding that a causal parameter in our experience, i.e., an invariant universal *time*, may not be correlated with the Minkowski or Einstein time, in which interactions notice past and future simultaneously. This means that the system admits quantum superposition of states in the total 4D picture where time acts as another spatial dimension and events can move forward and backward in time *t*.

The Steuckelberg formalism also implies the existence of a "fifth" *electromagnetic* potential, through the requirement of gauge invariance, and there is a generalized Lorentz force which contains a term that drives the particle off-shell, whereas the terms corresponding to the electric and magnetic parts of the usual Maxwell fields do not. The electromagnetic field tensor in the Stueckelberg model is analogous to the usual magnetic field, and the new field strengths, derived from the τ-dependence of the fields and the additional gauge field, are analogous to the usual electric field. Called "pre-Maxwell" theory by its originators, the standard Maxwell theory, with customary field strengths, potentials, and four-current, is recovered through a process called "concatenation", via integration of the pre-Maxwell potentials over the evolution parameter τ.

In this regard, the key significance of the fifth pre-Maxwell potential has been advanced in considerably more detailed exposition, both in connection with possible unprecedented "mass transfer" effects in some special electrostatic protocols [26], but also in terms of development of a full-fledged second-quantized theory of n-particle mechanics, incorporating both *t*–advanced and *t*-retarded propagators on the same footing [27]. This *time-symmetric* dynamics is, in turn, a function of the temporal monotonic evolution parameter τ. This remarkable edifice has been developed through the yeoman work of Martin Land in particular and others [28, 29], ably expanding/clarifying the original seminal work inaugurated by Stueckelberg.

For our purposes in the following specific examination of possible new paradigms in gravitational field physics, we note that this fifth pre-Maxwell potential has for its source the matter density, and therefore motivates the investigation into the connection between these dynamical equations and gravitation. We show that the fifth gauge field can be absorbed into a conformal metric and the Lorentz force. Then it is shown that the generalized radiation field passing through an optical medium with a dielectric tensor results in an analog gravity for the eikonal approximation for an arbitrary metric.

The conformal metric is not, however, even locally equivalent to a Schwarzchild metric. To arrive at a more general framework for achieving an underlying model for gravity, we will study the aforementioned eikonal approximation of the generalized electromagnetic equations in a medium with a non-trivial dielectric tensor. It has been known for many years that the Hamilton-Jacobi equation of classical mechanics defines a function which appears to be an eikonal of a wave function, and therefore that classical mechanics appears to be a ray approximation to some wave theory [25]. The propagation of rays of waves in inhomogeneous media appears, from this point of view (as a result of Fermat's principle), to correspond to geodesic motion in a metric derived from the properties of the medium [30].

Now, the SHP theory has the structure of Hamiltonian dynamics with the Euclidean 3-dimensional space replaced by 4-dimensional Minkowski space. Since all four components of energy-momentum are kinematically independent, the theory is intrinsically off-shell. When we include the influence of the scalar parameter τ describing wavefunction evolution, this theory leads to five dimensional wave equations for the associated gauge fields. In fact, for wave phenomena in general, in the eikonal approximation in the presence of an inhomogeneous medium, there is provided a basis for geodesic motion in four dimensional spacetime. Since the geodesics predicted by the eikonal approximation, with appropriate choice of $g_{\mu\nu}$, can be those of general relativity, this theory provides a quantum theory which underlies classical gravitation, and coincides with it in this classical ray approximation.

One of the key results which concern the SHP formalism [31] is that by employing the metric tensor in the kinematical terms of the Stueckelberg-Schrodinger equation one can obtain classical general relativity in eikonal approximation. Since the eikonal approximation lowers the dimension of the differential equations by one [33], the eikonal approximation to the 5D Stueckelberg quantum equation in a curved spacetime, characterized by the metric tensor $g_{\mu\nu}$ results in the 4D Einstein geodesic equations. For the eikonal (semi-classical) approximation, these equations lead to Einstein's geodesic flow on a curved manifold; general relativity then appears as an emergent phenomenon. In this case the eikonal approximation to the relativistic quantum mechanical current coincides with the geodesic flow governed by the pseudo-Riemannian metric obtained from the eikonal approximation to solutions of the Stueckelberg-Schrodinger equation. This construction provides a model in which there is an underlying quantum mechanical structure for classical dynamic motion along geodesics on a pseudo-Riemannian manifold [34]. This treatment is significant since it also predicts new unsuspected possible paradigm changing effects within the event horizon of a black hole [6].

Towards this end we here study an application of Stueckelberg's manifestly covariant quantum theory in general relativity. We deal with a simple case and compare the results to those expected in general relativity. We study the form of the wave equation of a test particle in the presence of a Schwarzchild gravitational field, assuming that the source is massive enough to ignore changes in the metric caused by the smaller mass of the test particle. Since the Stueckelberg formalism describes the evolution of the wave function of the body according to the invariant time of evolution (τ), we can treat the mathematical behavior of the wavefunction also around and beyond the horizon ($r = 2M$). Within the horizon, the interval classically effectively changes its signature and becomes spacelike; the distance from the origin singularity becomes timelike, but the description according to world time (τ) enables us to study the behavior of the wavefunction evolution in an absolute sense.

Essentially, we find that within the black hole horizon, the expectation value of the distance from $r = 0$ singularity has a strong gradient towards the horizon. This result can only be explained quantum mechanically, since classically, the particle should move towards the origin. Interference effects apparently induce results which are very different only when the particle has a wavelength of a stellar scale, $\lambda \sim M$. In the neighborhood of the horizon, the wavefunction is evidently spread, in analogy to the action of tidal forces. This result means, that for certain wave-packets, there is "gravitational repulsion" that prevents the test particle from falling towards the $r = 0$ singularity, and maintains it near the horizon in a manner that depends on the particle's angular momentum. Therefore this quantum gravitational model predicts that the test particle will move to the $r = 2M$ shell of the black hole, which means that matter should accumulate on the interior of the horizon.

To demonstrate these potentially revolutionary findings, we turn now to a study of a quantum equation which results in a gravitational physics in the ray approximation, providing a quantum theory which may underlie the observed classical gravitational fields. The rays are associated with the probability flow of particles. The eikonal eigenvalue condition is one dimensional in this case, since the field is scalar. For an analog of this structure (corresponding to a distribution of events periodic in both space and time) in four dimensions described by a relativistically covariant equation of the Stueckelberg-Schrodinger type, the metric obtained is a spacetime metric, and the geodesic flow is that of the quantum probability for the spacetime events (matter) described by the Stueckelberg wave function. Towards this end we start with the Stueckelberg-Schrodinger equation [34]:

$$i\frac{\partial\Psi}{\partial\tau} = \frac{1}{2m\sqrt{g}}\partial^{\nu} g_{\mu\nu}\sqrt{g}\partial^{\nu}\Psi \qquad (4.1)$$

where $g_{\mu\nu}$ is assumed to be symmetric, and is somewhat analogous to a gauge field – actually a *tensor gauge* field, and m is an intrinsic property of the particle with dimension mass.

In the eikonal approximation for the 5D equation above, we assume a wave function Ψ such that:

$$\Psi(x,\tau) = A(x)e^{-i(\frac{\kappa}{2m} - \frac{\sqrt{\kappa}}{m}S(x))} \qquad (4.2)$$

Where S is the eikonal phase, which, when substituted into (4.1) gives:

$$\frac{\kappa}{2m} A(x)e^{-i\left(\frac{\kappa}{2m}\tau - \frac{\sqrt{\kappa}}{m}S(x)\right)}$$

$$= -\frac{\kappa}{2m} g_{\mu\nu}\frac{\partial^{\mu}S\partial^{\nu}S}{m^2} A(x)e^{-i\left(\frac{\kappa}{2m}\tau - \frac{\sqrt{\kappa}}{m}S(x)\right)} \qquad (4.3)$$

In the eikonal approximation it is assumed that κ ($\kappa = p_\mu p^\mu = -\tilde{m}^2$ the dynamical measured mass squared; we use the signature (-,+,+,+) in the local flat space) is large compared to the square of the second derivative of S; the dynamical evolution of the system in τ is effectively frozen, and the theory reduces to a four dimensional eikonal form:

$$g_{\mu\nu}\frac{\partial^{\mu}S\partial^{\nu}S}{m^2} = -1 \qquad (4.4)$$

In the optical analogy of the eikonal approximation [30, 31], the functions $S(x)$ are the Fresnel surface of rays, and $\partial^{\mu}S = p^{\mu}$ is the momentum in the direction of the phase surface.

If we identify the momentum p^{μ} as $m\dot{x}$ where the dot corresponds to differentiation in τ, this equation becomes:

$$-g_{\mu\nu}dx^{\mu}dx^{\nu} = d\tau^2 \qquad (4.5)$$

the equation for the invariant line element of Einstein. The Schrodinger current associated with (4.1) is [29, 30]:

$$j_\tau(x)_\nu = \frac{1}{2im}(\Psi_\tau^* g_{\mu\nu}\partial^{\mu}\Psi_\tau - \Psi_\tau g_{\mu\nu}\partial^{\mu}\Psi_\tau^*) \qquad (4.6)$$

The Fresnel surface of the system's dynamics, analogous to the optical case is [24]:

$$K = g_{\mu\nu}p^{\mu}p^{\nu} + m^2 = 0 \qquad (4.7)$$

It is clear that ${\partial K}/{\partial p^{\mu}}$ is in the direction of the eikonal form of the Schrodinger current (j_τ^{μ}). This implies that K is the evolution operator for the dynamical flow of the particles which correspond to the Fresnel rays. K is therefore the covariant Hamiltonian of the system. It then follows from the Hamiltonian equations that the flow is geodesic, where $g_{\mu\nu}$ is the metric [33].

We note that according to the Stueckelberg theory, $ds/_{d\tau} = \widetilde{m}/_{m}$, where $\widetilde{m}^2 = g_{\mu\nu}p^{\mu}p^{\nu}$ and this change in variable in (4.5) brings the line element to the form using the proper time *ds* with a factor $\widetilde{m}/m$, and is said to be on-shell if this quantity is unity. Since our discussion will be a consideration of the mass as belonging to a continuum, the wave functions will retain their local interpretation. In the following section we show the affect of gravity on the evolution of a wave function in a curved Schwarzchild spacetime.

For the Schwarzchild metric we have (considering the purely radial case for which $\phi = 0$, and take $G = c = 1$):

$$g_{\mu\nu} = \begin{pmatrix} \frac{1}{1-\frac{2M}{r}} & 0 & 0 & 0 \\ 0 & \frac{2M}{r} - 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{csc\,\theta^2}{r^2} \end{pmatrix} \quad (4.8)$$

And also:

$$\sqrt{g} = r^2 \sin\theta \quad (4.9)$$

After substituting the metric into equation (4.1) and separating variables, we find that the solution to the problem is of the form:

$$\Psi(x^{\mu}) = e^{-ikr-i\omega t-i\alpha\varphi} P_{l,\alpha}(\theta) R_{\kappa,\omega,l}(r) \quad (4.10)$$

Where $\kappa = 2mk$ has the dimensions of mass^2, and equals m^2 on the particle's mass shell.

Substituting this form into equation (4.1), we get an equation for $R_{\kappa,\omega,l}(r)$:

$$R''(r) + \frac{2(r-M)}{r(r-2M)} R'(r) + \left(\frac{r^2\omega^2}{(r-2M)^2} - \frac{\kappa r^2 + L(l+1)}{r(r-2M)}\right) R(r) = 0 \quad (4.11)$$

and for $P_{l,\alpha}(\theta)$:

$$P''(\theta) + \cot\theta P'(\theta) + \left(l(l+1) - \frac{\alpha^2}{sin^2\theta}\right) P(\theta) = 0 \quad (4.12)$$

For the far gravitational field, where we take $r >> 2M$ and therefore neglect high orders of $2M/r$, after substituting $\kappa \to m^2$, then (4.11) has the form:

$$R''(r) + \frac{2}{r} R'(r) + \left(\omega^2 - m^2 - \frac{l(l+1)}{r^2} + \frac{2M(2\omega^2 - m^2)}{r}\right) R(r) = 0 \quad (4.13)$$

If we divide the whole equation by $-2m$ we find that this equation has the exact form of the Schrodinger central potential equation, where the term $\frac{2mr^2}{l(l+1)}$ represents the repelling *centrifugal potential*. The centrifugal term shall be omitted hereafter, since we shall keep only the first order terms of $\frac{1}{r}$.

Since, relativistically, ω is the energy of the particle including its mass and since we are not interested in tachyonic solutions, we take $\omega^2 - m^2$ to be positive. Therefore, as opposed to the nonrelativistic hydrogen problem, the eigenvalues of interest are positive. The tachyonic solutions exponentially decrease at infinity and therefore don't add any relevant amplitude to the far field solutions; however, they should be taken into consideration near the horizon.

The effective central potential for this equation has the form:

$$V(r) = \frac{-M(2\omega^2 - m^2)}{mr} \approx \frac{-M}{r}\left(m + \frac{2p^2}{m}\right) \approx \frac{-mM}{r}(1 + 2v^2) \quad (4.14)$$

The effective gravitational mass is altered by a small factor of $2v^2$. Moving back to ordinary units, the effective gravitational mass is:

$$\mu = m\left(1 + \frac{2v^2}{c^2}\right).$$

The solution to the radial equation is of the form:

$$R(r) = \frac{c_1 J_1(2\sqrt{\mu M r}) + c_2 Y_1(\sqrt{\mu M r})}{\sqrt{r}} \quad (4.15)$$

where $J_n(z)$ and $Y_n(z)$, give the Bessel function of the first and second kind accordingly.

The fact that the energy eigenvalues of the equation are positive means that the wave functions, as single momentum modes, are not square integrable. For the far fields, the massive object may be localized very tightly, and may have an uncertainty in its location which is very small compared to the scale of the gravitational distances. The situation where the object's wavelength usually doesn't play the role in the physics of the problem, changes when the metric causes the wavelength (or equivalently the location uncertainty) of the relevant scale, which will cause interference phenomena to be crucial.

We now discuss predictions close to and within the black hole horizon, and show the results of a complete and exact computation of the wave function in the entire interior region.

Taking:

$$R(r) = \frac{B(r)}{\sqrt{r}\sqrt{r-2M}} \quad (4.16)$$

and substituting in equation (4.11), we get for $B_{m,\omega,l}(r)$ a Schrodinger-like equation of the form:

$$B''(r) - \frac{r(r-2M)\left(l(l+1)+mr^2\right) - M^2 - \omega^2 r^4}{(r-2M)^2 r^2} B(r) = 0 \quad (4.17)$$

We first study the behavior of particles near the horizon ($r \to 2M^+$). Expanding the potential around $r \to 2M^+$ the potential in equation (4.17) takes the form:

$$U(d) = \frac{-\omega^2}{d^2} - \frac{2\omega^2 - m^2}{d} + \frac{l(l+1)+½}{4M^2 d} \quad (4.18)$$

where $d = \frac{r-2M}{2M} \to 0$. Solving the radial equation in this region we obtain for the dominant parts:

$$R(d) = a_1 d^{-2iM\omega} + a_2 d^{2iM\omega} \quad (4.19)$$

Finding the expectation value of $r \to 2M^+$, with normalization, we need to compute:

$$\langle r \rangle = \frac{\int_{2M}^{2M+\lambda} 4\pi r^3 R(r) R(r)^* dr}{\int_{2M}^{2M+\lambda} 4\pi r^2 R(r) R(r)^* dr} \quad (4.20)$$

where λ is the uncertainty in the location of the particle on the R axis or, equivalently, the width of the wave packet associated with the wavelength of the particle.

Changing variables around the horizon we get:

$$\langle r \rangle = \frac{M\left(a_1 a_2\left((3\varepsilon - 4iM\omega)\varepsilon^{8iM\omega} + 4iM\omega\right) + 16M^2\omega^2\varepsilon^{4iM\omega}\right)}{a_1 a_2\left((\varepsilon - 2iM\omega)\varepsilon^{8iM\omega} + 2iM\omega\right) + 8M^2\omega^2\varepsilon^{4iM\omega}} \to 2M \quad (4.21)$$

Where $\varepsilon = \frac{\lambda}{2M} \ll 1$.

What can be seen from the result is that the particle is very strongly captured by the horizon. The uncertainty in its location is now related to the phase in the term $\varepsilon^{iM\omega}$ which becomes exponentially small and concentrated near the horizon through normalization of the wave function.

Taking the same potential from equation (4.18) and solving the radial equation for $r \to 2M^-$ we obtain (only the first term of (4.18) applies to the interior solution):

$$R(d) = a_1 d^{-2iM\omega} \quad (4.22)$$

where $d = \frac{2M-r}{2M} \to 0$.

Finding the expectation value of r in a small neighborhood of the horizon, i.e., at $r = 2M - \rho$, where $\rho \ll 2M$ we need to compute:

$$\langle r \rangle = \frac{\int_{2M-\rho-\lambda}^{2M-\rho+\lambda} 4\pi r^3 R(r) R(r)^* dr}{\int_{2M-\rho-\lambda}^{2M-\rho+\lambda} 4\lambda\pi r^2 R(r) R(r)^* dr} \quad (4.23)$$

Changing variables around the horizon we get:

$$\langle r \rangle = 2M - \rho + \frac{2\lambda^2}{6M - 3\rho}. \quad (4.24)$$

Using λ in the expectation value calculation indicates a width for computing the local probability. Since we are using λ as the effective spread of the particle's wave function (and beyond that we have a zero probability of finding the particle) the calculation is a good approximation.

It can be seen that within a small distance of the horizon at the point $r = 2M - \rho$, the expectation value tends towards *r=2M.* This result means that effectively, a particle that is at the horizon, will most probably stay there and not fall into the center.

This is in stark contrast to the standard expected classical dynamics. Classically, within the horizon, the time component of the metric becomes spacelike and the distance from the origin singularity becomes timelike, suggesting an inevitable propagation of all matter within the horizon to a total collapse at $r = 0$. However, the quantum description of the wave function provides a different understanding of the behavior within the horizon.

Matter outside the horizon has a very small wavelength and therefore interference effects can be found on only a very small atomic scale. However, within the horizon, matter becomes totally "tachyonic" and is potentially "spread" over all space. Small location uncertainties on the atomic scale become large around the horizon, and different mass components of the wave function can therefore interfere on a stellar scale. The interference phenomenon, where the probability of finding matter decreases as a function of the distance from the horizon, appears as an effective gravitational repulsion.

The reason why the interacting particle behaves differently quantum-mechanically than is expected classically, is the fact that the expectation value takes interference into account. A similar phenomenon is the reason for which the electron in its ground state doesn't fall into the atom. One can think of this as the result of quantum effect of interference.

After solving numerically equation (4.13) we can see in Figure 2 the evolution of the wave function $\Phi(r, t)$ in the interior region $0 < r < M$. Because of the symmetry of the boundary conditions around $t = 0$, the function separates into two wave functions, one propagating in the t direction and the other propagating in the $-t$ direction as r decreases. When approaching to $r = 0$, the propagation "freezes" in t and the only thing that changes in the evolution of the wave function is the amplitude which decreases.

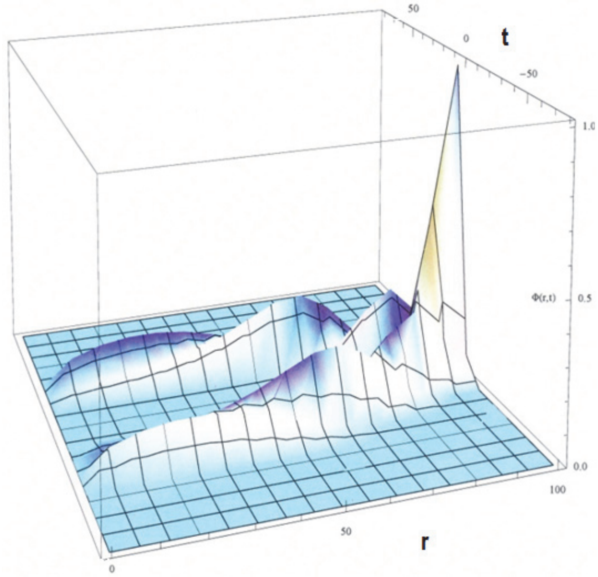

Figure 2. The probability density function $\Phi(r,t)\Phi^*(r,t)$ 3D "evolution" from $r = M$ to $r = 0$.

The evolution of the wave function from the horizon to $r = M$ is shown in Figure 3. It can be seen that the boundary conditions cause two major parts of the wave function at the origin to interfere at $r = M$ and create the Gaussian.

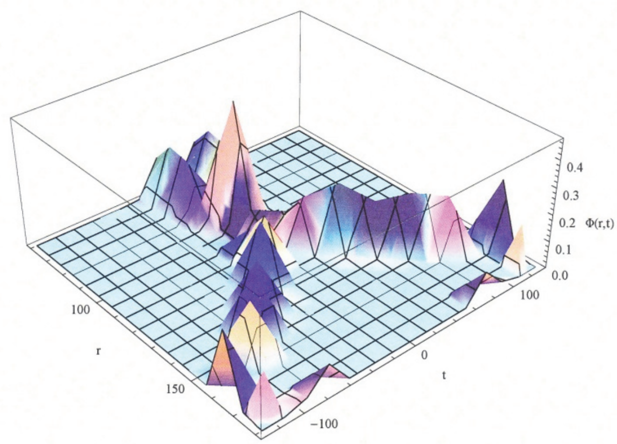

Figure 3. The probability density function $\Phi(r,t)\Phi^*(r,t)$ 3D "evolution" from the horizon to $r = M$.

The most surprising fact in the solution is the ongoing "disappearing" of the probability density function when approaching $r = 0$. According to the classical interpretation it looks as if the particle disappears in its future, something that is of course impossible. The dependence of the wave function in τ is therefore the only part in the wave function which is "non-localized" and therefore is the only variable that can be interpreted to be the 'time-evolution' of the particle.

Due to the similarity of the Stueckelberg theory to the structure of the KG (Klein-Gordon) equation, one might assume that we could have used the KG equation and have gotten the same results. The KG equation, however, as pointed out by Newton and Wigner [1] does not provide a wave function which can make local conclusions about the distribution of matter, which is our purpose here. The Stueckelberg theory, however, which is intrinsically off-shell, has the property that it provides a quantum theory with the correct properties of locality. The results of our calculations in this framework have a clear interpretation whereas from the point of view of KG theory, they are difficult to interpret. This is because Stueckelberg's 'world-time' does not necessarily propagate with "r" in the black hole, while in the KG equation "r" is the timelike axis where the particle must propagate towards its "future" at r = 0, and must not "disappear" as it seems to do so in a detailed analysis.

Since the numerical wave function that has been created isn't localized in r, it is hard to normalize it around the horizon and therefore, it is impossible to perform an accurate expectation value calculation for r. However, integrating over the probability density close to the horizon gives a bigger probability than the integration around r = M as opposed to the wave probability density function at r = 0 which is zero. The numerical solution therefore, also strengthens the previous analytic results that the particle is preferably found near the horizon.

5. Neutrino Flavor Oscillations and Stueckelberg Dynamics

The following three sections, through their theoretical development, offer some key experimental tests of the Stueckelberg off-mass shell theory in connection with sub-atomic dynamics: neutrinos neutral K-meson and tachyons in particular. This information has been culled from the prolific writings of John Fanchi, one of the leading researchers in this field. In his writings Fanchi has referred to the Stueckelberg theory using the term Parameterized Relativistic Dynamics (PRD) [35].

Neutrino oscillation by flavor mixing has been described by the mixing of either two of three neutrino flavors. Here we outline the two-state flavor mixing calculation within the context of Steuckelberg manifestly covariant quantum theory, showing that the result is an expression for the probability of transition from one neutrino flavor to another that has similarities with the result of the conventional theory, but differs in details that are experimentally testable.

The evolution equation for a state may be written in terms of the evolution operator as:

$$i\hbar \frac{\partial}{\partial \tau} \,|\, \nu_j \rangle = K_j \,|\, \nu_j \rangle \tag{5.1}$$

where K_j is the eigenvalue of the mass operator for mass state j. We restrict this discussion to two mass states $\{\,|\,\nu_j\rangle\}$ and two neutrino flavor states $\{\,|\,\nu_\alpha\rangle\}$, α=$e$(electron-neutrino), μ(muon- neutrino) both of which may be written as 2-component column vectors:

$$\{\,|\,\nu_j\rangle\} = \begin{bmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{bmatrix}, \{\,|\,\nu_\alpha\rangle\} = \begin{bmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{bmatrix} \tag{5.2}$$

We relate the mass basis $|\,\nu_j\rangle$ to the flavor basis $|\,\nu_\alpha\rangle$ with a unitary transformation U such that:

$$|\,\nu_\alpha\rangle = U \,|\, \nu_j\rangle \tag{5.3}$$

where:

$$U = e^{i\theta_j} \tag{5.4}$$

And θ is the mixing angle of mass states in vacuum. We hypothesized as in the conventional theory that the mixing angle is not zero so that it is meaningful to speak of flavor state mixing.

The evolution parameter dependent solution of equation (5.1) in the mass basis is:

$$\begin{bmatrix} |\nu_1(\tau)\rangle \\ |\nu_2(\tau)\rangle \end{bmatrix} = \begin{bmatrix} e^{-iK_1\tau/\hbar} & 0 \\ 0 & e^{-iK_2\tau/\hbar} \end{bmatrix} \begin{bmatrix} |\nu_1(0)\rangle \\ |\nu_2(0)\rangle \end{bmatrix} \quad (5.5)$$

where:

$$K_j = \frac{\hbar^2 k_j^\mu k_{j\mu}}{2m_j} = \hbar^2 \left[\left(\omega_j/c\right)^2 - k_j \cdot k_j \right] /2m_j \quad (5.6)$$

The energy-momentum four-vector is k_j^μ and m_j is the mass of state *j*. In SHP theory, the 4 components of the energy-momentum four-vector are observables. The treatment of neutrino oscillations as flavor mixing within the conventional theory assumes the 3-momentum equality $\mathbf{k_j} = \mathbf{k}$ for both mass states.

In an oscillation process, we begin with a pure beam of electron neutrino ν_e particles and calculate the probability for formation of muon neutrino ν_μ particles. For two flavors of neutrinos and the unitary transformation *U* we have:

$$|\nu_e\rangle = cos\theta\, |\nu_1\rangle + sin\theta\, |\nu_2\rangle$$

$$|\nu_\mu\rangle = -sin\theta\, |\nu_1\rangle + cos\theta |\nu_2\rangle \quad (5.7)$$

The evolution parameter dependent state for the electron neutrino is:

$$|\nu_e(\tau)\rangle = e^{-\frac{iK_1\tau}{\hbar}} cos\theta|\nu_1\rangle + e^{-iK_2\tau/\hbar} sin\theta|\nu_2\rangle \quad (5.8)$$

The above expressions are used to form the matrix element for the transition state ν_e to ν_μ:

$$\langle\nu_\mu||\nu_e(\tau)\rangle = [-\langle\nu_1|sin\theta + \langle\nu_2|cos\theta] \times$$

$$\left[e^{-i\kappa_1\tau} cos\theta|\nu_1\rangle + e^{-i\kappa_2\tau} sin\theta|\nu_2\rangle\right] \quad (5.9)$$

where $\kappa_j = K_j/\hbar$. Expanding and simplifying the transition matrix element gives:

$$\langle\nu_\mu|\nu_e(\tau)\rangle = sin\theta cos\theta\left(e^{-i\kappa_2\tau} - e^{-i\kappa_1\tau}\right) \quad (5.10)$$

where we have used the orthonormality condition:

$$\langle\nu_i|\nu_j\rangle = \delta_{ij} \quad (5.11)$$

For $\{i, j\} = \{1,2\}$. Using a trigonometric identity we write for (5.10):

$$\langle\nu_\mu|\nu_e(\tau)\rangle = \frac{1}{2} sin2\theta\left(e^{-i\kappa_2\tau} - e^{-i\kappa_1\tau}\right) \quad (5.12)$$

We calculate the square of the magnitude of the exponential term for later use in calculating the probability. From Euler's formula we have:

$$e^{-i\kappa_2\tau} - e^{-i\kappa_1\tau} = cos\kappa_2\tau - cos\kappa_1\tau -$$

$$i(sin\kappa_2\tau - sin\kappa_1\tau) \quad (5.13)$$

The square of the magnitude of this term is:

$$\left|e^{-i\kappa_2\tau} - e^{-i\kappa_1\tau}\right|^2 = 2[1 - cos(\kappa_2 - \kappa_1)\tau] \quad (5.14)$$

Consequently, the probability of forming state ν_μ is:

$$P(\nu_e \to \nu_\mu) = ½ sin^2 2\theta[1 - cos(\kappa_2 - \kappa_1)\,\tau] \quad (5.15)$$

The corresponding probability of remaining in state ν_e for the 2-state system is:

$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_\mu) =$$

$$1 - ½ sin^2 2\theta[1 - cos(\kappa_2 - \kappa_1)\tau] \quad (5.16)$$

Therefore, the probability $P(\nu_e \to \nu_e)$ may be written in the form:

$$P(\nu_e \to \nu_e) = 1 - sin^2 2\theta \left[sin^2 \frac{(K_2 - K_1)\tau}{2\hbar}\right] \quad (5.17)$$

Finally, from (5.16) we get for $P(\nu_e \to \nu_\mu)$:

$$P(\nu_e \to \nu_\mu) = sin^2 2\theta sin^2 \frac{(K_2 - K_1)\tau}{2\hbar} \quad (5.18)$$

The model developed above is specifically related to experiment by applying it to the disappearance of an electron neutrino produced by a source, such as a nuclear reactor. Evidence of electron neutrino disappearance is obtained by counting coincidence events in which positrons and neutrons are simultaneously created by inverse β decay of the neutron, i.e., the interaction of an electron antineutrino and a proton: $\bar{\nu}_e + p \to e^+ + n$. According to the flavor-mixing hypothesis, the electron antineutrino state may transform into a alternative neutrino species as it propagates. In principle, detectors can be placed along the electron antineutrino path to determine the probability of disappearance of electron antineutrinos as a function of distance *L* from their source.

The relativistic energy difference between the initial and final flavor states is:

$$K_1 - K_2 = \frac{\hbar^2 k_2^{\mu} k_{2\mu}}{2m_2} - \frac{\hbar^2 k_1^{\mu} k_{1\mu}}{2m_1} \quad (5.19)$$

In the case of negligible dispersion, we have:

$$K_2 - K_1 \approx \frac{m_2^2 c^2}{2m_2} - \frac{m_1^2 c^2}{2m_1} = \frac{(m_2 - m_1)c^2}{2} \quad (5.20)$$

For comparison with the conventional theory, we express $K_2 - K_1$ in terms of the rest mass difference such that:

$$K_2 - K_1 \approx \frac{\Delta m}{2} c^2, \Delta m = m_2 - m_1 \quad (5.21)$$

Substituting the above expression into the probability of forming the final state ν_μ from initial state ν_e gives:

$$P(\nu_e \to \nu_\mu) = sin^2 2\theta sin^2 \frac{(K_2 - K_1)\tau}{2\hbar} \quad (5.22)$$

To get an approximate value for the evolution parameter τ in order to compare the standard probability for this state transition with that calculated using the SHP ansatz, we consider the behavior of two hypothetical scalar particles: an interacting particle in the experimental system and a "τ-clock" particle. The "τ-clock" particle trajectory may be used to replace the evolution parameter τ with familiar observables. The non-interacting "τ-clock" particle is introduced because it serves to independently define the evolution parameter τ. The value of the evolution parameter is then used to parameterize the experimental system. The assumption here is that the parameter of evolution parameter clock can be equated to the parameter characterizing the experimental system.

In the limit of negligible dispersion (the classical limit), the most probable trajectory of the "τ-clock" particle is given by:

$$(\delta\tau)^2 = (\tau - \tau_0)^2 = \frac{1}{c^2} \delta\langle y_2^{\mu}\rangle \delta\langle y_{2\mu}\rangle \quad (5.23)$$

where τ_0 is the evolution parameter at which the rest frame clocks of both particles were calibrated. We can arbitrarily define the τ-axis so that $\tau_0 = 0$ and subsequently find from equation (5.23) an expression for the evolution parameter in terms of the measurable space-time trajectory of particle 2:

$$\tau^2 = \frac{1}{c^2} \delta\langle y_2^{\mu}\rangle \delta\langle y_{2\mu}\rangle \quad (5.24)$$

Neglecting statistical variations and writing $\{y_2^{\mu}\} = \{t, x\}$ for linear motion, we obtain:

$$\tau^2 = \delta t^2 - \frac{\delta x^2}{c^2} = \delta t^2 [1 - \beta^2] \quad (5.25)$$

where $\beta = \frac{v}{c}$, $v = \frac{\delta x}{\delta t}$.

If we write L as the distance δx traveled by particle 2 in the interval δt, then equation (5.25) can be written as:

$$\tau = \frac{L}{v} [1 - \beta^2]^{½} = \frac{L}{c} \frac{[1 - \beta^2]^{½}}{\beta} \quad (5.26)$$

If we substitute equation (5.26) into (5.22) we find:

$$P(\nu_e \to \nu_\mu) = sin^2 2\theta sin^2 \frac{\Delta m c^2}{4\hbar} \frac{L}{c} \frac{[1 - \beta^2]^{½}}{\beta} \quad (5.27)$$

For a comparison to the result obtained using SHP theory, we write the probability of forming the final state ν_μ from initial state ν_e is:

$$P_{SHP}(\nu_e \to \nu_\mu) = sin^2 2\theta sin^2 \left\{ \frac{(m_1 - m_2)c^2}{4\hbar} \frac{L}{c} \frac{[1 - \beta^2]^{½}}{\beta} \right\} \equiv sin^2 2\theta sin^2 \alpha_{SHP} \quad (5.28)$$

Dynamical factors are collected in the term α_{SHP}. We make a direct comparison of equation (5.28) with the conventional theory by substituting $E_\nu = m_1 c^2 / \sqrt{1 - \beta^2}$ into the conventional theory result to find:

$$P_{Std}(\nu_e \to \nu_\mu) = sin^2 2\theta sin^2 \left\{ \frac{(m_1 - m_2)c^2}{4\hbar} \frac{L}{c} [1 - \beta^2]^{½} \frac{(m_1 + m_2)}{m_1} \right\} \equiv sin^2 2\theta sin^2 \alpha_{Std} \quad (5.29)$$

Comparing P_{SHP}, P_{Std} and the dynamical factors $\alpha_{SHP}, \alpha_{Std}$ shows that the parameterized Stueckelberg relativistic dynamics model and the conventional theory have the same dependence on the flavor mixing angle θ, but their dependence on dynamical factors differs significantly.

In principle, it should be possible to test the validity of the conventional theory with the SHP theory as more experimental data becomes available. However, the possibility of experimental testing is corrupted by a systematic bias in which results of neutrino oscillation experiments are presented as plots of $\Delta m^2 \equiv m_2^2 - m_1^2$ versus $\tan\theta$. The factor Δm^2 allows a simplified presentation of the conventional theory as given by equation (5.29), but Δm^2 does not simplify the presentation of the data for the SHP relativistic dynamics result shown in equation (5.28).

An alternative plot that facilitates the comparison of competing theories without introducing a systematic bias is to plot $P(\nu_e \to \nu_\mu)$ versus L because both $P(\nu_e \to \nu_\mu)$ and L are directly measurable. We can have more confidence in the objective validity of masses and mixing angles obtained from theory-independent techniques if the masses and the mixing angles are calculated from a presentation of data that does not depend on a particular theory.

6. Experimental Test: K-Meson State Transition

Our purpose here is to present the formalism for describing a two-state particle, and then use the formalism to analyze the stability of the neutral K-meson (K^0). The results are then compared with the results of conventional theory.

Consider a system with 2 particles: an τ-clock particle and the 2-state particle. The 2-state particle is the primary system of interest in this treatment of particle stability. It is worthwhile to consider the τ state particle to illustrate how to use off-mass shell relativistic dynamics to relate the evolution parameter τ to a physical system. We begin with the N-particle Stueckelberg equation:

$$i\hbar\frac{\partial\Psi}{\partial\tau} = \left[\sum_{a=1}^{N}\frac{\pi_a^{\mu}\pi_{a\mu}}{2m_a}I + V\right]\Psi \qquad (6.1)$$

and rewrite it for a 2-state particle and a scalar particle, thus

$$i\hbar\frac{\partial\Psi}{\partial\tau} = \left[\frac{\pi_1^{\mu}\pi_{1\mu}}{2m_1}I + V_1\right]\Psi + \left[\frac{p_2^{\mu}p_{2\mu}}{2m_2}I + V_2 I\right]\Psi \qquad (6.2)$$

We have assumed for simplicity that the τ-clock particle, particle 2, is not experiencing an electromagnetic interaction $\{A_2^{\mu} = 0\}$ but may be subjected to a scalar potential V_2. The matrix potential experienced by particle 1 is assumed to depend on the coordinates of particle 1 such that $V_1 = V_1 I$ *where* $V_1(x_1^{\mu})$ is a scalar potential. Particle 1 does not interact with the τ-clock particle, particle 2.

The trial solution for this system is the column vector:

$$\Psi = \begin{bmatrix}\Psi_1(1,2,\tau)\\ \Psi_2(1,2,\tau)\end{bmatrix} = \phi(2,\tau)\begin{bmatrix}\psi_1(1,\tau)\\ \psi_2(1,\tau)\end{bmatrix} \qquad (6.3)$$

where we have simplified the notation by denoting the coordinates of particle 1 by index 1 and the coordinates of particle 2 by index 2. The inner product of the trial solution is

$$\Psi^{+}\Psi = \Psi_1^{*}\Psi_1 + \Psi_2^{*}\Psi_2 = \phi^{*}(2,\tau)\phi(2,\tau)$$
$$\{\psi_1^{*}(1,\tau)\psi_1(1,\tau) + \psi_2^{*}(1,\tau)\psi_2(1,\tau)\} \qquad (6.4)$$

The normalization condition is:

$$\iint \Psi^{+}\Psi dx_1 dx_2 =$$
$$\int \phi^{*}\phi dx_2 \int[\psi_1^{*}\psi_1 + \psi_2^{*}\psi_2]dx_1 = 1 \qquad (6.5)$$

which implies:

$$\int \phi^{*}\phi dx_2 = 1 \qquad (6.6)$$

and

$$\int[\psi_1^{*}\psi_1 + \psi_2^{*}\psi_2]dx_1 = 1 \qquad (6.7)$$

Substituting the trial solution into the Stueckelberg equation for this system gives the following equations:

$$i\hbar\frac{\partial\psi_j}{\partial\tau} = \left[\frac{\pi_1^{\mu}\pi_{1\mu}}{2m_1} + V_1\right]\psi_j; j = 1,2 \qquad (6.8)$$

For the 2–state particle, and

$$i\hbar\frac{\partial\phi}{\partial\tau} = \left[\frac{p_2^{\mu}p_{2\mu}}{2m_2} + V_2\right]\phi \qquad (6.9)$$

It is easier to understand the analysis of particle decay by first analyzing a non-interacting particle with 2 states. Equation (6.8) for the free 2-state particle is

$$i\hbar\frac{\partial\Phi}{\partial\tau} = -\frac{\hbar^2}{2m}\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right]\Phi \qquad (6.10)$$

where $\{x_1^\mu\} = \{\overrightarrow{x, ct.}\}$. It has the free particle solution:

$$\Phi=\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix}\sim\begin{bmatrix}exp(-i\omega_1 t + i\vec{k}_1\cdot\vec{x} - i\Omega_1\tau)\\ exp(-i\omega_2 t + i\vec{k}_2\cdot\vec{x} - i\Omega_2\tau)\end{bmatrix} \qquad (6.11)$$

The constants $\{\Omega_j\}$ multiplying the evolution parameters are:

$$\Omega_j = \frac{m_j^2}{2m\hbar} \qquad (6.12)$$

where m_j is the eigenvalue of state j and m is an effective mass that is determined below. The value of m_j is found by substituting $\{\Psi_j\}$ into the field equation to find:

$$\hbar\Omega_j = \frac{\hbar^2}{2m}\left[\frac{\omega_j^2}{c^2} - \vec{k}_j\cdot\vec{k}_j\right] \qquad (6.13)$$

and

$$m_j^2 = \frac{\hbar^2}{c^2}\left[\frac{\omega_j^2}{c^2} - \vec{k}_j\cdot\vec{k}_j\right] \qquad (6.14)$$

The normalization condition is:

$$\int\Phi^+\Phi\, d^4x = 1 = \int(\Phi_1^*\Phi_1 + \Phi_2^*\Phi_2)d^4x \qquad (6.15)$$

The effective mass is given by:

$$m^2c^2 = \langle p^\mu p_\mu\rangle = \int\Phi^+ p^\mu p_u\Phi\, d^4x \qquad (6.16)$$

The effective mass depends on the contribution of each state $\{\Phi_j\}$ to the total wave function Φ. For example, if the particle can be found in either state Φ_1 or Φ_2 with equal probability, then:

$$\int\Phi_1^*\Phi_1\, d^4x = \int\Phi_2^*\Phi\, d^4x = \frac{1}{2} \qquad (6.17)$$

and equation (6.16) becomes:

$$m^2c^2 = \frac{1}{2}\hbar^2\left(\frac{\omega_1^2}{c^2} - \vec{k}_1\cdot\vec{k}_1\right) + \frac{1}{2}\hbar^2\left(\frac{\omega_2^2}{c^2} - \vec{k}_2\cdot\vec{k}_2\right) \qquad (6.18)$$

Recalling that:

$$m_j^2 = \frac{h^2}{c^2}\left[\frac{\omega_j^2}{c^2} - \vec{k}_j\cdot\vec{k}_j\right] \qquad (6.19)$$

Is interpreted as the mass of state j, then the effective mass is:

$$m^2 = \frac{1}{2}(m_1^2 + m_2^2) \qquad (6.20)$$

If $m_1 = m_2$, then the effective mass m of the 2-state particle is equal to the mass of the particle either in state Φ_1 or Φ_2. If $m_1 \neq m_2$, then m is the average given above.

Expectation values can be used to relate the 4-position of the 2-state particle with τ. We begin by evaluating:

$$\delta\langle x^\mu\rangle = \frac{1}{m}\langle p^\mu\rangle\delta\tau \qquad (6.21)$$

Performing the operator calculation and again assuming that states Φ_1, Φ_2 equally probable, as in the previous example, we find:

$$\delta\langle x^\mu\rangle = \frac{\hbar}{m}\left(k_1^\mu + k_2^\mu\right)\delta\tau \qquad (6.22)$$

And

$$\delta\langle x^\mu\rangle\cdot\delta\langle x_\mu\rangle = \frac{\hbar^2}{m^2}\left(\frac{k_1^\mu+k_2^\mu}{2}\right)\cdot\left(\frac{k_1^\mu+k_2^\mu}{2}\right)\delta\tau^2 \qquad (6.23)$$

If $k_1^\mu = k_2^\mu = k^\mu$, then we obtain the familiar relationship:

$$\delta\langle x^\mu\rangle\cdot\delta\langle x_\mu\rangle = \frac{\hbar^2}{m^2}k^\mu\cdot k_\mu\delta s^2 \qquad (6.24)$$

That describes the special relativistic trajectory of the particle. If we measure time x^0 in the rest frame, equation (6.24) becomes:

$$c^2(\delta\langle t\rangle_{rest})^2 = \frac{\hbar^2 k^\mu k_\mu}{m^2}\delta\tau^2$$

or $\delta\langle t\rangle_{rest} = \delta\tau$ for a free particle with $m^2c^2 = \hbar^2 k^\mu k_\mu$.

We are now ready to consider the stability of a particle with two mass states. The field equations for the two states of interest Ψ_1, Ψ_2 here are represented by equation (6.8). Assuming $\{A_1^\mu\} = 0$, equation (6.8) becomes:

$$i\hbar\frac{\partial\Psi_j}{\partial\tau} = \left\{-\frac{\hbar^2}{2m_1}\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right] + V_1\right\}\Psi_j; j = 1,2 \quad (6.25)$$

where $\{x_1^{\mu}\} = \{ct, \vec{x}\}$. Equation (6.25) has the solution:

$$\Psi_j \sim exp\left[-\Omega_j\tau - \frac{\lambda_j t}{2} + i\vec{k}_j\cdot\vec{x}\right]; j = 1,2 \quad (6.26)$$

The term Ω_j is given by:

$$\Omega_j = \frac{M_j^2c^2}{2m_1\hbar} \quad (6.27)$$

where M_j is the mass state j. The 3-momentum of state j is $\hbar\vec{k}_j$, and

$$\lambda_j^2 = \frac{8m_1c^2}{\hbar^2}V_1 - \frac{4c^2}{\hbar^2}\left[M_j^2c^2 + \hbar^2\vec{k}_j\cdot\vec{k}_j\right] \quad (6.28)$$

The formalism presented above is readily applicable to the neutral *K*-meson. In the conventional theory the neutral kaon state K^0 and its antiparticle $\overline{K}^0$ are considered superpositions of states. We write the symmetric state K^0 and antisymmetric state $\overline{K}^0$ in terms of the states Ψ_1 and Ψ_2, thus:

$$K^0 \sim \Psi_1 + \Psi_2 \quad (6.29)$$

And

$$\overline{K}^0 \sim \Psi_1 - \Psi_2 \quad (6.30)$$

Now, the intensity of a K^0 beam is

$$K^{0*}K^0 \sim (\Psi_1 + \Psi_2)^*(\Psi_1 + \Psi_2) =$$

$$\Psi_1^*\Psi_1 + \Psi_2^*\Psi_2 + \Psi_1^*\Psi_2 + \Psi_2^*\Psi_1 \quad (6.31)$$

Substituting (6.26) into (6.31) gives:

$$K^{0*}K^0 \sim e^{-\lambda_1 t} + e^{-\lambda_2 t} +$$

$$e^{\frac{\lambda_1+\lambda_2}{2}t}e^{i(\Omega_1-\Omega_2)\tau}e^{-i(\vec{k}_1-\vec{k}_2)\vec{x}} +$$

$$e^{\frac{\lambda_1+\lambda_2}{2}t}e^{-i(\Omega_1-\Omega_2)\tau}e^{i(\vec{k}_1-\vec{k}_2)\vec{x}} \quad (6.32)$$

A similar result is obtained for the intensity of a $\overline{K}^0$ beam.

The expression for $K^{0*}K^0$ in equation (6.32) can be compared to the result of the conventional theory by writing (6.32) in the rest frame where three-momenta vanish and $t \to \tau$. The result is:

$$K^{0*}K^0 \sim \left[e^{-\lambda_1\tau} + e^{-\lambda_2\tau}e^{\frac{\lambda_1+\lambda_2}{2}\tau}2\cos(\Omega_1 - \Omega_2)\tau\right] \quad (6.33)$$

The difference in mass between states Ψ_1, Ψ_2 is contained in the argument:

$$(\Omega_1 - \Omega_2)\tau = \left(\frac{M_1^2c^2}{2m_1\hbar} - \frac{M_2^2c^2}{2m_1\hbar}\right)\tau \quad (6.34)$$

Now, the change in mass between state K_1 and K_2 in the conventional theory is:

$$\Delta m_{12} = \frac{\hbar}{c^2}|\omega_1 - \omega_2| \quad (6.35)$$

where $\hbar\omega_1$ is the energy of the long-lived state K_L^0 and $\hbar\omega_1$ is the energy of the short-lived state K_S^0.

The change in mass between state K_1 and K_2 in the off-mass shell theory is given by equation (6.34):

$$\Delta M_{12} = \frac{1}{2m_1}\left[M_1^2 - M_2^2\right] \quad (6.36)$$

where M_1 is the mass of the long lived state K_L^0 and M_2 is the mass of the short lived state K_S^0. The mass m_1 for the 2-state system is the average given in equation (6.20), or:

$$m_1^2 = \frac{1}{2}\left[M_1^2 - M_2^2\right] \quad (6.37)$$

Substituting equation (6.37) into (6.36), and defining the mass difference $\delta_{12} = M_1 - M_2$, we have, to first order in the mass difference δ_{12}:

$$\Delta M_{12} \approx \frac{2\delta_{12}M_1}{2\left[M_1^2 - \delta_{12}M_1\right]^{1/2}} = \frac{\delta_{12}}{\left[1 - \frac{\delta_{12}}{M_1}\right]^{1/2}} \quad (6.38)$$

Recognizing that $\delta_{12} << M_1$, we have:

$$\Delta M_{12} \approx \delta_{12}\left[1 + \frac{\delta_{12}}{2M_1}\right] \quad (6.39)$$

The mass differences predicted by the conventional theory in equation (6.35) and Stueckelberg manifestly covariant mechanics in equation (6.39) are very similar. Differences between the two theories arise because the theories treat time in a much different manner. This difference was highlighted in the analysis of neutrino oscillations presented in section 5.

7. Tachyon Physics and the Stueckelberg Model

An important feature of SHP manifestly covariant quantum theory is that all values of energy and momentum must be considered, and not just those values that yield on-shell masses. One consequence of this feature is that we can describe free particle motion for both bradyons (subluminal) and tachyons (superluminal) particles. This has significant physical implications.

Now, classical special relativity does not allow subluminal particles to become superluminal particles. The mass *m* of a classical particle moving at a speed *v* is related to the rest mass m_0 by the transformation:

$$m = m_0 / \sqrt{1 - \left(\frac{v}{c}\right)^2} \tag{7.1}$$

The mass *m* approaches infinity as $v \rightarrow c$ from below or above. However, in nonrelativistic quantum theory, time-dependent potentials allow for transitions between energy states, which are observed in a variety of systems, such as spectra and lasers. Some classically forbidden transitions can occur in nonrelativistic quantum theory because of the quantum tunneling effect.

Now in SHP theory we have seen that τ-dependent potentials allow for transitions between mass states, as was seen in section 5 on neutrino oscillations. Because of this feature we can now describe a system that allows for quantum transitions across the light cone.

Consider mass state transitions at a scattering vertex. The following hypothetical physical picture can be postulated:

$$\text{Projectile } (\Psi) + \text{Target } (\Phi_T) \rightarrow \text{Product} \tag{7.2}$$

A field equation for this system is:

$$i\hbar \frac{\partial \Psi}{\partial \tau} = \frac{\hbar^2}{2m_T} \frac{\partial^2 \Psi}{\partial x^\mu x_\mu} + g(\Phi_T + \Phi_T^*)\Psi \tag{7.3}$$

Where g is a postulated coupling constant. An iterative solution to the τ-dependent perturbation problem is obtained by analogy to the nonrelativistic *τ*-dependent perturbation calculation. In the nonrelativistic theory, the transition amplitudes refer to transitions between energy states, as in the laser. In Stueckelberg manifestly covariant dynamics, the transition amplitudes refer to transitions between *mass states.*

The field equation for a τ-dependent perturbation is:

$$i\hbar \partial_\tau \Psi = K_0 \Psi + K_1 \tag{7.4}$$

where K_0 refers to the unperturbed mass generator and K_1 refers to the τ-dependent interaction term. The perturbation calculation is subject to the constraint that assures the perturbation is Hermitian:

$$\int [K_1^* \Psi - \Psi^* K_1] d^4x = 0 \tag{7.5}$$

An approximate solution to the perturbation problem is obtained by writing the eigenfunction expansion:

$$\Psi(x, \tau) = \int a_\alpha(\tau) \psi_\alpha(x, \tau) d\alpha \tag{7.6}$$

where ψ_α are solutions to the unperturbed system and $a_\alpha(\tau)$ are expansion coefficients superposition of unperturbed mass states.

The transition probability amplitude in off-mass shell dynamics is:

$$a_\alpha = a_\alpha^0 - \frac{i}{\hbar} \int_0^\tau [\psi_\alpha^* K_1 d^4x] d\tau' \tag{7.7}$$

The transition probability to state α is:

$$P_\alpha = a_\alpha^* a_\alpha \tag{7.8}$$

And the transition rate density to state α is:

$$R_\alpha = \frac{\partial P_\alpha}{\partial \tau} \tag{7.9}$$

If we apply the τ-dependent perturbation theory formalism to the interaction in equation (7.2), we find the usual four-momentum constraints

Four-Momentum

$$k_\alpha = k_a + K_b \quad (7.10a)$$

$$k_\alpha = k_a - K_b \quad (7.10b)$$

where subscript *a* represents the projectile, subscript *b* denotes the target, and subscript α denotes the product particle. In addition, we find a set of equations that constrain the allowed masses. The mass constraint equations are:

Mass

$$q_\alpha = q_a + Q_b \quad (7.11a)$$

$$q_\alpha = q_a - Q_b \quad (7.11b)$$

where the free particle masses are:

$$q_n = \frac{\hbar^2 k_n \cdot k_n}{2 m_n \hbar} \quad (7.12)$$

Possible mass state transitions are given in Table 1. Letter B denotes bradyon and letter T denotes tachyon. The first row of the table, for example, says that a bradyon projectile interacting with a bradyon target can produce a bradyon under the Product-1 constraint. If the Product-2 constraint applies, a bradyon projectile interacting with a bradyon target can produce either a bradyon or a tachyon. The final product would depend on the properties of the target and projectile. Allowed interactions would also have to account for four-momentum constraints that represent four-momentum conservation.

Table 1. Possible Mass State Transitions

Target Q_b	Projectile q_a	Product-1 $q_\alpha = q_a + Q_b$	Product-2 $q_\alpha = q_a - Q_b$
B	B	B	B T
B	T	B T	T
T	B	B T	B
T	T	T	B T

According to table 1, tachyons may be created when two particles interact, even if the two interacting particles are bradyons. Furthermore, an existing tachyon may be annihilated in interactions with bradyons or tachyons. Thus, a bradyon interaction may produce a tachyon that exists for a short time and then is annihilated to produce more bradyons. These observations are still valid when four-momentum conservation is added to the kinematics of mass state transitions. Equation (7.11ab) is the one feature of off-mass shell dynamics that makes the formulation different from other tachyon kinematic systems. It serves to constrain the types of physically allowable processes. The additional constraint arises from an additional conservation law, given by equation (7.11ab) that is fundamental to the manifestly covariant Stueckelberg dynamics model.

A legitimate question to ask is whether the new mass constraint violates existing experimental data. Two types of experiments have been frequently performed in high-energy physics: scattering of a particle beam by a thin foil, and counter-circulating beam experiments. So, if tachyons exist, then why have they not been observed in these experiments?

The answer is that the constraints imposed on the allowed physical properties of tachyons by our relatively simple model are quite restrictive. This implies that an experimental search must be very specific. Some possible experiments are suggested by the above calculations. If the theory is correct, a projectile bradyon interacting with a thin foil of target bradyons should produce tachyons if the projectile energy and momentum are properly set and a non-zero coupling constant for a parameter-dependent potential exists between the projectile and the target. In this regard, the question of detectability becomes relevant since we have seen that tachyons may be annihilated as well as created. Thus, the observable signal of the produced tachyon may not be tachyonic, but be masked as a bradyonic signal. Specific predictions will depend on the particular physical experiment being considered.

Moreover, the incorporation of a new mass constraint does not contradict corresponding experimental results. There are mass relations in the literature that are used to compute the mass of a product particle as the sum of the masses of its constituents. These relations are usually accompanied by corrections for binding energy and other interactions. At this level of complexity, the new mass constraint is not in violation of known experimental data.

Perhaps the most significant physical point of this development is that tachyon creation within the context of off-mass shell dynamical models, parameterized by an evolution parameter τ, and where coordinate time (t) is time-symmetric, governed by a quantum time-operator, does not require continuous acceleration of a classical time-like entity through the light cone. Instead, tachyons are created or annihilated by quantum transitions. In other words, the additional temporal parameter τ allows expectation values to be defined over all space-time (or energy-momenta). As a consequence of these extensions, the SHP theory is an unconstrained theory in the sense that the probability amplitude receives contributions from both on-shell mass terms and off-mass shell terms. Moreover, the formulation of such a theory that uses a Stueckelberg-type equation can be viewed as a non-local theory because a relativistic quantum potential can be defined. Consequently, if τ parameter-dependent interactions are allowed, such a model provides a physically consistent description of tachyons, including a mechanism for crossing the light cone.

8. High Temperature Bose-Einstein Condensate Phase Transition in a Fully Relativistic Model of Statistical Mechanics

In most papers which discuss the quantum properties of the particles in an ideal relativistic Bose gas with non-zero chemical potential, particular attention has been given to the behavior of the Bose-Einstein condensation and the nature of the phase transition in d space dimensions. However, these works were all in the framework of the on-shell relativistic statistical mechanics. As we have learned, off-mass shell mechanics requires not only the standard chemical potential μ but an additional chemical *mass potential* μ_k. Here, we do not consider just the position of the particles as is done in standard nonrelativistic mechanics, but establish the statistical mechanics of a many *event* system, for which the points in Minkowski spacetime constitute the fundamental entities for which distribution functions must be constructed to achieve a manifestly covariant theory. In this model we describe the total number expectation of the canonical ensemble of particles as:

$$N = V \sum_k \left[\frac{1}{e^{(E-\mu-\frac{\mu_k m^2}{2MT})}-1} \right] \tag{8.1}$$

Here, it was shown that complementary to the usual (mass-shell) form of relativistic quantum mechanics that, as originally demonstrated by Haber and Weldon [37], taking into account both the particle *and* antiparticle distribution functions, a systems of bosons can undergo a high temperature phase transition. The introduction of antiparticles in the theory implies the existence of another term in the total number expectation, with a negative sign, carrying an opposite sign for the energy chemical potential:

$$N = V \sum_{k^\mu} \left[\frac{1}{e^{\left(E-\mu-\frac{\mu_k m^2}{2MT}\right)}-1} - \frac{1}{e^{\left(E+\mu-\frac{\mu_k m^2}{2MT}\right)}-1} \right] \tag{8.2}$$

The total number remains unchanged in the equilibrium state, but the presence of antiparticles implies annihilation and creation processes. Thus, in counting the total number of particles, antiparticle distribution must carry a negative sign, consistent with the interpretation of Stueckelberg. Since the sign of the energy of the antiparticle is opposite to that of the particle, the chemical potential μ must change sign for the antiparticle, but the mass squared of both particle and antiparticle are positive, and therefore the sign of μ_k does not change.

On the other hand, both terms in the sum in equation (5.2) must separately be positive, implying the inequalities:

$$m - \mu - \mu_k \frac{m^2}{2M} \geq 0 \tag{8.3a}$$

$$m + \mu - \mu_k \frac{m^2}{2M} \geq 0 \tag{8.3b}$$

resulting in the inequalities representing the nonnegativeness of the discriminants in the mass quadratic formulas (5.3ab),

$$-\frac{M}{2\mu_k} \leq \mu \leq \frac{M}{2\mu_k} \tag{8.4}$$

The bounds of the intersection of the regions satisfying the inequalities in (8.3ab) are given by:

$$\frac{M}{2\mu_k}\left(1-\sqrt{1-\frac{2|\mu|\mu_k}{M}}\right) \le \frac{M}{2\mu_k}\left(1+\sqrt{1-\frac{2|\mu|\mu_k}{M}}\right) \quad (8.5)$$

which for small $\frac{|\mu|\mu_k}{M}$ reduces, as in the no antiparticle case to:

$$|\mu| \le m \le \frac{2M}{\mu_k} \quad (8.6)$$

Replacing the summation in (8.1) by integration, one obtains the formula for the number density (for details see [31]):

$$n = \frac{1}{4\pi^3}\int_{m_1}^{m_2} m^3 dm \int_{-\infty}^{\infty} sinh^2\beta d\beta \ \times$$

$$\left[\frac{1}{e^{\frac{mcosh\beta-\mu-\frac{\mu_k m^2}{2M}}{T}}-1} - \frac{1}{e^{\frac{mcosh\beta+\mu-\frac{\mu_k m^2}{2M}}{T}}-1}\right] \quad (8.7)$$

where m_1 and m_2 are defined by the bounds (5.5). Integrating over the β variable, one finds for high temperature $\mu/T \ll 1$,

$$n \cong \frac{1}{\pi^3}\left(\frac{M}{\mu_k}\right)^2 \mu T\sqrt{1-\frac{2|\mu|\mu_k}{M}} \quad (8.8)$$

For T above a critical value, the range of admissible masses will be pinched down to zero, corresponding to a phase transition, where the dispersion: $\delta m = \sqrt{\langle m^2\rangle - \langle m\rangle^2}$ vanishes as $\sqrt{T-T_c}$, a second order transition, corresponding to a ground state with $p_\mu p^\mu = -\left(M/\mu_k\right)^2$. States with temperature $T>T_c$ correspond to the off-shell excitations of such a ground state.

The phase transition that we have described selects a definite mass for the particles, but this result is statistical. Although the mean fluctuations vanish, there is nevertheless sufficient freedom in the phase space for each particle to fulfill the off-shell requirements for the formulation of the Stueckelberg theory.

The high temperature Bose-Einstein phase transition with appropriate chemical potential, brings the system of particles to mass shell (statistically) providing further explanation, in addition to self-interaction of fields, that enforces the asymptotic stability of particle masses.

This mechanism provides an insight into a possibly more general formulation which would explain the stability of the asymptotic mass of a particle in the Stueckelberg theory in the presence of an arbitrary number of collisions; the existence of several solutions could give rise to what appears phenomenologically as mass spectra of observed particles.

Yet, as we have emphasized, to achieve a fully manifestly covariant theory, distribution functions must be constructed in order to elevate the statistical mechanics of *particles* to the statistical mechanics of a many "event" system. Assuming each event is part of an evolving world line, the counting of events is essentially equivalent to the counting of world lines corresponding to particles. Therefore, the statistical mechanics of events is closely related to the theory of the statistical mechanics of particles.

We have considered the ideal relativistic Bose gas within the framework of a manifestly covariant relativistic statistical mechanics, taking into account antiparticles. We have shown that in such a particle-antiparticle system, at some critical temperature T_c, a special Bose-Einstein condensation sets in, which corresponds to a phase transition from the sector of relativistic mass distributions to a sector in which the boson mass distribution peaks at a definite mass [38]. The results which can be computed from the latter coincide with those obtained in a high temperature limit of the usual on-shell relativistic theory.

With the understanding that the particles in the relativistic model of statistical mechanics possess a mass distribution, the observation of the predicted new high temperature Bose-Einstein condensation could represent an empirical signature that off-mass dynamics are playing a key role in the particle-antiparticle system.

9. Conclusion and Prospects

The evidence from the many branches of physics considered above have amply demonstrated that the standard yardsticks governing physical reality: space, time, energy and mass, may not be as immutable as customarily thought and might be in need for a re-evaluation. The Stueckelberg-Horwitz-Piron (SHP) off-mass shell model has been presented as the focal point from which several such associated future paradigm shifts can emerge.

One of these areas that has been presented for this purpose is the decades-old controversy surrounding the LENR issue, and how it could be resolved. One of the key reasons for lack of acceptance of this possible mode for nuclear fusion, is the unfortunate dearth of corresponding reasonable, tractable and viable theoretical models that would provide an adequate explanation for this phenomenon of low energy nuclear reactions. Here we have shown, through application of the SHP and similar related off-mass shell models for sub-atomic particle interaction, that there is the related promising theoretical edifice of this nature that can be implemented to explain all the various anomalous effects that have been observed to be associated with LENR dynamics. Unfortunately, so far this model has been afforded little consideration not only by mainstream nuclear physics, but even frontier seasoned LENR researchers as well, principally due to the required acceptance of the main radical assumption of the change in mass of sub-atomic particles (deuterons/electrons) during the associated nuclear reactions. Nevertheless, as we have outlined, Mark Davidson [16-18] has courageously presented just such an off-mass shell model for nuclear reactions, that if seriously considered in this context and suitably explored further theoretically and experimentally, might provide the template/springboard for leading to a greater respectability of the off-mass shell model itself in both classical and quantum mechanics. In the study of the physics of nuclear fusion, it might possibly propel the protocol and dynamics surrounding the cold fusion phenomenon into the role of "poster child" for off-mass shell sub-atomic interactions.

The example shown where the eikonal approximation of the solution to the Stueckelberg-Schrodinger equation in the context of the Schwarzchild gravitational field, yields a scenario in which a particle inside or outside the event horizon of a black hole, will most likely remain at the horizon, is a most phenomenal result in light of the current accepted orthodox knowledge surrounding gravitational field physics, which precludes such behavior. It implies that the standard classical model of general relativity and the Einstein 4D field equations, in connection with the dynamics of gravitational collapse, may actually be an emergent phenomenon. Accordingly, the field equations of general relativity might represent an approximation of a more general 5D quantum eikonal equation, which takes into account the nature of not only the Einstein time t, now represented by a time operator allowing for possibility of both directions of time, but of a new scalar time parameter τ representing the monotonic evolution of the wave function.

Yet we must acknowledge that we have considered just one example associated with only one aspect of gravitation. Consequently, to make a more definite assessment of the suitability of the SHP off-mass shell model as the quantum template underlying classical general relativity as a whole, it will be necessary to further to investigate the off-mass shell model for gravitational interaction in terms of other black hole metrics besides the Schwarzchild ansatz, such as Reissner-Nordstrom (charged black hole) and Kerr (rotating black hole) metrics. Also, if the Stueckelberg model can confirm the classical empirical scenarios associated with Einsteinian gravitational field models, such as the bending of light near a massive object (gravitational lensing) and the precession of the perihelion of the planet mercury, this would be further evidence to demonstrate the viability of the off-mass shell model in the context of large-scale gravitational interactions. Also, current controversy in connection with so-called black hole "firewalls", and the question of information loss in gravitational collapse that have plagued classical general relativity for decades, may indeed be afforded new insight from a quantum treatment through SHP formalism. Work is currently ongoing to verify the effectiveness of the Stueckelberg-Schrodinger equation in this context. For those who wish to further skirmish on this promising theoretical frontier, the following papers of Horwitz in collaboration with others are highly recommended [10, 31, 33, 34, 38, 39].

From the information presented in the current paper, it is highly conceivable that in the original development of the framework for relativistic quantum mechanics in the early 1930's, and in classical general relativity by Einstein as well, important pieces of the puzzle went missing as likely unfortunate sins of omission. These missing components, when properly accorded their due and incorporated into the edifice of scientific knowledge, might even have demonstrated the long sought-for compatibility between classical gravitational field physics and quantum mechanics.

Perhaps it all comes down to the ill-advised unquestioned acceptance of the original dictates by Wigner in his famous 1939 *Ann. Math.* paper [6] in which he asserted that objective properties of quantum systems must be Poincare invariant – the state space must be irreducible representations (irreps) of the Poincare group, in which the systems *must* be on-shell. However, starting with E.C.G. Stueckelberg, there has been developed an equally consistent quantum relativistic framework in which mass is considered as a dynamic variable. In other words, mass conservation is demoted from an a priori constraint, to the status of conserved Noether current for a certain class of interactions.

These questions have also been thoughtfully addressed by Caulton in his recent Powerpoint presentation entitled "Adventures off mass-shell" [40]. By posing the query as to what would constitute the correct possibility space for a relativistic quantum mechanics, he questions the associated orthodoxy which insists that the single-system Hilbert space support only one representation of the Poincare group. Caulton avers that this is false, made particularly relevant in the case of interactions.

Also, since Poincare invariance is a reality condition, this applies only to the entire system of fields, not to individual particles. An alternate approach that can be consistently applied entails expanding the state spaces to represent entire four-dimensional histories, most of which are off-mass shell, rather than instantaneous states.

Thus, concomitant with this requirement for a manifestly covariant quantum mechanics, via the relaxation of constant mass in the SHP model, is the necessity for a new worldview for *time* in quantum mechanics, in which the coordinate time t is promoted to the status of a quantum observable represented by a time-operator, reserving a separate scalar parameter τ for the monotonic evolution of the wave function.

On a more pragmatic note, putting all this together in regards to possible new technologies and forms of energy generation that could arise in conjunction with further development of the SHP off-mass shell model, one possibly could envision some quite mind-boggling ramifications. For instance, the possible change of mass that has been suggested to be a key feature of sub-atomic nuclear reactions in LENR (low energy nuclear reaction) scenarios/protocols [16-18], raises the important question about the actual nature of mass in general, beyond the microscopic so-called Higgs scenario. Particularly, in regards to macro-objects, could there be an hitherto unknown principle, which when applied through appropriate engineering protocol, would produce a mechanism that could change the very fabric of reality for that object, to reveal the actual malleable nature of the assumed properties of that object? Indeed, could a given piece of matter possess a *reality-index*, due to the ultimate intrinsic mutable nature of that object's mass? Such questions that one otherwise might think would be solely the province of science fiction, now possibly can be accorded serious consideration, after further application of the SHP model in many branches of physics.

Above all, on a grander panoramic scale, our expanding knowledge that may arise in connection with the Stueckelberg off-mass shell theory examined in both classical and quantum mechanics, will explicitly shape the future of society as well as science, especially concerning our openness to phenomena that challenge our current belief systems.

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Biological Organization as the True Foundation of Reality*

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The presumptions underlying quantum mechanics make it relevant to a limited range of situations only; furthermore, its statistical character means that it provides no answers to the question 'what is really going on?'. Following Barad, I hypothesise that the underlying mechanics has parallels with human activities, as used by Barad to account for the way quantum measurements introduce definiteness into previously indefinite situations. We are led to consider a subtle type of order, different from those commonly encountered in the discipline of physics, and yet comprehensible in terms of concepts considered by Barad and Yardley such as oppositional dynamics or 'intra-actions'. The emergent organisation implies that nature is no longer fundamentally meaningless. Agencies can be viewed as dynamical systems, so we are dealing with models involving interacting dynamical systems. The 'congealing of agencies' to which Barad refers can be equated to the presence of regulatory mechanisms restricting the range of possibilities open to the agencies concerned.

Keywords: Biological organization, Quantum mechanics, Agential realism, Karen Barad

1. Introduction – Physics and Mind

The following is where my research into how the mind fits into physics has reached at the present time. Let me say first of all that I think there is a need for a new physics, because there are a few clouds on the horizon with regard to ordinary physics. The standard model works well as far as it goes, but there are things beyond the standard model and it is proving difficult to reconcile theory and experiment. There's also the fact that standard quantum mechanics doesn't give a realist view of nature. An account that would talk about what is happening and not just give a statistical view would be preferable, since if you look just at the statistics you may end up not knowing what is really going on (for example, if you average what is said during lectures you would lose in the statistics the fact that something meaningful is happening). There are a number of reasons for going beyond the current consensus view. There's also the problem of quantum observation, where there are many different views as to what's actually happening: collapse, many worlds, transactions, and so on. That's another problem with the conventional view. The simplest approach I think is to say that one can work with *generalized life*, we are familiar with the phenomenon of life; the subject matter of biology, in the regular view that's really a special case of chemistry, thus life depends on chemistry to a great degree.

I should say that this is not really my own work: it's more a synthesis of what other people such as Barad, Bohm and Yardley have been writing [1-3]. A lot of people have been saying things along similar lines that are consistent with each other, but I'm not sure that anybody has created an integrated picture saying this is how it all fits together, which is what I am trying to do myself.

2. Generalized Life

The simplest approach to ground what I am saying is to say that one is working with *generalised life*. We're all familiar with the phenomenon of life: that's the subject matter of biology, but the life that we are familiar with is, from this perspective, a special case. It depends on chemistry to a considerable degree, in addition to which it is extremely complicated and issues like how does it

* Text based upon a recording.

really work and so on are unclear, and it might therefore be good to look at such issues from a general point of view. One view that I find useful is that of Karen Barad. In her book, "*Meeting the Universe Halfway: The Entanglement of Matter and Meaning*" [1], she talks about a concept called agential realism, the idea that agency, which I'll be defining in a moment, is the foundation of everything, and agencies work together to make phenomena. One can use this concept to unite three different perspectives: physics, biology and semiotics.

Of the three, I guess people at this meeting are most familiar with physics. The defining feature of physics, as we normally understand it, is its use of mathematics. It works with mathematical models: you do calculations, and you verify your theories that way. Biology is rather different: biologists don't normally do the precise computations that physicists do; they mainly focus on *processes*, and how things all fit together. Thus, in biology one approaches the problem of understanding nature in a totally different way.

Physicists in general are much less familiar with the third item in the trio, namely semiotics, the science of signs. This originated in the 19th century in the work of philosopher Charles Sanders Peirce [4], who studied signs and how they work in great detail. Signs are in some ways like pieces of information, which quantum physicists have started considering to be something very important. But signs are something different to this, a major difference being that while the basic characteristic of information involves it going from one place to another with two entities being involved (source and destination), semiotics involves three entities: a sign, the object that the sign relates to, and the interpretation process linking the two, mediated by what Peirce calls the *interpretant*. Further, information transmission is typically a linear process, whereas semiosis is by its nature a nonlinear one, in view of three entities being involved, a difference likely to be relevant to the detailed mechanics.

3. Standard Physics

So, regular physics, that is to say the physics found in the regular physics journals, is incomplete without the other two disciplines. These would typically be ignored by physicists, who would normally say that while biology is something they could perhaps contribute to in various ways it is not the main concern of physicists; and again, in the case of semiotics, the view would be that while this subject might be of interest to linguists it is not really the business of physics.

But I think in the new physics that will not be the case.

3.1. *Agency*

I am now going to show how it all works together, taking into account Barad's concept of *agency* [1]. Her approach is a development of Bohr's philosophy, but she criticises Bohr in that according to her Bohr was focussed on knowledge of reality, not what reality actually is. Bohr says we can't know what reality, is but Barad goes beyond Bohr by regarding *phenomena* as objective reality: we can agree that certain phenomena occur even if we're not too clear how to talk about them, and these are assumed to be the consequences of agencies working together, just as when we design a circuit and want particular phenomena to occur, we cause this to happen by having particular agencies (the transistors and their connecting wires) all work together to produce the phenomena of interest.

Now consider agency to be something that can be characterised mathematically in some way, like a Turing machine. When one has something in mathematical form one can explain what happens just by doing the mathematical analysis.

3.2. *Biology – Design Versus Mechanism*

Now there's a sense in which human designs are mechanistic while biological ones are not, a point emphasised in Robert Rosen's book *Life Itself* [5]. With human designs, there is normally an explicit statement regarding what your agencies do, whereas in biology the agencies responsible for a process such as balance change over time; the process develops and improves over time. In the process of development, the system itself finds ways for the component agencies to work together to produce the relevant phenomenon, a characteristic theme in biology, leading to what Barad refers to as *intra-action* (action within a phenomenon), as opposed to the usual *interaction*, which is less specific. This gives the situation a top-down character, with higher level processes influencing lower level ones as well as the reverse.

A point I want to make here is that biology uses very specific forms, forms that are universal, like balance and vision. Thus, in biology we find very general schemes, common to many species. Now you do have these very general schemes also in physics, for example crystals with their periodic lattices, but in biology the schemes are extremely complicated. When we ask why is biology like this, the answer is that for an organism to survive at all it has to do specific things and these require complicated machinery: if organisms were less complicated they would not survive.

This leads us to the problem of how can it happen at all. This is the origin of life problem, and I think the answer to it is to generalise biology, as I said in my introduction. Our usual picture of life derives from the life that we see around us, but perhaps there is something more general, which does not depend on chemistry, on very specific molecules. If life exists in a very simple form, it can perhaps evolve to become gradually more complicated. There is a good illustration of this with weather, which is based on the simple equations of fluid dynamics. That does feature various kinds of phenomena, such as hurricanes and clouds, some quite complicated, which we are aware of just because they persist. Such forms survive in a similar way to the way that organisms survive.

So where can we go from there? The key idea is that systems of this kind may be able to evolve and become complex even in the absence of the chemistry that plays an essential role in ordinary life. What is required basically is for them to be able to evolve strategies for survival, which would depend on the ability of structures to be able to specify (in accord with semiosis) particular behaviours, as we know does happen in the special case of the nervous system with its information processing capacities. Effective evolution requires also two further factors: reproduction, and an encoding system similar to DNA (semiosis enters at this point also). In the case of ordinary life, very complex mechanisms are involved, but these might begin in a simple way (for example, reproduction does not have to be the very general process utilised currently by life, and merely requires particular organisms to be able to produce copies of themselves), and evolve to forms with more advanced capacities, with new ones being developed from combinations of existing ones, while at the same time existing strategies get implemented in more and more effective ways. There are parallels with language, which becomes progressively more powerful over time.

So that basically is the idea. I defined a basic form of life, illustrated by the weather phenomenon, and mechanisms by which it could become more advanced and complicated. You may say that is all very well: that's a separate discipline; what's it going to do with ordinary physics? One could make the connection by postulating that this deeper form of life might be able to utilise the laws of physics as a means of survival. This implies some ability to control nature, but once you have a language system that can *refer* to what is going on, that can be used to *control* what is going on: we do that all the time. One kind of thing that could happen with enough steps of evolution is a system that can impose more and more order upon nature and produce a subset of nature which looks like our ordinary world, so there is no difficulty in principle in explaining the laws of physics. So, this is a project for the future, which may very well link with more conventional ideas, as well as the kind of ideas being talked about in this conference. Hopefully there will be cross-fertilisation between the two, as your approaches become more biological, and this one more physical.

4. Controversy

And now a controversial idea. One possibility is that if this kind of system can control the laws of physics then it may also have a role in the evolution of life, thereby providing a mechanism able to support the idea of intelligent design. In this connection, there is no *quantitative* proof that Neo-Darwinism is correct; it's not like physics. If somebody did a computation which really did show convincingly that human life might have evolved *without* having to suppose that relevant information is fed in from a deeper level, fine, but the calculation is not there, and we have instead what Popper called *promissory materialism*. It is quite possible instead that some version of the intelligent design concept is correct. Also, less controversially, it is quite possible that this kind of approach will be needed to account for such human capabilities as mathematics, where again there is no clear explanation for such skills in conventional terms. I don't think we even have any explanation for how thinking works: we can write programs that simulate thinking, but detailed connections with the neurosciences are not there. I suspect that the truth of the matter is that evolution at this deeper level of physics produces efficient systems of this kind on the basis of natural selection, and a subset of people have the ability to connect with that level, and thereby gain the inspiration needed to do advanced mathematics. Similarly, I think for music (cf. Josephson and Carpenter [6]), where we don't appear to have any good explanation for musical aesthetics, which involves very specific forms that seem to have a special creative power.

Acknowledgements

I am indebted to Ilexa Yardley and Alex Hankey for stimulating discussions of these topics.

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The Emergence of Neurocosmology: Evolution Physics, Consciousness, Physical Reality and Our Experiential Universe

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Many scientists have come to believe that any true unification theory in physics must include a concept of consciousness as well as a model for the mind that interprets the external physical/material world. And, that number is growing. This physical model goes even further. The single field theory includes a physical model of the neural net and explains how mind and consciousness can emerge from the physics of living organisms. Yet it is general enough to assimilate more intuitive models such as Andrews' 0-D point Void which witnesses and co-creates higher-dimensional Riemannian geometrical realities as well as other more generalized physical models of consciousness to form a truly synergistic model of reality. In other words, physical reality and the consciousness that perceives and interprets that reality both come from the same source, they are co-created at the very beginning of the universe. A singular discrete 0-D point/twist Void emerged within the absolute spaceless-timeless Void of nothingness that preceded everything and through a logical sequence of events produced everything that now exists as our universe. This synergistic model goes well beyond the simple notion of mind and consciousness as mere human bound perceivers and interpreters of the external material/physical world by placing the physical origin of consciousness within every geometrical point in the universe itself. Although full blown consciousness itself is not everywhere in the universe–it is not a property of every bit of matter we observe or event that we detect–the universal will of consciousness to emerge in all forms of life is everywhere in the universe. This physical model clearly demonstrates that the precursors to our experience of consciousness are fundamental elements and active participants in creating the physical world that we perceive and scientifically interpret through the application of physics.

Keywords: Single field theory, Unified field, Consciousness, Mind, Quantized space-time, Einstein, Schrödinger, Standard Model, Big Bang, Cosmic inflation, Evolution

1. Introduction

Intuitives are often the very people who have intimately and directly experienced consciousness, giving them the power to access consciousness directly at later times. They are typically Near Death Experiencers (NDErs), people who have reached mystical enlightenment and other higher states of consciousness through other means, although a greater number of people have had similar experiences that changed them mentally (rewired their neural nets in a beneficial manner) without ever consciously realizing it. They generally believe that consciousness is an active participant in creating our world rather than just deciding between choices our world offers to us. Their views of the role of consciousness with regard to the inner workings of the world in general greatly differ from those of ordinary people and especially scientists.

Even the word 'intuition' has been looked down upon in science as recently as a few decades ago. Scientists consider their own strictly logical worldview above reproach, believing their insights have been and are presently based upon strict and accurate observations of the external world around them. In general, scientists have even looked down upon all types of intuitive knowledge and have obstinately refused to consider intuitive knowledge of consciousness and how it works in relation to the world as a whole. They view intuitive data with suspicion, as no more than anecdotal evidence without any scientific validity.

Yet intuitive knowledge of the world, anecdotal or not, can provide valid observations of consciousness and how consciousness interacts with the world at large. So to those few scientists willing to seriously consider information from intuitives, knowledge of the world that did not come from direct observation of the world, it can seem as though conservative scientists are missing a large part of the world in their theoretical models. Conservative scientists also seem overly biased if not acting completely in an unscientific manner within the broader meaning of science itself by neglecting all intuitive knowledge out-of-hand.

The intuitive Sperry Andrews [1] has proposed a speculative theory of how everything in the world originated from nothing and how such theories as general relativity and quantum mechanics might account for a conscious universe if not a more formal physical Consciousness space. Andrews has suggested that the solution to the consciousness question can be found in Bernhard Riemann's original conception of (spaces) surfaces with metric curvature, since an n-dimensional space (surface) is embedded in an $n+1$ dimensional manifold all three-dimensional points are united at one point in the four-dimensional embedding space or manifold.

Andrews suggested geometrical model corresponds to the Riemannian geometry of a three-dimensional double-polar spherical surface or space embedded in a four-dimensional single-polar spherical manifold (space) as utilized in the single field theory. In the case where $n = 0$, the 0-D point (with point/twist in single field theory), which still represents a dimensionless Void, could be embedded in all higher dimensional embedding spaces.

So all point-centered events (such as quantum events) would share a dynamic relationship with each other corresponding to the relationship that every discrete point/twist in three-dimensional space shares with every other such point via their connectedness at the single-polar point in the single field model. Andrews also believes that consciousness is "a re-creative witness of what is shared inter-subjectively, coalesces with the structure of the universe as a whole by acting on physical space-time through a 0-D point-centered Void". [2] In this and other respects, his intuitive insights fit and add to the Riemannian geometric structure of the physical space-time continuum as expressed by the single field theory quite well.

The single field model unites general relativity, electromagnetic theory, quantum theory and consciousness by utilizing an interpretation of points in space as 'twists'. Each point in three-dimensional physical space-time is also a 'twist' [3] (Penrose's concept of a 'twistor' is based on Clifford's 'twist' [4]) or must have the property of 'twist' due to its natural tendency, ability or innate potential to act as a center of rotation or circular motion. Every 0-D point/twist in three-dimensional space acts as the beginning point of a vector (virtual torque) that stretches into the fourth dimension (Riemannian embedding manifold) of space in a five-dimensional space-time framework. This point-centered vector can be identified in common physics with magnetic vector potential (whereby special patterns in three-dimensional space constitute an individual organism's consciousness) and gravnetic vector potential (DE in free space and inertial mass inside material particles).

In all cases the vector potential at discrete geometrical 0-D point/twists can also be associated with the state vector Ψ (Schrödinger's wave function) which is represented by a similar or analogous dualism in the form of the quantum probabilities ψ (analogous to metric or 3-D extension space) and ψ* (analogous to anti-symmetric or 3-D point space) in quantum (matrix) mechanics. According to this interpretation of the quantum, probabilities (and indeterminism) only enter nature after the dualistic split, which means that the state vector Ψ in its role as the Schrödinger wave function is not necessarily indeterministic in itself.

In so far as the Schrödinger wave equation (simultaneously) describes physical reality of an individual observation and the superposition of all wave functions representing every possible interaction and observation in the universe simultaneously or what David Bohm called the quantum potential field. [5] This function can easily be equated to the quantized curvature (a curved sheaf or 'sheet' of parallel three-dimensional surfaces stacked in the fourth dimension of space) in Beichler's single field theory. [6, 7] Since consciousness can collapse the wave function to determine physical reality as well as play a pivotal role in the emergence/evolution of the material universe, consciousness and the single field theory together form a branch of science that should henceforth be called Neurocosmology.

2. Neurocosmology and the Single Field

In the single field theoretical structure, the role of consciousness has fundamental importance as it should in any unifying theory of physics. Twists manifest electromagnetically in the space-time continuum as the fundamental components of the magnetic vector potential field, but special multileveled magnetic (domain) structures of varying single field density patterns (complexities of memories) form individual consciousnesses such that they play out in the overall single field as separate holomovements in time. These magnetic vector field potential patterns (four-dimensional imprints of our three-dimensional living bodies) emerge in the overall single field from the originally chaotic structures of new memories to form the mental context and complexity of consciousness that we perceive in our 'selves'.

Memory structures (multi-leveled magnetic vector potential patterns) are formed through the interaction of microtubules (bio-magnetic induction coils) and surrounding water molecules (whose spins are quantized by interference patterns resulting from electromagnetic pulses emitted by the microtubules) in our neurons. In fact, the whole neural net, including plasticity, can be explained on this basis. [8, 9]

The single field theory itself is an extended, and thus completed, version of Einstein's unified field theory. [10] It completely incorporates the Standard Model of point particles and quantum fields, although the philosophical interpretation of the quantum theory differs from the normally accepted Copenhagen Interpretation and similar interpretations. Within this context, the discrete 0-D point/twists also manifest gravitationally in the space-time continuum as gravnetic (normal gravity's counterpart analogous to the electric/magnetic relationship) vector potential fields which accounts for what are mistakenly called Dark Matter and Dark Energy in modern physics.

In other words, Dark Matter is just an additional (non-local curvature) effect of the same normal baryonic matter that causes normal (local) gravity effects. This non-local gravnetic effect can be expressed by the Heaviside equation (gravitational equivalent of the Lorentz equation in electromagnetic theory) in classical Newtonian physics or the anti-symmetric tensor (Einstein, Cartan [11] and later Schrödinger) in relativity theory. The fourth spatial dimension, which acts as the embedding dimension of our normally perceived three-dimensional reality, can be geometrically modified (to account for point-elements or twists) and defined to allow the unification of gravity and electromagnetism in a five-dimensional space-time framework (Kaluza-Einstein-Bergmann).

The resulting macro-extended embedding spatial dimension can then be quantized into parallel three-dimensional 'sheets' (a quantum sheaf of three-dimensional Riemann surfaces) with an 'effective width' along the fourth spatial direction, literally quantizing the space-time curvature of the continuum. Our three-dimensional material reality corresponds to the n = 1 or lowest energy quantum ground state 'sheet'. Higher quantum energy or possible 'excited' state 'sheets' (n = 2, 3 …) are stacked in the fourth direction of space like pages in a book.

The real existence of the fifth dimension of space as an embedding dimension for our four-dimensional space-time of experience and the single field density variations that constitute other fields, material bodies and life, mind, and consciousness implies a further sixth embedding dimension whose geometry and physical characteristics are yet to be 'specified'.

This sixth embedding dimension could possibly be the 'place' where a 'cosmic consciousness', universal collective consciousness or a Consciousness space exists that could directly affect and influence all of space-time in the manner suggested by Andrews [12] and others.

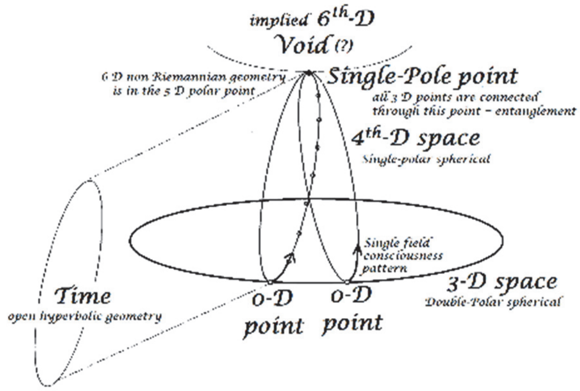

Figure 1. Physics of the 4th-D of space implies a 6-D manifold, but the physics of the 6th-D is almost completely unspecified.

The 1938 research of Einstein and Peter Bergmann [13] implied that utilizing a higher-dimensional embedding space should be the proper course for unifying gravity and electromagnetism if the physical characteristics of the embedding space could be completely specified, but they did not completely specify the geometry of the higher embedding dimension and thus failed in this earlier attempted unification utilizing a five-dimensional space-time structure.

Einstein eventually gave up on the five-dimensional approach because he could not justify using a hyperspace without any observational or detectable evidence that the higher dimension actually existed. Unfortunately, he never suspected that consciousness and intuition interacted with the universe as a whole through the higher dimension in what we normally call intuition and sometimes paranormally refer to as our sixth sense, otherwise he may have taken the higher-dimensional approach to unification more seriously.

Our material bodies can be represented in relativity theory as a complex matter/energy pattern (a three-dimensional curved surface that undulates over time) equivalent to a complex quantized curvature pattern (four-dimensional) that varies internally over time. Within this relativistic context, our mind can be modeled as a corresponding three-dimensional complex electric field pattern within the quantized curvature pattern. Mind, as such, is ephemeral and would disappear (decohere or destabilize) after a few moments as individual electrical potentials neutralized or canceled each other out, positive for negative, unless some higher form or level permanent 'pattern' were not at work to lend more than temporary stability and even 'permanence' to the electrical field patterns that constitute the action of mind. Such a higher level pattern could only be magnetic in nature according to electromagnetic theory.

Thus, an individual consciousness could only emerge from and be equated to the multi-leveled magnetic (domain structure) pattern made up of vector potential points in three-dimensional space that extend into the fourth dimension of space. So every living organism has its own individual consciousness, not just humans and other highly evolved animals, and that consciousness extends into the higher embedding dimension of our commonly experienced four-dimensional space-time, representing each living organism's own unique experiential existence within the material/physical world context of the whole universe.

The scientific theories that we use to explain our external physical/material three-dimensional world and how it varies over time are constructed within the mental context provided by this mind and consciousness, which are themselves products of the physical structure of the same world which is being perceived. Consciousness, and only consciousness, differs radically enough (locally) in its individual structure from the surrounding physical/material structure of the world (non-locally) enough, but not so much, that it could also be a physical and yet non-material extension in the higher-dimensional embedding space. Only a consciousness of this type could think of its 'self' separate enough from the physical/material world while still remaining part of that physical world to perceive its 'self' as separate from its surrounding external material world.

Within this context, the philosophical debate between quantum discreteness and relativity's continuity, which has poisoned real advances in physics over the last century, are actually a misstatement and misrepresentation of the geometrical problem of simultaneously accounting for a point-space (Riemann's point-element) and an extension-space (Riemann's metric-element). [14] Placing this problem within its correct context and recognizing the problem in its true form, as just the simple geometric dualism of physical space (point as quantum versus extension as metric curvature), resolves the physical problems between quantum and relativity that have previously plagued physics. Both of these problems, geometrical and physical, reduce to our conscious interpretation of space and time as perceived by the brain/mind and interpreted by consciousness.

In the end, physical space is one constant thing, neither point or extension based, but actually neither and/or both simultaneously. This situation is hard to envision and thus harder to understand at the present three-dimensional level of theoretical physics, which is an abstraction that merely reflects the way we experience and thus commonly think of our world. Quite clearly, our mental picture of the world is three-dimensionally biased due to the preponderance of our simple three-dimensional sensations of the physical world. This duality is therefore a product of our brain/mind that only a pure reference to consciousness and its higher-dimensional reality can ultimately solve. We must perceive three-dimensional space as a unitary or holistic conceptual 'thing', not as the dualistic reducible 'thing' that geometry tells us it is, to fully develop science and advance physics.

So the unresolved problems of unifying physics naturally come right back to understanding consciousness and its interpretative relationship with the natural world of perception and how the natural world is represented by a particular geometrical model of space and time. When this truer and more accurate physical reality is realized, the determinism/indeterminism debate reduces to no more than "much ado about nothing" since neither viewpoint actually represents physical reality, just human vanity as related to the whole of physical reality. Nature tells us how nature acts in any given physical situation through our observations of nature, we do not tell nature how to act based upon our philosophical and mathematical interpretations of how we think nature 'should' act. In other words, we should not project our mental and philosophical biases onto the external natural world in our attempts to understand how nature works. This means that the quantum and relativity are not incompatible as has long been thought, but are in fact totally and completely compatible.

Completing the Einstein unified field theory by combining the anti-symmetric approach of Erwin Schrödinger [15] and its equivalent non-symmetric equivalent pursued simultaneously by Einstein [16] (to account for DM and DE) with the higher embedding dimension approach of Theodor Kaluza (to account for a unified EM and GR), [17] and accepting the consequences of doing so by accounting for points given this this new geometrical structure, leads to a full unification of quantum and relativity in the form of a quantized space-time curvature. The curvature is quantized by utilizing Oscar Klein's implied suggestion [18] that quantizing the embedding dimension (in this case the fourth spatial dimension), even though it is now macroscopically extended and closed, quantizes three-dimensional space (the embedded surface/manifold).

Each three-dimensional 'sheet' (stacked like pages in a book in the fourth direction of space) is actually a quantized group of parallel three-dimensional (infinitesimally thin) Riemannian surfaces intersecting and perpendicular to four-dimensional extensions of three-space points as described by Einstein and Bergmann. In other words, it is our three-dimensionally moderated conscious geometrical interpretation, or rather misinterpretation, of space and time that is presently delaying the progress of physics, which is exactly why an intuitive approach to the problem has now become necessary to overcome the deadlock and begin to advance science once again.

3. A 0-D Point/Twist Void Replaces the Original Singularity

Everything in our scientific model of reality changes by adopting the 0-D point/twist Void as the original Riemannian point-element from which our more advanced Riemannian space-time structure of physical reality evolved. For example, the original singularity in the form of a dimensionless point-centered process from which everything (or every 'something') in our universe evolved (according to the Big Bang or other models) has specific qualities that separate it from the absolute Void of 'no-thing-ness' from which it emerged. Establishing how these 'differ' defines how the evolution of our experienced material/physical universe has proceeded over the 'life' of our universe, including the evolution of life, mind and consciousness, within the originally 'no-living-thing' or normally inanimate nature of matter and energy.

The Riemannian geometry that expresses this unification starts with the discrete 0-D point/twist Void which creates our commonly experienced three-dimensional physical space, embedded in a fourth dimension of space. From this nothing of zero dimension, our three-dimensional matter/field/energy reality emerged from the potential of the single field in the four-dimensional embedding space. While this geometry accounts for and describes the creation of the four-dimensional space-time continuum, it also accounts for the dynamical substantiality of our world that is solely a product of the single field potential. The twist portion of the three-dimensional discrete 0-D point/twist maintains and guarantees the integrity of this fundamental unit of re-creation as it creates the 'virtual torques' (pre-force) in both directions of the fourth dimension, which are collectively the precursors for the potential and anti-potential of the single field.

In this new post-Riemannian geometry, which is based upon both Riemann's original metric- and the added point-elements, the higher embedding fourth dimension must be single-polar spherical and this geometric pre-requisite is fulfilled by the simple fact that the virtual torques and negative virtual torques (having oppositely directed twists) come together at the polar point. Their oppositely directed twists meet at the central single-pole point to give a full half twist over the full extension of a closed-loop line drawn from a point in the three-dimensional surface into the fourth dimension as required by the Riemannian geometrical structure.

However, these differences imply the existence of a further sixth embedding dimension whose geometry is completely unspecified except possibly at the single-polar point where the next embedding space comes into contact with the lower embedded dimensions of space. So the sixth embedding dimension is physically open in that it can be used for any physical or semi-physical characteristics that are as yet unaccounted for with the one stipulation that whatever characteristics are adopted for the sixth dimension, they must manifest universally throughout the lower embedded dimensions.

The discrete nature of the 0-D point/twist Void also allows for the quantization of the single field and formation of quantum fields to be rendered in terms of Riemannian geometry, further allowing quantum (matrix) mechanics and wave mechanics to be adequately explained as physical characteristics of the geometrical point/twists (discrete quantum field centers) within the context of the single field (which is equivalent to Bohm's quantum potential field). The single field can also be interpreted as the superposition of all possible Schrödinger wave functions for all possible quantum events while the 'collapsed' wave function corresponds to the four-dimensional extension of any particle from its three-dimensional center of mass.

The single field also serves as the precursor to all classical three-dimensional fields, such as gravity, electricity and magnetism, as well as matter/energy, life, mind and consciousness, all of which can be described [19, 20] as a continuous spectrum of single field density patterns in five-dimensional space. These structures form our complete external reality (external to mind and consciousness) which is essentially a three-dimensional projected (intrinsic reduction) surface of the extrinsic four-dimensional space-time curvature in an overall five-dimensional continuum.

The inanimate matter that we perceive in our three-dimensional brain/minds (through three-dimensional sensations) is no more, nor less, than temporal and spatial variations of curvature of the three-dimensional surface ('sheet') as it is extrinsically extended into the higher embedding fourth dimension of space. These are accompanied by the normal electric and magnetic fields associated with inanimate matter as perceived by us, while the emergence of life, mind and consciousness through the evolutionary process proceeds from the development over time of specific complexities of matter/energy, electric and magnetic fields.

The evolution of life and consciousness itself has been influenced by and proceeded from a primordial or primal awareness based on the reciprocal relationship between the absolute Void of nothingness that preceded the Big Bang (or other event that created or began the clock ticking for our present universe) and the 0-D discrete point/twist Void that emerged from that absolute Void as the original singularity. The 0-D discrete point/twist Void thus implies the possibility of something coming from nothing which introduces a way to explain how the 'some-thingness' of our perceived physical/material universe emerged and evolved from the 'no-thingness' of the assumed Void that existed before the Big Bang within the Riemann geometric context of the single field theory.

4. Abstracting the Point as an Infinitesimal Sphere

The only way that a higher-dimensional embedding dimension could be envisioned or imagined by our three-dimensional mind/brain is if each and every discrete geometrical point or 0-D point/twist Void is constantly trying to enfold into itself and thus back into the Void from which it originally emerged. Instead, it just pushes itself by duplication further into the fourth dimension of space before unfolding three-dimensionally back into itself where it again duplicates itself and thus expands and creates the three-dimensions of normal space.

However, physical reality (and logic) would dictate that such a 0-D point/twist could only (or must) be stable since our space, which is made of such point/twists, does not 'collapse' into itself (by enfolding), but remains constant.

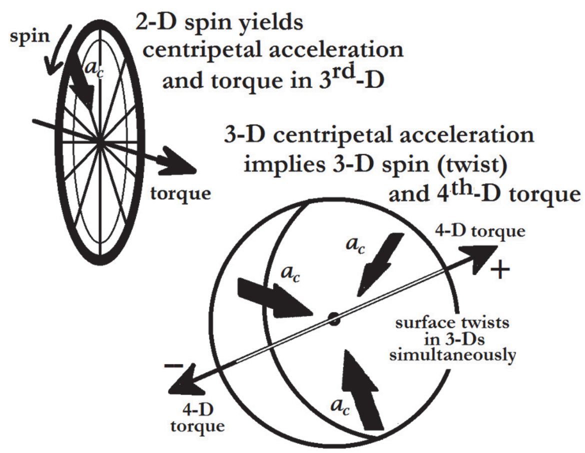

Figure 2. Centripetal acceleration inward implies 'twist' in the 3-D surface by analogy to the 2-D spinning wheel.

Therefore, the 0-D discrete point/twist Void must be a dynamical object–a stable object whose stability depends upon a dynamic equilibrium–in that it would constantly and continuously be trying to enfold into itself, more-or-less like an object spinning three-dimensionally toward its center point in three-dimensional space, while an equal and opposite unfolding outward 'virtual force' occurred to stabilize it. An enfolding of this type could be abstractly described as a three-dimensional 'virtual spinning', or 'twist', of a three-dimensional object into itself in four-dimensional space.

A 0-D discrete point/twist can thus be approximated, or pictured, as a two-dimensional spherical surface in three-dimensional space 'spinning' three-dimensionally inward, toward its center. A discrete geometrical point can thus be imagined as a dimensionless point-centered spherical surface in three-dimensional space of (or approaching) zero radius ($\Delta r \rightarrow 0$ analogously to the case of $\Delta S \rightarrow 0$ in Riemannian metric geometry). In other words, we can determine the internal structure and physical attributes of this spherical point by decreasing its infinitesimally small but not quite zero radius (measure of its extension Δs in the three-dimensions of space) to zero (a dimensionless point) in a Riemannian manner using our imagination. Doing so gets rid of the extension in space along all radii, but not the enfolding spin that endows each dimensionless point with its characteristic 'twist'.

So, a 0-D point/twist is a sphere-like structure whose radius has been reduced to, or approaches, its infinitesimal limits of zero (simultaneously) in each of the sphere's three dimensions. Yet its three-dimensional spin, or twist, would still result in it enfolding into itself, creating a 'virtual torque, as well as expanding by duplication of new 0-D point/twists in each direction into the fourth dimension of space.

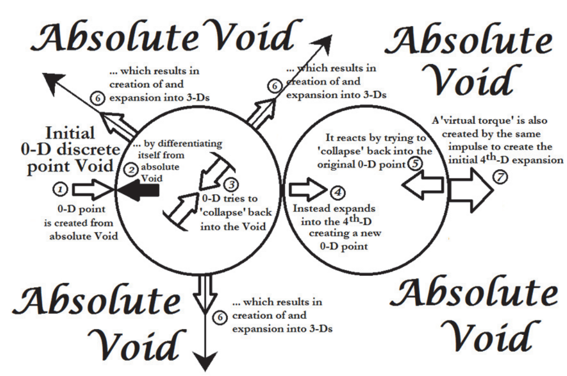

Figure 3. The 0-D point/twist Void causes expansion in all 4-Ds due to its 'desire' to collapse inward, when it cannot.

These new 0-D point/twists in both directions of four-dimensional space would form as an equal but opposite reaction to any action implied by the 'desire' or 'need' of the original 0-D point/twist Void to completely 'collapse', or 'implode', back into the absolute Void as this would be prevented by the 'twist'. The virtual torque in the fourth embedding direction (#7 in the diagram above) of our real physical space is thus a product of the 'twist' of every 0-D point/twist Void.

The 'twist' also creates a 'virtual torsion' in the three-dimensional space (sometimes referred to as 'torsion space' by other physicists) surrounding each and every 0-D point/twist Void. So all the discrete geometrical points that constitute our 'real' perceived four-dimensional space-time continuum are actually physically real 0-D discrete point/twists attempting to collapse back into an absolute Void, but they are prevented from doing so since they are maintained (or stabilized) in a dynamic equilibrium by the 'twist'.

The resulting 'torsion' field in the expansion direction of the surrounding three-dimensions of space results in the creation of new discrete 0-D point/twists and the subsequent expansion of three-dimensional space that is made up of all such 0-D point/twist Voids. This point-centered 'virtual torsion' in three-dimensional space also accounts for the point-centered nature of magnetism and gravnetism, the gravitational equivalent of magnetism, in the material universe. This (action/reaction) co-creative process takes place, and repeats itself, during every infinitesimal moment-to-moment of time, which leads to an explosive expansion (commonly called cosmic inflation) of three-dimensional space coupled to an equivalent expansion into the fourth direction of space that continues until an infinite number of moments have passed, such that (true) measurable extensions of space (length, area and volume) and time (duration) come into being.

The first 10^{-36} seconds after the Big Bang, which cosmologists speak of as the shortest amount of time after the initial singularity event, would just amount to the duration of time equal to an infinite number of moments (infinitesimal points of time) during which an infinite number of discrete 0-D point/twists were created to allow the first measurable extension (volume) of three-dimensional space. The expansion was just as 'explosively' rapid during that period as after, but after that period each moment's expansion 'trebled' the extent of each of the four-dimensions of space in what has become called 'cosmic inflation'. During this period the larger part (volume) of our universe was created at more than the speed of light, which is a ridiculous notion since there was not yet any speed of light nor anything else but what was virtual, semi-physical or potential, until approximately 10^{-32} seconds had passed and cosmic inflation ended abruptly, or so we are told.

The various 'virtual torques' in the fourth dimension correspond collectively to a pure physical 'potential' and thus form the beginning of the single field that corresponds to a geometrically structured four-dimensional space with varying internal density within a five-dimensional space-time continuum. These virtual torques collectively form pure potential as the single field, not energy or matter themselves, but the potential to later form matter and energy, given both the quantum and geometric restrictions of the space-time continuum and single field, by which matter/energy and other physical fields are defined. No energy existed, just pure potential, before the period of cosmic inflation ended since there were no real material particles to carry the energy, just as there was no speed of light since the electromagnetic and gravito-gravnetic fields had not yet formed in and around material particles as stresses and strains in the curvature of the space-time continuum.

A large part of the story that modern cosmology, especially quantum cosmology, tells about this period of time are just imagination-driven false speculations fed by a misinterpretation of what constitutes material particles and energy. The expansion continued until an as yet undefined moment in the process when either quantum anomalies, some form of anomalous single field fluctuations, or geometric conditions caused a 'blow-out' at some points in the balloon-like three-dimensional surface of our universe. These 'blow-out' points formed the first protons after the surface ('sheet') counteracted and closed (or capped) them off.

The 'problem' of too rapid an expansion was still not fixed with the initial 'blow-out' forming protons and a new series of 'blow-outs' began, but this time the counteracting surface tension (of the parallel three-dimensional surfaces) was enough to stop the local point-centered curvature from blowing-out thus creating electrons with the equal and opposite electrical charge of protons. Any other excess (virtual) 'momentum' of the inflationary expansion outward only resulted in small (the minimum local amount of point-centered curvature distinguishable and thus measurable to the surface or quantum 'sheet' of parallel three-dimensional surfaces) puckers, or bumps, that science now detects as free neutrinos.

This process ended the inflationary period and slowed down the runaway expansion, locking the more slowly expanding 'three-dimensional surface' of the universe into what we detect today with only small variations. No anti-particles were created at this time (which is why they have not been observed or detected by science) since the 'blow-outs' were all directed in the favored direction (outward for positively curved surfaces) of the fourth dimension instead of inward.

From this moment onward, our present day universe has continued to evolve according to thermodynamical principles and the other theories of physics into the universe that we now perceive.

The physical/material universe that thus evolved is presently characterized by discrete geometrical points that appear physically as discrete 0-D point/twists of Void in all four dimensions of space. From the very beginning singularity and onward, there have existed certain immeasurable and vaguely defined 'qualities' that eventually led to (or even 'forced' or 'pushed' although 'influenced' might be a better term) the emergence and evolution of life, mind and consciousness. In other words, the potential for life, mind and consciousness already existed in every geometric point in space, whether it was inhabited by matter or not. The original 0-D point/twist Void (a 'some-thing') was differentiated into existence (and thus began time) from the absolute Void of 'no-thing-ness'.

The very fact of some type of 'differentiation' implies some form of primal awareness between the absolute Void that was before creation, the initial singularity that was created and the 0-D point/twist Void into which the singularity evolved before the Big Bang began. That differentiation process, whatever it was or whatever form it took, created the 0-D point-twist 'tendency', 'desire', 'need', 'instinct', 'memory', or whatever it can be called, for a primal awareness that differentiated it from the absolute Void, as a primary quality (or 'qualia') of the 0-D point/twist Void.

Every time that the discrete 0-D point/twist Void duplicated itself during the expansion process, the newly created discrete 0-D point/twists carried with them the same primal awareness and thus its very own distinction of its 'self'. Each geometric point in space thus 'senses' its 'self' as not being another such geometrical point or they would all collapse and become a single dimensionless spaceless-timeless nothingness of the Void. Physically the twist keeps them from such a 'collapse' as well as reabsorbing each other, guaranteeing the discrete nature of the geometrical points of space as well as the physical 0-D point/twists themselves.

So the 'sense' of a primal awareness is related to the physical property of the 'twist' that is associated with each geometrical point. It allows them to remain contiguous but separate so that they can form a continuous extension while remaining discrete within their dimensionless selves. Just as all of the 'virtual torques' of each point in four-dimensional space collectively yield the potential of the single field, the collective nature of this primal awareness lends or imparts space as a whole with a pre-consciousness potential in the form of a semi-physical field.

The single field potential is the precursor for all matter, fields and energy in the universe while the corresponding pre-consciousness field potential is the precursor for the later emergence, evolution and further development of life, mind and consciousness that is associated with or coupled to inanimate matter.

Just as the collective effect of the 0-D point/twist Void's 'virtual torque' in 4-D space is the establishment of a single field potential that is the precursor to matter/energy, the collective effect of the 0-D point/twist Void's inherent quality of 'primal awareness' is the establishment of a corresponding pre-consciousness field potential in physical space that is the precursor to the emergence of life and evolution of consciousness

Figure 4. The pre-consciousness potential field is necessary.

In other words, the universe itself is imbued with the potential for the emergence of consciousness in every infinitesimal geometrical discrete point from which it is constructed. This structural property or quality could be described as a Consciousness space, universal collective consciousness, cosmic consciousness, or even an absolute space such as the "sensorium of God", as Isaac Newton called it. Technically, all of these descriptive words work with the concept to one extent or another and only a better and more advanced physical theory can distinguish between them or offer a better alternative than this.

5. The Point of Unification

The worldview of physics has just consciously changed, so the physics of the world must change to compensate and remain relevant. The long neglected geometrical point, which has barely had a place in physics at all except to cause problems (at singularities), has now claimed a new relevance and champion in the discrete 0-D point/twist Void and corresponding Riemannian point-element. So, while its contribution to physics has been largely dismissed in the past, the geometrical point of physical space can no longer be ignored in Einstein/Riemannian relativity theory.

The concept of a point-element has allowed mathematics a chance to define and characterize the higher embedding manifold/space of a three-dimensional surface, which has, in turn redefined relativity theory. [21] How it changes quantum theory relative to unification must also be explored and the best place to start is with the Heisenberg uncertainty principle (HUP) since the HUP enunciates and interprets the basic formulas of quantum mechanics.

Given the different formulations of the HUP, which basically defines everything that follows in the quantum theory, several ways to proceed that allow other physical models of reality to be included or unified with the quantum theory can be demonstrated. By setting these two equations equal, as they are equal to the same quantity (h bar over two), we get for the simple case of a material interaction at the quantum level of reality, which localizes an event in space and time, the equation

$$\Delta x \Delta p = \Delta E \Delta t \,. \quad (1)$$

From this equation, it would seem from HUP's expression of uncertainty that bringing space and time together, as represented by the different uncertainties, suppresses the quantum effect.

This suppression is solidified and exemplified by the disappearance of Planck's constant, rendering the event physically real for consideration by classical physics.

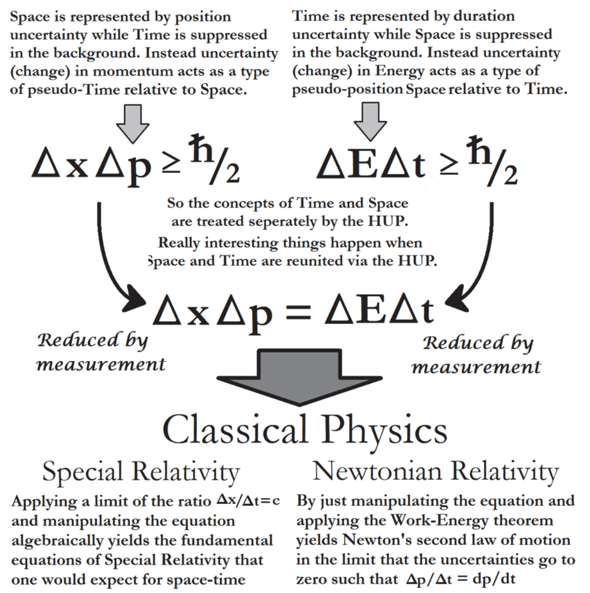

Figure 5. Classical and quantum physics are fully compatible.

For example, when the condition that the ratio of the uncertainties in position to time is less than or equal to the speed of light ($\Delta x/\Delta t \leq c$), Einstein's equations for special relativity can be easily (algebraically) derived. On the other hand, when that condition is relaxed such that the speed of light is not considered at all in applying the uncertainty principle, Newton's second law of motion ($F = dp/dt$) can also be derived. [22]

In other words, suppressing Plank's constant by combining the different quantum expressions for space and time results in a reality described by Newtonian physics and general relativity, or rather classical physics. When quantum restrictions are suppressed in this manner, quantum theory becomes closed with respect to classical physics, meaning that quantum theory can never be derived from relativity theory, just as Plank's constant could never just pop up out of any normal relativistic considerations of material reality in either Newtonian three-dimensional space or Einsteinian four-dimensional space-time as Einstein hoped.

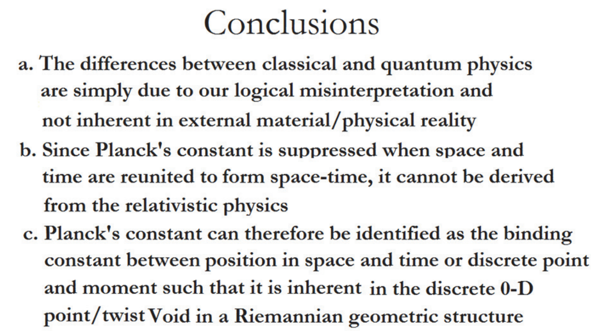

Figure 6. Interpreting the correct role of Planck's constant in space-time physics leads to a simple unification of physics.

The false belief that the relativity and quantum theories will always be mutually incompatible has dominated theoretical physics for the past century, when in fact they are only mutually incompatible with regard to three-dimensional space. So they cannot be unified intact, as they now exist in physics, while retaining the major characteristics and concepts of each theory. It is thus true that quantum indeterminism has no place in a continuous world, just as a discrete point cannot exist along a continuous line (it would from a discontinuity) or surface, yet an infinite number of discrete 0-D point/twists of Void still make up a continuous space-time manifold.

The quantum theory and relativity, in physical reality as well as in purely mathematical geometry, are in truth mutually compatible. When each is fully understood they can be easily unified. Moreover, the continuous world of relativity can remain deterministic while the quantum world of the discrete point remains indeterministic. Under these circumstances, it is safe to conclude that the HUP is merely a limiting condition that applies when circumstances (specific physical conditions) are established to artificially separate changes in time and three-dimensional space by experimental means. Doing so invokes Planck's constant, which vastly limits the physical possibilities of what might occur or transpire in any material interaction under consideration, thus reducing the determinism of nature and the real world into the indeterminism of mathematics within the very discrete points of space which constitute a virtual absolute space. This means that it makes the most sense for the Planck constant to be interpreted as the binding constant for space and time, to yield space-time.

So, given this interpretation of the HUP, a specific quantum event, as specified by the collapse of its wave function, can be localized at a specific known point in space-time as opposed to all other non-localized quantum point-events. This means that another path can be followed that leads to a complementary interpretation of the quantum and this path implies the physical reality of a higher embedding dimension of space. In the original equations of the HUP, when Δx and Δt are simultaneously forced to go to zero (by measurement or observation), an exact discrete point location in space and time results.

This point could then be considered (equivalent to) the point of origin in a space-time diagram that represents a specific quantum event in space-time, wherein both Δp and ΔE become infinite (undefined). This may seem a trivial concept, but it is instead full of useful information since the localized point in space-time corresponds to Andrews' concept of a 0-D point Void (or a discrete 0-D point/twist) and the event can be interpreted as a point-element with respect to a three-dimensional Riemannian surface curved In a four-dimensional manifold or embedding space.

6. Quantizing Four-Dimensional Space-Time

This shared point of view between an intuitive and scientists can be better illustrated using a common (Herman) Minkowski space-time diagram. The origin of the space and time axes coincides with Andrews' 0-D point in a Riemannian geometry as well as with a localized discrete point that marks a specific event in the quantum theory.

The 'absolute elsewhere', which appeared in Minkowski's original development of the space-time continuum, has never been considered viable or even meaningful in modern relativity physics, it is considered a useless archaic concept. Yet, it still implies that something can exist beyond the purview of relativity (underneath or in the background of our physical space reality). So, it really should be of interest in fundamental physics now that the origin of the space-time diagram, literally the zero-point of a space-time event, has been related to the quantum theory and point location of the collapse of the wave function.

Minkowski's 'absolute elsewhere' can now be interpreted as relevant in a combined quantum/relativistic five-dimensional space-time framework with respect to the discrete 0-D point/twist Void, not just the point location in space-time at its origin, which completely alters its traditional non-role in relativity physics. Quite simply the 'absolute elsewhere' can be identified with a higher embedding dimension of space-time that is physically real even though beyond direct observation and detection, yet necessary to unify the different theories (modern paradigms) of physics. The concept also unites all four dimensions of space-time as a whole by providing a role for the formation of 'qualia' in our experience as three-dimensional beings due to the fact that it can also be identified with the semi-physical pre-consciousness field.

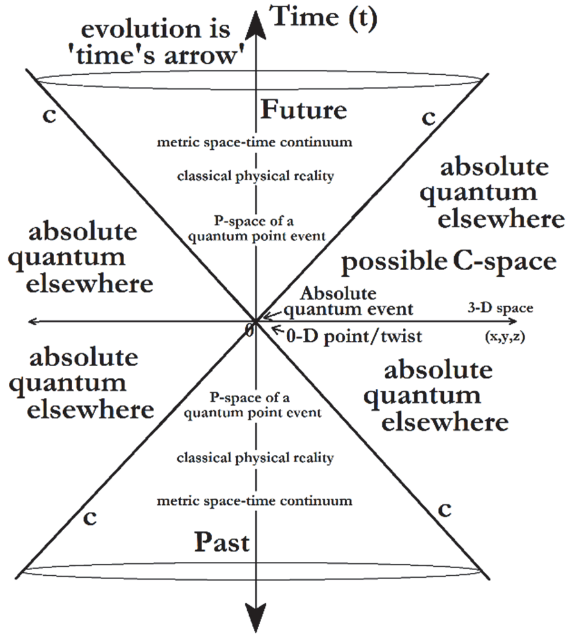

Figure 7. The origin of a space-time axis coincides with the point event created from a quantum wave collapse.

In other words, it can be related to consciousness even though we cannot directly perceive an event occurring outside the light cone, implying that we can know of the event indirectly without directly observing it. Since single field theory utilizes a five-dimensional space-time model with a single polar point through which all points in three-dimensional space are directly connected to each other and a sixth dimension is implied by the physics, the single polar point could be equated to a point in a further sixth embedding dimension of space that could thus take the form of Andrews' all-encompassing witnessing consciousness, acting or co-creating physical reality through the individual discrete 0-D point/twist Voids in space-time.

If we localize the quantum event to a discrete point (a discrete geometrical point particle or a simple 0-D Void point/twist in terms of Riemannian geometry), then what's left of space-time outside of or beyond the light cone (the so-called normally irrelevant 'absolute elsewhere') can be interpreted as physically equivalent to a region (or a volume in three-dimensional space) of infinite uncertainty. At least the fact that mathematically $\Delta E = \Delta p =$ infinity that results from an absolutely certain measurement of a discrete geometrical point in time and/or space can thus be equated to the 'absolute elsewhere' as an indeterministic infinite region of reality. Or rather $\Delta E/\Delta p =$ infinity/infinity = 𝟙 (some form of unity or oneness), a single unique reality of an infinite number of discrete points, according to Andrews [23] that corresponds to the region of the space-time diagram beyond the physically possible limits set by the speed of light c, where $\Delta x/\Delta t > c$.

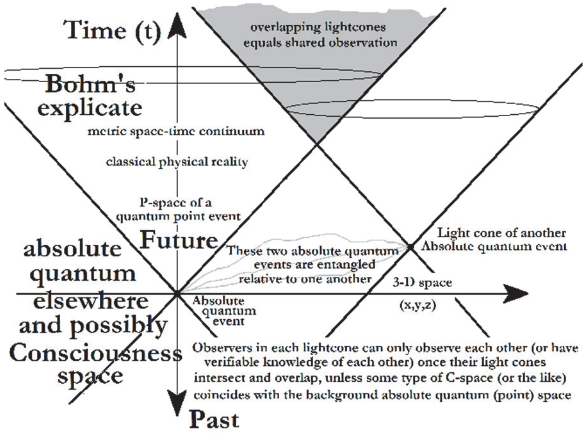

Figure 8. Time cones for different quantum events overlap and only in the overlap will one event be 'aware' of the other.

His 'unity' would amount to a single 'whole' space in itself constructed from the infinitely uncertain number of discrete points. This region is thus complementary and even necessary to fully understand the region inside the light cone that is classically deterministic with regard to both Newtonian and relativistic worldviews. The 'absolute elsewhere' thus represents the part of the diagram where infinity means 'undefined' rather than 'a number too large to count'. So on a space-time diagram, the infinite, or indefinite, nature of Δp and ΔE would clearly correspond to the region outside of the light cone as the range of physical possibilities for any particular discrete quantum point event potentially occurring in the 'absolute elsewhere'.

This region of the space-time diagram could also correspond to a higher embedding ($n+1$) dimension of our n-dimensional Riemannian surface, since the speed of light only applies in our normal three-dimensional space. Since the spread of light outward from a point-source is only a limit in three-dimensional space, it has no significance along the fourth direction of space, so a virtual photon, which corresponds to a discrete point in three-dimensional space as a classical spherical electromagnetic wave front moves through that point, would follow or move along a straight line instantaneously in the corresponding fourth direction of space and back into itself along a closed loop (path) in four-dimensional space. A virtual photon, and even a real photon that emerges when the virtual photon becomes real during some material quantum event, could thus carry information between distant locations in three-dimensional space instantaneously via the higher embedding dimension of our three-dimensional space in a process that is otherwise called quantum entanglement.

We could therefore actually learn about and possibly observe events outside of our personal light cone along this four-dimensional closed loop or path by means other than our normal senses, for example via some form of collective consciousness in the higher space. Consciousness could easily utilize the hyper-dimensional connections between discrete 0-D point/twists in our common three-dimensional space and those that are non-local (outside our light cone) in the 'absolute elsewhere'.

When this explanation is considered within the context of a background collective 'absolute elsewhere' that is associated with consciousness in some physical manner, all events in the universe are simultaneously known to consciousness and fully capable of being known by a higher enough level individual consciousness even if the individual's brain is not consciously aware of that knowledge.

7. The Absolute Nature of Q-Space

Yet the above space-time diagram is still incomplete and misleading since it only refers to the reference frame of one particular quantum point event as consciously collapsed or localized by the HUP equations to the origin of a standard space-time diagram. In reality, the real universe consists of an infinite number of other quantum point-centered events (that are just as real) which lay outside of any one point particle's light cone (*i.e.* within its own 'absolute elsewhere'), wherein all point particle events (taken together) constitute the whole of our experienced physical universe.

This collective background of all individual discrete quantum point events, including the quantum point events both inside and outside of any one quantum point event's unique 'absolute elsewhere', could just as well be related to Bohm's quantum potential field or even his implicate order. However, it is actually a discrete point-generated absolute Quantum-space or Q-space, hiding or suppressed in the indeterministic background by the determinism of relativistic classical space.

All real quantum field points in physical space (points that exist after the collapse of the wave function into an apparent classical reality) are entangled by the geometric restrictions of the five-dimensional space-time continuum, even though they may be unobservable and (materially) non-interactive within any given 0-D point/twist's 'absolute elsewhere' (outside of its light cone) until a future time when their light cones overlap. Only then could any information be gained about events in the 'absolute elsewhere' relative to our light cone by normal means and only then can the occurrence of events outside the range of our normal senses and scientific instruments be confirmed as real (having actually occurred) or not.

So the complete collective 'absolute elsewhere' of all possible real events is a point-by-point generated background four-dimensional space-time continuum that is simultaneously relative as a whole to the whole material universe of extrinsically curved relative four-dimensional space-time continuum.

This hypothetical background space-time is the collective effect of all the infinite number of differently located discrete 0-D point/twists that constitute our commonly experienced physical reality and thus constitutes an absolute relative space-time that can only lie somewhere behind (in the background) of the whole of normal relative physical space, whether it is located within our particular light cone or not. It would be a commonly shared virtual 'absolute elsewhere' that is reduced by each and every discrete quantum 0-D point/twist event to suit that event and that event only when the psi function describing the possibility of that event collapses. Therefore, the complete 'absolute elsewhere' is not simply beyond the light-cone of any one particular quantum event, it is 'absolutely everywhere' beyond all possible events that occur in the four-dimensional space-time continuum all the time.

Since a specific 'absolute elsewhere' is isolated, and thus defined by each and every discrete quantum point event in relative space, out of the whole virtual and infinite collection of discrete points that constitute all of relative space, a specific 'absolute elsewhere' must require a collection of corresponding 'absolute elsewheres' that constitutes a virtual background 'space' of its own. This absolute background space structure is absolutely necessary to complete the relativity of experiential material/physical three-dimensional space so it can even exist. This virtual 'absolute elsewhere' space must exist somewhere that is not the relative four-dimensional space-time constituted by discrete quantum points. This then implies a higher-dimensional co-space that maps point-by-point onto our normal four-dimensional space-time of experience.

Even a 'Newtonian-like' absolute space, which was associated by Descartes with mind, consciousness and God, and described by Newton as the "sensorium of God", could be used to represent this virtual background 'absolute elsewhere', depending on any given scientist's philosophical point of view. But this new single field model implies that the 'absolute elsewhere' is a semi-physical pre-consciousness space-time–it is filled with a semi-physical pre-consciousness potential field–or embedding manifold that both co-creates and witnesses events in the three-dimensional physical world of matter/energy fields through the individual discrete 0-D point/twists.

As such, it would both influence and define the evolution of life, mind and consciousness, as natural processes in the material/physical universe. Since it is a complete space-time model in itself, it could not be equated to the fourth dimension of space or with five-dimensional space-time, but could be equated to a complete six-dimensional manifold in which the whole of five-dimensional space-time is embedded.

8. Synergy with Other Conceptual Models

In Federico Faggin's model [24] as well as more speculative models, Consciousness-space or some form of super consciousness (rather than Physical or P-space) is the actual reality that generates P-space through Information- or I-space. Faggin's model at least forms some mechanism to account for this action in that his C-space acts through I-space to create P-space utilizing the quantum theory of point-particles on a point-by-point basis. However, it could also be viewed as the final evolutionary state of our real consciously perceived and interpreted material/physical universe with respect to the Beichler-Andrews model. [25] Faggin's and the whole range of speculative models more-or-less offer a look at the ideal and/or final state of the universe toward which the universe as a whole is presently evolving: An ideal and/or final state of pure being relative to our present state of both being and becoming.

Still other physical models our world and existence that claim reality is just information, a hologram, a computer program, quantum bits or some other such device are more metaphysical speculation than science. So they offer little if anything to science other than a form of escapism and an excuse not to do real physics and develop a new and better theoretical paradigm to replace modern physics. This category is filled with non-sensed realities (literally non-sensed since our sensations of the material world are themselves material), that are being mistakenly interpreted by our consciousnesses as material reality, thus rendering our sensed material reality as somehow not real or un-real. These speculative models claim that the material reality we experience is nothing but a mental illusion, a trick played on us by our individual consciousnesses or some greater Consciousness.

Within this context, questions regarding God, a Supreme Being or Supreme (advanced) Consciousness are often raised, since they take advantage of a logical loophole of sorts in the scientific arguments, *i.e.*, it is philosophically impossible in science to even prove that what we sense really exists. According to a generalization of Gödel's Principle, science must go outside of any physical system, such as our universe, to prove the existence of that physical system. All that can be proven from within a system, such as our universe, is the logical consistency or nature of that system. So all that can be 'proven' about our experienced physical universe is the logical consistency of events in our universe, which is as good a definition of physics and science in general as has ever been made.

So the best science can do, according to its own doctrine, is develop theories that explain what we sense as existing and verify those theories, but never 'prove' them. Under these conditions, the reality of God or a Supreme Being is not within the realm of science to either confirm or deny even though almost everyone senses that there is far more to our world than we sense or even can sense through our three-dimensionally limited senses. God is neither definable and thus measurable nor verifiable by any possible scientific standards. The concept is simply not falsifiable and therefore not scientific.

Under these circumstances, there always exists the possibility that things beyond science do really exist and they can still be validly discussed and debated, just not within a scientific context. For example we sense some type of a non-material force at work in evolution. Being non-material and even non-physical, it is not normally considered good science, but according to the single field theory such a semi-physical pre-consciousness potential field is necessary as is the implied point-by-point 'virtual torque' that collectively yields the single field potential from which matter and energy are derived.

Being semi-physical, the pre-consciousness potential field interacts with matter to create a 'virtual force' that does not move matter (lie a real material force), but instead influences matter to evolve into more complex systems. This 'virtual force' manifests in other ways in physics, but it is also directly sensed by consciousness and given many other names, not just that of a 'force of evolution'. There are many things that we intuitionally sense that are not completely scientific, but still exist at some undefined level of reality and existence. Concepts such as this are not just related to our intuitionally sensing the presence of some form of God or Supreme Being, but they say nothing about the actual reality of that Being, which is still open to non-scientific belief.

So as real as these things may well be, depending on any given person's personal definition or specific knowledge of reality, their reality is a matter of opinion and belief, not verifiable science. Having said that, scientists and other academics are not automatically atheists as many non-scientists claim, they just separate their belief systems from their scientific endeavors. Newton was faced with this same problem through criticism of his new physics, which did not mention God. He was a very religious man and replied to those criticisms by describing his concept of absolute space as the "sensorium of God" and that statement is still a wise position for scientists to take. Even if they do not believe in an absolute space as did Newton, there is still the 'absolute elsewhere'.

However, concepts of consciousness are still important in this respect since consciousness is a physical construct, although in all likelihood not a material construct or thing. Consciousness must be physical at some level of reality since it interacts with and within the physical world. So consciousness must be a physical construct, as it is when it is explained as a complex multi-leveled magnetic vector potential field pattern within the single field. Many people question whether the universe itself is conscious, but since the universe is comprised of the whole single potential field with internal variations in density and density patterns, any one consciousness pattern of any living organism within the single field would render the whole single field conscious.

Yet in reality our experienced universe is filled with a seemingly infinite number of individual consciousness patterns and that number is growing all the time, both in complexity and quality (qualia), so it would also seem that the universe as a whole is evolving toward a point in time when it will become aware of itself, if that moment has not already arrived, given the simple fact that both life and consciousness continually evolve by moving toward greater complexity within the universe. So it is safe to logically (and even scientifically) conclude that a universal or cosmic consciousness is an evolutionary endpoint, either individually or collectively, and probably both, without our direct experiential or observable knowledge of that possibility. Hence, idealistic models such as Faggin's are not just metaphysical and/or speculative, but at least scientifically legitimate to one degree or another depending on their own inherent scientific practicality as descriptions of a future state of the universe.

In Andrew's original model, the point-centered dimensions of our (commonly experienced) three-dimensional physical space are emergent properties of a spaceless-timeless Void. Every point-centered process would therefore emerge from a 0-D embedding dimension corresponding to the quantum point at the origin of the space-time diagram. This notion of a six-dimensional embedding manifold could also be related to a cosmic consciousness, collective consciousness, super consciousness or even more specific models such as Faggin's concept of C-space as well as other more speculative theoretical models. It could even be related to metaphysical models and those that deal with spiritual matters such as the Tao, Great Spirit, Demiurge, Brahman and/or *Ein Sof*. A universal collective consciousness of this type that acts through each and every point in our three-dimensional space of experienced reality could easily correspond to the implied sixth embedding dimension.

9. An Age-Old Dualism Revealed

The age old metaphysical paradox of the duality between transcendence and immanence, highly debated in the Middle Ages of European history when there was little difference between religious philosophy and real science within the study of Natural Philosophy, can now find its resolution in physics of the single field theory. The duality breaks down to nothing more than a misunderstanding of how consciousness manifests itself in our real material world within the context of the extended Riemannian structure of the universe utilized by the single field theory. The debate over transcendence and/or immanence breaks down to another facet of the dualistic relationship between extension- and point-geometries (whether considered metric-element/point-element, relativity/quantum or continuous/discrete) that has served no less than to de-unify (derail attempted unification) and mystify physics over the last century.

Any apparent duality of nature, such as this, can be resolved in that it must have a singular solution because nature is a unitary and not a dualistic or multiple thing. The duality is really a product of our mental interpretation of nature, rather than nature itself, so it can be resolved at a higher level of consciousness which is more in tune with nature. Just as a universal or cosmic Consciousness can be both transcendent and immanent, it can manifest itself in two different ways through the 0-D point/twists from which our physical reality emerged.

In common everyday physics the interaction of a potential field (gravity, electric or magnetic) and a piece of matter can be interpreted in two different ways: in terms of a force or an energy, which are themselves intimately related through the work-energy theorem. The action or influence of the pre-consciousness potential field (which is the precursor of consciousness) on matter can also be interpreted with respect to both of these physical concepts. It can only influence matter because it is a semi-physical field, but it cannot move or accelerate matter as can a fully physical field. When animate matter interacts with the pre-consciousness potential, it can be interpreted as a 'force' such as the 'evolutionary force' that causes an organism to seek higher and higher levels of consciousness (a transcendent or holistic interaction) through biological evolution.

However, when the pre-consciousness potential interacts with animate matter it can also develop a form of virtual internal 'energy' (the immanent or point-by-point interaction) controlled by an individual's consciousness that many people have sensed in themselves and talked about over the centuries, but that science does not normally recognize as existing. This virtual 'internal energy' is usually referred to as Chi, Ki, Qi, *Prãna*, *Mana*, *Orenda*, or *Od*, depending on the local culture, and some scientists also refer to it as 'subtle energy'. The concept has even been fictionalized in literature as 'the force' of the Jedi, an idea which was based upon the real concept sensed by many people of various ethnic backgrounds.

These virtual or potential 'energies' are normally described as a 'cosmic energy' that exists everywhere, an all pervasive 'organic energy', a 'life force' or 'life energy'. In any case, the concept seems universal to all cultures independent of their early development and the concept predates historical records in many cases. The concept is related to consciousness, or at least spoken of in the same way as consciousness, in many cases. Enlightened beings, those who have experienced the higher dimensions of space and/or Consciousness itself, have the ability to manipulate this 'energy' for material purposes. In other words, at high enough levels of consciousness, the pre-consciousness potential can be utilized much as physical fields are utilized to move and/or change matter.

Although there are many tales of people utilizing these energies throughout history, manifesting these virtual 'internal energies' or potential energies to kinetic energies, normal science has only recently become interested in them. Observations and measurements have been made of the effects of these energies in some individuals, but science lacks any underestimating of where the energies come from or how they are produced and utilized by individuals, at least until now. Yet the knowledge of these energies and how they relate to our common material universe has completed the emergence of the new science of Neurocosmology.

10. Physics from Our Sense of a Higher Space

On the purely theoretical physics side of the matter, the relationship to Faggin's P-space, whereby C-space (transcendent) creates the reality of P-space through the individual discrete points (immanent) described by the Standard Model of quantum theory, is not as different from the Beichler or the Beichler-Andrews models as one might expect. Both Andrews and Faggin suggest three-dimensional point-by-point quantum processes are mediated or made real geometrically by the Amplituhedron, which greatly simplifies standard model calculations using Feynman diagrams. But if the Amplituhedron is interpreted a real geometrical object, or otherwise a representative of a real geometrical aspect of physical space, it could easily correspond to a non-Riemannian geometry that acts physically in lower spaces through the single-polar point via its capacity as a link to the otherwise undefined, yet implied, six-dimensional embedding manifold, which in turn acts through the individual 0-D point/twists in the embedded three-dimensional space.

More weight can be offered for this interpretation since the Amplituhedron can be connected to the twistor/gauge theory of the quantum, implying that it has much broader meaning within the quantum theory and should be interpreted as a possible geometrical reality incumbent in the discrete quantum points of our commonly experienced three-dimensional physical space.

> When the volume of the amplituhedron is calculated in the planar limit of $N = 4$ $D = 4$ supersymmetric Yang–Mills theory, it describes the scattering amplitudes of subatomic particles. The amplituhedron thus provides a more intuitive geometric model for calculations whose underlying principles were until then highly abstract. The twistor-based representation provides a recipe for constructing specific cells in the Grassmannian which assemble to form a positive Grassmannian, i.e. the representation describes a specific cell decomposition of the positive Grassmannian. [26]

In fact, any geometrical device that gives physically proper answers for the quantum theory could be used as a non-Riemannian geometry within the single-polar point as an expression of the physical geometry of the discrete 0-D point-twist Void structure of three-dimensional space.

In any case, whether the Amplituhedron defines that reality or not, the reality of the quantum event emerges from the quantum collapse to a distinguishable discrete point-event according to the HUP and did not exist before the event. The conscious 'collapse of the wave packet' determines the reality of the material event according to the classical quantum theory explained by the Copenhagen Interpretation. Therefore, reality would emerge first at the origin of the space-time axes, at least for a singular discrete event, at a specific point in time and space. This concept could be extended to material reality from this time forward, excluding other quantum events that might alter the future of this event. This is essentially the point made by Einstein and his colleagues in EPR.

So each point quantum event has a future that is part deterministic and part indeterministic as defined by the light cone and the corresponding 'absolute elsewhere', respectively. But it is indeterministic materially, which means that the determinism is materialistic, but also materially limited to the curvature of three-dimensional space as limited by the speed of light. Indeterminism, as a characteristic of the 'absolute elsewhere', allows consciousness to abstract ideas and concepts whether they are historically real (follow along the events time-line) or not. So consciousness allows us to think beyond the limits on the brain/mind and associated sensations that are restricted by our experiences which are material reality oriented within what we perceive and have perceived in our personal light cones. This is an important feature of consciousness often associated with imagination, abstraction, thought, intuition and other 'qualia' which are all important facets of consciousness.

The 0-D point/twist Void at the origin is also instantaneously connected to the single-pole in the embedding dimension and thus mirrors its many physical aspects, as are all the discrete 0-D point/twists that constitute our three-dimensional world of experience at any moment in time. In this respect, all 0-D point/twists are intimately connected to each other throughout the universe, even the ones that constitute our consciousness. So, in a sense, the primal awareness associated with the original singularity point still plays out through each 0-D point/twist in the universe and the universe itself must be aware of itself at some level of consciousness derived from the pre-consciousness on a moment-to-moment basis.

Collectively, the primal awareness inherent in each and every 0-D point/twist creates the semi-physical pre-consciousness field that permeates all of space-time and is associated with the whole of the 'absolute elsewhere', which forms the background to our commonly experienced relative space (inside all possible light cones of all events in the universe) that directly influences real physical events inside the light cone.

Our experienced universe is now developing toward that ideal and/or final evolutionary end according to the Beichler-Andrews model, which is still a work in progress. As such, any speculation about the reality of an Information-space could only refer to a partially filled vessel that is presently being constructed, and filled, by all sentient beings that have evolved past the inanimate matter stage of a universal physical system of evolution. This would include all living beings and perhaps someday, at a much higher level of evolution, we will have evolved into non-material beings that are part of and contributors to a fully functional Consciousness-space that is creating Physical-space through an Information-space that we each helped to create by individually evolving.

11. The Absolute Necessity of Universal Evolution So the Universe Can Know Itself

The concept of a pre-consciousness potential field completely changes the way that science should regard our physical reality. This semi-physical (virtual) field would fully complement the singe field. It acts through individual discrete 0-D point/twists by way of the geometrical point-by-point three-dimensional field patterns of magnetic vector potential that form complex internal surface patterns in the four-dimensional single field. These physical but non-material patterns represent and are the individual consciousnesses of living beings.

Yet this semi-physical field would also act collectively as a non-material but still semi-physical ‘force’ for order and increasing complexity in the universe. This ‘force’ affects or influences the action of matter, but this ‘force’ alone is not able to move matter in the manner of a true fully physical/material force such as described in normal physics. However, it can move matter or otherwise consciously influence the material world through the action of any individual consciousness of high enough awareness of our complete physical reality.

Physics has always been confronted with the problem of something as basic and fundamental as simple ‘order in the universe’, let alone the complex order required for the existence of life, mind and consciousness. But no one has ever been able to make any logical sense of how they emerged (*i.e.*, were they created?) after the Big Bang (or the creation of the universe). So the question was formerly relegated to the domain of the supernatural and/or metaphysics if logically considered by default, or otherwise ignored altogether.

Physics only came close to even considering this problem and that consideration came from the branch of science called thermodynamics. However, even thermodynamics has failed, or has at least been proven inadequate, to finally solve the problem of how order has become manifest in our universe. Thermodynamics merely circumvents the problem of order by introducing more disorder (entropy) than order within a larger system than the orderly system originally considered. However, a radical change in the laws of thermodynamics, that balance disorder (entropy) and order (evolution), would now seem to be in order to fix physics and allow science to explain order in the universe by other means than calling order a chance process.

The four normal laws of thermodynamics still hold true (and do not change) for the idealized situation of closed systems, even though a truly closed physical system is only an ideal that technologists use to design human-made machines that only approximate natural processes, *i.e.*, diesel engines, air conditioners and refrigerators. Entropy is still favored over order by the universe in the large, but only because the volume or total size is increasing as the universe expands, while the number of material particles remains roughly (except for pair production and spontaneous particle/nuclear decay) constant.

The combination of increasing size and a constant amount of matter/energy yields a net increase in randomness over time in the universe as a whole, maintaining the validity of the original four laws of thermodynamics. So the underlying order of the universe implied by the hypothetical existence of the pre-consciousness potential field supports and even ‘forces’ the need for additional new thermodynamical ‘laws’ to balance the current theoretical model rather than completely replacing the old laws by a new theory of thermodynamics.

Over the past century and a half of its existence, ways have been developed to overcome the shortcomings of thermodynamics, which only enforces the validity of retaining thermodynamics as is in spite of its shortcomings. Prigogine’s Principle is already used quite extensively in conjunction with the second law because it clears up many of the problems associated with the idealization of a closed system as suggested by the second law. In reality, there is no such thing as a closed system, which is the thermodynamical ideal used for computation, since a closed system is impossible.

This makes it necessary to invoke Prigogine’s principle or various mathematical methods in conjunction with the second law to actually describe and analyze real situations. In general, Prigogine's Principle states that a dissipative energy system, whose equilibrium destabilizes through a loss of energy, moves toward a maximum chaotic state before falling into another more stable equilibrium state at lower internal energy. It is so commonly used in thermodynamics that Prigogine's Principle should be elevated to the status of the fourth law of thermodynamics.

The mathematical system of chaos theory (non-linear dynamics in physics) has also been used to supplement thermodynamics because entropy is a form of chaos. So the fifth law should introduce the concepts of chaos and the emergence of complexity as fundamental physical processes, rather than just mathematical methods, in the universe. It could be stated in such a way that 'under the proper environmental conditions (such as a system's interaction with external natural forces) complexities would naturally emerge to form new orderly systems'. These newly emerged complex physical systems would have characteristics that could not have been predicted from the characteristics of the chaotic (entropic) system before the complexity emerged, one being the principle of internal organization of the emergent system. So to improve efficiency of the system as well as improve internal function, once formed, complexities reorganize the chaotic systems from which they emerged for their own benefit and continuity.

The sixth law would combine the previous two laws–Prigogine's Principle and the emergence of complexity–yielding a physical law of material system evolution. System evolution occurs when chaotic (entropic) mixes of complex emergent material systems move toward higher and higher levels of complexity over the course of as time due to the influence of external natural forces, *i.e.*, they utilize internal potential fields for their own means once they have emerged.

Under these circumstances, system evolution must be ubiquitous, open-ended and continuous throughout the universe. Animate matter (or rather biological systems) is just a specialized form of complex material systems which experience biological evolution as presently described by Darwinian evolution and modern genetics, but can also be affected by some form of top-down evolution (as an internal organizational principle) that is not even implied by the normally accepted theory of evolution.

The next and final law of thermodynamics, Murphy's Law, states that 'anything that can go wrong will go wrong'. It will always be the next and final law because something new, unexpected and completely unsuspected could always materialize. Murphy's Law could also be described as the 'law of unintended consequences' in that it would introduce some of the fundamental uncertainty of quantum theory into thermodynamics, since it is impossible to know absolutely everything (all of the possible influences) about an event, or system, according to the quantum theory. Murphy's Law also seems a good balance for the Zeroth law (in its vague generality), while the other new laws balance the three basic classical laws of thermodynamics.

And finally, since evolution is occurring in all material things, everywhere and all the time, it would be more accurate to say that evolution, rather than entropy, is time's arrow. Only evolution is every bit as ubiquitous as time in our universe. So it certainly makes far more sense to think of and perceive the world around us, and even interpret nature as a whole, within the context of evolution rather than within any entropic principle, especially since the observed order presented to us by evolution seems to be the end product of an entropic (chaotic) material system. Evolution itself is just the manifestation of the pre-consciousness potential field on matter in our world.

12. The New Physics of Biological Evolution

The presently accepted scientific theory of evolution is completely biological in nature and thus very straightforward, although it seems to depend on some undefined and/or non-specific form of 'force' in nature that pushes, or favors, evolution–against constancy and a non-changing world–except for simple motion as explained by physics.

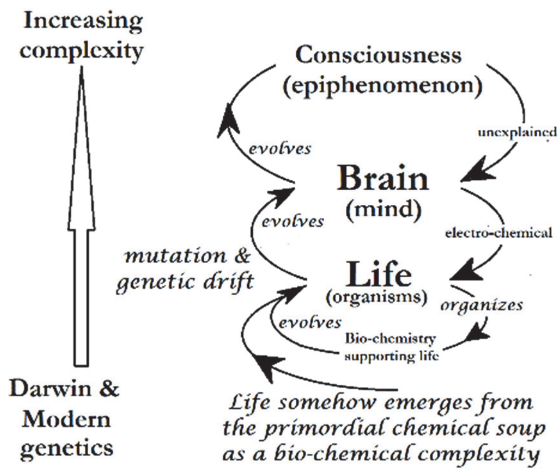

Figure 9. Darwinian and modern genetic evolution theory.

As such, the modern theory of biological evolution, as good as it is, still has gaps in it when compared to the archeological and geological evidence for evolution.

It is generally thought that biological evolution depends solely on the agencies of natural selection (Darwin), genetic mutation and genetic drift (modern genetic evolution), but these agencies always proceed from the bottom up, from the genome to the organism as a whole. Biological evolution thus ignores any contribution of organisms as a whole, or in part, to organize or reorganize themselves internally in an evolutionary manner, so it would seem that evolution can only come from chance outside interaction of the organism (Darwinian natural selection) with its environment.

Yet many people still sense a more fundamental organizational principle at work in evolution within themselves as well as the world in general and thus continue to question the validity of the present biological theory of evolution. People sense this 'force' of evolution at play in the world, in part and as a whole, but the present theory of evolution provides no answers or clarification about the character or identity of this 'force'. This 'feeling' of scientific inadequacy forces many people (non-scientists) to invent such alternatives as Creationism or Intelligent Design to fill the perceived logical gaps in modern evolution theory, even though these inventions are not necessary.

The 'force' which people 'sense' is merely the action of the pre-consciousness potential field within themselves and our physical world of experience. That non-material but still semi-physical 'force' acts, or interacts, with specific material bodies to create order in the inanimate world as well as top-down (from consciousness to mind and then to life) evolution within animate matter through the exigency of the emergence of greater complexities and complexities of complexities.

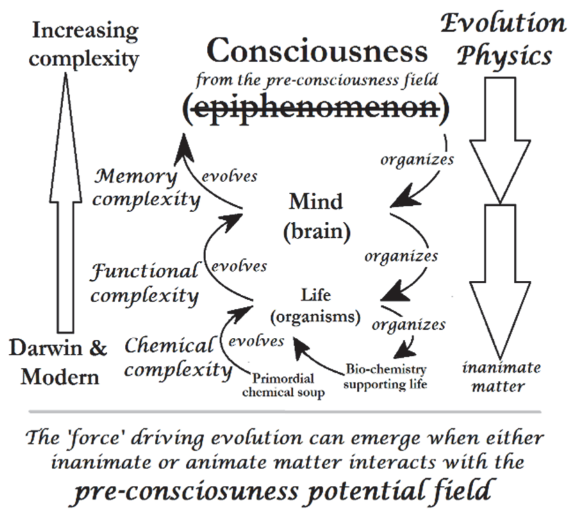

Figure 10. Physical systems evolution allows for top-down evolution from consciousness to mind to life.

In other words, the principle of physical evolution that emerges from the new thermodynamics can now be considered to supplement normal bottom-up evolution (Darwinian and genetic) by including top-down evolution from consciousness to mind to the living organism. This existence of top-down evolution answers many of the difficulties facing the older versions of evolution theory.

This form of top-down evolution easily accounts for and explains the pernicious problems faced by ordinary biological evolution, such as the Cambrian Explosion two-hundred million years ago. During the Cambrian Explosion, simple single-celled organisms very rapidly evolved into extremely complex multi-celled organisms over a vastly shortened evolutionary period of a million years or so. The Cambrian Explosion was caused by the top-down evolution from mind to body. This leap was soon followed by the split between plants and animals, which reflects the natural dualism of form and function, respectively. The plant kingdom followed form, which allows the external appearance of its members to be outwardly modeled by chaotic complexities (iterated function systems such as the Mandelbrot and Julia sets), while animals followed function, which allowed them to evolve brains and complex nervous systems.

In science, animate and inanimate objects are usually classified according to specific biological principles, *i.e.*, self-locomotion, pro-creation, cellular structure, and so on. But in nature, animate and inanimate organisms can also be distinguished by their internal levels of complexity, as defined by specific physical models of life, mind and consciousness, all of which are more fundamental physically than self-locomotion, pro-creation, cellular structure and so on. In other words, both groups follow the same basic physical principles and laws, as described by the physical theories that are interpretative explanations created by the human mind, but only animate matter has reached a high enough level of complexity to be considered alive as opposed to the non-life of inanimate matter. Within this context, life, mind and consciousness can only be defined in physics within the larger sense and context of the universe, but normal modern physics has not yet risen to the level of explaining them. That fact of reality has now changed.

Life, the proverbial 'life force' or biofield as some call it, is the complex matter/energy field pattern that corresponds to a living organism. It is essentially an independent and individualized pattern of quantized space-time curvature in the physical worldview that corresponds to the physiology (bio- and electro-chemical interactions in the organism) and the anatomy studied by biologists and bio-chemists. Mind is the complex electrical scalar potential field pattern associated with the whole living organism, literally the three-dimensional complex electrical pattern of the living organism which would include all bio-chemical interactions, as well as purely electrical interactions, that maintain life in the organism. This includes all of the specialized electrical activity in the brain and the nervous system as well as those between every cell in the body and different organs.

And finally, as already stated, consciousness is the multi-leveled complex magnetic vector potential field pattern associated with the mind of the living organism. Bio-scientists of all types seem completely oblivious to the simple physical fact that every electrical interaction in the body (and in nature as a whole), even those that are no more than simple electron exchanges between atoms and molecules, generate magnetic fields of higher order than the electrical interactions themselves. These magnetic fields form semi-permanent domains which result in a more permanent multi-leveled magnetic field structure that influences and organizes later electrical interactions in the body and that is the role of consciousness relative to mind.

So the body of every living organism has a larger collective magnetic field that results from the combined effect of all of the many levels and types of individual magnetic domain structures in the body. The brain has the most complex and complicated domain structure due to the existence of vast complexes of neural nets while the heart has the strongest magnetic field of all the internal organs. Magnetic fields commonly direct electrical flow in the same manner that consciousness directs mind and only magnetic fields form structural levels called domains to form permanent field structures, which compares well with the concept of levels of consciousness.

Within this context, living organisms originally evolved as Darwin and modern genetic biologists have claimed, internally from the bottom up, but with reservations because current evolution theory is inadequate and incomplete. The action of a pre-consciousness potential field on matter is necessary to explain the initial origin of life in the chemical soup from which it emerged. As animate organisms became more and more complex over time, bottom up evolution (from within) has become more and more difficult, while top-down evolution (from within) has slowly come to dominate the most complex organisms, simply because mind and consciousness represent the whole context of a living being and not just one internal aspect of its being. Every living organism is thus a product of the interaction of both top-down and bottom-up evolutionary processes with the ultimate goal of developing higher levels of consciousness as well as a more diverse group of consciousnesses.

The evolution of physical systems, which now supplements earlier theories of biological evolution, is a natural part of our physical universe, an expression of the pre-consciousness potential field, rather than just a biological process. In fact, extremely complex physical systems that are presently considered inanimate, such as stars and planets, may ultimately prove to be animate at some level or another due to their own vast complexity. Life is only differentiated from inanimate (non-life) matter by its level of complexity, yet everything beyond individual material particles is complex to one degree or another. So some inanimate objects may ultimately be found to have their own unique forms of mind and consciousness that are not presently recognized as such.

'Life' (the biofield) is not matter and energy, mind is not electricity and consciousness is not magnetism, but rather intricate internal patterns of these natural physical fields and the forces they produce. Life, mind and consciousness are the complex multi-leveled matter/energy, electric and magnetic field patterns that have emerged and developed into ever more complex patterns over the course of history. Once living organisms emerged, they began to reorganize their own internal matter/energy interactions (field structures) by modifying electric/chemical and magnetic interactions to run more efficiently, thus enhancing further development and evolution.

All material objects are constructed from these same three different physical fields–matter/energy, electric and magnetic–imprinted upon one another. In all cases, these three fields must act, or react, in concert with one another as specified by our scientific theories. Yet the animate matter of living organisms is defined by a much higher-level of complexity within the field components that renders these particular field patterns in living organisms different from their inanimate material counterparts, so it is more difficult for the different fields to interact with each other the more complex the living organism. This difficulty requires a continuing natural evolution of higher and higher complexities of mind and consciousness to organize the more complicated internal systems of organisms.

So the complexity of interaction between these three types of field patterns acts like a positive feedback system that naturally progresses living organisms toward greater consciousness. All of these patterns must work together to create a living organism, which means that all living organisms have the same complex mix of patterns. But different living organisms have evolved both higher level patterns (for example paramecia versus humans) and different types of complexities (for example plants versus animals) than others as well as greater complexities within each type of field pattern and between each type of field. In other words, all life is conscious to one degree or another, but only in more highly evolved organisms has awareness of consciousness emerged as a chaotic complexity of memories within mind.

Within this much greater universal context, the brain/mind system stores memories whose pattern complexities form individual consciousness, or at least conform to the context already present in consciousness (already existing inherited patterns) as preordained by the influence of the pre-consciousness field that acts through every 0-D point/twist in space. As the new multi-leveled (domain structures of) complex magnetic vector potential patterns stored in mind change, the context established by existing consciousness for perceiving and interpreting new data input from the external physical world (through the five senses) also changes. But when changes in the complexity patterns are extremely profound and thus strong enough (such as those experienced during spiritual enlightenment or NDEs) they directly affect neuronal genomes. If ensuing plasticity changes are that intense within the overall context of and individual mind and consciousness (such that they are important for preservation and enhancement of the species), they are passed on genetically to offspring and become part of the overall genetic pool of the species.

It is through such processes that the human species might soon be reaching a tipping-point in its own evolution, catalyzing a new leap in evolution that ends with the emergence of a new Hominid species. Since the magnetic vector potential acts through individual discrete points in three-dimensional space, or rather the 0-D point/twists that constitute the three-dimensionally curved surface (or 'sheet') that is our experiential material space (affecting the whole single field), the memories and thought patterns of individuals become permanent density pattern subgroups stored at the 0-D point/twist level (in the single field) due to the activation of the pre-consciousness potential field as a whole. This all means that the single field acts as an infinite permanent storage bin for memories, thoughts and experiences, as well as countless consciousnesses, all of which are semi-independent of the living organism and mind to which they are originally connected.

The most complex memories that we easily recall and remember are stored and recalled by that part of the mind that correlates to the brain, because only the brain has the density of neurons and more important the complexity of neural nets that have the ability to render storage and allow for recall. This is why we mistakenly believe our mind and consciousness exist in the brain alone. Our memories are both stored and recalled through the interactions between and among microtubules (nano-sized bio-magnetic induction coils) and the electromagnetic interference patterns they create in the surrounding water medium. These interference patterns quantize the nuclear magnetic spins of the water molecules in specific patterns to match incoming sensations from the external world (through either, and or both three-dimensional space or the fourth dimension), imprinting those as memories composed of various magnetic vector potential patterns on a 0-D point-to-point basis within the single field.

Recent developments in neuroscience indicate that the neural net patterns in the brain rewrite themselves (an alteration called brain plasticity) according to new learning and experiences. These newer and more complicated complexities slowly, but sometimes radically, alter the context of the overall consciousness pattern. Since human knowledge is increasing so rapidly, far more rapidly than ever before, and we are experiencing new phenomena (a greater breadth and variety of phenomena) at ever increasing rates due to technological and scientific advances, the (basic) complexity structure of human consciousness (that we all inherit) is currently under a great deal of stress which leads to mental chaos against the background mental context of previously stored memories in which new memories are interpreted as relevant or even meaningful. Such mental chaos could be a prelude to the formation and emergence of new higher level complexities and thus consciousness.

Add to this the present-day social, cultural, political and economic stress that we are forced to deal with mentally, all of which were non-existent just a few decades ago, and it is easy to conclude that the human species is forging its own evolutionary path whereby the overall nature of our pre-consciousness potential field and its proclivity for advancing the consciousness of the universe as a whole will soon initiate a new evolutionary leap for the human species that overcomes, or rather integrates, these mental stresses, giving us greater access to, and knowledge of, the single field and the higher dimension of space where the single field exists in its pure form. Since spiritual and/or mystical enlightenment (the ultimate intuitive processes of consciousness) results from the direct interaction of consciousness with the higher-dimensional single field potential and the conscious waking awareness of this interaction, it is highly probable that the human race is presently standing on the verge of becoming spiritually enlightened as a whole with our next evolutionary leap.

13. A New Synergy Emerges

The only physically real 'place' that can be described geometrically (and thus scientifically describable) that can fulfill the basic requirements for storing memories in multi-layered domains to form an individual's consciousness is a higher dimension. It need not specifically be an embedding dimension (in the strict sense that an embedding space is represented by an extrinsic Riemannian metric- or extension-geometry), but it must still be inseparable from our normal three-dimensional space of experience through the individual discrete 0-D point/twist Voids in five-dimensional space that can analyzed by at least a non-Riemannian point-geometry.

A non-Riemannian geometry in the surface points whose extension is represented by a Riemannian geometry is intrinsic to the n-dimensional surface (or space) and thus does not require an $n+1$-dimensional embedding space. Any higher-dimensional Riemannian metric geometry, whose existence is required by the associated higher-dimensional non-Riemannian (or tangent) point geometry, could easily be considered spaceless and timeless since it technically lies outside of both our normal four-dimensional space-time continuum, or rather inside the discrete points that are not 'contained' within the continuum, but are tangent (Wolfgang Pauli first used this descriptive term in 1921 [27]) to the three-dimensional manifold 'surface' at any given point under consideration and also the embedded physical fifth dimension.

In other words, these would be the discrete points in a six-dimensional non-embedding space where such points are 'tangent' at every point in the surface to each and every point in our five-dimensional (extended) metric surface (manifold or space). This higher dimension could be thought of as a Consciousness space, providing for the collective consciousness or cosmic consciousness that is generated by the four-dimensional pre-consciousness potential field in three-dimensional space, just as four-dimensional space is filled by a single field that yields our material reality in physical three-dimensional space.

A Consciousness space of this type could represent all quantum possibilities for three-dimensional physical space (our commonly experienced material and physical reality) represented by wave functions before their collapse (not just those realities resulting from the collapse which create our classically experienced relativistic world), except for those wave functions that are collapsed by the conscious choice of conscious beings in three-dimensional space. This would guarantee the continued existence of three-dimensional space and all of its material inhabitants even when conscious three-dimensional beings are not witnessing it, *i.e.*, before life first evolved.

This aspect of the consciousness problem invokes Andrews' concept of 0-D point Voids as witnessing the unfolding of physical reality without the intervention of human or similar consciousnesses. In this way, a higher-dimensional Consciousness space could be thought of as creating our four-dimensional space-time reality, or physical space, through a corresponding discrete quantum 0-D point/twist Void space, generating our perceived four-dimensional (metric extended extrinsic) relative reality from the whole 'absolute elsewhere' background (a spaceless and timeless nothingness which would correspond to a Newtonian-like absolute space) by way of some as yet to be defined non-Riemannian point-geometry, such as the Amplituhedron suggested by Andrews and others.

Something like Faggin's Consciousness units (CUs) could then be likened to the multi-leveled consciousness complexity patterns (within the single field) in five-dimensional space-time, which manifest in the brain/mind of an individual as the awareness of human consciousness via (magnetic) vector potential patterns (domains) throughout the whole three-dimensional material living (animate) body.

A Consciousness space of this type, known by this or any other name, need not (specifically) be a sixth embedding dimension for our five-dimensional space-time continuum when just a sixth tangent, or perpendicular manifold, that manifests physical reality would suffice. It could act in our four-dimensional space-time continuum and either create of just be aware of our already existing material reality through each and every one of the individual discrete points, throughout the embedded dimensions of physical space within it.

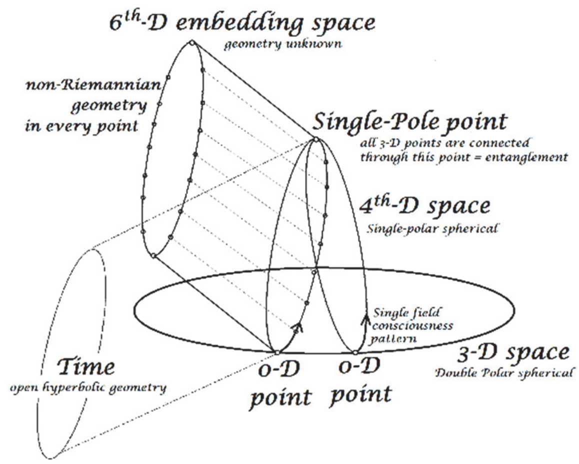

Figure 11. 6-D with combined metric and point embedding.

It need not be a full embedding metric space itself that would require either mathematical identification or physical justification, if not both.

This would mean that the geometrical physicalness of our experienced world emanates from and is causally present every moment in the individual points that constitute an embedding space (similar to Newtonian concepts of absolute space). This would correspond to the background collective 'absolute elsewhere' framework (or space) described above. It would be causally ever-present if for no other reason than because each of the 0-D point/twist Voids (that constitute space) are constantly re-creating four-dimensional space through the discrete quantum points as explained by modern quantum theory, but still expressed relativistically by the field concept. So the material objects that define the three-dimensional relativity of our commonly experienced space can themselves be identified as extended field density patterns in the four-dimensional single field that appear as individual quantized local curvatures relative to the whole of the three-dimensional surface that is our material world.

This whole physical system finds its origins in 0-D point/twist Voids which can also be geometrically identified with and equated to the Riemannian point-elements that constitute all of our extrinsically curved physical reality. The 0-D point/twist Voids are themselves individual physical 'things' simply because space, time and the single field only emerged as the collective nature of these individual 'things' began to manifest themselves as real.

Since this single field coexists with a pre-consciousness potential field, also associated with the collective nature of the 0-D point/twists, all phenomena remain indivisible, which supplies a rationale for how an all-embracing 'Consciousness' or C-space could have arisen spontaneously from the primal awareness of the absolute Void. This also explains why an all embracing 'Consciousness' can be represented mathematically and scientifically as a higher-dimensional geometry, whether or not it is a non-Riemannian point geometry or an all embracing Riemannian metric geometry, or perhaps even both simultaneously.

14. This Synergy Enhances Single Field Unification

This model works well (as far as it goes) with respect to special relativity and the corresponding space-time diagram system with its concept of an 'absolute elsewhere'. But what about the unification of general relativity and electromagnetism as well as their expression in quantum theory in the single field theory? The single field (of potential) occupies four-dimensional space and varies over time, or rather its internal patterns of varying density occupies five-dimensional space-time. The consciousness associated with living organisms in three-dimensional space appears as a complex of multi-layered magnetic domain structures that are physically tied to both an organism's electric field structure (mind) and matter/energy field structure (life force or the biofield corresponding to the body/brain) in three-dimensional space.

So consciousness, mind and life (biofield) are whole body field structures (complex patterns), but only consciousness has a specific domain (the ability to form internal interacting variational levels) structure since common gravity/matter and electric fields do not form domain structures. We commonly, and falsely, believe that mind and consciousness 'exist' only within the brain because the complexity of neural nets that form our fundamental logical networks, by which we become consciously or mentally aware–our waking awareness–of consciousness and mind, only exist in the brain.

Given the complete single field structure of individual consciousnesses, Andrews' theoretical models fit quite well. Andrews' model is a near perfect Riemannian match for the single field model developed by Beichler, while Faggin's and those consciousness models that posit other forms of Consciousness spaces could also be assimilated into the combined Beichler-Andrews model. The single field is based upon a four-dimensional Riemannian geometry, as is general relativity, but with extrinsic and thus real curvature (of a three-dimensional 'sheet' or 'effective width' of infinitesimally thin parallel three-dimensional surfaces) bent or warped–curved extrinsically–into the fourth embedding dimension of space. So both the fourth dimension of space and curvature are physically real, even though we do not normally observe or detect them through our normal senses, which are decidedly three-dimensionally biased.

The three-dimensional spherical Riemannian surface that is extrinsically curved into a higher-dimensional manifold (space) and the ensuing space-time structure are not just mathematical gimmicks or artifacts that happen to describe gravity fields in three-dimensional space better than Newton's theory, as in Einstein's original version of general relativity. What we commonly sense as our three-dimensional material reality, our perceived world, lies within the curved three-dimensional 'sheet' that is perpendicular to the fourth direction of an overall four-dimensional embedding manifold/space, which is, in itself, a surface in a still higher embedding manifold/space. Our 'sheet' is the n=1 quantized portion of the single field in the fourth direction of space (n is a quantum number in this case, not to be confused with *n* when it denotes the number of dimensions when referring to Riemannian spaces and manifolds). All subsequent 'sheets' (n = 2 and higher) are stacked like pages of a book into, and throughout, the full extension of the fourth dimension of space.

From the perspective of another position in the four-dimensional space, outside of our surface or 'sheet', our three-dimensional world is just the densest portion of the single field and thus forms our matter/energy world of experience, while the three-dimensional gravity, electric and magnetic fields, as well as life, mind and consciousness, are all just specific single field density patterns (with varying levels of internal complexity defined by varying single field density that distinguish them) within the overall single field that occupies five-dimensional space-time. Our three-dimensional (n = 1) 'sheet' could also be perceived and interpreted as the quantum mechanical superposition of all possible Schrödinger Ψ-wave functions or, alternately, as David Bohm's quantum potential field.

The density of the single field is greatest (maximized) in our three-dimensional 'sheet' (or rather from the infinitesimally thin primary three-dimensional surface at the four-dimensional center of the 'sheet'), as 'viewed from the higher-dimensional perspective. Single field density decreases exponentially as the distance from our 'sheet' in the fourth direction of space increases, which means that an even higher and fully specified embedding sixth dimension is once again implied if only to account for the changing single field density and individual field variation patterns in five-dimensional space-time [28].

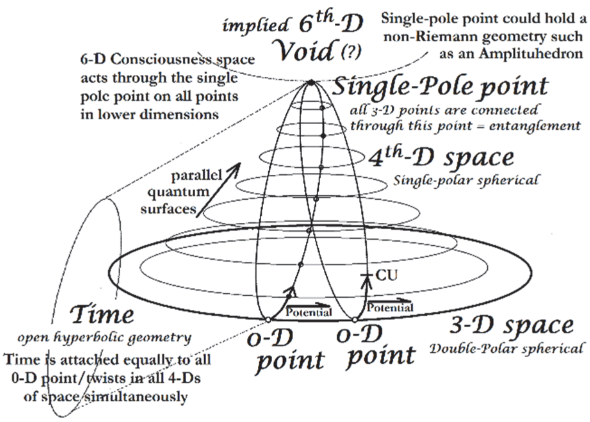

Figure 12. 6-D manifold might be only one point-Riemannian.

This six-dimensional embedding manifold/space for our own five-dimensional space-time surface is completely undefined beyond its mere suggested existence given the five-dimensional single field theory. In fact the implied sixth dimension could be a fully embedding Riemannian but undefined manifold or space (as shown above) or it could be an empty Void (which is Euclidean flat by default as was Newton's absolute space) with only a non-Riemannian geometry at the polar point where the fifth and sixth dimensions coincide as shown below.

In either case, some form of overriding Consciousness space that can be equated to the sixth dimension can now be physically and not just mathematically and/or metaphysically justified.

This high dimension would directly influence all lesser embedded physical dimensions through the single pole in five-dimensional space-time and its direct connection to the individual discrete 0-D point/twists. In effect, the individual 0-D point/twists in three-dimensional space would mirror the single-pole point and its conscious influence on material/physical reality as a witnessing co-creator as explained by Andrews. [29]

Very few basic physical characteristics of the sixth dimension, if any, can be inferred from the single field inhabiting the lower embedded dimensions. In a sense, the overall sixth dimension (in that it could be extended and thus Riemannian) is 'transcendent' over all of the embedded five dimensions (four of space and one of time) yet it is also immanent through the 0-D point/twists Voids through which it acts in the lower embedded dimensions to either physically influence the material world or possibly even create it.

The concepts of transcendence and immanence are usually only invoked or spoken of with regard to some form of Supreme Being or another, so it is unusual, although geometrically justified by the point/extension duality of space and time, that they are here used to describe how whatever inhabits the sixth dimension could act on and/or react to events in our normal three-dimensional space of experience. However, they are still open to interpretation and speculation unless further physical characteristics of the dimension can be found and scientifically verified.

Otherwise, the sixth dimension would have a dualistic point/extension structure as do the other lower embedded dimensions of space and time. Beyond that, little is known within modern science about higher dimensions so any physical characteristics of higher embedding dimensions (six and above) are up for debate within normal science. However, that fact still leaves the higher embedding dimension open to serve as the witness of qualia in the fifth dimension of space which is inseparable from all lower dimensions of normally experienced space. [30] The geometry itself is ambivalent to the manner in which it is interpreted, so the higher dimension could also be equated to a cosmic consciousness, universal collective consciousness, Faggin's C-space or similar concept, or even the Tao, Eru Ilúvatar or Great Spirit.

Any one model does not yet favor another in this regard and they cannot be chosen between by the geometry alone without any other new information that would be deemed scientifically acceptable. The extension or metric view of the theoretically implied sixth dimension could also have any possible geometrical structure since its structure (even whether it is geometrically open or closed) cannot be inferred from either the five-dimensional embedded geometry or the physical characteristics of the single field that fills that geometrical framework.

15. More to the Point

So the point structure of the sixth dimension offers a different case altogether in the form of action/reaction within our normally perceived three-dimensional reality. The six-dimensional discrete point geometry structure would necessitate direct connection with all discrete points in the lower four dimensions of space as they vary over time in a form of physical immanence. So any geometrical property inherent in the discrete points in the sixth dimension could be non-Riemannian and directly affect, influence or emanate through the discrete points in all embedded dimensions of physical space even though they need not be physical in the normal sense of the word in the sixth dimension.

Since we know that all discrete six-dimensional points must have some kind of an internal or tangent geometry and we know that the Standard Model of the quantum is based on point-particles suspended in quantum fields, it would seem quite natural to postulate that the non-Riemannian geometry in the 0-D point/twists must be able to generate our quantum reality and is related to quantum mechanics, wave mechanics and the Standard model at some fundamental physical level.

Many of the speculative consciousness models, including information spaces or just quantum Information, computer programs, bits of information and various holographic models, are no different and could be applicable in the same sense of ideal final states. However, they offer no explanation how their models play out in the real world, they are non-falsifiable hypothetical speculations, so they cannot yet be taken seriously by science.

In the view of these other scientists, the extended relative space described by metric geometry (our three-dimensional material world of experience) is generated quantum mechanically through the discrete quantum points and may or may not be physically accurate in spite of the many overwhelming successes of relativity theory. They base their assumptions on the simple fact that the proposed unification of general relativity and electromagnetism as well as with the quantum has so far failed miserably, so new and sometimes completely radical ideas are required to explain the failure of unification, but they are no closer to unification than the older attempts.

On the other hand, Faggin and Andrews have both suggested the Amplituhedron as the discrete point bound quantum mechanical generator of our extended world. This interpretation of physical/material reality could be considered the internal geometrical (non-Riemannian) structure of the discrete points in six-dimensional space that manifests through three-dimensional discrete point space over time, *i.e.*, it is a falsifiable model. In fact, an important historical precedent already exists that links the non-Riemannian geometry of the discrete point to a possible unified field theory based on the metric geometry of relativity.

Within a short time after Einstein first enunciated his general theory of relativity (1915/16), Gerhard Hessenberg, [31] Tulio Levi-Civita [32] and Hermann Weyl [33] independently began to develop non-Riemannian geometries to fill the theoretical gap (the discrete point) left by Riemann himself in his metric geometry, *i.e.* Riemann purposely left out the concept of a point-element and constructed his geometry of surfaces on metric-elements alone.

Weyl developed his concept of a 'gauge geometry' to unify gravity and electromagnetism in a unified field theory, [34] but Einstein and others demonstrated that Weyl's geometry led to inconsistencies with known physics and observed reality. So Weyl withdrew his gauge theory from contention for developing a unified field theory but continued to develop his gauge theory as a strictly mathematical venture. Sometime later, Weyl and other scientists succeeded in applying Weyl's gauge theory to the quantum field, [35] where it remains an important contribution to overall quantum theory even today. This adventure, or perhaps misadventure, of Weyl's clearly demonstrates that there should be an intimate connection between the non-Riemannian geometry of point-elements in relativity theory and discrete points in the quantum theory. In other words, quantum theory represents a point generated absolute space with time that resides in the background behind Einstein's relative space-time.

Quantum theory itself has a similar and related dual structure in the matrix mechanics used to apply the HUP and the wave mechanics of Schrödinger. Matrix mechanics deals with discrete discontinuous objects or events localized to discrete points in either space or time (but not space-time), while wave mechanics deals with the continuity associated with individual events and observations. Thus quantum matrix and wave mechanics represent essentially the same point/extension duality as is found in geometry, even though quantum mechanics is supposedly a non-geometrical form of physics.

This also means that the indeterminism that quantum theory is associated with is internal to the discrete points and does not directly affect the extended space-time that results after the collapse of the wave packet (at a specific discrete point in space-time), such that related events in the past and future can be classically determined after the initial Ψ wave has been collapsed to the certainty of the event occurring in space-time.

This argument also confirms that Planck's constant h is a binding constant for space and time resulting in a space-time continuum at each and every point. But more importantly for a generalized theory of consciousness, this means that the non-Riemannian geometry of a discrete point in six-dimensional space can take any form that generates quantum or quantum-like physics within our normally experienced three-dimensional space or four-dimensional space-time, including the geometry of the Amplituhedron. This occurs without any relativistic restrictions on the non-Riemannian geometry since a point geometry or geometry tangent to a point in the surface does not constitute an embedding criterion for a metric geometry. This revelation fits quite well with both Faggin's and Andrews' theoretical models since both use the Amplituhedron to determine the effects of consciousness on our commonly experienced world. The Amplituhedron is also related to the twistor/gauge theory, which lends more support to this argument.

16. Experiential Consequences

An intuitive experiencer, a person who has directly touched or has somehow become consciously aware of having come into contact of a higher-level consciousness, if not Consciousness itself (the higher-dimensional embedding space or manifold), may readily recognize this theoretical physical model, but describe his or her experience in a completely different manner. For example, many NDErs have said that they cannot find the words or language to describe their experience, or what they sensed about their location during the experience, because the geometry that they sensed (experienced) is different from the geometry of our three-dimensional material world. That is primarily why science has only been able to access the higher-dimensional world mathematically or by logical inference and finds it necessary to speculate, to some extent, on its physical nature. Humans do not normally have a waking sense or awareness of the higher dimension.

Those individuals who have attained some level of spiritual awakening, whether spontaneous, due to some (usually tragic) event, or through deep meditation and religious practices, also find it difficult (if not impossible) though absolutely necessary, to describe their feelings about the experience because the terminology does not exist within our normal language structures or communicative skills. The concepts needed to describe our higher-dimensional reality do not fit the logical (neural net) structure of the brain. This makes attaining higher levels of consciousness both difficult and rare, let alone bring them into conscious awareness after they have been experienced, since our normal mind is limited to interact all the time with the three-dimensional world of experiences and interactions.

Our minds automatically place and interpret experiential events within the context of a commonly-sensed physical reality in an external three-dimensional material world. So any person who has experienced an NDE, at least one that is strong enough to break into conscious awareness afterwards, will absorb the experience mentally by internally rewiring some basic neural nets in a manner that changes the personality of the ND experiencer, sometimes quite radically.

Spiritual practitioners who intentionally choose to awaken their individual consciousnesses though philosophical enlightenment and/or spiritual practices, but cannot do so until their neural net has rewired itself through contemplation to a sufficiently advanced level to bring the experience into their awareness. Even if this higher state of consciousness is attained and they reach true enlightenment, they still interpret their experience as a higher 'self' without realizing that the higher dimension of Consciousness they have attained is just a waking experience or awareness of their own natural physical extension into the higher dimension of reality.

Everyone is always in contact with that higher dimension, we already sense it intuitively at a very low level of consciousness, but directly experiencing it and becoming aware of that experience in the waking state is another matter altogether. Philosophically and scientifically understanding the process may or may not be of help in attaining the goal of direct experience and enlightenment, but it is definitely of helpful in understanding and rationalizing the enlightening event after it occurs.

In accordance with the notion that Consciousness acts through the individual discrete quantum points (Andrews' 0-D point Voids) to co-create our three-dimensional experience of space, a new interpretation and relationship between quantum theory and relativity is at hand. Single field theory has already accomplished this unification, yet it has not previously taken Consciousness into account as a universal aspect of our common physical reality, although the reality of a Consciousness space is implied by the existence of the semi-physical pre-consciousness field potential.

The extended metric space of matter, in which we exist, corresponds to the superposition of all possible Ψ-waves (wave functions) prior to consciousness collapsing an individual wave function that creates the apparent certainty of discrete (0-D) quantum points. This superposition of all possible waves is reminiscent of Bohm's concept of a quantum potential field. Henry Stapp [36] has also stated that he is leaning toward such a philosophical conclusion, whereby our universe is a complex superposition of all possible Ψ waves. This notion would also include the background 'absolute elsewhere' as described above.

In the case of an experiencer, rather than that of a scientist, this theory can be seen and interpreted in the mind's eye a bit differently. In the words of one of Andrews' co-authors, Steven Salka, *"an effective way to view consciousness would be as a 'superposition' of existence and nonexistence, producing an indivisible experience of 'nonlocal being', plus who and what we perceive ourselves to be (local observers)."* [37] This relationship between an observer-based localization and the nonlocal whole has been examined and expressed in Andrews' theoretical model. Using ideas from general relativity and quantum mechanics, he suggests how a space-time continuum can also include quantum mechanical potentials and probabilities, arising as complementarities, as properties of consciousness. He investigates opportunities to contemplate the origins of existence, offering falsifiable experiments.

But this 'superposition' can also be (and has been) interpreted subjectively and identified with some form of deity or Supreme Being, ranging from Yahweh, to Allah, Eru Ilúvatar, the Great Spirit or Brahman, and even Plato's Demiurge, characterized by its Cosmic Consciousness and acting immanently and/or transcendently in our real world of experience. This particular interpretation is made possible by the fact that immanence and transcendence are properties usually associated with a God or Deity rather than physics. In any case, the possibility of a higher-dimensional space or manifold in which our five-dimensional space-time continuum is embedded is a purely scientific and mathematical notion and could have nothing whatever to do with religious beliefs. It is only a matter of personal interpretation and choice, with choice representing the concept of 'free will' as a characteristic of individual consciousness.

17. Conclusion

The idea or scientific concept of evolution can be expanded to include all material objects, not just living organisms, by balancing the laws of thermodynamics to include not only disorder and entropy, but emergence (formation) and order. Doing so is implied by the simple observation that order is found everywhere in the universe and therefore must be a fundamental characteristic of physical reality within the universe, but also by the logical process of explaining the origin of the universe from a single (singularity) 0-D point/twist Void from which both the single potential field that eventually leads to the emergence of matter and energy as well as a semi-physical pre-consciousness potential field which actually 'forces' and guides the complex evolution of life, mind and consciousness from simple inanimate matter and energy sources.

Within this context, both Prigogine's principle and chaos theory (the emergence of complexities from chaos) are commonly used as a counterpoint and correction to the second law of thermodynamics because the second law is based on thermodynamically closed systems, even though such closed systems appear nowhere in nature. Therefore, Prigogine's principle and the concept of complexities emerging from chaos should be made the fourth and fifth laws of thermodynamics, respectively. Yet when they are put together, they imply the sixth law of thermodynamics which could be described as the natural evolution of more and more complex physical systems. Under these circumstances, biological evolution becomes a special case, and a universal necessity, within physical science, rather than a standalone philosophically ridden anomaly in biology. Evolution, rather than entropy, is the real 'arrow of time'.

Moreover, the evolution of life in general and the continuing progressive evolution of mind and consciousness in all living organisms after life first evolved from some undefined primordial soup–whatever that initial evolutionary mechanism may ultimately prove to be–seems to have become the primary purpose of the universe. We Homo sapiens are just part of the greater universe realizing and becoming aware of itself from within itself because there is nothing of itself outside of itself to differentiate between itself and something else, which fits in quite well with Beichler's single field theory and both his and Andrews' model of Consciousness evolving from a spaceless-timeless Void.

On the other hand, the single field model of a neural net and brain plasticity not only implies that mind and consciousness can drive evolution (top down) as opposed to the modern Darwinian and genetic models which points to a bottom-up driven (evolution) mechanism, acting through genetic mutation and genetic drift. The notion that evolution can be consciousness driven (from the top-down) further implies–given the social, economic, cultural, technological, educational and scientific conditions of a chaotic and rapidly expanding information/rote knowledge base–that the human race is nearing, if it has not already reached, a tipping point for a vast evolutionary leap that will result in the emergence of a new human subspecies at a much higher level of consciousness than now exists.

This new level of human Consciousness will allow humans to actually think in terms, and directly experience the effects, of a four-dimensional space. This new subspecies of the Homo genus will even emerge fully enlightened at birth, or so we can hope. A large and growing number of scientists already believe that a new scientific revolution, which will be as much about the Mind and Consciousness (that perceive and interpret our common material/physical reality), as it is about the physics we will develop to better describe nature. Still, few even suspect that the next scientific revolution will be part of a much wider and far more comprehensive human-wide evolutionary leap in consciousness.

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A Mathematical Model of Free Will Based on Experience Information in a Quantum Universe

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In the past two years, I have shown how to define a new kind of information that fits all known conditions to be applied to experience, and demonstrated many applications and advantages. The information is different from both digital information and quantum information, and presents information science with a completely new type of information. The systems to which it applies are instabilities, which are neither classical nor quantum systems, but which have been shown to be universally selected for the loci of control of complex biological systems. A simple reason justifies this seemingly strange fact: in complexity biology, criticality, the condition of being at a critical instability, optimizes sensitivity of regulatory response, and thus regulation itself. Increased levels of performance in challenging environments then ensures the universal adoption of this unusual condition. Information operating at critical instabilities is neither quantum, nor classical, i.e. digital. It must be of a new and different kind. This article draws on previous work demonstrating that it has a Double Aspect structure, depicted as **O**=========>, with its first aspect, **O**, being a feedback loop internal to its structure, and its second aspect, =========>, being the information content. The internal feedback loop enables the subjective states of mind associated with experience information, i.e. states of subjective experience, to reduce wave packets, and thereby to make choices. Using this mode of action, the mind can use the brain's motor cortices to inject desired information into its surroundings. In a quantum universe, choices are made. In this way, a simple model of freedom of choice, and free will, follows from the phenomenon of experience information. An appendix shows how the work on Yardley's circular theory and Elizabeth Barad's concept of agency is brought together by this approach.

Keywords: Complexity, Criticality, Quantum Universe, Wave-packet Reduction, Event Selection, Free choice

1. Overview

The idea that the human being is capable of free will was dealt a body blow by Descartes [1], when he proposed to apply Galilean kinematics to a supposed world of classical particles at a microscopic level. Such a world, he stated, would follow an inevitable evolution in time like a clockwork machine, which neither its creator who had initially set it in motion, nor any human observer could alter. The human mind could do nothing to change the outcome, which was divinely determined. The human soul was a mere impotent observer trapped in the body. Newton's Laws of Motion [2], and subsequent contributions to the science of mechanics [3] did nothing to alter this prognosis. With the advent of the mechanical theory of heat in the 19th century [4], and the advent of thermodynamics [5], and statistical mechanics [6], Descartes' position became more deeply entrenched. Even Einstein was deeply enamored with the idea of a rigidly evolving mechanical universe, and fought against the uncertainties of quantum theory. [7]

Since the time of Clausius [4] and Gibbs [5], many factors in theoretical physics have challenged the idea that the future of the universe is predetermined. First, quantum theory with its uncertainty principle [8], emphasized that the future is not determined, to Einstein [7] and Bohm's [9] chagrin, with Einstein's hated quantum correlations [7] being finally established by Aspect's demonstration [10] that Bell's inequalities [11] are indeed violated. Quantum theory holds; hidden variable theories [9] do not.

The 20th Century produced other demonstrations that the model of the universe according to classical mechanics is wrong; Chaos theory [12] showed that adjacent trajectories in phase space become separated by singularities; for which quantum uncertainties implied that no initial condition could be well enough defined to predict on which side of a singularity an initial condition would place a trajectory. Complexity theory [13] showed that biological systems produce (1/f) noise [14], characterizing fractal physiology [15], and behaving distinctly differently from mechanical systems. [16]

More subtly the 19th Century discovery of the liquid-gas critical point by Van der Waals [17], generalized to other fluids and fluid mixtures, and to magnet systems by Pierre Curie [18], spawned a huge field of research that really took off in the mid-20th Century [19], with extensive possibilities for highly complex critical systems emerging. [20-24] What Descartes and his later materialist admirers could not have foreseen was that the revolution created by the discovery of instabilities would cause a revolution in our understanding of what an organism is capable. Slow elucidation led to their discovery at the heart of complexity biology. Examples now show that the physics of 'Strange Attractors' is involved. [26] It may well be required to understand refined aspects of biological function.

Brian Ford has written eloquently of the intelligent behaviors of many kinds exhibited by single cells [27], deploring the reductionist perspective, which leads his fellow biologists to dismiss such observations as not fitting their mechanical reductionist world view. Single cells are seen to vigorously pursue prey – lymphocytes chasing invaders; paramecia hold courtship 'dances'; algae rescue neighboring cells in distress; amoebae build protective shells. Not the stuff of genetic maps and molecular biology, but explicable by the theory of free will that we shall now present based on complexity.

2. Complexity and the New Order in Biology

Complexity has slowly assumed a position of leading importance in biology. From humble beginnings in the 1960s, twenty years on the Santa Fe Institute came to be dedicated to its furtherance, and forty years on even the President of the Royal Society was saying that scientists had to get to grips with it. [27] As stated above, phenomena in complexity biology lead to the conclusion that organisms are not mechanical systems. [28] At the heart of organism regulation is criticality – a mathematical singularity causing instability lies at the heart of regulation in all biological systems. The roles played by such critical instabilities are manifold. They have been suggested by Kauffman and his colleagues, to maximize information processing [29], and to be the key to understanding macrophage dynamics. [30] Another group suggests that, cortically, they increase variability, and optimize system performance. [31] To Summarize: criticality optimizes system responsiveness to stimuli [32], an idea justifying their universal adoption as Loci of Control for biological regulatory systems.

But the locus of control of a biological regulatory system is the place where all the action begins: if our minds are able to initiate actions, then they must do so from the locus of control of our musculoskeletal system. The point is irrefutable: the key message of complexity biology in its present state of maturity is simply that: the mind functions from criticality. Its internal information system is therefore the form of information that occurs at criticality. To analyze the inner functioning of mind, whether in humans, or animals, amoebae or even algae, we need only identify the kind of information operating at critical instabilities – the information that came when biology originally bought into them (as it were), when biology adopted criticality for its manifold benefits.

3. Information at Criticality

Information science has a long history, stretching back to encoding of messages by Kings and Queens and Generals in the ancient world. The forged message that brought Mary Queen of Scots to the scaffold was written in a simple code. Arab scholars of ancient times were the first to break letter substitution codes. Today, we find crossword style puzzles in many newspapers where finding solutions requires an understanding of the natural frequencies of letters in the language. The second world war was won by using computers to break the German mechanical system of cyphering their high command messages. The story is well known to all. [31]

Possibly less well known today is the story of Claude Shannon, the encoding theorist who first proved that the only way to render a cypher unbreakable is to use it only once. Shannon developed the general mathematical theory of encoding [32], which he recognized to be parallel to the theory of entropy in thermodynamics. Jon Von Neumann suggested, when asked, that it be given the name 'Information Theory'.

It may be surprising for some to learn that Shannon's theory was not the 20th century's first mathematical theory of information. The credit for that goes to the Cambridge statistician Ronald Fisher, who developed the first theory of information [33], later dubbed Fisher Information by physicist Roy Frieden who demonstrated its importance to understanding the foundations of physics. [34] Frieden's point, to which we shall return, is that the entire structure of classical mechanics in its form due to Lagrange, is that of an information system, like the one elucidated by Fisher.

Much later in the 20th century a mathematical group at Oxford started analyzing quantum phenomena from the perspective of information. They arrived at a theory of *quantum information*. [35] None of these mathematical theories applies at critical instabilities. So, if biological control systems universally operate at criticality, how can the system of information on which they operate be identified? Does it have any special properties? The very least that can be said is that the information must be unusual not to have been identified previously, and must have a new and different structure.

4. Information Structures at Criticality

For reasons that will become clear by the end of the next section, the name proposed for information occurring at criticality is Experience Information – it's structure is ideal for the description of subjective experience. In order to derive its structure, we first hypothesize that, because of the similarities between them, instabilities in different fields of physics will present a similar structure in the information, which they support, and by which they communicate. Based on this plausible assumption, an information structure at criticality will be derived in a particular field. Its generalization to other fields of physics will then be justified by the plausibility of the structure discovered. The key question will be, 'Does the result of these assumptions seem to work?' The final justification of the derivation will be the plausibility of the results attained.

Approach under the first hypothesis: one kind of instability is as good as any other: The field of physics where the structure of information at criticality was first derived [36] was Fluid Dynamics. In this field, a critical instability occurs at the transition from laminar flow to turbulent flow. This case is helpful, because, being able to be represented pictorially, the information structures are simple to derive. They are easy to visualize; as in the case of theorems in school geometry, where the idea of a mathematical proof is often first presented. It first appears in mathematics learned in primary or middle school levels.

First, let us consider the kind of information that is relevant when describing the flow of a fluid. Fluids undergo flows of two kinds, the first steady, smooth and regular, and the second fluctuating and constantly changing. Clearly, the first, known as *Laminar Flow* should be simple to describe, while the second kind of flow regime, known as *Turbulent Flow*, being more complex, will be more difficult to understand. As it happens, there is a critical instability that divides the first, smooth and constant, *Laminar Flow* regime from the second, entirely different and far more complicated, *Turbulent Flow* regime.

When a fluid undergoes laminar flow, the flow lines form a vector field, with the flow at each point represented by a single, constant vector, →.

As the flow rate increases, and the flow regime begins to approach the transition to turbulent flow, the possibility of vortices forming begins to arise, but none do, because the amount of dissipative energy is too low, so the single vectors, →, still provide a good description.

At the critical point itself, however, when the fluid's critical Reynolds number is reached, the flow is still not turbulent, but the microscopic character of the fluid flow vectors becomes altered. At the critical point, vortices are (desperately) trying to form, but ae just (only just) unable to do so. Imagining the microscopic structure of the fluid flow vectors suggests that the following transformation takes place: *each flow vector gains an attachment of one or more infinitesimal vortices*. While these do not alter either the speed or direction of flow, the vector velocities, they completely change their microscopic structure.

A finite vortex, even a tiny one, produces time varying changes in all the fluid flow vectors surrounding its location. An *infinitesimal* vortex will do something similar, but *the changes in vector length and direction will be infinitesimal*, and will influence *only an infinitesimal region of vectors*.

We therefore propose that, at its critical Reynolds number, a fluid's flow vectors transform into *a mixture of an infinite number of infinitesimally different flow vectors*, all sewn together by *an infinitesimal vortex*. The flow vectors possess a completely different structure. Their information characteristics are thus completely different, as hypothesized. So far so good.

The new flow vectors can be conveniently depicted by a double arrow =========> representing the mixture of vectors, with a loop, **O**, at one end representing the infinitesimal vortex attached to it, and which is now integral to the structure of the new flow vectors, i.e. **O**===========>.

These new flow vectors constitute the new information states, which apply at the fluid's critical Reynolds number. Hence, the new arrow, **O**==========> can be considered to represent the structure of information states in a flowing fluid at its critical Reynolds number, i.e. information states in this kind of system at criticality.

Now consider the generalizability of this result. The key question is whether, having derived the structure of the form of information at a critical instability in a flowing fluid, the same structure applies to other kinds of system. In a ferromagnet, for example, the order parameter is the magnetization, which also has a vector form, →. At the critical point, critical fluctuations may be expected to produce a mixture of magnetization vectors, which will be sewn together by fluctuation(s). Again, the arrow, **O**===========>, seems a reasonable representation.

Critical instabilities were originally discovered in Van der Waals' CO_2 gas-liquid phase transition system, where the order parameter is the density, which is a scalar. Here, vectors do not initially seem appropriate. Nevertheless, at the critical point, the density is no longer stable and fixed, but is transformed by density fluctuations into a mixture of different densities, all mixed by the fluctuations. If density is represented by a vector in one dimension, the mixture may again be represented by **O**========>. Here, the loop is mixing vectors of different lengths, while in the fluids it was mixing vectors with different directions as well.

The generalizability of the result, case by case, seems to suggest the following approach: in any given system, its critical point will occur at a particular point in its space of intensive (usually thermodynamic) variables, with a line of phase transitions in a specific direction from the critical point. Along the line of phase transitions, the order parameter has a physical structure specific to that kind of phase transition, and represents the kind of information in the system. Physically this will be represented by an element of some linear algebra →, because, *away from its critical points, a system's physics can be represented in some linear form*.

At the system critical point, however, the linearity of the physics breaks down completely (this also happens in the 'critical region), and the structure, →, becomes mixed by the fluctuations into a mixture of its values, which may be represented by the combination of the double arrow and its attached loop, **O**=======>.

From an abstract, mathematical perspective, a critical point is a mathematical singularity, where physical values tend to be locally undefined, though averages of fluctuating values may still apply – averaged over time or space (volume). The 'loop', **O**, now represents a singularity i.e. completely non-linear behavior, while the arrow, =======>, represents the mathematical structure into which the linear algebra is transformed by the singularity. For example, in earlier treatments, I have suggested that if the linear algebra comes from a quantum variable that forms a Hilbert space with elements →, the mixtures of elements, =====>, form Banach spaces. Associated treatment of the elements of cognition suggests, however, that things may be more complicated.

However, that may be, for present purposes we shall take the basic structure of information at criticality to be a singularity represented by a feedback loop, O, coupled to an attached mixture of linear elements, =====>. The mixture possesses a high degree of internal coherence due to close juxtaposition of all its elements. The physics is non-linear, because of the presence of the mathematical singularity causing physical instability. A non-linear system can be represented by a circle, the infinitesimal vortex loop, **O**; while the encoded information is represented by =====>, the double arrow mixture.

Such information clearly cannot be represented digitally. The individual vectors can be so represented, but not an infinite mixture of them. The animal is far too complex for digital modelling to be able to preserve its characteristics, as we shall see in the next section.

5. Experience Information

To justify the name, experience information, requires seeing how this information structure fits 'experience' as we subjectively know it from our time in this world. Several profound thinkers have made deeply perceptive statements about experience, starting in ancient times, and then through the 18th, 19th, 20th, and 21st centuries.

The ancient Vedic literature describes the quietest and deepest states of mind open to human awareness. [37] The Mandukya Upanishad [38] describes it as a 4th state of consciousness, beyond waking dreaming and deep sleep, while the great philosopher and teacher, Adishankara, says that in that state of consciousness, 'The Self knows Itself by Itself'. [39] A person thoroughly familiar with the state, recognizes it as present even when the mind is in the first three states of consciousness [38], becoming aware that, like a bird in a tree watching its friend enjoy pecking some fruit, the Self is only a witness to activity in which mind and body engage. [40]

All this deep structure of the Self-knowing-itSelf can be represented by the self-observing loop, **O**. [36] In other words, the deep structure of the Self, its property of knowing itself, and the property of Self-Knowledge that is the essence of consciousness, is presented in the loop, **O**. The rest of the picture, the double arrow, ====>, must represent the information content of the mind. This shows that, in this physical model, the information content of the mind is superimposed on the experience of the 'Self', which underlies all other experience. Most satisfactorily, that is the central message of the Upanishads. [37, 38, 40] Another name for that state of awareness is 'pure consciousness'. [41, 42]

Philosophers who commented most deeply on the nature of the information superimposed on the state of pure consciousness are Plato [43] in his theory of 'forms' as the fundaments of human cognition, and Immanuel Kant [44] in his Critique of Pure Reason, In that work Kant states that the mind apprehends objects of perception as 'wholes', further reasoning that it is the 'Self' that holds them together as wholes.

As we saw in the last section, an infinitesimal vortex, **O**, holds together different vectors in the mixture, ======>. This now translates into the statement that the 'Self', represented by **O**, holds together components of objects of perception, →, into wholes represented by ======>. The structure of experience information, **O**======> reproduces Kant's insight with perfect accuracy, a serendipitous result that could not have been expected at the outset. It can only mean that something is going right.

6. How Consciousness Reduces Wave Packets

It is still not clear that the structure presented as that of experience information applies to anything other than cognition. How can it be turned into a basis for action as well? How can its incoming information be transformed from cognitions into outgoing information?

To answer this question, another role for the loop, **O**, must be spotted. The clue comes by putting together two facts about excited states, excitations, at criticality. First, they are not quanta, but critical point correlations; and, second, the correlations have replaced the quanta that stabilized values of the phase transition order parameter.

The role of quanta of any kind is to stabilize the value of physical properties of a system oscillating in the physics of the quanta. Quantum fields stabilize values of a system parameter. Sound waves stabilize fluid density, electromagnetic waves stabilize electromagnetic field values; similarly, quantum fields of elementary particle physics stabilize the vacuum state of their unified field.

At instability, a parameter becomes unstable, since stabilizing quantum fields are not present. Something has removed them. From the perspective of quantum physics, the non-linear structure presented by **O** is a Perfectly Self-Observing System. Observation destroys quantum fields, so self-observation annihilates system stabilizing quanta. [45] The system becomes unstable.

The new role of the loop is therefore to bring about wave packet reduction. From the perspective of quantum theory, reduction of wave packets has always been regarded as the key way that consciousness enters into physics. An entire theory of Orchestrated Reduction (**O.R.**) of wave packets has been formulated by Penrose and Hameroff [46] based on quantum gravity applied to microtubule proteins in the cell cytoskeleton.

The theory of quantum wave packet reduction based on the loop, **O**, considered as a perfectly self-observing system seems more realistic and practical. The loops are definitely present; they definitely annihilate stabilizing quantum fields; and they are available to act on incoming wave-packets when cognition takes place.

The loops can also reduce wave packets presented internally. This gives them the power to make different kinds of choices in different brain cortices. In the motor cortex, they can select particular muscle groups to activate in chosen ways.

The power of the loops to reduce wave packets thus leads to an ability to activate the body's motor systems and implement chosen actions in the outside world. Brian Ford's paper in this issue [26] implies that networks of genes operating at criticality in single cells do the same. All that remains is to show how the time evolution of the universe incorporates such processes.

7. Quantum Theory and the Perceived Universe

Classical physics yielded a picture of a mechanical universe with a predictable time evolution. Quantum theory yields an entirely different picture because of the non-predictability of precise destinations for individual quanta once they have been emitted. However, when dealing with overall patterns of distribution, effects become more predictable, if not exactly so, as quantum uncertainties will always leave residual uncertainties in the details of actually observed distributions.

In quantum theory, wave packet reductions produce information. An instrument like a bubble chamber can induce a single quantum to produce a self-consistent series of wave packet reductions, because, as Stapp points out [47], an observer is not needed for wave packet reduction to occur, or in this case, not for one of a self-consistent series of wave-packet reductions, which are later going to be observed as a whole by a human operated device like a camera.

The kind of world revealed by particle tracks in a bubble chamber is one of successive interactions, each determined by previous wave packet reductions on the path of the quantum in question. Each path looks as if it obeys classical dynamics because momentum transfer at each interaction event is small. By considering quanta of increasing complexity and numbers, a description of an appearance like the macroscopic, seemingly classical, world can be built up. [48] Quantum theory predicts that correlated information production events at the quantum level, will produce self-consistent information patterns generating the illusion of a classical, macroscopic world.

Here the quantum correlations are responsible for the world's seemingly objective appearance. The great French physicist, Bernard D'Espagnat used them in his proofs [49, 50] that quantum theory denies that the universe we perceive is a strongly objective reality.

Taking its appearance as an objective reality, which it seemingly portrays, leads to the delusion that the world in which we live is strongly objective with each object existing in its own right, independently of all others. When the understanding of quantum theory is extended by Alain Aspect's experimental proof [10] of Bell's theorem [11], quantum correlations are seen to guarantee that the illusion of a purely objective reality is consistently maintained. [48] Quantum correlations have an entirely natural place in our universe.

Any remaining doubt that the universe we inhabit is built out of information production is dispelled by the work of Roy Frieden [34], who demonstrated that the entire structure of Lagrangian mechanics, whether applied to classical mechanics, or quantum field theory, is built out of Fisher Information.

In quantum theory, the account given in this section makes the latter obvious, but the theory also holds true for classical mechanics. It is as if, when the appearance of the classical world is generated, its mechanics cannot escape vestiges of the information production processes that gave rise to it.

The resulting universe is one in which the future builds up as quantum information production processes of wave packet reduction generate information 'telling the story' of creation. Those observing the process can participate in it in ways that seem completely real, particularly because of their abilities described next.

8. A Theory of Free Will

Having established that our universe can evolve in time based on information injected into it by processes of quantum information production [48], in accordance with principles stated by Stapp [47], it is now possible to add the mechanism of information production by biological control systems stated in section 6. Such reduction of wave packets is closely related to the Hameroff-Penrose orchestrated reduction (**O.R.**). [46] Both depend on non-linearities, generate information at a quantum level *and can thus make choices in biological systems' internal environments*.

Action by minds can be at mental or physical levels, though the former precedes the latter. In some cases, if the mind concerned possesses powers of reflective responsibility, it may not progress that far. But when the mind injects information into the motor cortex, selecting particular actions by particular muscle groups, then physical information is transmitted into the outside world, as the animal or person performs their action of choice, moving in a particular direction at a chosen speed, emitting a cry or other sound, changing expression or focus of attention.

It is noteworthy, that a similar range of processes is open to single cells, given that their epigenetic, and other systems controlling responses of various kinds similarly function from criticality. A powerful means to explain the fascinating range of phenomena recounted by Brian Ford in his Special Issue article [26] is thus available.

This explanation of actions generated by minds contrasts with the radical behaviorism of BF Skinner [51] who proposed that all actions are merely reflexes. It adds a nail to behaviorism's coffin from another, new perspective. Skinner's fundamental assumption was, of course, denied by the observations of variable, (1/f) responses [14], seen in Fractal Physiology [15], but it is interesting to see how the (1/f) response condition refuting radical behaviorism can be developed into a psychology of free will, the possibility of which, as a thorough going materialist, he emphatically denied.

9. Conclusions

The sequence of ideas based on experience information, and its relation to quantum theory presented in the last five sections, demonstrates how it is possible for the new theory of experience to encompass a theory of selected actions, i.e. a theory of free will. The theory has a myriad of further possible developments, which will be presented in later papers.

Regarding wave packet reduction, the theory is far more economic than the Hameroff-Penrose approach, since it employs the form of information universally used in biological systems where complexity locates loci of control at criticality. Experience information then applies, rather than digital or quantum information, or any other kind. All attributes of experience then apply to all biological organisms, even down to single cells.

Minds generate information internally at criticality and inject it into the outside world. There it joins other information determining the evolution of creation. The resulting picture suggests that human and animal minds are co-creators of our universe's future, alongside the power that reduces other quantum wave packets in the rest of creation. As time passes, all together create our universe's history: a complex pattern of seemingly real, correlated information.

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Exploration of the Fundaments of Oncogenesis: A Unified Field Approach to Aetiology

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Oncology, a complex multifactor etiology has eluded all-inclusive prevention and remission by a lack of comprehensive understanding of a singular causation at the most fundamental physical level. Current physiological models address the myriad *branches* but not the global biophysical *root* of aetiology. Current thinking claims that underlying quantum field dynamics are the 'basement of reality'. Einstein emphatically stated comparably to his General Relativity, that 'it was merely a convenient stopping place on the road to a more unified theory'. We propose that oncogenesis (for the several hundred tabulated cancers) occurs from one single causative factor at the root of the three considered branches of causation. Physical science (which includes chemistry, biochemistry or biophysics) has evolved from 3D Newtonian classical mechanics to the current vogue (albeit experimentally verified) of 4D quantum mechanics. Empirical access to the 3rd regime of unified field mechanics (UFM) with inherent additional degrees of freedom is imminent. In regards to that perspective, it is proposed, that a 'telergic stressor' mediated by noetic action of the unified field produces conformal change in protein molecules that can cascade into a system of oncogenic specificity. Although the action of this fundamental stressor appears extracurricular to the current investiture on the 'branches of causation': Environmental (radiation, chemical), genetic/epigenetic, or psychosomatic; this paradigm shift is in process as threshold violations of QED (Quantum Electrodynamics) are occurring in more than one arena. The epigenome is involved in regulating gene expression, development, tissue differentiation, and suppression of transposable elements. Unlike the underlying genome which is largely static within an individual, the epigenome can be dynamically altered by external conditions. Numerous mutations occur on the pathway to the onset of a cancer; we quantify a single Unified Field noetic effect that applies to the aetiology of all cancer.

Keywords: Casimir effect, Catastrophe theory, Noetic effect, Oncogenesis, Unified field mechanics, Van der Waals

Le microbe n'est rien, le terrain est tout – Pasteur.

1. Introduction

More than 200 types of cancer are catalogued. Currrently, cancer is said to be caused by external factors, such as tobacco, infectious organisms, and an unhealthy diet; and internal factors, such as inherited genetic mutations, hormones, and immune conditions.

In this work, we obtusely propose that all cancer arises from one fundamental cause. This is not obvious by current thinking steeped myriad multiple gene mutations as the primary causation. While true, this represents the branches (each cancer type) and does not represent the fundamental root of the problem. The definitional role of endogenous and exogenous agents of DNA damage is that DNA damage is considered to be the primary cause of cancer. True, there is a 'force' transforming normal cells into cancer cells, but we intend to examine the stressors that cause DNA damage and demonstrate that there is a single stressor involved; not at the level of biochemistry where conformal molecular changes lead to DNA mutation, but deeper than what is currently called the 'basement of reality' - Quantum Mechanics. The global stressor has been hidden behind the uncertainty principle. But now that 3rd regime physics of Unified Field Mechanics, will be available; a new energy of the unified field will be shown as the fundament of oncogenesis.

1.1. *Conformational Isomers*

There are a number of classes of isomers that can undergo conformal change, with the possibility of causing damage to DNA molecules

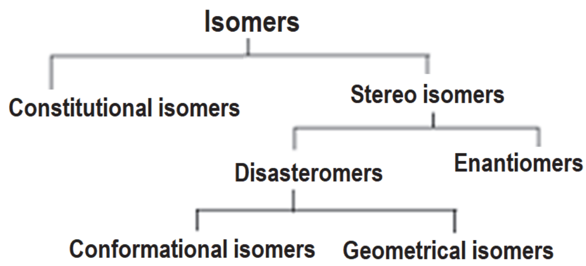


Figure 1. The various types of isomers.

Different structures of a molecule resulting from rotation about sigma bonds are known as conformal isomers or conformons.

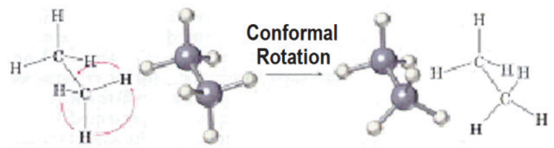


Figure 2. Conformal isomers produced by bond rotation.

More than one mutation is necessary for carcinogenesis. In fact, a series of several mutations to certain classes of genes is usually required before a normal cell will transform into a cancer cell.[1] On average, for example, 15 "driver mutations" and 60 "passenger" mutations are found in colon cancers.[2] Mutations in those certain types of genes that play vital roles in cell division, apoptosis (cell death), and mutations and epimutations in DNA repair genes will cause a cell to lose control of its cell proliferation.

There are more than 200 different types of cancer. Cancer starts when gene changes make one cell or a few cells begin to grow and multiply too much. Gene mutations that occur after birth. Most gene mutations occur after you're born and aren't inherited. A number of forces can cause gene mutations, such as smoking, radiation, viruses, cancer-causing chemicals (carcinogens), obesity, hormones, chronic inflammation and a lack of exercise.

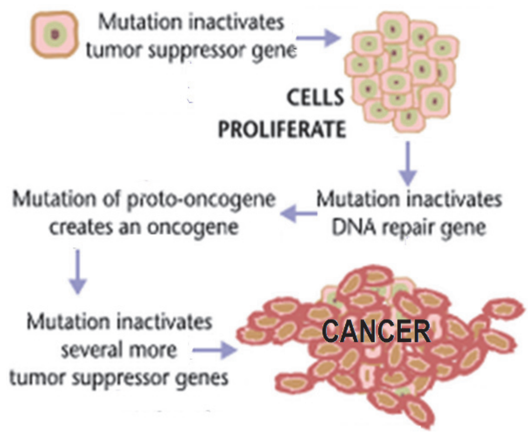


Figure 3. Cancers are caused by a series of mutations. Each mutation alters the behavior of the cell.

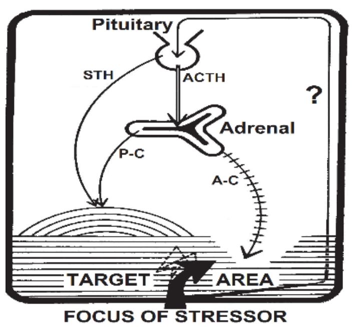
2. Selye – Pioneer of Physiological Stress

In the science of biology, a mechanism is a system of causally interacting parts and processes that produce one or more effects. Scientists explain phenomena by describing mechanisms that could produce the phenomena.

Hans Selye is considered first to demonstrate the existence of the physiological stress response of an organism to stressors.

Selye conceptualized the physiology of stress as having two components: a set of responses which he called the "general adaptation syndrome" [x2], and the development of a pathological state from ongoing, unrelieved stress: *Each individual, well-defined disease … has its own specific cause.*

Through some unknown pathway (labeled by a question mark), the "first mediator" travels from the directly injured target area to the anterior pituitary. It notifies the latter that a condition of stress exists and thus induces it to discharge adrenocorticotrophic hormone (ACTH).



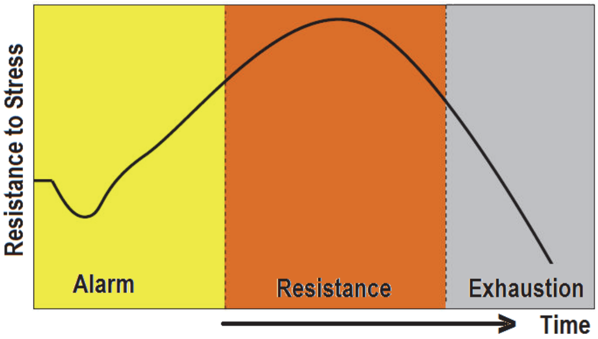


Figure 4. a) Selye's generalization of physiological stress. STH is somatotrophic hormone, ACTH, adreno-corticotrophic hormone, P-C, prophologistic corticoids and A-C, antiphlogistic corticoids. Fig. adapted from [x]. b) Selye's General Adaptation Syndrome. Fig. redrawn from [x].

Glucocorticoids (steroid hormones), bind to the glucocorticoid receptor present in most vertebrate animal cells to regulate glucose metabolism for certain aspects of immune function, such as reduction of inflammation. Thus, they are to treat overactive immune system diseases, interfere with abnormal mechanisms in cancer cells and amelioration of side effects of anticancer drugs. Cortisol (hydrocortisone) is an extremely important glucocorticoid essential for life mediating a numerous cardiovascular, immunologic metabolic, and homeostatic functions.

Glucocorticoids affect cellular function by binding to glucocorticoid receptors (GR). The GR complex, when activated up-regulates the expression of anti-inflammatory proteins in the nucleus (transactivation) which represses the expression of proinflammatory proteins in the cytoplasmic matrix (intracellular fluid) by preventing the translocation of other transcription factors from the intracellular fluid into the nucleus (transrepression).

General Adaptation Syndrome (GAS), developed by Hans Selye, is a profile of how organisms respond to stress; GAS is characterized by three phases: a nonspecific mobilization phase, which promotes sympathetic nervous system activity; a resistance phase, during which the organism makes efforts to cope with the threat; and an exhaustion phase, which occurs if the organism fails to overcome the threat and depletes its physiological resources.[84]

Selye discovered and documented that stress differs from other physical responses in that stress is stressful whether one receives good or bad news, whether the impulse is positive or negative. He called negative stress "distress" and positive stress "eustress". The system whereby the body copes with stress, the hypothalamic-pituitary-adrenal axis (HPA axis) system, was also first described by Selye. He also pointed to an "alarm state", a "resistance state", and an "exhaustion state", largely referring to glandular states. Later he developed the idea of two "reservoirs" of stress resistance, or alternatively stress energy.

3. Mechanism Initiating Protein Conformation in Prion Propagation

We use prion conformation to illustrate a possible test case for examining the basis of noetic medicine in topological phase transitions in 3rd regime brane interactions. This introduction is only a primitive slice introducing the anticipated new field of integrative Noetic Science revolutionizing medicine and psychology and implementing myriad *conscious* technologies like sensory bypass prosthesis or ℵ-wave (eternity–wave) accelerated healing for example. Experimental work is underway to isolate and utilize the noetic field for these tasks.

> *When the great innovation appears, it will most certainly be in a muddled, incomplete form. To the discoverer himself it will be only half-understood; to everyone else it will be a mystery. For any speculation which does not at first glance look crazy, there is no hope.* [101]

Transmissible prion based spongiform encephalopathies propagate by conformational change of the prion's protein, PrP structure. An experimental design, relying on the utility of a new fundamental teleological action principle inherent in the topological geometry of a covariant polarized Dirac vacuum putatively driving self-organization in all autopoietic complex living systems, is developed to elucidate the fundamental nature of this conformational change. Further, PrP propagation is considered a mechanical action that can be described by 'interactive computational modes' of 'topological switching' driven by incursive oscillations occurring in the bioenergetics of the prions physical chemistry when improperly coupled to the long-range coherence of the noetic action. The experimental apparatus, a multi-level interferometer, is designed to focus this noetic field in a manner that simulates the mechanism driving PrP conformation to pathological form.

An extensive body of literature exists for phenomena related to the zero-point field; but relative to noetic theory this work is considered descriptive metaphorically of only the 'fog over the ocean' rather than the structural-phenomenology of the ocean itself.

Instead the deep structure of a real covariant Dirac polarized vacuum is utilized [1-3]. The Casimir and Zeeman effects are considered evidence for a Dirac vacuum. New assumptions are made concerning the Dirac polarized vacuum relating to the topology of spacetime and the structure of matter cast in a twelve-dimensional (12D) form of Relativistic Quantum Field Theory (RQFT) in the context of a new cosmological paradigm called the Holographic Conscious Multiiverse (HCM) [4-6]. In this anthropic cosmology, the observed Euclidian-Minkowski, $E_3 - \hat{M}_4$ spacetime present is a virtual standing wave of highly ordered Wheeler-Feynman-Cramer retarded-advanced future-past parameters respectively [7, 8]. See Figs. 4 & 11 for a graphic illustration of this paradigm. An essential ingredient of HCM cosmology is that a new action principle synonymous with the unified field arises naturally and is postulated to drive self-organization and evolution through all levels of scale [9-11].

In this context, an experimental design [12] is introduced to isolate and utilize the new noetic action to test empirically its putative ability to effect conformation in prion protein. The Prion, PrP [13-15], the infectious protein responsible for degenerative spongiform encephalopathies like Mad Cow, Scrapie and Creutzfeldt-Jacob Diseases is designated as 'system zero', the most primitive known system with anthropic properties, albeit purely mechanistic [9, 10, 16]. Noetic Theory postulates that prion protein, PrP is 'animated' by the self-organizing properties of the long-range coherence [17, 18] of the *élan vital* or unitary noetic field [9, 10, 19-32]. In addition to manipulating conformational change, from the experimental results we attempt to calculate the energy Hamiltonian required to initiate the misfolds.

4. Structural-Phenomenological Micromagnetics of Proteins and Prion Conformation

Biological molecules contain coupled coherence domains with long-range resonant interactions extending throughout the entire living system [17, 18] from and into the surrounding spacetime [9]. This resonant coupling produced by the teleology of the noetic field driving its hierarchical self-organization has local, nonlocal and supralocal (complex HD) parameters [9]. The Schrödinger equation, extended by the addition of the de Broglie-Bohm quantum potential-pilot wave mechanism has been used to describe an electron moving on a neural manifold [33, 34]; but this is not a sufficient extension to describe noetic aspects of living systems which requires further extension to include action of the noetic unitary field in additional dimensions. The following is a brief review of quantum properties of water illustrating one regime in the noetic hierarchy [35, 36].

Properties of water, the fluid medium supporting life, result from the structure of individual H_2O molecules and intermolecular forces between the molecules dominated by Hydrogen bonds. The capacity of H_2O molecules to from diverse 3-D networks (hexagon, square, & pentagon) of H bonds, while maintaining 4-fold bonding at each molecule is structurally significant. Liquid H_2O is a structurally random network of strained and broken *H*-bonds. This network is labile; bonds break in one place and reform nearby. Isolated H_2O molecules act as though each *H*-bond bore a 1/3 proton charge, and as if the *O* bore –2/3.

Dipoles of neighboring H_2O molecules partially align and act in concert under polarizing influences of an *EM*-field. This alignment and molecular polarizeability stems from a large static dielectric constant, relating to H_2O's ease in dissolving ionic crystals like alkali halides. Also the H_2O molecules small size allows close approach to ions. At room temperature ~1 in 55 million H_2O molecules dissociate into H^+ and OH^- ions readily incorporated into the liquid's random *H*-bond network. The high mobility of H^+ and OH^- ions causes a net transfer of ionic charge along chains of *H*-bonds. Crucial conformations in the hydrophilic and hydrophobic chemical groups of complex biochemical molecules are caused by H_2O solvation. In liquid phase atoms are disordered and free to move.

The key to understanding protein folding diseases lies in the arrangements of their amino acid structure. Virtually all proteins consist of two periodic structures called α and β sheets whose conformation is derived from the hydrogen bond [37]. Protein folding usually occurs spontaneously as a structural property of the protein itself. If unfolded a protein typically refolds properly without assistance; but some are aided during the folding process by enzymatic proteins called molecular chaperones [38, 39] because intermediary structures often have the tendency to aggregate deterring the end result. Chaperones prevent aggregation by keeping chaperoned molecules sequestered inside cavities within their structure. Occasionally a protein will misfold; and recently it has been realized that misfolds are a more common property of proteins than previously suspected [40]. Most proteins fold into one shape only; this is not true of the prion protein, PrP which is also said to act as its own chaperone.

Prion protein whose misfold aggregations damage nerve cells in PrP encephalopathies is constantly produced by the body. Normally it folds properly, remains soluble, and is disposed of without problems. But if misfolded encephalopathic prion protein, PrP^{Sc} 'bumps' into the normal-folding intermediate, PrP* it shifts the folding process and the protein, despite a normal amino acid sequence, ends up as more pathological prion protein. This process continues as long as the body keeps producing the normal protein. Thus the encephalopathic prion self-replicates itself without precursor material or nucleic acid of its own by a pathological chaperone mechanism disrupting the normal conformation pathway. Recent research demonstrates that Alzheimer's Disease, Cystic Fibrosis, an inherited form of Emphysema and many cancers although apparently unrelated all result from protein misfolds [41].

About 250 amino acids comprise the normal cellular form of the prion protein PrP^C found in all mammals which in humans is produced on gene 20 with evidence that the gene is evolutionarily pre-mammalian [41]. Whereas PrP^C is soluble, the infectious form PrP^{Sc} is hydrophobic producing aggregates causing neuropathology; however both PrP^C and PrP^{Sc} have the same chemical makeup, differing only in conformation. The normal cellular isoform PrP^C has three α - helices and two small β strands. PrP^C is α rich; whereas the PrP^{Sc} isoform is β rich. That PrP^C is the required precursor for PrP^{Sc} propagation has been demonstrated by $Prnp^{0/0}$ genome studies where disruption of both alleles on mouse chromosome 2 blocks PrP expression such that no prion encephalopathy occurs [41].

PrP^C is produced in the endoplasmic reticulum before it is brought to the cell surface where it can be drawn into a caveola, subcellular cavernous sites. In these cavities if the intermediate conformation, PrP* occurs in the presence of PrP^{Sc} normal cellular PrP^C is converted into more of the infectious form, PrP^{Sc}. It is in this context that the prion acts as its own chaperone or that another protein dubbed protein-X catalyzes the misfold; but so far the search for protein-X has failed [41].

The tenets of Noetic Field Theory (NFT) [20-32] suggest that the X-factor is not a protein but a spacetime-coupled cavity-QED effect of a coherence force inherent in the continuous-state parameters of the unified noetic field. Therefore the etiology of PrP encephalopathies could be generalized by developing this model. The Noetic Field [20-32] produces periodic symmetry variations with long-range coherence [9, 10, 17, 18] that can lead to a critical Noetic Effect[1] [20, 21, 27] of consciousness. This can be described by a form of double-cusp catastrophe dynamics (Fig. 3). Operationally the plane of equilibrium experiences sustained hyperincursion by the noetic field. The coupled modes of this process rely on a special form of the harmonic oscillator called the incursive oscillator [42-48]. There is a force of coherence [49]. For example, for an Earth observer's temporal perception, railroad tracks recede into a point at the horizon. For an atemporal eternal HD observer, the tracks remain parallel. This is the origin of the coherence force which forms a kind of logic gate driving equilibrium of the Casimir boundaries to parallel or degenerate modes thus giving rise to the possibility of effecting conformational states.

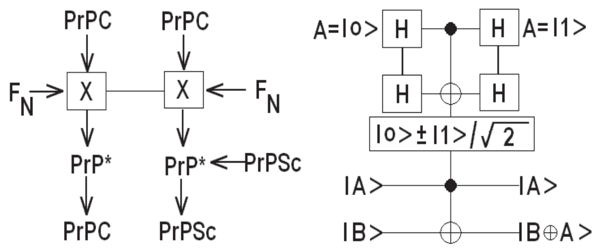


Figure 5. a) Flow chart for Prion propagation, where factor-X is postulated to be the action of Noetic Field, F_N . b) Circuit representation for a possible quantum logic gate configuration for PrP^C Propagation. Two Hadamard gates, H generate a superposed intermediate conformation of PrP^C called PrP* in state $|0\rangle \pm |1\rangle / \sqrt{2}$, illustrating the possibility that the Prion's pathological process acts a quantum Hadamard Controlled-Not Gate; $|A\rangle$ is the control qbit and $|B\rangle$ is the target qbit.

Normal prion protein biochemistry is operationally defined by usual time dependent metabolic quantum fields; but noetic theory postulates that the encephalopathic conformation, probably in conjunction with the PrP* intermediary, in some manner couples to the atemporal realm where a 'force of coherence' creates a telergic 'chaperone effect' acting on the coherence gap created by the presence of the 'stronger' PrP^{Sc} molecule driving conformation in the encephalopathic direction. In this approach, pondering Fig. 2 suggests that

[1] Noetic Effect – The various resultant effects of the unitary noetic field as its flux enters spacetime and the mind and body of complex living-systems.

molecular serendipity has gifted the fundamental structural-phenomenology of this prion state as a "Rosetta Stone" of anthropic cosmology [50, 51]. Soon after this insight we came across a somewhat parallel thought: "…The prion protein thus contained, whether by happenstance or homology, a natural mechanism for dimerizing about the symmetric tetrapyrrole" [52]. What is meant by this, for noetic theory, is that the PrP* caveola have Cavity-QED resonant properties [53] in synchrony with the noetic field such that the inherent PrP^{Sc} dominance is able to drive PrP^{C}, when present, to the PrP^{Sc} form.

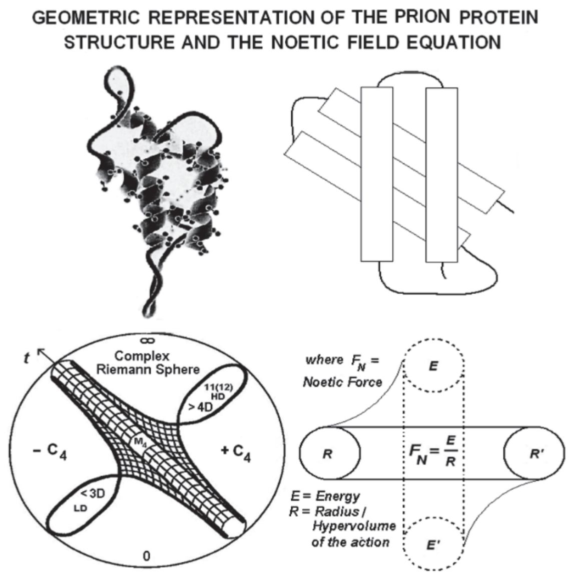


Figure 6. a,b) Best guess putative model of the prion's protein structure gleaned from over ~300,000 possible choices. a) Ribbon model showing α, β sheets. b) A simplified geometry of a). In c,d) Topological and geometric idealizations of the noetic field equation describing an action of the noetic field, called the 'noetic effect', on a biological or spacetime manifold.

This noetic postulate is compatible with Prusiner's view that prion propagation appears to occur by a form of what Prusiner's group calls 'Dominant-Negative Inhibition' [54-56]. They postulate that PrP^{Sc} interferes with PrP^{C} function in conjunction with an auxiliary molecule called protein-X because PrP^{Sc} exhibits more avid binding properties [41]. However as stated our interpretation for a protein-X differs; we postulate instead that QED cavity dynamics within the canella where PrP* binding occurs can be described as a form of logic-gate for interactive computing [57, 58]. This is a boundary condition problem; here probably of the Born-von Karman type where the boundary conditions restrict the wave function to periodicity on a Bravais lattice of hexagonal symmetry, stated simply as $\psi(r + N_i a_i) = \psi_r$, where *i* runs over the dimensions of the Bravais lattice, a_i are the lattice vectors and N_i are integers [59]. In this model, the presence of the periodic spherical rotation effects of the cyclical coherence-decoherence modes allow the action of the noetic field [9, 10, 60]. This Noetic Processing is governed by the fundamental equation of Consciousness $F_N = E / R$. Cyclotron resonance states may maintain homeostasis of the noetic field or induce an electromotive force, the Noetic Effect, on proteins leading to conformational change.

The structural-phenomenology of atoms and molecules is full of domain walls amenable to description by combinations of Gauss' and Stokes' theorems ordered in terms of Bessel Functions where boundary conditions create resonant cavities built up by alternating static and dynamic Casimir conditions [21, 61-63]. As frequency increases central peaks occur with opposite or zero polarity at the domain edges. These properties are relevant to Ising Model [64] spin flips of the domains of the Riemann-Block Spheres effecting homeostatic planes of equilibrium (Fig. 3). The noetic effect can maintain equilibrium or produce catastrophes causing conformational change in protein structures [65].

5. Catastrophe Theory and the Noetic Formalism

Recently the fundamental basis of complex self-organized living systems has been redefined in terms of a new noetic action principle beyond the limitations of 'Biological Mechanism' [9, 10]. This model can be utilized to call for a new field of Noetic Medicine [66] based on the structural-phenomenology of the noetic field and whether resultant action of the noetic effect is positive or negative. Living systems exhibit complex self-organization. The noetic field is the factor driving self-organization [9, 10]; therefore hyperincursion and anticipatory properties are inherent in the fundamental hierarchical basis of the self-organization which can be formally described in terms of Double-Cusp Catastrophe Theory.

The structural-phenomenology of Double-Cusp Catastrophe (DCC) Theory in $\geq 9D$ appears homeomorphic to the Riemannian manifold of both 10(11) dimensional M-Theory and the topological geometry of the continuous-state spin exchange dimensional reduction compactification process inherent in the action of the corresponding scale invariant least unit of noetic superspace as cast in HCM cosmology [4-6]. In this general framework the double-cusp *equilibrium surface* is analyzed in terms of a hierarchy of Ising-like *jumps in state* [64] providing a framework for expanding the basis of allopathic medicine and psychology [66] for which the prion is utilized as a fundamental test case. One can say that the noetic least-unit tiling [67, 68] the fabric of the Planck backcloth is a complex HD catastrophe manifold with Dirac spherical rotation symmetry mediated by the unitary action of the noetic field.

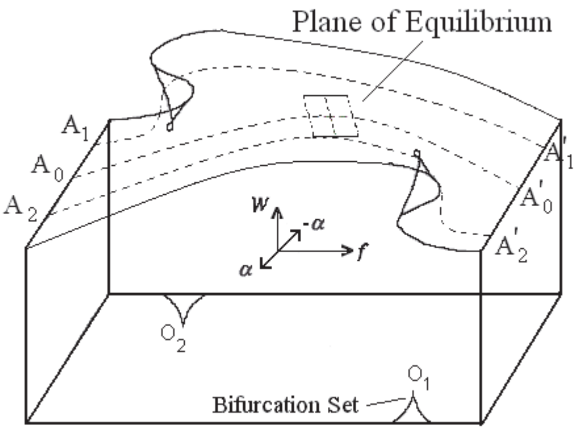


Figure 7. a) The DCC is illustrated showing cusps at each end of the plane of equilibrium. The DCC is said to occur in ≥ 9 dimensions and thought to be the catastrophe form most compatible with the symmetry of NFT. The spacetime component of the plane of equilibrium is a topological manifold tiled of noetic *least units*. The equilibrium manifold undergoes a 'conscious' quantum computation best described by interactive computation. b) Graphically illustrates the fundamental scale invariant noetic equation $F_N = E / R$ of conscious action, the basis of the noetic effect on the plane of equilibrium. c) The hysteresis loop of the Hamiltonian mapped out by the future-past parameters of noetic spacetime. The area E represents the energy of the noetic force F_N .

Any internal or external stress or change in life energy, *E* is a nonlinear dynamical process producing stability or instability in the boundary conditions of *R*; a causal instability in *E* $\rightarrow$ stress $\rightarrow$ displacement $\rightarrow$ catastrophe $\rightarrow$ Ising jump…whereas stable flux is homeostatic. The hysteresis loop of the noetic field (Fig. 3b) is scale invariant; the same processes occur in HCM cosmology and domains of living systems. The area represents the energy of the string tension or élan vital. This energy, E_N is measured in *Einsteins*, the fundamental physical quantity defined as a 'mole' - Avogadro's number (6.02×10^{23}) of bosons, defined here as noeons of the unitary field [9, 10].

Equation (5) describes the equilibrium surface of the DCC [69, 70] as modeled in (Fig. 3); where $B \pm Q$ is the state variable and μ_d and υ_d are the control parameters.

$$(B+Q)^3 + (B+Q)\mu_d + \upsilon_d = 0 \qquad (1)$$

The position of the two cusps is found at $\mu_d = 0$ and $\upsilon_d = 0$. At any moment temporal permutations of the noetic catastrophe cycle evolve in time from future to past and higher to lower dimensions in the same manner as the spacetime present of the least-unit of HCM cosmology for the spatial domains: $R^{12} \supseteq ...R^4 \supseteq R^3 \supseteq R^2 \supseteq R^1 \supseteq R^0$; followed by an Ising rotation where the cycle repeats.

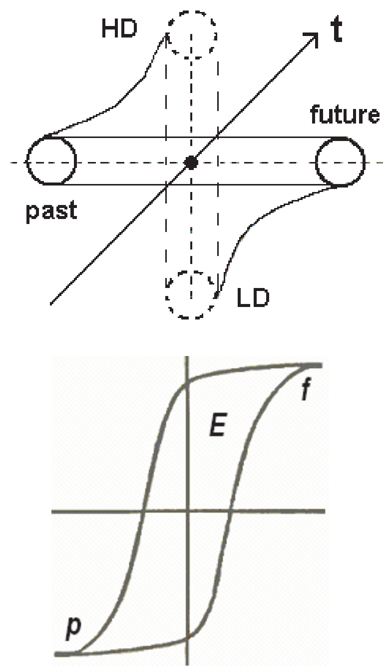


Figure 8. Noetic Action on the Equilibrium Plane of a Double-Cusp Catastrophe.

6. Extending Definition of Matter

Discovery of the electron in 1897 by Thompson demonstrated that atoms are not indivisible. The advent of quantum mechanics showed that matter has wave properties. Now the discovery of additional dimensions

(XD) extends our understanding of the structure of matter to include Calabi-Yau mirror symmetric topological brane phenomena behind the veil of the uncertainty principle in a 3rd regime of Unified Field Mechanics (UFM).

6.1. *Point-Particle Infinite Mass-Energy*

The term point particle is not rigorously defined causing inconsistencies in usage, but is generally used to denote a spherical 0D object with no spatial extension and as described by the inverse square law has all its matter concentrated at the 0D point. From classical electromagnetism (em) we know that the energy of a charge configuration increases as the distance between them decreases. For two general charges q_1 and q_2 separated by a distance r, the electrostatic energy is:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}\,. \tag{2}$$

Ignoring the 1st term, a constant, it is in the 2nd term where we see that the energy of the charges q_1 and q_2, increases to infinity the closer we bring them together. Assuming that electrons are *point particles* carrying a charge density as a charged spherical shell; if we shrink its size to 0D, it is supposed to mean the electron has *infinite* electrostatic energy.

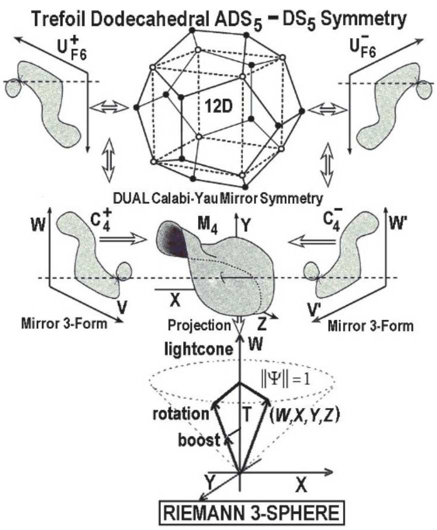


Figure 9. The complete 12D UFM superimplicate order space, not enfolded. The Euclidean cube embedded in the 12D dodecahedron is causally free of the Euclidean shadow in 3-space and thus beyond the semi-quantum limit of the manifold of uncertainty.

From the equation for a spherical shell of charge, e with radius, r *we obtain a result similar to that for the two charges in (4.1)*:

$$E = \frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}\,, \tag{3}$$

where similarly if the spherical shell is brought to a point, the electron's self-energy goes to infinity. Since QED predicts that virtual electron-positron pairs can emerge from the vacuum for up to the Planck time, one can violate the conservation of energy with quantum mechanics at the Planck scale, $\delta t \delta E \leq \hbar$ allowing renormalization techniques to provide an *ad hoc* solution to this conundrum by letting quantum effects cancel out classically infinite contributions to electron mass at a scale similar to the scale of electron mass [1].

The 'Heisenberg Microscope' at CERN's LHC is said to 'see' to 10^{-16}. Quantum mechanics appears as the 'basement of reality' only because the Uncertainty Principle acts as a gating mechanism limiting observation. With the imminent advent of 3rd regime UFM we can take the next step in understanding the structural-phenomenology of matter beyond the traditional 3-space arena of current observation [2, 3].

6.2. *Space-Antispace as a UFM Intermediary*

The Standard Model, a quantum field theory, is incomplete, while it seems theoretically self-consistent, with some phenomena unexplained (dark energy, non-zero neutrino mass); as well-known, it is not yet a complete theory. At the fundamental level Physics reduces to the structure and interaction of fermions. In 3-space Fermions appear to be 0D singularities rather than extended objects, a sufficient rendition until now.

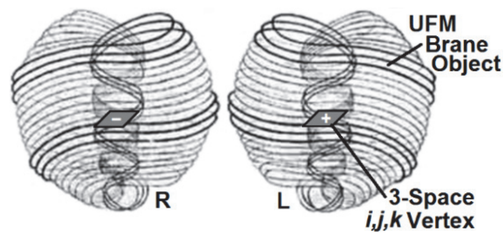


Figure 10. Leadbeater & Besant 'ultimate physical atom' [44] which we tweaked with a traditional 3D fermionic singularity in the center to symbolize a global picture of matter. The two R-L forms are identical to each other, differing as an object and its mirror image. Mirror symmetric components are essential to the UFM particle dual regime model.

Among these are Bohm's description of an *implicate and superimplicate order, with observed particles not fundamental, but 'forms' produced by a continuous convergence and divergence of waves*; and Vigier's concept of matter where a *particle is a complex structure associated with a pilot wave guiding its* motion by exerting a potential force, which we begin to formally describe in terms of an inherent action (guiding force of coherence) of the unified field [2, 3].

To allow the cycle of 'extension' to operate properly (completely), such that the 'mirror image of the mirror image in 12-space is causally free of the CQED quantum shadow in 3-space a 2nd duality is required. This is conceptualized in Fig. 9.

7. Future Development

According to the American Cancer Society: Gene mutations that occur after birth. Most gene mutations occur after you're born and aren't inherited. A number of forces can cause gene mutations, such as smoking, radiation, viruses, cancer-causing chemicals (carcinogens), obesity, hormones, chronic inflammation and a lack of exercise.

It is well known that ~40% of cancers can be prevented by lifestyle changes causes of cancer include genetic factors (but most cancers are not clearly linked to the genes we inherit from our parents, however 'faulty' genes can increase risk; lifestyle factors such as tobacco alcohol use, diet, and physical activity; certain types of infections; and environmental exposures to different types of chemicals and radiation. Known causes of cancer, including genetic factors; lifestyle factors such as tobacco use, diet, and physical activity; certain types of infections; and environmental exposures to different types of chemicals and radiation, and a 'telergic stressor' mental energy, teleological 'telergic stressor', psychosomatic both self-induced or externally induced.

Allopathic medicine denotes three classes of carcinogenic agents:

1) Chemical

2) Radiant energy

3) Microbial/viral agents

These three provide only the branches of oncogenesis, not the fundamental unitary root of all cancer.

8. Noetic Force

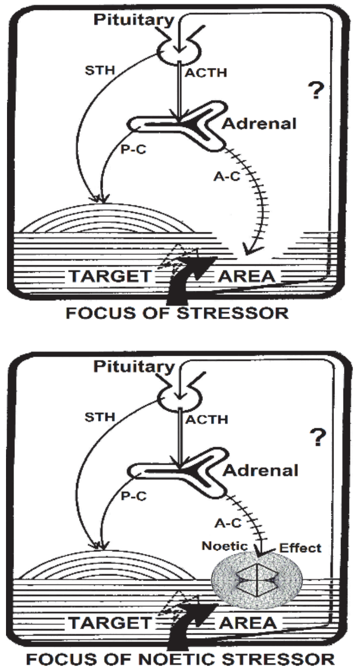


Figure 11. Noetic Effect addition to Selye's physiological stressor model called the General Adaptation Syndrome (GAS). Figs. adapted from [x].

9. Hysteresis, Noetic Hysteresis

When the inherent noetic field is couples to a physiological substrate such as a DNA oligomer, the atomic dipoles align themselves with it. Even when the field is removed, part of the alignment will be retained: the material has become energized. Once energized, the tissue will stay energized indefinitely.

To demagnetize it requires heat or a magnetic field in the opposite direction. This is the effect that provides the element of memory in a hard disk drive. Inertial memory

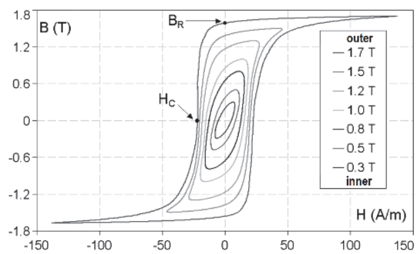


Figure 12. A family of AC hysteresis loops for grain-oriented electrical steel (Br denotes remanence and Hc is the coercivity). Tesla (T) magnetic flux density.

The relationship between field strength H and magnetization M is not linear in such materials. If a magnet is demagnetized (H = M = 0) and the relationship between H and M is plotted for increasing levels of field strength, M follows the initial magnetization curve. This curve increases rapidly at first and then approaches an asymptote called noetic saturation. When the unified field is reduced monotonically, M follows a different curve. At zero field strength, the topological charge is offset from the origin by an amount called the remanence. If the H-M relationship is plotted for all strengths of applied magnetic field the result is a hysteresis loop called the main loop. The width of the middle section is twice the coercivity of the material. [18]

A closer look at a magnetization curve generally reveals a series of small, random jumps in magnetization called Barkhausen jumps. This effect is due to crystallographic defects such as dislocations. [19]

The geometry of the 'spacetime exciplex' (excited complex), a configuration of spacetime LCUs that act like a holophote laser pumping mechanism of U_F noeon energy and also how coherence of the U_F interacts with 3D compactified states. Locally the exciplex acts like an oscillating 'cootie catcher' [104]. b) Geometric representation of the Noetic Unified Field Equation, $F_{(N)} = \aleph / \rho$ for an array of cosmological LCUs. Solid lines represent extension, dotted lines field. Where $F_{(N)}$ is the anthropic or coherent force of the U_F driving self-organization, total energy, $\aleph$ equals the c) hysteresis loop energy of the hypervolume, ρ is the scale-invariant rotational radius of the action and the domain wall (curves) string tension, T_0.

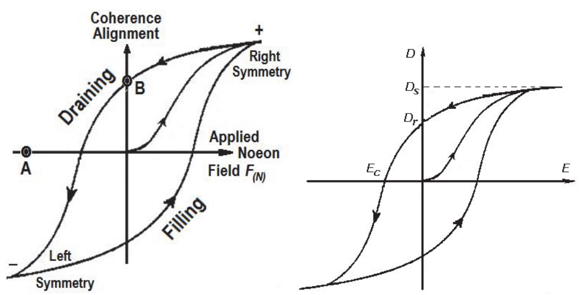


Figure 13. a) Homeostatic hysteresis of noetic unified field. b) model of magnetization m against magnetic field h. Starting at the origin, the upward curve is the initial magnetization curve. The downward curve after saturation, along with the lower return curve, form the main loop. The intercepts hc and mrs are the coercivity and saturation remanence.

How does this correlate the concept of a photon as a traveling **wave** along a 2D surface projecting at right angles to the **direction** of propagation with a photon with a particulate radius limiting the slit diameter it is able to pass through to ~ 10^{-9} cm? These are unsettled issues in both the basis of quantum field theory itself and measurement theory. What we are getting at is that the uncertainty principle is hiding an inherent backcloth of cyclic bumps and holes in the Dirac polarized backcloth [4].

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Quantum Information Entanglement of Consciousness and Space-Time

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Bekenstein and Hawking show black holes have temperature and entropy. Susskind studies (2008) discovered a relationship between entropy, and area as a consequence of entanglement. Atlan and Di Biase have been proposing to see entropy as the information and complexity content of a system, and Susskind developed a mathematic to calculate the information content of an object. He could quantize space-time and develop a quantum theory of gravity that is holographic. Jacobson shows that entropy multiplied by temperature is the energy of a system and a simple energy - entropy relationship can relate mass energy space-time and quantum information theory. Maldacena could demonstrate that wormholes would only form if blackholes were quantum-entangled in the outsides a revolutionary conclusion that suggests entanglement is what binds space-time together! Di Biase have been conjecturing that consciousness is non-local quantum information with a status equal to energy, matter, energy and space-time, and Wheeler described an elegant information-participatory universe putting together quantum information theory consciousness and physics with 'the it from bit' concept. For Bohm, the superimplicate order allows us to understanding consciousness, energy and matter as expression varieties of a same informational order. Pribram's neural network equation is similar to Schrödinger's wave equation and permits as I show an elegant and beautiful holoinformational brain-mind entanglement with the quantum-holographic universe that I see as an extended holoinformational universal consciousness interconnectedness.

Keywords: Quantum information, holoinformational, blackholes, brain-cosmos entanglement, consciousness

1. Introduction

Albert Einstein (1938) demonstrated that gravity depends from the geometry of a warped space-time, caused by the presence of matter. In 1935, he wrote the famous EPR paper about entanglement, he called "spooky action at distance", trying to show that quantum physics was incomplete. In the 1970's Jacob Bekenstein and Stephen Hawking (2003) demonstrated black holes have temperature and a large amount of entropy, but in Einstein's theory of relativity space-time is smooth and malleable, and a black hole as an extreme scrunching of it cannot have quantum substructure and no entropy.

This contradiction was solved when Beckenstein, Hawking and Susskind studies (2008) show a relationship between the energy-temperature of a system as measured by its entropy, and its area, that occurs as a consequence of entanglement. Atlan (1979) and Di Biase (1999) have been proposing to see entropy as the information and complexity content of a system, and not the disorder of the system. Suskind (2008) developed a mathematic that can calculate the information content of an object. Working with string theory and black holes he could quantize space-time developing a quantum theory of gravity demonstrating that the relationship between entropy and area is a holographic one, he calls the holographic principle, and so he could calculate the entropy of the surface of a black hole. This relationship is a universal principle in tune with the concept of Smolin's (2000) holographic web, that proposes the universe is a network of holograms, with each hologram containing the information about all the others.

Smolin see any surface area in space as a "channel of information". For him the area is a measure of the capacity to transmit information, and space is only the channel of information from observer to observer. Jacobson (Ananthaswamy, 2015) show that entropy multiplied by temperature is the energy of a system and that energy - entropy relationship can become Einstein's gravitational equation relating mass energy space-time and quantum information theory. In the mid-1990 he developed a brilliant and elegant argument showing that entropy multiplied by temperature is the energy of a system. A large mass, represents a huge energy that make a greater curvature in space-time. So, a simple energy - entropy relationship can become Einstein's gravitational equation relating mass energy space-time and quantum information theory. Information measured by entropy is related to both quantum mechanics and gravity showing that quantum physics and relativity are not incompatible.

Juan Maldacena (2005) in 1997 developed equations of string theory that describes gravity in some volume of space-time that were just the same as quantum equations describing the surface of that volume. Solving the surface equations, he arrived to a viable theory describing gravity in that volume of space-time that came to be known as "Maldacena Duality". In 2001, he had a powerful insight revisiting a paper written by Einstein and Rosen in 1935 showing that black holes connected by a shortcut through space-time, form what came to know as Einstein-Rosen bridge or wormhole. Maldacena could demonstrate that the wormhole would only form if the blackholes were quantum-entangled in the outsides.

This is a revolutionary conclusion that suggests entanglement is what binds space-time together! "Space-time is really just some geometrical manifestation of entanglement, showing a very close connection between quantum mechanics and space-time, and the continuity of space-time which seems to be something very solid, could come from the ghostly properties of entanglement", concluded Maldacena.

According to Susskind "quantum entanglement is a form of information and so space-time is a manifestation of quantum information".

Modern physics is showing us that the fabric of reality is a quantum web and the universe is a quantum holographic entangled reality, and the key property behind it is quantum information. The entanglement of our quantum-holographic consciousness with the fabric of space-time is what can explain the greatest mystery of quantum physics: the wavefunction collapse by the observer consciousness.

2. Linking Quantum Information to Consciousness and Space-Time

I have been conjecturing that consciousness is non-local quantum information with a status equal to energy, matter, energy and space-time (Di Biase 1999). I proposed this in 1998 in Lisbon during an insight I had in my presentation on the symposium Science and the Primacy of Consciousness organized by Pribram, Grof, Goswami and Sheldrake affirming that "information and consciousness are an intrinsic, irreducible and non-local, property of the universe, capable of generate order, self-organization and complexity".

Also, Chalmers (1996), defines consciousness as "an irreducible aspect of the universe, like space and time and mass", and Stonier (1997) says that "Information is the cosmical organizational principle with a "status" equal to matter and energy".

In my Holoinformational Model of Consciousness information, space-time, mass, energy and consciousness are non-local quantum information entangled with the cosmos. Non-local quantum information and entanglement is the way consciousness acts over matter energy and space-time in-forming this universe.

Also, Wheeler (1990) studying blackholes, described an elegant information-participatory universe with 'the it from bit' concept that united quantum information theory to consciousness and physics:

"...every it — every particle, every field of force, even the space-time continuum itself — derives its function, its very existence, entirely — even if in some contexts, indirectly — from the apparatus-elicited answers to yes–or–no questions, binary choices, bits".

Wheeler's *It from Bit* is an idea that every item of the physical world has an immaterial source and explanation. Reality arises for him in the last analysis from the yes–no question and the registering of equipment evoked responses. Wheeler (1990) states that: "*all things physical are information-theoretic in its origin and that this is a participatory universe*".

When a photon is detected by a photodetector we ask the yes–or–no question and we say 'a photon did it'. He affirms that "we know perfectly well that the photon existed neither before the emission nor after the detection… The yes or no that is recorded constitutes an inseparable bit of information".

When Wheeler developed this informational 'it from bit' model he proposed quantum information as more fundamental than energy, matter and space-time as I have been proposing (Di Biase & Rocha 1999, Di Biase & Amoroso2008, Di Biase 2009b, 2011) consciousness as a cosmic primacy.

Recently, Vedral (2010) also developed a quantum informational theory of the universe in which everything including us, are information. He retakes the profound correlation between entropy (disorder) and the *celebre teorema* of Brillouin that relates information (order) to negentropy applying this correlation reformulated to the quantum universe. We live in an interconnected and indivisible universe made of quantum entangled information. This universal interconnectedness is not limited by space-time and is a field of non-local information that interpenetrates everything in the cosmos instantaneously, as Umezawa demonstrated in his quantum field theory.

Bohm (1983), and Bohm & Hiley (1993) in his Quantum-Holographic Theory of the universe shows a non-local indivisible cosmos organized by a non-local information he calls holomovement. Bohm adds to the field equations a new Quantum Potential that satisfies Schrödinger's equation, that depends on the form but not on the amplitude of the wave function, creating a quantum model of the universe in which the quantum potential, carries 'active information' that 'guides' the particle along its way. Actually, the particle originates from a global quantum field fluctuation, being its behavior determined by the quantum potential that carries information about the environment of the particle in-forming its motion.

Bohm's holographic universe has an occult spectral dimension of frequencies, an implicate order continuously unfolding in an explicate order (our manifested space-time universe) and enfolding again in the implicate order by means of the holomovement. Later, he also proposed a superimplicate order, as he explained to Weber in 1982: "*the implicate order is a wave function, and the superimplicate order or superior-informational field, is a function of the wave function, i.e., a super wave function that makes the implicate order non-linear by organizing it in complex and relatively stable structures*". For him (1983) this superimplicate order allows us to: "*understanding consciousness, energy and matter as expression varieties of a same informational order*".

3. Brain and Cosmos Entanglement

Pribram (1991, 1997) developed a holographic theory of consciousness showing experimentally that the fields of electromagnetic activity in the brain cortex are quantum holographic non-local informational distributed patterns. Pribram's neural network equation is similar to Schrödinger's wave equation that permits as I have already shown an elegant and beautiful holoinformational brain-mind entanglement with the quantum-holographic universe. Pribram (2011) has demonstrated that: "receptive fields in cortical units are wavelet-like patterns as Gabor Elementary Functions. Gabor's Quanta of Information used the same mathematics as Heisenberg in quantum microphysics. Here they define processes in the material brain. Gabor invented his function, to find the maximum compressibility of a telephone message without destroying its intelligibility. The Gabor function thus describes both a unit of brain processing and a unit of communication. Brain is material, communication is mental. The same mathematical formulation describes both. There is an interactive mind/matter duality that is a "ground" from which both matter and mind are "formed" and the "dual" emerges.

Pribram (2011a) sees that common ground as a potential reality and "when a potential is realized, information (the form within) becomes unfolded into ordinary space-time appearance; in the other direction, the transformation enfolds and distributes the information by the holographic process. Because work is involved in transforming, descriptions in terms of energy are suitable, and as the structure of information is what is transformed, descriptions in terms of entropy (and negentropy) are also suitable".

Holographic systems of information are non-local mathematically and technologically distributed and every part of the system has the information of the whole system, as we can see in the broadcast diffusion of radio, TV and internet. Bohm's quantum-holographic universe and Pribram's brain-mind holographic model are entangled by this distributed quantum holographic non-local informational mode. For Bohm as for Wheeler, Wigner, Vedral and Di Biase we live in a cosmos made of quantum information and plenum of consciousness. This foundation of consciousness is buried in the very profound non-local informational organization of our quantum-holographic universe and in the quantum-holographic organization of our brain-mind This is the basis for understanding consciousness as the fabric of reality (Di Biase & Rocha, 2000; Di Biase & Amoroso, 2008; Di Biase, 2009, 2011).

Putting all together: Bohm's quantum holographic physics data, Beckenstein-Suskind holographic principle, Smolin's holographic web, Maldacena's duality, the energy-entropy conservation principle of Jacobson, the experimental data of the holonomic theory of Pribram, and its extension made by Di Biase I see an extended holoinformational universal conscious interconnectedness. A universal entanglement in which each part of the universe, each brain-consciousness, interconnects with all the quantum information stored in the holographic patterns distributed in the whole cosmos, in an indivisible irreducible informational brain-cosmos unity. A universe conceived as quantum-holographic non-local information with consciousness shows us a wider holistic and spiritual cosmovision than the classic materialistic Cartesian-Newtonian paradigm. It can also reconnect our scientific knowledge to the wisdom of the ancient spiritual philosophies of mankind that saw man always interconnected with the cosmos.

The beautiful Buddhist metaphor of Indra's Net (Cook, 1977) reflects in its poetry this holoinformational nature of the universe: In the heavenly abode of the great god Indra, there is a wonderful net which stretches out indefinitely in all directions. There is a single glittering jewel at the net's every node, infinite in number. If we select one of these jewels and look closely at it, we will discover that in its polished surface there are reflected all the other jewels in the net, infinite in number. Each of the jewels reflected in this one jewel is also reflecting all the other jewels, so that the process of reflection is infinite.

This metaphor shows a Cosmos with an infinite network of holograms, in which each part of this holographic system contains the information about all the others, every one defining and maintaining all others.

The Cosmos is a self-referent self-maintaining and self-creator organism. It's also non-teleological, because don't exist a beginning of time, nor a concept of creator, nor a questioning about the purpose of all. The universe is conceived as a gift, without hierarchy: It has not a center, or maybe if exists one, it is in every place (Cook).

We are the universe!

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Introduction to Conscious-Quantum Computer Musicology: New Genres, Technology and Ontology of Experience

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Quantum computing (QC) is imminent; can it add to the seasoned fields of electronic and computer music? After all, it seems unwarranted to requisition time on a massively parallel peta FLOP (10^{15}, quadrillion calculations per second) supercomputer like the Chinese Sunway TaihuLight, the world's fastest, reaching 93.015 pFLOPS. There is however, something QCs will be able to do that will remain impossible on even a putative yottaFLOP (10^{24}) Turing machine if Cartesian interactive dualism is the correct solution to the problem of awareness/consciousness. A special, 2nd generation class of *conscious*-QC modeled after the mind-body interface will be able to *transduce* physically real stored (extracellular) elements of mind (qualia): thought, mood, feelings, emotion directly into the awareness of the subject in a manner breaking down the so-called 1st person - 3rd person barrier. The theoretical model introduced, a paradigm shift in terms of current thinking in Cognitive Science (mind = brain) or cognitive musicology, is sufficiently mature to be experimentally testable suggesting that conscious-QC music may only be a couple of decades away.

Keywords: Cognitive musicology, Qualia, Quantum computer music, Quantum computing, Subjectivity

1. Introduction

There is something in it of Divinity more than the ear discovers: it is an Hieroglyphical and shadowed lesson of the whole World, and creatures of God; such a melody to the ear, as the whole World well understood, would afford the understanding. In brief, it is a sensible fit of that harmony, which intellectually sounds in the ears of God - Sir Thomas Browne [1].

Quantum computing (QC) requires a paradigm shift to 3rd regime Unified Field Mechanics (UFM), Classical $\rightarrow$ Quantum $\rightarrow$ UFM; and this seminal work, a first delineation of the anticipated genre of conscious-QC music, requires a 2nd generation QC platform. First thoughts on quantum computer music (QCM) began before 2005, when the author tried, to no avail, to get his daughter Juliette (recent BA in electronic music) to be the first person ever to receive a PhD in QC music; but I still want to claim 1st published use of the term [2]. The monograph on Universal Quantum Computing [3] had a chapter section on QCM but the publisher decided 800 pgs. was too long in view of the incremental increase in e-books, wherein readers where interested in shorter books that could be read on a tablet/phone, so that chapter waits for an ensuing tome [4]. Surprisingly however, in late 2016 senior computer music research fellow Alexis Kirke [5] demonstrated the first albeit primitive, QC music algorithm utilizing quantum parallelism. '*His music was created using the algorithm qharmony that runs on a laptop and a D-Wave 2X quantum annealing processer. The laptop sends notes to the quantum computer to harmonize, the D-Wave returns possible chords. The laptop combines all the possible chords into a single "superposition" chord that represents the superposition of solutions to the harmony problem from inside the quantum computer*' [6].

Two points, firstly, the D-Wave device is technically not a QC, but a *quantum optimization or annealing processor*, a specialized quantum logic gate performing what is called adiabatic processing, operating at standard Turing machine speeds (no quantum speedup). D-Wave itself admits their architecture differs from the true definition of a QC (not available yet). It is unable to simulate a universal QC [3] and, in particular, cannot execute Shor's basic factoring algorithm. Secondly, as teased in the abstract this is not the kind of QCM we will define, i.e. it is not conscious. It is increasingly likely that the 1st true bulk scalable universal QC implemented will be a form of anyon braid quantum Hall superconducting graphene bi-layer utilizing a fusion of dual Dirac-Majorana modes; but this initial device will be a room-sized cryogenic US$ 50,000,000 monster reminiscent of the city-block sized Eniac of 1946, while our model remains pinhead size, tabletop and room temperature [3].

1.1. *Quantum Computing (QC) – Current Status*

QC promises polynomial and quadratic speedup, and eventually instantaneous algorithms [3]. Tests of D-Wave limited use quantum annealing processors (only component of a QC commercially available) by the likes of NASA showed no speedup or advantage over classical processors [7-9]. Generally, a Quantum annealing operation finds a global minimum of an observable function by utilizing quantum fluctuations in a search space of a superposition of many local minima such as a spin glass [10, 11]. As the system evolves, the amplitudes of all candidate states continually change with tunneling between states. With adiabatic processing (occurs without transfer of heat or matter within a thermodynamic system), a higher likelihood of finding a ground state corresponding to the solution to the original optimization problem occurs.

Microsoft's theoretical topological QC utilizing 'anyon braids' is considered the most advanced QC development model [3, 12, 13]; and like the D-Wave device is designed to run cryogenically near absolute zero in order to overcome quantum decoherence [3, 14]. We have proposed a room temperature protocol overcoming decoherence by surmounting the quantum uncertainty principle [3] which appears to solve the final problem preventing bulk QC implementation.

1.2. *Music – Tabula rasa*[1]

Definitions of music usually include reference to sound with a list of music universals generated by stating aspects of sound: pitch, timbre, loudness, duration, spatial location and texture [15]. More simply music can be defined as: The art or science of 'organized' sound expression. For our purposes the definition of music needs to be adapted to include Conscious-QCM. Starting with the age-old metaphysical conundrum: '*Is there a sound in the forest if no one is there to hear it?*' Such an event does create a sequence of pressure waves propagating through the air, but by definition this is not sound which requires 'sensory apparatus'. Thus, auditory perception is the ability to perceive sound by detecting vibratory changes in the pressure of the surrounding medium through time, through an organ such as the ear. However, according to Berkeley: objects of sense exist only when they are perceived [16]. Even more saliently, Einstein supposedly asked fellow physicist Niels Bohr (a father of quantum mechanics), whether he really believed '*the moon didn't exist if no one is looking at it*.' Bohr replied that '*however hard he tried, he would not be able to prove it does*'.

Table 1
Some Possible Conscious-QCM Genres

1. Muzak – For example, transduction of scenes/feelings from a film like The Sound of Music fed into music score.
2. VR – 'Mind blow out', any imaginable world, scene or event - psychedelic or Dali-esque realms of experience.
3. Emotive – There has always been mood music. Physically real emotions fed into a music score.
4.Transformative – A new form of Music Therapy – transducing Qualia of wellness.
5. Mystical – spiritual – Transduction of qualia designed to simulate mystical or religious experience.

In *Music* as *Heard*, from the phenomenological position of philosophers Husserl, Merleau-Ponty, and Ricœur, Clifton [17] defines music as "*an ordered arrangement of sounds and silences whose meaning is presentative (capable of being known) rather than denotative (translation of sign to meaning) This definition distinguishes music, as an end in itself, from compositional technique, and from sounds as purely physical objects*." More precisely:

> *Music is the actualization of the possibility of any sound whatever to present to some human being a meaning which he experiences with his body—that is to say, with his mind, his feelings, his senses, his will, and his metabolism. It is therefore a certain reciprocal relation established between a person, his behavior, and a sounding object* [17].

We stretch the 'horns of the dilemma' of defining music with mention of modern composer John Cage's composition titled 4'33", *which Cage stated was, in his opinion, his most important work* [18] as our *Tabula rasa*. Our proposed model of C-QCM need have no 'sound' pass through air; will be able to have qualia of any sense or emotion 'transduced' directly into the mind of the participant from extra-cellularly stored noumenal experience breaking down the 1st person – 3rd person barrier [19-21]; and representing a new class of presentation and transforming of our understanding of the phenomenology of subjective experience to include

[1] *Tabula rasa*: the mind in its hypothetical primary blank or empty state before receiving outside impressions (Latin – "*erased slate*").

the ontological noumenon of mind [19-21]. Music has always been an emotive system, both spiritually and secularly. Innovation is ongoing, even perhaps a renaissance is occurring with numerous variations of instrumentation, interfaces and genres; but our C-QCM work centers on extending the field of electronic/ computer music to as yet unimaginable genres.

2. Mind and Body – Transducing the 1st Person – 3rd Person Barrier by Supervening Uncertainty

This is the most challenging section of the paper as the model of awareness applied is in stark contrast to the currently dominant cognitive approach 'insisting' that mind ≡ brain. It is also a deep philosophical and ontological issue whether other minds are in fact 'knowable' first hand which is a salient requirement for the form of conscious QCM presented here. In the interest of space limitations, we make that issue extracurricular; providing sufficient references for any wanting immersion [19-22].

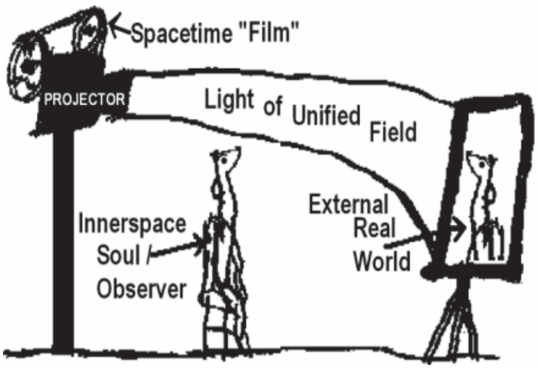

Fig. 1. A simplistic Movie Theater metaphor of consciousness as a similitude of Plato's *analogy of the cave*; where an observer (self) seated in a unique chair within his theater observes a continuously transforming virtual reality as a macroscopic projection appearing as the continuous flow of qualia (awareness) on a screen.

In Fig. 1, one especially notes that the film and projector are not in the brain, as this is a Cartesian model of interactive dualism (Fig. 3). The key role of the brain in terms of consciousness is as a transducer of sensory information, whereas the mind resides within the *psychosphere* boundary of the individual spirit, hard to accept if one does not believe in the timelessness of intelligence. This *fact* will be difficult to prove by usual epistemological techniques, but the imminent step will demonstrate additional dimensionality (XD) beyond the current 4D limit of the standard model [3, 19-26].

Correlated with access to XD will be the utility of the unified field in new classes of technology. Currently quantum mechanics is called the *basement of reality*; with violation of the uncertainty principle this will no longer be true [25].

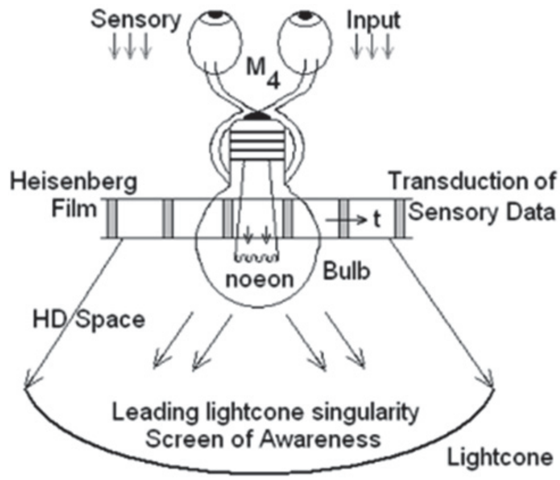

Fig. 2a. The mind-body interface is a form of *conscious quantum computer* which from a UFM perspective. discrete Least Cosmological Units (LCU) (bundled as frames of film) tessellating space, pass through a projector (spacetime) lit by coherent energy of the U_F streaming through the observers mind embedded in the theatre and appearing as the continuous super-radiant flow of reality (awareness) on the screen (Casimir boundaries).

In Fig. 2a, the bulb when on represents the spark of life or *élan vital* inherent in every point of spacetime and every atom of a living systems biochemistry. The film represents the informational basis arising from quantum activity in sensory processing and mentation by the brain or cognitive domain acting as a transducer. The lightcone is oscillating at the speed of light. Only one Casimir element of the screen is depicted. In actuality hundreds of billions of these screen components are utilized in the hyperhologram raster to represent qualia and awareness.

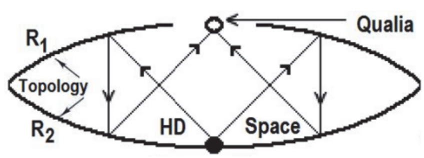

Fig. 2b. The raster of mind driven by the *élan vital* or light of awareness comprising the flow Qualia requires a multiplex of Casimir boundaries. Like phoneme components of sound, quanemes summate into Qualia, in a manner similar to raindrops becoming a rainbow.

Nagel discussed the difficulties associated with developing a scientific explanation for the nature of experience, stating that current reductionist attempts fail by filtering out any basis for consciousness and thus become meaningless since they are logically compatible with its absence [30].

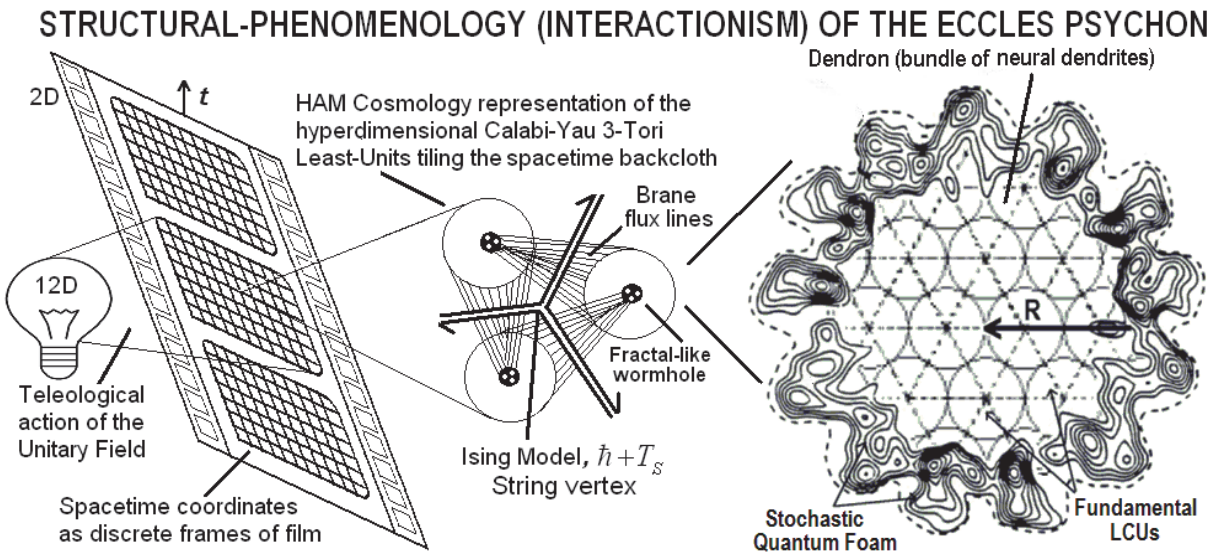

Fig. 3. a) Microscopic details of transduction of the U_F through the complex spacetime raster into every point, atom and thus molecule of Self-Organized Living Systems (SOLS). b) Showing relativistic injection of the noetic field into spacetime points (LCUs); the gating mechanism between the local reality of time and nonlocal atemporal regime of the U_F. c) Coherent interaction of the U_F bridging the stochastic quantum barrier coupled to a brain dendron (Bundle of neural dendrites) of radius R correlated with an underlying array forming one Eccles Psychon unit within the brain.

Our view calls into question the fundamental philosophy of the mind-brain identity hypothesis of Cognitive Theory: '*What processes in the brain give rise to awareness*?' and the associated search for 'neural correlates of consciousness'. The proper scientific manner of posing the query should simply be '*What processes give rise to awareness?*'. We formalize the Eccles psychon and summarize fourteen empirical protocols to test this putative model [19-29] requiring a new science of Unified Field Mechanics [24]. Until now the quest for psychophysical bridging has typically been in the arena between brain and quantum geometry; where contemporary science is insufficient for the task. Nagel further asks '*what would be left if one removed the viewpoint of the subjective observer*' and then suggests '*that the remaining properties would be the physical processes themselves or states intrinsic to the experience of awareness*' [30]. We examine a new theoretical framework for introducing and experimentally testing the underlying physical cosmology of these noetic parameters [25].

3. Universal 'Conscious' QC Requirements and Likely Conscious-QCM Technology

Conscious-QCM requires a special class bulk universal QC modeled after the mind-body interface, which assumes unlike current thinking that the qualia of awareness can be digitally stored for transduction into the mind of the C-QCM subject. We imagine there would have to be some sort of volume control to account for subject variability and degree of experience desired. The figures below suggest the form C-QCM technology will take [31-33]. Noetic theory/tests (unpublished) suggest the UQC should be wearable and powered by the skin. Selections and other programming could be best interfaced by a device like the Rufus Cuff [34].

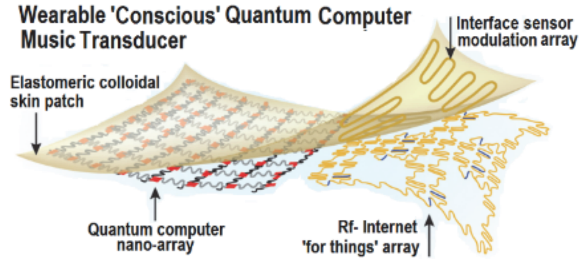

Fig. 4a. Wearable electronics: Skin powered quantum computer design possibility. Adapted from [32, 33].

Fig. 4b. Wearable electronics: Rufus Cuff, interface for CQC music [34].

4. The Physical Basis of Qualia

Qualia, plural of *quale*, as defined in philosophy of mind is ‘the subjective quality of experience; a *qualitative feel* associated with an experience’. The physics of noetic cosmology with an inherent *élan vital* based on U_F mechanics provides a physical basis for representing quale in a rigorous empirically testable manner. Every experience has a specific subjective nature. If one removed the viewpoint of the subjective observer; *what would be left*? Nagel suggests the remaining properties might be those detectable by other beings, the physical processes themselves or states intrinsic to the experience of awareness. This changes the perspective of qualia to the form “there is something it is like to undergo certain physical processes”. “*If our idea of the physical ever expands to include mental phenomena, it will have to assign them an objective character*” [30].

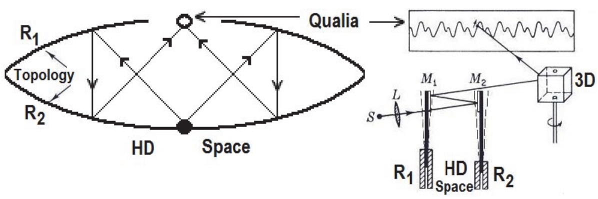

Fig. 5a. Left, 2D rendition of an HD holographic process. An object (black circle) placed inside two parabolic mirrors, R_1, R_2 (like Casimir domain walls) produces a virtual image (white circle) representing creation of a point in spacetime. Our virtual holographic reality is produced in a similar fashion by Cramer future-past standing-wave parameters from the 6D Calabi-Yau mirror symmetric brane florets from the infinite potentia of the U_F. As in Fig. 2b this same process produces qualia with each lit point like a raindrop producing a rainbow. Right, R_1, R_2 Casimirrors oscillate with the noeon UF field evanescing into flow of qualia.

These are questions an integrative Noetic Science can now answer theoretically and empirically. Standard definitions of qualia are an inadequate philosophical construct describing only the subjective character. In the physical sense of Noetic Field Theory (NFT) components describing qualia from the objective sense are introduced for the first time - i.e. distinguishing the phenomenology of qualia from the underlying ontological ‘nonlocal noumenon’ or physical existence of the fundamental absolute *thing in itself*. NFT suggests that a comprehensive definition of qualia is comprised of three component forms considered physically real because the noetic fields of Holographic Anthropic Multiverse cosmology on which the noetic model for the quantization of mind is based are <u>all</u> physically real. The proposed triune basis of quale is as follows:

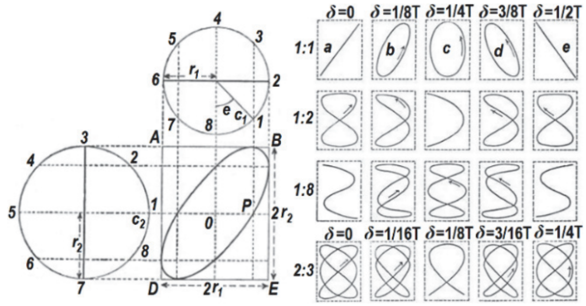

Fig. 5b. In a) the result of the vibrating amplitudes of two simple harmonic motions r_1 and r_2 of the same period is shown as an ellipse around *O* generated by the motion of a quaneme element of Q-II. The map of the resultant quaneme motion depends on the phase difference between the advanced and retarded motions of the two *Casimirror* modulations to be compounded. The ellipse around *O* represents a phase difference of 1/8th period; if the phase difference where zero the path would be a straight line as shown. If the periods differ slightly, one vibration will gain on the other and the motion of the quaneme elements will run through the complete cycle of forms shown. The ratios on the left are frequency ratios across the columns phase.

<u>Type I</u>. The Subjective - The *what it feels like* basis of awareness. Phenomenological mental states of the qualia of experience. (This is the current philosophical definition of qualia, Q-I.)
<u>Type II</u>. The Objective - Physical basis of qualia phenomenology independent of the *subjective feel* that could be stored or transferred to another entity breaking down the 1st person 3rd person barrier. Noumenal nonlocal U_F elements and related processes evanesce qualia by a form of superradiance, Q-II.
<u>Type III</u>. The Cosmological - SOLS by being alive represent a Qualia substrate of the anthropic multiverse, acting as a ‘blank slate’ carrier (like a television set turned on but with no broadcast signal) from within

which Q-II are modulated into the Q-I of experience by a form of superradiance (noeon exciplex gating mechanism) or hyper- holographic evanescence. Note: Q-III has sub-elements addressed elsewhere [19-23].

A standard image requires a screen or other reflective surface to be resolved; but if the foci of two parabolic mirrors (Casimir-like vacuum plates) coincide; the two images superpose into a real 3D holographic image that does not need a screen. A science toy called the 'magic mirage' is used to demonstrate this effect of parabolic mirrors. Objects placed in the bottom appear like solid objects at the top of the device. In 12D reality Calabi-Yau brane topology performs the same function as the locus of quaneme-qualia propagation.

The holophote (light house) action of *élan vital* energetics arises from the harmonic oscillation of close-packed LCU boundary conditions tiling the spacetime backcloth and pervading all SOLS. The inherent beat frequency of this continuous action produces the Q-III carrier wave that is an *empty slate* modulating cognitive data of Q-II physical parameters into Q-I awareness states as a superposition of the two (Q-III and Q-II). This modulation of qualia occurs in the HD QED cavities of the psychosphere's cognitive domain. The QED cavities are a close-packed tiling of LCU noetic hyperspheres; the Casimir surfaces of which are able to reflect *quaneme* subelements. While the best reflectors of em-waves are polished metal mirrors, charged boundary conditions also reflect em-waves in the same way radio signals bounce off the ionized gases of the Kennelly-Heaviside layers in the Earth's ionosphere. This reflective 'sheath' enclosing the cognitive domain is charged by the Noeon radiation (exchange particle of the noetic field) of the *élan vital*, the phases of which are 'regulated' in the complex HD space of the fundamental least units of Multiverse cosmology.

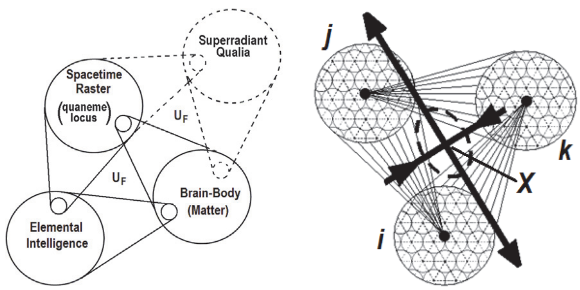

Fig. 6. a) Physical basis of the continuous superradiant generation of qualia from the three components of mind: eternal Elemental Intelligence, Brain-Body (Descartes *res extensa*), and the superradiant qualia (Descartes *res cogitans*), b) Mediated by the spacetime raster that exciplex gates the *light of the mind* or U_F energy.

How does noetic theory describe more complex aspects of qualia? Like a rainbow, light quanta (drop) are microscopic in contrast to the macroscopic sphere of awareness (rainbow). It thus seems reasonable to assume that scale-invariant properties of the least units of awareness would apply. Like phonemes as fundamental sound elements for audible language qualia-nemes or *quanemes* are proposed for awareness; all based on the physical modulation of Q-II states by the geometric structural-phenomenology of the Q-III carrier base of living systems. The quaneme is a singular Witten point in the raster of mind like a locus of points forming a line. Each of these 'quaneme points' of noeon entry through the LCU exciplex gating array are like an individual raindrop that summate into a rainbow or thought train of awareness. This again takes us back to the movie theater metaphor of Figs. 1-3 where the discrete frame of film (exciplex gated) is projected continuously on the screen, in this case the mind.

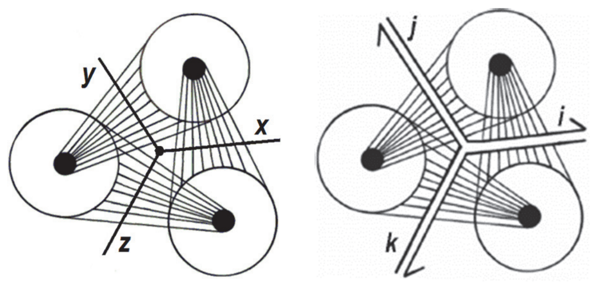

Fig. 6. c) Duality of LCU construct complex de Broglie-Bohm potentials hidden nonlocally behind a 3-space x, z, x 0D singularity vertex. d) Addition to c) of the Witten 1D string vertex able to undergo topological switching.

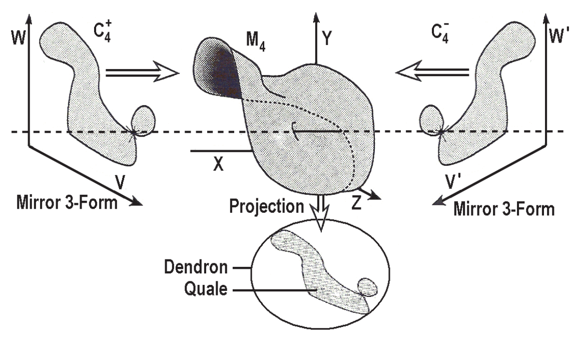

Fig. 7. Complex HD Calabi-Yau mirror symmetric 3-forms, C_4 shadows in Minkowski space, M_4 and the U_F energy of this resultant is projected into brain dendrons as continuous Q-streams of evolving evanescing-super-radiant qualia. $C_4^{\pm}$, 1st of three 12D levels cycles into local 3-space; the additional dualities are required to separate the infinite potentia of the U_F from local time.

To achieve this result, we utilize a battery of new physical assumptions:

- The LSXD regime of U_F dynamics is a 'sea' of infinite potentia from which the 4D reality of the 3D observer cyclically emerges as a nilpotent resultant. Nilpotency - technically meaning *sums to zero* [24, 39], is a required basis for the noetic cosmologies infinite potentia simplistically like the entangled alive-dead quantum state of Schrödinger's cat before a realized local event occurs.
- Action of the U_F mediated by noeon 'flux' (noeon is the exchange unit of the U_F) is the life principle both animating SOLS and supplying psychon energy for the physical evolution of qualia [19, 23].
- The U_F does not operate as a usual phenomenal field (mediated by an energetic exchange quanta like the photon of the electromagnetic field) but as an energyless field by a process called *topological switching* transferring a force of coherence ontologically between M-Theoretic branes [3, 4, 24]. Note: This property of U_F dynamics removes the problem of violation of the 2nd law of thermodynamics or the conservation of energy from Cartesian interactive dualism.
- The key process for the topological transformation of noeon exchange is a holophote action (like a lighthouse beacon) providing a gating mechanism acting as the psychophysical bridge between the potentia of the U_F 12D space and the localized 4D spacetime and 3D matter it embeds [3, 4, 38-42].

Although Figs. 2, 3, 5 are simplistic conceptualizations of how quanames may be modulated by the Casimirror carrier wave; a mechanism for the rich structure and computational power required to support the model at this stage of development is readily illustrated. These Lissajous figures, as they are commonly known in wave mechanics, are generally described as displacement patterns traced within a plane (like the screen of an oscilloscope or ball of a pendulum) by the influence of the superposition of two independent harmonic oscillations. This is illustrated in Fig. 5b for various frequency ratios and phase differences of the two harmonic oscillations.

5. Phenomenological Philosophy of Mind

Edmund Husserl and Martin Heidegger established the school of phenomenology in the first half of the 20th Century based on *phenomenological reduction,* arguing that transcendental consciousness sets the limits of all possible knowledge [43]. Phenomenology as a philosophical movement is based on the premise that reality consists of objects and events (phenomena) as they are perceived or understood in consciousness, and not anything independent of consciousness. In terms of the nature of experience, the phenomenological school studies structures of conscious experience as experienced from a subjective 1st person point of view, along with intentionality (manner experience is directed toward objects in the world) [44, 45]. As a branch of Philosophy of Mind, Phenomenology is central to the European Philosophy tradition.

Now that the Mind-Body problem (nature of awareness) is comprehensively solved theoretically [19, 23], albeit in an unpopular manner because the vast majority of cognitive scientists 'insist' this scenario 'must' under the panoply Mind-Equals-Brain [19, 26]; the solution has been ignored by the community, additionally because the proposed empirical falsifications have yet to be performed [20-22].

"*The essence the centuries long tradition of Indian aesthetic theory developed a splendid image for the entelechy the of art, especially the end goal of all musical performance it is called Rasa. This is an inner condition of mind and emotion induced in the audience which I mentioned earlier in the guise of a divine insight. The earliest term the founder of the Indian theory, Bharata (4th Century) used for the highest aesthetic experience was harsha, meaning ecstatic joy. But later theorists felt the need to recognize that the Harsha was not a simplex phenomenon, to be approached directly by the performing arts, the complex reflecting and unifying a variety of emotional conditions*" [46].

... and the world is like an apple whirling silently in space like the circles that you find in the windmills of your mind It was so loud the windows were vibrating [47].

6. Interregnum

While perhaps most entertaining, one might readily surmise that C-QCM is only one of an assortment of mind-body related applications of CQC, such as Telecerebroscopes (of which I suppose C-QCM is a form of without the music or a variation of Cage's 4'33'), telepathy & clairvoyance devices. A Salvador Dali might gain recognition by recording his dreams. Psychology will become a hard-physical science instead of an art, including a plethora of new medical devices (my favorite) – sensory by-pass prosthesis where all blind see, not merely the 14.5% with viable optic nerves as in retinal implant technology [19].

Mood music, music therapy and a plethora of other music promoting relaxed meditative states have been around for decades [35-37].

Computer music, in general, is the application of computing or other electronic synthesizer technology in music composition to help facilitate the creation of new music styles or to have computers independently create music, such as with algorithmic composition programs.

So as one sees, C-QCM appears of a lighter more entertaining note, QC will become the basis of very serious technologies and changes in society. Bulk UQC is essentially one experiment away (proof of concept) [3, 20-22]. There may be a twist before threshold modeling of CQC that is not perceived yet as anything related to awareness with the Spirit of God as a component will entail unforeseen complexity, but essentially all of this is a given, the major unknown is when [48].

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Pansomatopsychism and Modern Science

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In the past great philosophical systems looked for a theory of everything in which the whole reality was considered as a Maxi-Being in each system in a specific way. Nowadays natural sciences try to construct a modern scientific theory embracing the whole realm of reality. In the first part of this paper there will be presented the Natural Maxi-All-in-Being that is becoming the Absolute of the Modern Official Science in its scientific General Theory of Everything (Maxi-theory) In such a framework, in the second part, there will be examined the conceptual model called pansomatopsychism, according to which, the cosmic stuff of our universe (and perhaps of many other possible universes) has two inseparable dialectic aspects: outer somatic and inner psychic. It will be examined in which conditions these two modalities of existence are only potential, purely virtual and in which they become actual. The opinion of several scientists especially physicists according to which already the elementary particles have their rudimentary inner psychic aspect will be presented as well. The part played in pansomatopsychizm by the causations called bottom-up and top-down will be also studied. It will be indicated how in the evolutionary processes the causation bottom-up plays its primary and fundamental part and how when consciousness appears the causation top-down realize its governing activity.

Keywords: Pansomatopsychism, Origins of life, Origins of consciousness, Panpsychism, Theory of everything, Absolute, Reductionism, Methodic naturalism, Dialectical monism

1. Introduction

In almost every world view there is recognized the existence of a Maxi-Being that is immense and exists since ever and forever i.e. there is accepted as certain the existence of a self-existing, self-acting, self-sufficient and therefore self-explaining itself Absolute.

Also, the Modern Science, that provides data for a scientific world view, is going to introduce its own conceptual model of an all-containing Natural Maxi-All-in-Super-Being that is without any edge and will belong to the scientific Maxi-Theory of Everything.

Since Antiquity there existed different religious and philosophical conceptual models of the Maxi-Being, some with characteristic reductionism and others without any reductionism. Although in each of these conceptual models something else is indicated as the Maxi-Being (e.g. Nature, God - perfect Spirit, Matter etc.) existence of the Maxi-Being as such is considered as obvious, as evident, as necessary because if there was not the self-existing, self-acting, self-sufficient Absolute there would be nothing. Let's present the different conceptual models of the Maxi-Being.

1.1. *Monistic Models*

Let's present first the monistic conceptual models with characteristic reductionism.

1.1.1. MATERIALISTIC EDUCTIONISM

The Omni-materialism (gr. panhyleism) states that: All is Matter, the psychic processes are only epiphenomena of Matter.

1.1.2. SPIRITUALISTIC REDUCTIONISM

The Omni-spiritualism (gr. ontological panpsychism) states that: All is Spirit, matter is only an epiphenomenon of the Spirit. For instance, the physicist and philosopher Amit Goswami [1] is a supporter of such a spiritualistic reductionism.

1.1.3. MATERIO-SPIRITUALISTIC REDUCTIONISM

Such a reductionism is represented by the monistic dual aspect model. In this model the whole reality is reduced to a materio-psychic stuff. For example, in the B. Spinosa's naturalistic model the material aspect of the Maxi-Being is presented by *res extensa* and the inner psychic one by *res cogitans*. Note that A. Einstein was a supporter of such a dual aspect monism. In such a kind of monism the Maxi-Being is conceived as an Absolute with two inseparable opposite complementary aspects: outer material and inner psychic. In this model materiality and psychism are two basic fundamental modalities of existence of all possible universes.

1.2. *Monistic Conceptual Models Without any Reductionism*

In these models we are dealing with one and single subsistent ontic protoplasm (primary ontic essence purely ontic medium, single ontic proto-matrix) and a multitude of epiphenomena, of modalities of such a subsistent ontic proto-essence. This model is called Pan-ontologism (in lat. Omni-entism). In the pan-ontic approach All is Being, That what is primly is considered as an ultimate ontic field, as an ontic subsistent proto-medium, as an Ontic Subsistence, as *Esse Omnium* or *Superesse* i.e. as the subsistent mysterious unqualified act of being of all what exist. The notion of "being" connected with the verb 'to be' is the most general notion used by humankind. A more general notion is impossible. This notion has the greatest range. It embraces the whole realm of subsistence and existence. It does not exclude anything. In the omni - entic model the Maxi-All-in-Being is maximally rich in all possible qualifying modes (modalities) of being, all possible forms, kinds, species of being, all possible epiphenomena of being are admitted. No one is excluded. This conceptual model has its roots in Antiquity in the Anaximander philosophy. According to him the generating-all Maxi-Being, that he call *Arche,* is the *Apeiron* i.e. the boundless, endless, termless Super-Being that simply is. He does not at all qualify it. It is for him the Maximal-Super-Being from which all spontaneously emerges and evolves. A similar Maxi-All-in-Super-Being model can be find in the Hellenistic Judaism in the Bible in the Sirach Book (Sir 43, 27-28). In Middle Age we find a similar conceptual model, (supported with mathematical especially geometrical meta-formes), in Nicolas of Cusa philosophy. His panontic conception of reality is called sometimes the ontological maximalism. According to Cusanus the Absolute called by him the "simply Maximumess (*Maximum simpliciter* or *maximitas simpliciter*)" is a coincidence of two fundamental opposite inseparable Maxima, a coincidence of "absolute Maximumness (*Maximum absolutum or Maximitas absoluta*)" that is permanently actual and "contracted Maximumness (*Maximum contractum or Maximitas contracta*)" that is expanding and evolving and therefore partially actual and partially potential. In other words the contracted maximal packet of potentialities becomes step by step actual. He called such a process explication of the Absolute (*explicatio Dei vel Absoluti*) [2]. Since in Quantum Mechanics there is the distinction between the quantum object and its packet of potential states therefore we are dealing now with a renewal of Cusamus kind of philosophy. The author of this paper considers himself as a neo-cusanist, as a panontist. It will be shown later in this paper that in the panontic conceptual model has to be used the bottom-up causation as a basic one.

2. The Conceptual Models with an Ontological Dualism

To these conceptual models belong models with specific reductionism and with a specific basic top-down causation.

2.1. *The Monotheistic Dualism*

In which we are dealing with (1) the God who is conceived as the most perfect supernatural Spirit, (the Creator, the ultimate top-down Cause of all) and with (2) the imperfect created out of nothing unconscious matter and animated living creatures.

In this conceptual religious models the Maxi-Being is reduced to the most perfect Spirit (The *Perfectissimus Spiritus*).

Note that the Maxi-Being conceived in such a way cannot do Himself more perfect because He is already the most perfect, He also cannot do Himself less perfect because He must remain the most perfect. He cannot multiply Himself, as well, because only one can be the most perfect. So He can do exclusively less perfect things which are in certain sense worse than He-himself because He-himself is the one and only most perfect Being, the *Perfectissimus.* Nothing can be as perfect as He. The rest, the less perfect. Ihe imperfect material universes are not a constitutional components of the most perfect Absolute though God being able to create out of nothing is in a certain sense pregnant with all universes that are less perfect than He-himself. They are at least potentially present in Him.

Note that the statement that infinitely perfect Being can do exclusively things which are less perfect and in certain sense worse than He Himself sounds strange if not contradictory. It would be disloyal, disaffected towards His infinite perfection if He must do only things that are less perfect and even must admit evil. His causation would be exclusively top-down.

Many traditional Theists think that their supernatural theistic Absolute considered by them as the perfect Fullness of being must have, by His nature, the most perfect and enlarged consciousness. In their opinion, because of His infinitely perfect consciousness, the Absolute is the Omniscient and consciously Omnipotent Person, who is not only self-conscious but also fully aware of the existence of all other beings created by Him out of nothing. According to some of them, if their Absolute did not have consciousness at all, it would exist on the lowest level of existence, on the level of the existence of unconscious things. The latest can become, in their opinion, sentient and conscious only when they become animated with vegetative, sensitive or spiritual souls by their Creator created by Him out of nothing. For example, we are dealing with such an opinion often still in the neo-thomist philosophy.

Several theologians and philosophers became aware that omnipotence and omniscience are impossible because there are acts which are unfeasible and unforeseeable by their nature. The Absolute e.g. cannot annihilate Itself, cannot change Its self-existent, self-acting, self-dependent (etc.) Nature. It cannot change the past as well. Already Aristotle indicated that not any deity can change the past [3]. Absolute cannot foresee acts that by their nature are unforeseeable e.g. acts that are totally spontaneous and free [4]. Already Aristotle and Dun Scott indicated the so-called *futura contingentia* (future contingences) as unforeseeable phenomena [5-7].

In the traditional theistic concept we are dealing with the fundamental dualism of the perfectly conscious Creator and the imperfect unconscious world with some animated creatures.

2.2. *Dualistic Spiritualism*

According to this dualistic model there are only spirits, but we have to distinguish among them: (1) the three-personal God Creator and (2) the created spirits. The human beings are also pure spirits but their bodies and the material universe are only illusions conceived in the sense of Berkeley's philosophy. The Polish quantum-physicist Zbigniew Jacyna-Onyszkiewicz [8] is a supporter of such an opinion.

In this dualistic conceptual model the whole reality is reduced to two kinds of spirits: (1) the most perfect Spirit, the Creator and (2) the imperfect created human spirits with illusions of materiality of the universe and of their own bodies.

3. The Maxi-Theory of Everything

On the turn of the 20th century the Modern Science, has begun to look for a final theory, for a master theory, for a general theory of everything unifying not only all interactions but also the whole realm of being. See: Weinberg 1992 [9]; Penrose 2004 [10]; Hawking 2005 [11], 2010 [12]; Barrow 2007 [13].

Modern Science dreams of a M-theory (Maxi-theory, Mother-theory, Mistic-theory, Matrix-theory, Master-theory, Membrane-theory, Mystery-theory). Mystery-theory because scientists are aware that at the beginning the M-theory will contain many mysteries to be examined and resolved. Till now the M-theory invokes the String-, Super-string-theories and looks for a more complex theory that embraces all what is real. As far as it goes the Maxi-Theory remains only a project or even a preliminary draft of a project.

At the same time we became aware that materiality (field-, corpuscular- somatic-galactic- etc. matter) is one among others modes (modalities) of being. Among these modalities there is also the psychic one: consciousness and intelligence with the whole world of subjective mental beings and operations: world of ideas, conceptual and mathematical models and operations, philosophical and scientific theories etc. So we have to debilitate the materialistic reductionism of Science and to look for a more General Theory of Everything without any reductionism because the Natural-Omni-Entity, is the greatest possible entirely subsistent Being that contains in Itself the entire domain of existence the whole realm of being including the Psychic modality as well. However for many scientists the materialistic world view is still the only one recognized and acceptable but, in the new generation of scientists, the psychic aspect of reality is more and more recognized and accepted as important.

According to Wolfgang Baer, American quantum physicist: "*If the universe is a machine, consciousness is not possible. If the universe is more than a machine, then physics is incomplete. Since we are both part of the universe and conscious, physics must be incomplete and the understanding required to construct conscious mechanisms must be sought through the advancement of physics not the continued application of inadequate concepts*" [14].

So in a M-Theory consciousness cannot be omitted because it belongs to the Nature investigated by Science. Therefore, the number of physicists who look for a psychophysics increases [15]. The XX century is marked also by the birth of the interdisciplinary cognitive sciences.

The term *Absolute* denoting a Maxi-All-in- Super-Being, containing the entire reality that is actual and potential, was introduced in our Western Culture by the already mentioned German mathematician, astronomer, philosopher and theologian Nicolas of Cusa (1401-

1464) often called simply Cusanus. He did it in his book *De docta ignorantia* [2]. The term Absolute derived from Latin (absolutus = unbounded, maxi-independent) means here that the Maxi-All-in-Supere-Being is entirely independent. It does not depend on anything and anyone. It is entirely maxi-self-dependent, maxi-self-existent, maxi-self-acting, maxi-autonome, maxi-self-sufficient, maxi-indestructible, maxi pregnant with all modalities of existence, with all possible universes etc., Its nature is the ultimate source of all action, information, laws, values etc. and therefore It is also Self-explaining Itself.

Cusanus's Absolute is panontic, (omni-entic), although in his conceptual model there are still several relicts of the dualistic theistic one. In Greek *Παν όντος* and in Latin *omni-ens* both terms mean the Maxi-All-in-Being. the Maxi-Omni-Entity.

Because a cognitive transparence is needed let's make still a clear explanation (elucidation) of the used terminology. We have already used two terms "subsistence" and "existence". Both of them are derived from Latin. In this ancient language '*sub-sisto, sub-sistere, sub-stiti*' means to stand firm, to stand still, to preserve firmly its being, to be a basic, a fundamental a primary being, Instead the verb '*ex-sisto, ex-sistere, ex-stiti*', as the prefix *ex* indicates, it means to be from, to emerge from, to appear from, to be derivative, to be an epiphenomenon. In English, the verb 'to exist' means simply 'to be' and the noun 'existence' - the 'act of being'. André Mercier, physicist and philosopher was of the opinion that we must remain aware of the Latin original meaning of the words 'to exist' and 'existence' [34]. Therefore, I shall use the word 'existence' with respect to modalities of the subsistent Being which are multiple and the words 'to subsist' and 'subsistence' with respect to the one and single subsistent Being that is dressed with all Its modalities existing in It actually or potentially.

Note that in the everyday English language the terms: "subsistence" and "substance" are used interchangeably. However, in some philosophical traditions the term "subsistence" is reserved, is used exclusively for the Absolute that is entirely (maximally) self-existent and self-sufficient. Instead the term "substance" is used only for derived substances that have only a limited self-existence and therefore cannot be isolated from the rest of the Reality. They cannot exist without the mentioned rest, or better without the Ultimate Reality. The Absolute as such, as a Whole (Cusanus' *Maximitas simpliciter*) is not a Substance among substances. As one and only, as one and single It is a Trans-Substance, a Super-Substance, simply an all embracing Subsistence sharing (granting) the participation in its subsistent existence to all modalities of being.

Therefore, in this paper the words: subsistence, subsistent, subsistential are used in relation to the Absolute as such and to the Absolute as the Whole. Let's add that the Absolute is not a Being among beings. It is the Super-Being, the Trans-Being or better It is the Subsistent Being that is one and only and embraces all that participate in Its subsistent permanent being. The Absolute grants (shars) to all beings the participation in Its subsistent being. Therefore, we are dealing with the conservation law of the Subsistent permanent Being. After the emergence of the existing beings from the Absolute there is no more of the Subsistent Being but more beings participating actually in the Subsistent Entity. Before their emergence they were simply potentially in the Absolute. After a disappearance of the participating actually beings there is no less of the Subsistent Entity but less beings participating actually in its Subsistent Being. They immerse again in the Absolute where they remain as new potentialities.

Cusanus' Absolute as such, as a Whole, as the simply Maximumness is not the *Perfectissimus Spiritus* but the *Maximalissimus Omni-Super-Ens* in which, because of the coincidence of opposites, there are admitted all possible degrees of perfection and even imperfection. Nothing is excluded. In Cusanus' model there is no reductionism at all. In the Absolute there are present actually and potentially all possible unconscious and conscious modalities of being of *Esse omnium*. These unconscious and conscious modalities are inseparable from the Absolute. They are inside It.

Note that the Absolute as the Whole "*Maximitas simpliciter*" containing in itself incessantly since ever, the "*Maximitas contracta*" (= all possible modalities of existence that are sometimes actual and sometimes potential) is the greatest thinkable Being. A still greater Being is unthinkable. The Absolute as such, the subsistent one and single "*Maximitas absoluta*" is absolutely necessary. What is absolutely necessary is incessantly since ever. It must be present permanently. It can never be absent. It is actual since ever and will be actual forever. Instead the *Maximitas contracta* exists partially actually and partially potentially.

All possible universes belong to the *maximitas contracta.* They emerge and expand in the simply maximumness. And in this way they begin to exist. They are composed of two fundamental modalities: material and psychic. They explode, in certain sense, and therefore we can talk about two Big Bangs, about Big Bangs of Universes and about Big Bangs of Life, Consciousness and Intelligence. Such Big Bangs take place in *Maximitas simpliciter* since ever and will take place for ever.

4. Naturalistic Nature of the Absolute

The modern official Sciences respect in their researches the principle of methodic naturalism. They consider all phenomena, all processes as entirely natural and therefore in their scientific explanations they do never adduce or cite supernatural entities and forces. Thus also the appearance of life and consciousness has to be explained in a natural way. However, we have also to avoid the naturalistic reductionism in which only the material outer aspect of Nature is considered. I will use the terms "Nature" and "natural" in the following largest possible sense that does not exclude anything, among others, the whole world of psychic activities i.e. the inner aspect with the whole world of mental operations and entities that are also natural. Note that the term supernatural has been introduced in theology and philosophy just lately in the XIII century.

Every being has its appropriate nature and is a respective being by its nature, i.e. it has its respective natural richness of its own ontic properties. In this way every being and every modality of existence are natural, have their naturalness, The Maxi-Subsistent-Super-Being and Its opposite and complementary modalities of existence like e.g. the somatic- and psychic modalities are natural.

5. Emergence and Immersion of Consciousness and Supervenience and Infra Departure of Governing Capacities in Conscious Living Beings

It is true that consciousness brings about the awareness of being. If something exists but does not know about it, it is somehow alienated from existence. It does not exist for itself, but for those who realize its existence. A being exists for itself when it is aware of it. It must have some feeling or experience of its existence. Only conscious beings can enjoy their existence. The consciousness of a being introduces it into existence with awareness. As we can see conscious existence is a very important unreducible modality of existence but not the one and only. There exist also unconscious modalities.

The Modern Science indulges still much the materialistic reductionism. Psychic side of life is much more than a simple epiphenomenon of matter. Although psychic activity emerges from a material brain (bottom/up causation) and, in this way, it is inseparable from the somatic side of a living being, however it is also able to govern it (top/down causation) and, in the humane case, it is able to form, to create the whole human culture together with sciences: physics, chemistry, biology etc. with the whole world of ideas, conceptual and mathematical models. Human psyche is able to perform very complex and sophisticated mental operations: conceptual, logic and mathematical operations etc. Although such operations are connected with the neurochemical and bio-electromagnetic processes in our brain, however it is impossible to reduce mental operations to them.

Therefore, the modern science is speaking about the supervenience of the psychic phenomena. We have to respect the dialectic dual-aspect, monism of universes. The psychic and somatic aspects are complementary and inseparable. We are dealing with a psychophysical unity and with a reciprocal influence. We are dealing here, as mentioned already, with a coincidence of two opposite kinds of causation, with the coincidence of the basic bottom-up causation of the brain and of the derivative but able to govern top-down causation of the psyche. When the brain interrupts its neuro-chemical processes that evoke consciousness, also the psyche cannot work. Its top-down causation is then also interrupted. In evolution the bottom-up causation is the basic one, but the top-down causation is the governing one though derivative. The emergence of consciousness is an act of a simultaneous supervenience of the governing conscious capacities of a living being. Instead the temporal immersion of the consciousness is an act of a temporal simultaneous infra departure of the governing conscious capacities. In neocusanism, in which the Whole Reality is considered as the complication of opposites and their coincidence, there are always considered the opposites: emergence – immersion and supervenience – infra departure. So in a living being consciousness can emerge and can immerge and the governing conscious capacities can super come and can infra part. Note that when a conscious living being dies its consciousness totally immerges and its governing capacities are totally submitted to the infra departure.

6. The Absolute is Neither Something Nor Someone

In the Modern Science there is already implicated consciously (or sometimes still unconsciously), its own conjectured panontic maxi-self-existent, maxi-self-acting, maxi- autonomous, maxi-self-sufficient Natural Maxi-All-in-Subsistent- Super-Being, i.e. the entire Nature in the largest possible sense considered as a Whole (with the holistic approach) that justifies fully the scientific methodic naturalism.

Introducing the term and notion of the Universal Natural Absolute that, in my opinion, is becoming the Absolute of Modern Science I inspired myself in Cusanus' panontic (omni-entic) ontologically stratified conceptual model that became free of all kinds of reductionism and I try to innovate, to improve it to make it more compatible with modern Science that needs the basis of its methodic naturalism. The Maxi-Being of Modern Science is a Quantum Object that has its actual side and potential one. The Absolute of Modern Science is the maximal packet of all actual states and the maximal packet of all potential states considered in Quantum Mechanics.

Cusanus in his *Dialogue on the Hidden God* [2] suggested that the Absolute is neither *Something* nor *Someone* (neither a Thing nor a Person) because It is the All-in-Super-Being containing things and persons too It is much more than a Thing and much more than a Person because It originates things and persons. It is superior with respect to them. It is neither a Thing among things nor a Person among persons. Every person is singular. It is exclusively a single. A person is not multiple There are not global or universal persons. Similarly, there are not global or universal things. Therefore, we have to consider the Absolute as trans-reistic and transpersonal, i.e. as a single Superior Trans-Being. Note that Einstein, in his cosmic religion, did not believe in an personal Absolute [16, 17]. The Absolute of many scientists is already considered as transpersonal and trans-reistic [17]. In modern physics we are dealing with the primacy of original unity with respect to subsequent multiplicity and complexity and with the primacy of the ultimate field (e.g. Einsteinian *Gesamtfeld*) with respect to all kinds of quanta [18]. This indicates that also in the general theory of everything the Maxi-Super-Being that is one and only will be considered as trans-reistic and transpersonal though we are dealing inside It with a multitude of things and persons.

Let's add that Cusanus' conceptual model is not dualistic, it is dialectic. The basic principle that governs in it sounds: *coincidence of opposites.* Cusanus defines, in Latin, the Absolute as the Whole as *complicatio oppositorum et eorum coincidentia* [19, 2] i.e. as an immense coiled up complexity of opposites and their complementary coincidence. like e.g. absolute maximumness (Act) and contracted maximumness (Potentiality), maximum and minimum, one and single and multiple, original and derivative (Cause and Effect), objective and subjective, trans-spatiotemporal and spatiotemporal, order (cosmos) and disorder (chaos), simplicity and complexity, indivisibility and divisibility, indestructibility and destructibility, nonlocal and local, un-individualized and individualized etc.

7. The Dialectic Pansomatopsychic Nature of the Cosmic Stuff of Our Universe

In the Nature, in the cosmic stuff of our universe (and perhaps also in the cosmic stuffs of other universes) there is a very important coincidence of opposites: (1) the outer somatic aspect and (2) the inner psychic one. This opinion is often called "the dual-aspect monism". These two dialectic aspects are inseparable. In other words, the somatic outside and the psychic inside are complementary and can co-exist potentially and actually. Probably the individualized somacity and psychism existed long time i.e., billions of years only potentially and began to exist actually when the conditions became competent. In the individualized somato-psychic beings there exists a coincidence of two opposite kinds of causation: the basic bottom-up causation and the derivative governing top-down causation. Let's shortly consider this coincidence of causations in the human case. Our somatic side, the neuro-chemical and bioelectrical processes in our brain cause the emergence of our consciousness. That's the basic bottom-up causation. The emerged consciousness begins to govern our body. It decides e.g. where we will go. That's the governing top-down causation. Thanks to the coincidence of the two kinds of causation we are active in our lives.

8. The Origin of the Term Panpsychosomatism

In Poland Bolesław Józef Gawecki (1889-1984) physicist and philosopher has introduced the term panpsychosomatism [20]. I prefer the term pansomatopsychism because the somatic aspect is primary and basic. The psychic aspect is derivative though also fundamental because when it emerges from the brain it can govern over bodies. In Nature, we know only consciousness connected with material background, i.e. animal and human brains. For consciousness to exist brains alone are not enough – there must be also appropriate neuro-chemical and bioelectrical reactions undergoing within them.

Today, every anaesthesiologist knows what chemical should be given to an animal or human being to make them unconscious, and what chemical will make them regain consciousness. Applied chemicals change the chemical state of the brain. Naturalists state that the kinds of consciousness we know are natural phenomena, and not super-natural ones. It must be added that there is also a pansomatopsychic concept of the physical All-in-Being, according to which the whole natural reality is double-aspect, subject-object, psychosomatic or better somatopsychic. According to the supporters of this concept, even elementary particles have, apart from their objectivity, their elementary subjectivity, elementary consciousness, ovular, stem or proto-consciousness.

This view is also called dual-aspect monism or panpsychosomatism. According to the supporters of panpsychosomatism, elementary physical beings are characterized not only by elementary external properties, but also by elementary psychical characteristics. Every elementary particle is regarded then as having its own "interior", as a piece of psychophysical energy. In the process of evolution, more and more complex organization is accompanied by growing interiorization, thus both aspects – internal and external – inner and outer become more and more complex. Therefore, there is neither pure matter nor pure psyche, only matter becoming psychical. Such an opinion was supported by P. Teilhard de Chardin in France [21].

Today we know the evolution of the nervous system: it starts with coelenterates. There still exist coelenterates called sea anemones. In some of them the whole nervous system consists of a single neuron (or neuron-like cell). The final stage of the evolution is man, whose brain consists of about a hundred billion neurons, complete with countless synapses with axons and dendrites [22]. Perhaps we should go back as far as to unicellular organisms. Perhaps their nucleus is a carrier of the germ of consciousness of proto-consciousness. If you stab a paramecium with a pin, it reacts. Is it only a pure chemical reaction, or perhaps some kind of feeling of existence and threat?

9. Different Kinds of Psychosomatism

According to B. J. Gawecki who, as was already mentioned, introduced the term panpsychosomatism, every elementary particle is a quantum of psychosomatic energy and every physical process is of a psychsomatic nature. But, according to him, it means not only that the cosmic stuff is psychosomatic, but also the Maxi-Being is of psychosomatic nature. God is the Soul of the Maxi-Being and the Universe is his Body [20]. Gawecki is not the first supporter of such an opinion. In the past there were several philosophers that supported the opinion of an incarnated in the universe God.

In France, similar opinion was propagated by P. Teilhard de Chardin. He was of the opinion that elementary particles have their psychic “dedans” (= inside) and spoke about a Cosmic Christ i.e. about the second person of God becoming incarnated in the Universe [21, 23].

Today, however, such opinions sounds strange and entirely unrealistic. If the Universe was God’s Body then we could ask if in it there is also God’s Brain and weather, perhaps, the galaxies are maxi-neurons of such a Brain. It is sufficiently clear that such speculations belongs to philosophical and theological fiction.

Nowadays philosophers, scientists and even theologians are disposed to state that the notions ‘somacity’ and ‘consciousness’ cannot be applied to the Absolute as such. It is considered by them as trans-somatic, trans-conscious and transpersonal. The Absolute cannot be neither a Body among bodies nor a Person among persons nor a consciousness among consciousnesses. In the Maxi-Super-Being we are dealing with the coincidence of opposites with non-somacity and somacity, with unconsciousness and consciousness, with impersonal beings and persons. The Absolute is the subsistential Entity as such of All of them. Bodies, consciousness and persons are modalities of the omnipresent and omni-penetrating Subsistence, that is one and single. They are local expressions, local modalities of the omnipresent nonlocal Absolute, of the Ultimate Reality embracing and penetrating all. Such an opinion is supported also by the Nobel Prize Laureate Christian de Duve [24].

Professors of medicine: Tadeusz Bilikiewicz, psychiatrist [25] and Tadeusz Kielanowski, pulmologist [26] are more moderate in their pansomatopsychic opinions. According to them (both were professors of the Medical University of Gdańsk), as regards the prebiotic cosmic stuff we can say that life, consciousness and intelligence are there present only in a potential way. The cosmic stuff is able to become alive, conscious and even intelligent when competent circumstances and conditions will appear in it like e.g. on our Earth.

More radical opinion is represented by professor of medicine in Warsaw Juliusz Reiss. In his opinion: *Matter is alive and conscious in dependence on its level of complexity and organisation* [27].

There are several physicists, for instance, Jean Charon. Czesław Białobrzeski, and others who are of the opinion that we should go even further back in the evolution of consciousness and attribute a proto-consciousness to elementary structures. According to them in order to create a Grand Unification Theory (GUT) it is necessary to develop psychophysics. For example the French physicist Jean E. Charon in his books on the Theory of complex relativity. One of his books has the subtitle: Introduction to psychophysics [28].

Charon in his trial of formulating a GUT containing a description of the psychic side of the universe has introduced beside the four spatiotemporal coordinates of General Relativity another four-dimensional coordinates superposing them on the GR coordinates by using the complex numbers. According to him the GR coordinates serve to describe the outer side of the cosmic stuff (and of its components) and the coordinates with complex numbers serve to describe the inner psychic side of the physical reality. Elementary particles are carriers of information in their psychic inside [28-30].

The Polish physicist Czesław Białobrzeski (1878-1953), has tried to formulate a quantum-mechanical version of pansomatopsychism [31]. According to him the quantum-mechanical function ψ that evolves in time in the configurative Hilbert space is a mathematical tool to describe the inner psychic side of the atomic structures and when we superpose on it operators of observables then also the outer side of these structures is described [31-33].

The inner side described by the function ψ was called by him "potentiality". In his opinion, if we could write a Schrődinger equation for the Maxi-Being then the Universal All-in Being could be called the Maxi-Potentiality.

According to him the atomic systems composed of elementary particles react, when we do experimental observations upon them, like living organisms to save their integrity. Therefore, he introduced into QM the notion of "organicity" [31-33].

10. Einstein's Pansomatopsychism

The impact of Spinoza's philosophy on Einstein thinking was decisive for his own worldview. He wrote: "*For Spinoza that what is psychic and that what is physical are different forms of manifestation of the same Reality which is one and only. This conviction is recognized as a knowledge scientifically proven by the majority of scientists. The better we understand the activity of our universe the closer we are to God*" [35].

Einstein's theism like Spinoza's was transpersonal.

11. The Eternity of Pansomatopsychism

I am convinced that pansomatopsychism as a natural phenomenon exists in the transpersonal All-in-Super-Being since ever and will exist for ever in It. In my opinion such a conviction is fully reasonable. The Absolute as the Whole (Cusanus' *Maximitas simpliciter*) was never deprived of life, consciousness and intelligence. *Esse Omnium* the Existence of All (Cusanus' *Maximitas Absoluta)* was never nude, never naked. It was never without the Multiverse. It was always dressed of actual and potential universes. The potential universes (Cusanus' *Maximitas contracta)* emerged and expanded by means of Big Bangs (Cusanus' *explicatio Absoluti, explicatio Dei)*. In many universes (may be in all of them) life, consciousness and intelligence are normal phenomena, they are the second Big Bangs, Big Bangs of life, consciousness and intelligence.

At the end let's remind the fundamental statements of the moderate and strong pansomatopsychism:

(1) Moderate statement: The cosmic stuff has an inscribed in itself pansomatopsychic potential capability to become alive, conscious and even intelligent which becomes actual when competent conditions and circumstances appear.
(2) Strong statement: The cosmic stuff is alive and conscious from the beginning according to its organization and complexity. The evolution of the stuff begins with a proto-life and proto – consciousness present already in the elementary particles and in the rudimentary structures composed of them.

12. Conclusions

Concluding we can ask the question: What is really present? The answer, in my opinion, is the following.

(1) It is certain that there is the Absolute as the Whole which by definition contains in Itself the entire domain of existence nothing excluding i.e. the Cusanus' "simply Maximumness" It is also obvious that in such an Absolute we are dealing with a fundamental coincidence of opposites. From the one hand we have all what is primary, causal, originating and from the other hand all what is secondary, effective derivative.
(2) It is certain that there is permanently present the Absolute as such the subsistent source of all what exist, Cusanus' *Esse omnium.* If there was not the Absolute as such there would be nothing because from the absolute zero of being and of all manifestations of being i.e. of information, acting, laws etc. nothing can emerge.
(3) It is also certain that there really exists the maximal packet of all possible modalities of existence, the maximal packet of all objective potentialities. This maximal packet constitutes the Cusanus' contracted Maximumness. The Absolute as such carries with and in itself the maximal packet of possible modalities of existence. The Absolute as such is pregnant with the maximal packet of potentialities.

In my opinion as well:

(1) the concept of the trans-reistic and transpersonal Natural Maxi-Being (*the Natural Maximalissimus*).
(2) and the conception of the potential and actual pansomatopsychic nature of the cosmic stuff of our universe and perhaps of outher universes can serve for scientists as a good philosophical background when looking for (1) a general theory of everything and for (2) origins of life, consciousness and intelligence.

Note that if the Absolute as such is dressed only with materio-psychic universes then materiality and psychism are the most basic modalities of the Subsistent Entity and then the indicated above materio-psychic reductionism cannot be included among the models with reductionism. But we do not know the whole packet of possible modalities of existence which can emerge and develop from and in the single Subsistent Entity. So the absolute Maximumness will remain for us the Maximal Mystery.

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The Homogeneity of Nature Principle and the Conviction That Life is a Cosmic Imperative

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The homogeneity principle states: what the laws of nature (as described by physics, chemistry, biology and other sciences) admit here and now, spontaneously happens elsewhere, under competent similar conditions. In other words: the same physical phenomena given the same circumstances run the same way in the whole universe. The homogeneity principle plays a fundamental part in scientific thinking. The purpose of this paper is to show that it conducts us even to the statement that life is a cosmic imperative. It will be shown that the principle in consideration is already inscribed in the mathematical structures of the different branches of natural sciences: classic and relativity physics, quantum physics and chemistry, quantum biochemistry etc. It is also called universality of natural laws principle.

Keywords: Homogeneity of Nature principle, life as cosmic imperative, universality of physical laws

1. Introductory Remarks

Astronomers have not yet any sur direct or indirect observational evidence that in our galaxy there is extraterrestrial life. In looking for such an evidence bio-astronomers and bio-cosmologists choose especially the transiting planets as they cross the disc of their host star because, when they transit, the atmospheres of these planets can be easier and better investigate.

Why do they investigate their atmospheres? Because on the basis of the homogeneity principle, consciously or perhaps still unconsciously but certainly instinctively, they are convinced that, if we will discover in the atmosphere of such an exoplanet the same chemical elements and compounds like on our Earth then we can be sure that there is life and that these components have to be considered as biomarkers.

The homogeneity principle states: what the laws of nature as described by physics, chemistry, biology and other sciences admit here and now happens other sciences admit here and now happens spontaneously elsewhere, under competent conditions. The homogeneity principle presupposes the universality of the laws of Nature. Therefore, the homogeneity principle can be defined as follows: the same physical deterministic and in-deterministic phenomena given the same circumstances run the same way. This statement concerns also statistical phenomena and in such a case it sounds: the same statistical physical phenomena given the same statistical circumstances run the same statistical way.

The homogeneity principle plays a fundamental part in scientific thinking. The purpose of this paper is to show that it conducts us even to the statement that life is a comic imperative. It will be also shown that the principle in consideration is already inscribed in the mathematical structures of the different branches of natural sciences: in classic and relativity physics, in quantum physics and chemistry, in quantum biochemistry etc. The important part played in the scientific debate by the homogeneity principle will be analyzed not only from the scientific but also from philosophical and logical point of view.

2. Nicolas Copernicus Revolution Results with the Homogeneity Principle

Modern science has come to the conclusion that the Nature is homogeneous. That's one of the results of the Copernicus principle, according to which, the Earth holds no central, specially favored position. This developed into mediocrity principle; our planet and solar system are no greater nor lesser that other planets and systems and, finally, in modern cosmology, into the principle of homogeneity and the cosmological principle: each indicating different aspects of the same principle.

Before Copernicus presented his new vision of the solar system, a majority of scholars were convinced that the universe was divided into two parts: the perfect super-lunar world and the imperfect sub-lunar world. When using his telescope, Galileo Galilei discovered spots on the surface of the sun, scholars gradually came to the conclusion that the super-lunar world is imperfect as well. Thus, step by step, the principle of the homogeneity of nature was born.

3. The Presence of the Homogeneity Principle in the Mathematical Structures of Physical Sciences

The homogeneity principle found its mathematically idealized expression in physics thanks to Emmy Noether (1882-1935) in her theorems showing the universality of physical laws [1]. According to her theorems, the laws of physics are invariant (1) with respect to the displacement in space what finds its expression especially in the law of momentum conservation, (2) with respect to the displacement in time what finds its expression especially in the law of energy conservation (3) and with respect to rotation what finds its expression especially in the law of angular momentum conservation.

Shortly: Noether's Theorems state that to each continuous symmetry group of the action functional there is a corresponding conservation law of the physical equations and vice versa. In such a way, thanks to Emmy Noether, the homogeneity principle entered into the mathematical structure of the classical physics.

The known quantum physicist Richard Feynman (1918-1988) expressed his doubts whether the homogeneity principle can be used in Quantum Mechanics because the Noether's theorems concern just the physical quantities that appear in the Heisenberg uncertainty relations. As it is well known the Heisenberg uncertainty relations [2] are products (1) of uncertainty of momentum and uncertainty of space location and (2) uncertainty of energy and uncertainty of time location and (3) of uncertainty of angular momentum and uncertainty of angle displacement.

However, as it was shown in subsequent researches there exists also a proper invariance of quantum mechanical laws with respect to the displacement in space and time and with respect to rotation (see e.g. Ramamurti Shankar [3]). Thus the homogeneity principle finds its expression not only in the mathematical structure of the classical physics, relativity theory including, but also, in a proper way, in the mathematical structure of Quantum Mechanics and in this way in the structure of Quantum Chemistry, Quantum Biochemistry and so on. It means that the principle under consideration finds its expression in the biological sciences as well. This fact shows that the opinion, according to which, life with consciousness and intelligence, is a cosmic imperative has a reasonable and meaningful basis.

4. Life as Cosmic Imperative

Although we have not yet any sure observational evidence that life, consciousness and intelligence are cosmic phenomena we are, to a certain extent, in a situation similar to that which exists after political elections but before the computing of the votes. Thanks to good samples of observational data and statistical calculations we know already in the evening which political party won the elections though the computing of the votes has not yet begun.

Christian de Duve (1917-2013), cytologist, biochemist, Nobel Prize Laureate, Member of Pontifical Academy of Science, professor of the Catholic University in Louvain Belgium and of the Rockefeller University in New York, indicates many observational data and uses the probability calculus to show that we can already state that "Life is a cosmic imperative". He presents the empirical data and his statistical calculations in his books: (1) *Vital Dust. The Origin and Evolution of Life on Earth* [4], (2) *Life Evolving: Molecules, Mind, and Meaning* [5], (3) *Singularities. Landmarks on the Pathways of Life* [6], As regards the empirical data he indicates that 20% of the cosmic dust (which constitutes 0,5% of the mass of our galaxy) is composed of organic compounds [4]. According to him, our universe is a great melting pot of organic synthesis because the amino acids can be found in meteorites and are found, using spectroscopy, in the comet tails [4-6]. Therefore, one of his books has the title *Vital Dust.* Note that peptides and proteins are composed of amino acids. All these compounds are shares of life.

De Duve opposes to the Einstein's statement: "God doesn't play dice with the world" his own statement: "God plays dice with the universe because he is sure to win".

He proves the truth of his statement using the following equation of probability calculus [5, 6]

$$P_n = 1 - (1 - P)^n$$

Where

n = number of opportunities (trials)

P = probability

P_n = probability after n opportunities (trials)

According to the modern science in the process of the physical, chemical biological evolution there play part three constituents

1. Chance (spontaneity)

2. Necessity (regularities, laws)

3. Opportunities (number of trials)

Many people when hearing the words "chance" and "probability" exclude automatically "inevitability" and "certainty". According to de Duve that's a great mistake because chance does not exclude inevitability and probability does not exclude certainty. If the number n of opportunities is very great and the process of evolution lasts many billions years then inevitably P_n becomes 1 with full certainty. According to de Duve, in the process of evolution the rule of congruence plays an important part. The chemical elements fit to each other better or worse. When they fit well they form chemical compounds. For instance, hydrogen and oxygen fit to each other and in favorable conditions form the chemical compound called water. The chemical element carbon fits to several other chemical elements and in favorable circumstances form organic compounds. According to de Duve there exists a certain chemical determinism in which the congruence rule plays its role. Nature arrives step by step to more and more complicated compounds, amino acids, proteins, RNA, DNA and so on. For him the appearance of life is a natural chemical process.

Therefore, de Duve concludes: If on an exo-earth there are favorable conditions and the necessary key events have happened then life and consciousness will appear and evolve. Therefore, according to de Duve: Life is a cosmic imperative.

The mentioned key events must be looked for, studied and their part played in the homogeneity principle must be taken into account because they play often the role of the *conditions sine qua non.*

The homogeneity principle provides the grounds for affirming that life and consciousness are truly cosmic phenomena. We can even say that the appearance of life and consciousness, given the required conditions and key events, is a law of nature. Of course, the origins of life and consciousness pose a much more complicated question than a simple matter of physics or chemistry.

Let's first consider two simple experiments: one from physics, one from chemistry [7]. When one sends electrical current (the flow of electricity) in the same directions through two parallel wires, an electromagnetic repulsive force appears between them. However, if one sends the flows of electricity in opposite directions, an attractive force appears instead. When an unbound piece of potassium or sodium comes into contact with water, fire (violet for potassium, yellow for sodium) and great heat results. Should one repeat these experiments in different places and times, under identical conditions, one can assume *with certainty* that they will run the same way. A countless number of experiments over the last three centuries have convinced the scientific community of the merits of the homogeneity principle also in all electromagnetic and chemical processes, Note, that all chemical links are of electromagnetic nature.

5. The Part Played by Electromagnetic and Chemical Processes in Living Beings

Electromagnetic and chemical processes are central to all living beings. The biological evolution is simply a new kind of chemical evolution, and organisms are sophisticated chemical laboratories. Gerald Francis Joyce writes: "life is a self-sustained chemical system capable of undergoing Darwinian evolution" [8]. The evolution of consciousness is likewise connected with electromagnetic and chemical processes. We know that consciousness is intimately connected with neuro-chemical processes in our and animals brains. The consciousness is connected with bioelectrical and chemical activities of the brains. Anesthetists know which chemical substances to apply to induce unconsciousness and which will induce to consciousness. When a brain does no longer show bioelectrical activity it is dead.

6. The Epistemological and Logic Status of the Homogeneity Principle

Italian philosopher and scientist Filippo Selvaggi was one of the defenders of the homogeneity principle [9]. He called it the *fundamental principle of induction* and indicated its different aspects (1) *the principle of physical causality*, by which the same causes in the same circumstances produce the same effects; (2) *the principle of the constancy of the laws of nature*, by which their corporal nature is determined to one and operates always in the same way, obstacles notwithstanding; (3) the principle of physical determinism, by which, ones one knows the state of a system and the laws that regulate it, it is always possible to foresee future events produced within the system. In this way, Selvaggi's formulation is always possible to foresee future events produced within the system. In this way, Selvaggi's formulation of the homogeneity principle reduces it to deterministic phenomena and laws. However the ubiquity of indeterministic quantum mechanics own laws shows that they, too, are valid throughout space-time as it was shown e.g. by R. Shankar [3].

Czeslaw Bialobrzeski has shown that in Quantum Mechanics we are not dealing with determination to one but to many possible effects. The quantum mechanical causation realizes one of the packet of potentialities. Its causation is not univocal but multi-vocal [12].

Therefore, Selvaggi tried to enlarge the homogeneity principle to quantum mechanical phenomena in his book Causality and indeterminism [10].

In Poland, the logic status of the homogeneity principle was investigated by the logician Leopold Regner. Let's first present his own formulation of the principle. "The conviction that what happens in determined circumstances W, will happen again exactly and unfailingly everywhere and always where and when there will be circumstances totally similar to W, is based on the recognition of the principle of homogeneity of nature. The homogeneity of nature consists on this that the course of a phenomenon does not depend on the circumstances of place (where) and time (when)." [11].

The logic status of the principle is presented by Regner as follows: "The principle of the homogeneity of nature is not a kind of a major premise that is present in an implicit way in every inductive inference but it is something that can be called preliminary assumption (praeambula) of induction. The principle of the homogeneity of nature is not a proved affirmation but it is a certain kind of postulate or presupposition about the properties of the Universe" [11].

In our opinion, the homogeneity principle constitutes, to a certain extent, a major implicit premise in majority of inferences in scientific practice. The results obtained so far have convinced the scientific community that nature must be considered homogenous, and this homogeneity permits human reasoning in the natural sciences to be to a certain extent unfailing. The homogeneity principle provides good and reasonable grounds for the conclusion that life, consciousness, and intelligence are all cosmic phenomena. We are convinced that future observations will prove the truth of this conclusion i.e. the Big Bang of Life.

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On the Origin of Sexual Preference

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Current science is inadequate for describing the complex framework for the origin of sexual preference because science has not had until now either a comprehensive model of living systems or 'consciousness' able to delineate the correspondence between biophysics and the noetic effect of the 3rd regime of unified field mechanics. This work begins reviewing aspects of psychology, biology and cognitive science, then develops an anthropic telergic teleology of mind-body interaction (physically real Cartesian interactive dualism) as the context for developing a pragmatic scientific model for the fundamental origin of sexual preference. The model utilizes archetypes originating in Jung's concept of a collective unconscious which are also presumed to be physically real elements of 'mind'. This so-called Noetic Theory (relying on spirit (chi, prana) as an inherent self-organized aspect of a 'vital field', as a physically real action principle) predicts a prenatal stressor acting during a key stage of embryonic development typically under a panoply of one or both parents exhibiting a threshold (gradient of severity) personality disorder(s). The resultant action of this 'noetic effect' orients the anima and animus archetypes as they are coupled into the biophysical substrate of the psyche (soul) and reverses, for the case here, the normal orientation hierarchy of the noetic field within the individuals 'psychosphere'. Initially, because of conceptual similarity, the periodic reversal of the Earth's geomagnetic field by the force of solar wind on the dynamo at the Earth's core is utilized as a metaphor to axiomatically illustrate inversion of the Jungian anima and animus. This scenario is followed by a more technical and experimentally testable scientific description utilizing pertinent new principles related to the discovery of physics of awareness.

Keywords: Awareness, Biophysics, Collective unconsciousness, Archetypes, Noetic field, Sexual preference

"It does not matter if a principle is popular or not; it only matters whether or not it is true. And if it is true; it does not matter whether I stand alone in it." - Joseph Smith [1].

"To conclude, therefore, let no man out of a weak conceit of sobriety, or an ill-applied moderation, think or maintain, that a man can search too far or be too well studied in the book of God's word, or in the book of God's works; divinity or philosophy; but rather let men endeavour an endless progress or proficience in both." - Francis Bacon [2].

1. Introduction

It may seem an immense journey to describe the scientific origins of sexual preference at this point in the evolution of human epistemology because we are considering complex issues that the current state of science is incapable of adequately addressing. Current thinking in psychology, genetics, biology, medicine, philosophy, physics, cosmology and theology have proven insufficient to definitively handle the issue of the fundamental origin of sexual preference; and the dominant model of consciousness based on 'biological mechanism' (no life principle) insists Mind = Brain [3]. Noetic theory might seem off base to those considering the issue to be at most confined to the biological/ psychological/sociological arenas. This work, while somewhat 'a cart before a horse' at this writing, is nevertheless empirically testable [4]; however, it is not easy to assess what impact, if any, the work might have on the political climate in the near term. Progress in science is typically made up of a myriad of continuous small advances; but occasionally, as in Einstein's theories or the advent of quantum theory in the early part of the last century, paradigm shifts occur that revolutionize thinking. We are on the brink of one of those moments.

Those considerations aside, the discovery of a comprehensive model of mind or awareness as illustrated in terms of noetic field theory utilized [5-8]; this is the point where the real voyage to new understanding begins. Often a new model seems overly complex when it is first introduced and takes years before satisfactory discourse occurs at the more general level. If the author is to be critical of his own work; it is obvious that portions of this paper are too general and some too technical which may leave both audiences somewhat unsatisfied. In defense, all that can be said is that this is a seminal work; and as is typical in such cases there will be proficiency in the future. To ease into the scientific origins of sexual preference a series of three metaphors is used before entering into a more technical discussion of the noetic stressor that can induce a prenatal polarity reversal of the Jungian anima and animus archetypes under certain familial conditions. This is preceded by a review of the following pertinent psycho-biological issues.

1.1. *Early History of the Origin of Psychoanalysis*

Sigmund Freud, the father of psychoanalysis is known also for developing additional theories relating to the psychology of human sexuality and dream interpretation in the late 1800s. His most important contributions to clinical psychology dealt with the connection between abnormal behavior and the unconscious mind. Freud also developed a model for the theory of transference, the process by which attitudes developed toward parental figures in childhood are transferred to others later in life playing a significant role in the quality of interpersonal relationships.

Freud coined the term *psychoanalysis* in 1896. Analytic therapy was different in those early times, rather than '50 minute hour' sessions in a therapists office today; a therapist often came and lived with the patient in his home during diagnosis and treatment. First hand observations made it much easier to observe the true basis of the condition; but this is not practical in modern times where an analyst can easily sit with a half dozen or more clients per day, consult with other therapists and be an expert witness in legal proceedings.

It is not widely known; but Freud's original inspiration for the development of psychoanalysis came from his studies of Jewish mysticism - The Kabbalah. The term Kabbalah comes from the Hebrew word קַבָּלָה which literally means *receiving* and refers to the Jewish esoteric school of thought forming the foundations of mystical religious interpretation [9, 10]. The Kabbalah includes a discipline and method for obtaining enlightenment used as an aid to explain the relationship between a mysterious, eternal, unchanging universe and the temporal mortal and finite world that God created as recorded in the book of Genesis.

The main Judaic text for studying the Kabbalah is called the Zohar, which teaches that studying the Torah proceeds along four levels of thought. These levels called the *pardes* were derived from the initial letters of their Hebrew names:

- *Peshat* (meaning simple) - The most direct interpretation of the meaning.
- *Remez* (hint or hints) - Allegories alluding to the meaning.
- *Derash* (from the Hebrew darash meaning inquire or seek) - And the Midrashic or Rabbinic meanings with repetitive words or verses making imaginative comparisons.
- *Sod* (secret or mysterious) - The most esoteric or metaphysical meanings expressed in the Kabbalah.

One can easily see how the techniques of Kabbalism might have inspired Freud to invent the introspective and therapeutic aspects of psychoanalysis.

Freud's theory of psychoanalysis was based on a number of stages of psycho-social development, a sort of evolutionary path of the psyche that each individual passed through with varying degrees of success on the way to adulthood. One of these stages Freud called the 'Oedipal stage' which he considered of central importance in his theories of the origin of homo-sexuality. The Oedipus Complex refers to unresolved sexual feelings of a child to the parent of the opposite sex.

2. Freudian Inversion Theories of Homosexuality

The term psychoanalysis was coined by Freud in 1896. Freud's theory of psychoanalysis was based on a number of stages of psycho-social development; a sort of evolutionary path of the psyche that each individual passed through with varying degrees of success on the way to adulthood. One of these stages Freud called the 'Oedipal stage' which he considered of central importance in his theories of the origination of homosexuality. Freud first began writing essays on homosexual inversion in 1905 [11]; and he was never able to completely resolve in his own mind whether homosexuality was a form of psychopathology or merely a statistically abnormal variation. Freud found little success in the psychoanalytic treatment of homosexuals; "to convert a fully developed homosexual is not much more promising than to do the reverse". He believed that homosexuals were not motivated to be 'cured'; that they were unwilling to give up the object of their pleasure. Freud thought the motivation for treatment was a vehicle used by the homosexual to assure himself that he tried everything he could to change, and failing could resign himself in good conscience to his pleasures [12].

Freud developed four theories of homosexuality:

1. From the Oedipus Complex - A young male has a typical early erotic bond to his mother, but there is an excessive amount of tenderness on the part of the mother which over sexualizes the bond in the mind of the child at a time when the distinction between self and other has not yet formed. Later when the child's ego begins to separate he feels guilty and develops a fear of castration as a punishment for his erotic feelings toward his mother. He develops hatred toward his mother and severs the erotic bond. A compromise sexual object is chosen, an effeminate boy [12].

2. Also of Oedipal Origin - The child maintained a particularly long sensitive relationship with his mother which the child refuses to give up. In order to preserve the erotic bond he subconsciously identifies with his mother and selects love objects that resemble himself. In loving them he experiences the erotic bond he had with his mother [12].

3. Inverted Oedipus Complex - Freud considered this the most common form of homosexual causation. The young boy has an identification with his father; but instead of identifying with him as a role model or father figure the child wants to be romantically loved by him and surrenders his masculine identity in order to be loved as a woman by another man. It is statistically common among both Lesbian and Homosexual couples that one partner usually the more submissive or 'feminine' (whether a male or female) plays the role of the wife and the other partner more dominant the role of the husband. An example of this in public life is the American comedienne / talk show host Ellen De Generes who has a butch haircut and generally acts very masculine in contrast to her 'wife' Portia de Rossi who remains gorgeously feminine.

4. Intense Love of the Mother - Leads to extreme jealousy of other siblings and the father. The jealousy is very extreme and leads to a death wish and sadistic fantasies of extreme violence. In what Freud termed 'reaction formation' the child transforms the repressed feelings into inclinations for homosexual love [12].

In summation, it can be seen that Freud attempted to derive a theory of homosexuality from an inherent personality disposition that he considered to be triggered by both random statistical occurrences and abnormal developmental personality factors appearing in the nuclear family beginning during the early years of childhood development. The psychoanalytic perspective as the cause of homosexuality has become increasingly unpopular today, especially because of the political issues surrounding homosexuality as a politicized human rights issue rather than a personality disorder.

However, Freud's theories of homosexuality have remained at the core of clinical theory for the last hundred years. The controversy has continued over the last several decades as to whether sexual preference is Psychological, genetic, environmental or as we intend to show here part of a much more complex structure of the soul.

3. Jungian Theory of Homosexuality

Jung did not write prolifically on sexuality in part to distance himself from what he felt was Freud's over-emphasis on the subject. Even today critics say according to Freud everything was based on sex. This is one reason Freudian psychotherapy has fallen into disfavor. This is not completely fair to Freud because his concept of the 'Libido' can be considered to refer to 'drive' more generally that just the sex drive. But like Freud, Jung considered homosexuality as a deviation from the sexual norm but not necessarily a pathological condition:

> *"If we regard sexuality as consisting of a fixed heterosexual and a fixed homosexual component, ... the assumption of fixed components precludes any kind of transformation. In order to do justice to it, we must assume a great mobility of sexual components, which even goes so far that one component disappears almost completely while the other occupies the foreground .... we need a dynamic hypothesis, since these permutations of sex can only be thought of as dynamic or energetic processes"* [13].

and further:

> *"For a man, a woman is best fitted to be the real bearer of his soul-image, because of the feminine quality of his soul; for the woman, it will be a man. Wherever an impassioned, almost magical relationship exists between the sexes, it is invariably a question of a projected soul-image. Conversely, it may also happen that the soul-image is not projected but remains with the subject, and this results in an identification with the soul because the subject is then convinced that the way he relates to his inner*

processes is his real character. In that event the persona, being unconscious, will be projected on a person of the same sex" [11].

In Jungian psychology, this conceptualization of homo-sexuality is defined in terms of the male-female archetypes called the anima and animus. A man's identification is with the contra-sexual archetype of the anima, "*with his unconscious femininity, thus leads to a projection of his persona, that is, his 'outer' masculinity, onto another man*" [15]. As Jung suggested we will describe how the fixed components of sexual archetypes may undergo an 'energetic process' of transformation.

3.1. *Jung's Concept of a Collective Unconscious*

Jung is considered the first modern psychiatrist to view the human psyche as "*by nature religious*" and make it the major focus for exploration [16, 17]. Jung considered the Collective Unconscious to be a nonlocal cosmic domain that stored for all time a finite collection of universal archetypes with infinite combinatorial possibilities relating to personality structure. These 'racial memories' are shared in the makeup of the psyche of every human individual. The collective unconscious includes the concept of archetypes which are the mode whereby the collective unconscious expresses itself in the individual. This is a deeper level than the more personal unconscious that Freud postulated.

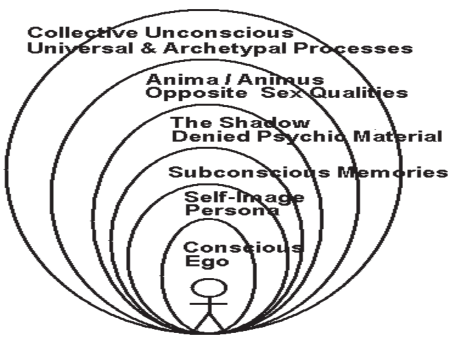

Figure 1. Conceptual model of Jung's Collective Unconscious. The set of concentric circles begins with the figure of a person at the bottom then proceeds inward to layers of his conscious and subconscious mind through the archetypes to the deepest level called the collective unconscious which Jung proposed to be universal and part of the psyche of every individual. Most psychologists currently consider the collective unconscious to be physically real.

The figure above shows a hierarchical conceptual depiction of the domains Jung considered to house the archetypes of the Collective Unconscious.

Contemporary medical psychiatry and therapeutic psychology is based solely on a personal unconscious. Jung's system adds a second psychic system of an eternal universal impersonal nature he defined as the Collective Unconscious. An archetype in this system as defined by Jung is a pre-existent thought form that can become conscious or facets of the personality. He believed that there are as many archetypes as there are life situations or personality factors. Jung experimentally demonstrated (by subjective reporting) the existence of archetypes in analyzing dreams, imagination, psychotic delusions, and fantasies produced in hypnotic trance.

The archetypes that we will be concerned with in this volume are the male/female opposite gender archetypes contained in every person called the anima and animus. The anima/animus archetypes are susceptible to personification and transformation expressing the process of individuation itself.

Jung thought this duality represented what he called a 'mythical syzygy'. The term syzygy is most commonly used in astronomy to refer to a straight line configuration of three celestial bodies in a gravitational system. Syzygy usually involves the Sun, Earth & either the Moon or a planet, with the latter either in conjunction or opposition. Solar & Lunar eclipses are times of syzygy, as are transits and occultations. The term is also applied to every new moon or full moon when the Sun & Moon are in conjunction or opposition.

As will be shown in a later chapter the crux of our SOSP model can be said to relate to a compound syzygy. A complex threefold syzygy structure (each component syzygy itself being a triad also) between the both of the parents and a certain susceptible developmental phase in the prenatal embryo.

Figure 2. Conceptualization of a balanced normal *syzygy* (like north and south poles of a magnet) between the anima and animus or male-female archetypes in an individual heterosexual adult psyche or personality structure of the soul.

As an archetype manifests itself and penetrates consciousness it influences the experience of normal and neurotic people. An archetype that becomes too powerfully manifest can totally possess the individual and cause psychosis or as we intend to demonstrate a reversal of sexual preference. One can suspect because of the psychic conflicts that Freud and Jung proposed as causative agents for homosexuality, similarities in the mechanism that causes any psychological disorder and why it is often the case that homosexuals also have associated psychological problems.

4. Psyche, Soul and Mind

The term psyche in general historically and in contemporary psychology and philosophy is used to refer to the totality of the conscious and unconscious human mind of a particular individual. Psychology is often defined as the study of the psyche. In psychoanalysis and other forms of depth psychology, the term psyche refers to the conscious and unconscious forces in an individual that influence thought, behavior and personality.

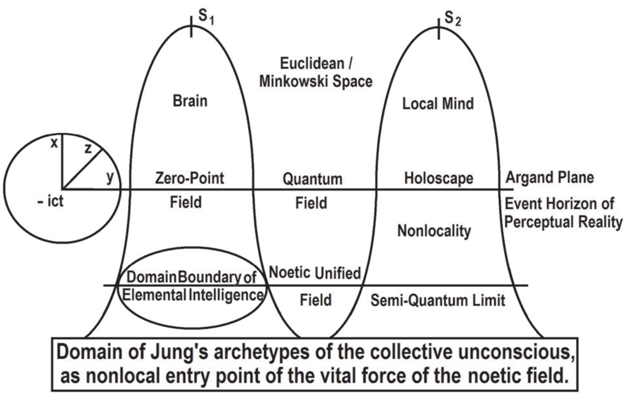

Figure 3. Local-nonlocal space/spacetime model of two individuals (S_1, S_2) showing how their psyches are imbedded in a physically real Jungian collective unconscious which is the source of racial archetypes forming the persona. This unification is associated with the teleology of the noetic unified field, $N_{(f)}$ which is an essential component of the extra-corporeal duality of a Cartesian mind-body dualism.

Sigmund Freud, the father of psychoanalysis, believed that the psyche was composed of three components:

- The id, which represents baser instinctual drives of an individual and remains largely unconscious.
- The super-ego, which represents a person's conscience and their internalization of societal norms and morality.
- The ego, which is conscious and serves to integrate the drives of the id with the prohibitions of the super-ego. Freud believed this conflict to be at the heart of all forms of neurosis.

Jung was very careful to define what he meant by the distinction between psyche and by soul:

> *I have been compelled, in my investigations into the structure of the unconscious, to make a conceptual distinction between soul and psyche. By psyche, I understand the totality of all psychic processes, conscious as well as unconscious. By soul, on the other hand, I understand a clearly demarcated functional complex that can best be described as a "personality"* [13].

Since the birth of the field of Consciousness Studies in recent decades cognitive psychology (the currently dominant school) has replaced psychoanalysis as the dominant model of psychology in academic circles. The word *mind* is now preferred by cognitive scientists to the term *psyche*; and the term awareness is preferred over the word consciousness which is perceived as too general.

In Noetic Field Theory (NFT) the school of thought used here, the word mind is also preferred over the term the psyche. The main difference between NFT and cognitive psychology is that physical principles of mind have been formally discovered. This historical event allows its principles to be applied to problems such as SOSP. Another important fact for NFT is that since mind is physically real it can be experimentally manipulated and used to engineer a new class of medical devices. This also means that since NFT can be empirically tested eventually the noetic theory of SOSP can be experimentally tested.

As will be described in more detail in a later chapter the content of mind or action of mind is not limited just to the brain but also pervades not only every atom of the body but is extended beyond the body into local and nonlocal regions of surrounding spacetime and eternity (something like the corona of the sun). This represents the sum total of the domain of individuality called the psychosphere in noetic theory. The detailed structure and phenomenology of the psychosphere will also be developed further in later chapters. We could have chosen to call the Psychosphere 'the nooephere' because it also has a Greek root stemming from the word noetic but we thought a term with the root 'psych(e) closer to psychology would be more immediately intelligible.

Briefly NFT defines the soul as the 'spirit and the body'. This refers to life on Earth. When a person dies he becomes a disembodied spirit and must wait for resurrection or reincarnation before his intelligence can be considered a soul again. Formally introducing the spirit into psychology and mind is important because it is related to the life principle which is purposefully removed from the basis of cognitive psychology. Most importantly as we shall see because mind and spirit are physically real concepts with field properties; it is this fact that has not only allowed the discovery of mind but what allows us to understand the causative agents reversing the dominance of the anima and animus.

5. Contemporary Psychological Issues Regarding the Homosexual Matrix

In the United States the main medical reference for the American Psychiatric Association (APA) is called the Diagnostic and Statistical Manual of Mental Disorders (DSM) [18]. It describes and classifies all known mental illnesses and emotional disorders. It was first published in 1952. Until 1973 homosexuality was classified as a mental disorder but in 1980 dropped from the DSM-III by a decade of relentless pressure from gay activists.

One of the reasons the APA administration stated for allowing the change in classification was the belief that this change would tend to keep employers from using the APA classification as justification for discrimination in hiring policies. This seems like a weak reason because there is no box in an employment application to check things like political party, religious affiliation or sexual preference. Marital status is usually queried however because employers want to know if they can expect a person to work nights and weekends. But the APA announced that it was also motivated to acknowledge that many gays and lesbians showed no signs of dysfunction and were satisfied with their lives and their sexual orientation.

Figure 3 illustrates the domain of Jung's archetypes of the Collective Unconscious, as a nonlocal entry point of the vital force of noetic field into living systems.

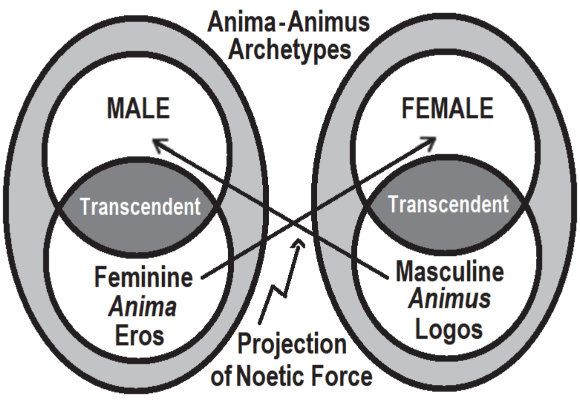

Figure 4. Cross section through center of Fig. 3 showing nonlocal interrelation of the male-female archetypes and structural framework for noetic stress.

Historically prejudice against homosexuals has been deeply rooted in both Eastern and Western society. In Muslim nations the penalty can still be death; and some of those governments will proclaim that homosexuality does not exist in their country. The beginnings of a shift in opinion is said to have occurred with the publication of two well-known reports by Kinsey, The first in 1948 - *Sexual Behavior in the Human Male* and in 1952 - *Sexual Behavior in the Human Female*. The beginning of gay rights organizations started after what was called the Stonewall riot in New York City in 1969. This was the first public protest by homosexuals against harassment by police. In California, oral sex has been a crime carrying a maximum penalty of 15 years. Anal sex could result in a life sentence if prosecuted to the full extent of the law. Interestingly both of these laws apply equally to partners of both the opposite and same sex.

6. Homosexuality as Neither Mental or Biological/Genetic Disorder

We begin to see after examining the psychological and biological/genetic correlates of homosexuality that homosexuality is a complex multifactor matrix [19] that until now has never been completely understood. It is easy to see why difficulties in understanding the homosexual matrix have arisen on all sides of the issue because there are a number of biological and psychological components associated with homosexuality. These conditions have acted as red flags suggesting that they are causative. But those factors turn out to be peripheral i.e. not part of the root cause of homosexuality but occurring because of a more general trauma associated with the abnormal setting itself hat caused the homosexual inversion in the first place. If the cause of homosexuality as neither of psychological or genetic origin it might seem that all the viable possibilities for understanding the condition are used up in terms of the tools available to contemporary science.

The position to be taken up here is in apparent agreement with the APA's profession that homosexuality is not a 'mental disorder' nor is it a biological condition. But as will be shown later this is a somewhat misleading suggestion of a false sense of normalcy. Because while noetic theory agrees that homosexuality has not been shown to be of genetic or psychological origin per se, the cause of homosexuality is representative of a whole new class of medical conditions relating to consciousness itself. The cause is an imbalance in the function of the newly discovered physical basis of the life principle! It was not possible to fundamental basis of the life principle had not yet been formally discovered.

This profound new discovery as introduced in this volume will eventually lead to psychology becoming a hard science instead of an art. Most psychologists think of Psychology as a science because it employs the scientific method in a variety of tests. But because many of these tests rely on subjective reporting rather than objective results, by definition this kind of measurement is not scientific. For example witnesses at the scene of a traffic accident virtually all report different views for example even to the extent of “seeing” different colors of the automobiles involved.

7. The Conundrum of Conversion Therapy

From similarities in the discussion of handedness in (next section) groups associated with religious movements like Exodus International in Seattle, WA USA have claimed a high success rate for the conversion of homosexual men and woman choosing to become heterosexual. As reviewed briefly above we have seen that the main founders of psychotherapy both Freud and Jung not only noticed the difficulty associated in performing conversion therapy (homosexuals wanted it to fail so they would feel free in remaining gay) but felt that homosexuality by itself might not be a mental disorder. Their main evidence was that historically a number of humanities most creative minds like that of Leonardo Da Vinci were homosexual and other than their so-called 'statistical sexual deviation' were considered well-adjusted individuals leading normal lives. However mental or emotional disorders are often associated with homosexual individuals and this was one of the main reasons that it had traditionally been classified as a psychiatric condition for most of the last hundred years since the invention of clinical psychology.

The APA has since, for over 20 years now, affirmed that homosexuality is not a mental disorder. In this guise they have recently passed a resolution warning that societal ignorance and prejudice combined with family pressure can cause some gays to seek conversion therapy that may do them serious harm. But this criticism is not fully justified as it is generally known that this is true of any psychotherapeutic regimen if the problem is deeply rooted and the analysis is not carried out properly or for a sufficient length of time. For in general all neuroses are believed to be caused by unconscious or repressed psychic trauma; and if these ‘wounds’ are laid bare without proper resolution and control a serious psychotic breakdown can occur.

Although Psychology utilizes the scientific method in various forms of experimentation and psychometric testing it is still only an art! Personality disorders which are considered the most serious and deeply rooted of psychoanalytic conditions are most often not curable even after decades of therapy. The best a therapist can hope for is to teach the ‘victim’ how to sufficiently cope with the condition through understanding and behaviour modification so that they can learn to lead a semblance of a normal life. The APA resolution has added fuel to the fire of gay and lesbian political rights groups who think of reparative therapy as “psychological terrorism”.

Proponents of conversion therapy claim that there is no genetic or biological component to homosexuality and the condition stems from dysfunctional family conditions in early childhood. The claim is that men who do not have a strong masculine identity or are very shy and timid in their interactions with women will readily lose their same-sex attractions if they can be taught to become more comfortable, proactive and confident with their manhood.

8. Historical Transmutation of Handedness

Handedness is considered a deeply rooted individual characteristic with about 2% to 11% of the general population being left handed depending on the study performed with about 1% being ambidextrous. People who are ambidextrous often have it to degrees; having some skills with one arm and different ones with the other. Definitional disparity is one reason for the difference in statistical range among different researchers. For nearly a hundred years, biologists and psychologists have debated whether or not handedness is genetic or a product of or socialization. If handedness is not genetic it remains a mystery why only a small percentage of the population should be left handed.

Probably as in the case we are making for sexual preference, handedness is a combination of genetic disposition and conditioning. In that respect the point in terms of transmutation made here is that some parents make an emphatic decision that their children will not be left handed in a predominantly right handed world and train them rigorously until they become right handed for all practical purposes. This is not an overtly natural progression and historically was thought to entail a degree of psychological trauma; now shown to be unfounded. Transmutation of handedness does occur successfully especially when begun at a sufficiently young age.

The cause of handedness still remains a complete mystery. There is little more than a confusing and conflicting array of statistical data subject to various interpretations suggesting that handedness is genetic or not genetic because in 18% of monozygotic identical twins one is left handed and the other is right handed. A recent theory by Coren [20] states that human beings are naturally right handed and that birth stress or prenatal brain trauma produces left handedness. He considers this to be the reason why a higher percentage of left handed individuals have psychological and emotional problems. This seems highly speculative with little empirical evidence in support; however, our reason for bringing up the apparent plasticity of handedness is in relation to historical periods where parents intervened during the handedness formation period (1 to 4 years, Fig. 5) as left handedness was considered undesirable, i.e. right handedness could be trained [21, 22].

The purpose for utilizing the handedness metaphor is to illustrate the current similarly confusing situation as to whether sexual preference is genetic or not. Also, to demonstrate the feasibility of transmutation for similar conditions.

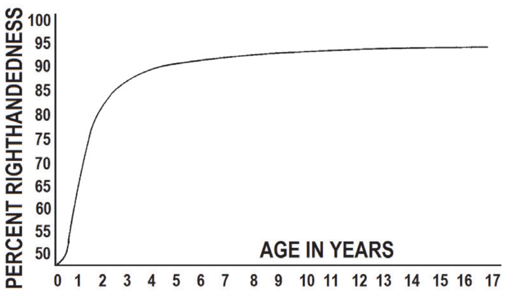

Figure 5. Handedness becomes increasingly determined after birth dramatically in the preschool years.

9. Polarity Reversal of the Earths Geomagnetic Field

Reversal of the Earth's geomagnetic field provides a profound metaphor for gaining insight into the origin of sexual preference. Based on two salient assumptions: 1) That the *life principle* is a physically real noetic UFM field, and 2) correspondingly, that Jung's collective unconscious is likewise physically real; we can paint a picture of the dynamics of field reversal as it as it pertains to sexual orientation.

The core of the Earth is mostly molten nickel-iron acting as a self-exciting dynamo which is believed to be the source of the Earth's geomagnetic field. (Figure 5) The polarity of the Earth's geomagnetic field reverses relatively often in geologic terms, averaging about 250,000 years between reversals. It has been shown that approximately 50% of the rocks in Earth's crust have a magnetic polarity that is opposite to the 'normal' or present-day polarity [23].

All of the several models describing reversal of the geomagnetic field seem to suggest that direct or indirect extraterrestrial influences precipitate the reversals: periodicity in violent solar activity, galactic effects such as cosmic ray intensity or supernova, changes in activity of the dynamo of the Earth's core, episodes of violent volcanism, or the impact and explosion of extraterrestrial objects [24].

A rocks magnetization is defined by three values: angles of declination, inclination, and magnetic intensity. The declination is a locally defined angle in the horizontal plane measured clockwise from 0 to 360 degrees with reference to true north. The inclination is the angle in the vertical plane between the magnetic direction and the horizontal [25, 26].

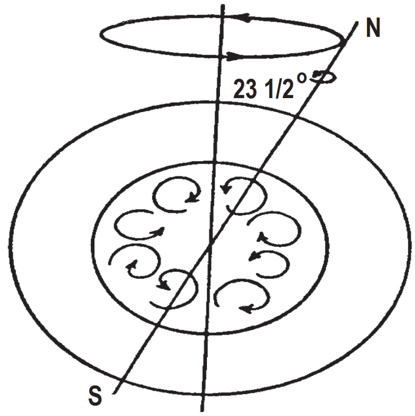

Figure 6. Precession of the Earth's axis creates a turbulence in the molten iron core effecting the geomagnetic field - Geomagnetically induced currents.

80% of the Earth's magnetic field is geocentric - meaning that this portion of the dipole field originates at the center of the Earth. The remaining 20% of the field, the non-geocentric portion, called the 'restfield' originates in external and internal non-dipole fields, remnant magnetization in the Earth's crust, or of extra-terrestrial origin. This so-called restfield can display rapid variations (Fig. 6), with the external portion varying greatly in only a few hours during a solar storm; and the internal field varying sufficiently in five years that world maps of magnetic declination and inclination field strength have to be remade for exact navigation purposes. In A hundred years this 'secular variation' can change as much as 10 degrees [25].

In addition to the external and internal forces that seem to precipitate polarity reversal there are a number of interdependent conditions required in the dynamo of the Earth's core before a polarity reversal can occur. If the position of Earth's axis changes from the influence of an extraterrestrial magnetic field several things can happen:

1) The external field would create eddy currents in the surface layers of the Earth that would counteract the normal external field of the Earth.

2) Thermal effects of the electrical currents would liquefy rock.
3) The molten rock would require the magnetic orientation of the prevailing field.

All three effects have been observed [27].

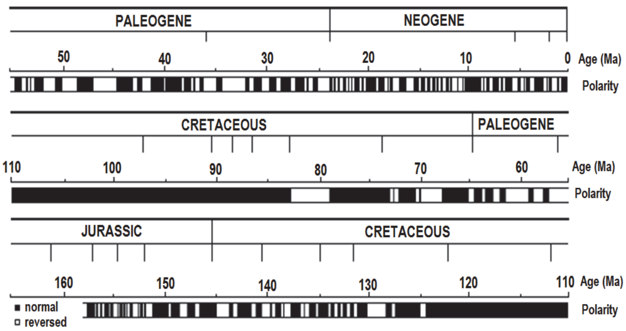

Figure 7. Chart of Earths Magnetosphere polarity reversals over last 160 million years (Tertiary to Permian). Black = normal polarity, White = reversed polarity.

Liquid rock is not magnetic until cooled to its Curie point of about 580 degrees C. It acquires a magnetic field oriented with the declination and inclination of the current field of the Earth which it which it retains after solidifying. Rock formations are found everywhere on Earth with reversed polarity. Reversed polarity rocks are significantly more strongly magnetized than can be accounted for by the Earth's geomagnetic field - ten times; and often up to a hundred times stronger than the magnetic charge they could receive from terrestrial magnetism. This intensity depends on the velocity which the lava cools and on the form, size and composition [23, 27-32].

In Fig. 9 below and the associated commentary we learned about the normal position of the Earth's geometric field and the external and internal effects that are involved in periodic reversals.

Figure 10 below illustrates the dramatic change in the position of geomagnetosphere when a strong external influence is applied.

Secular variation describes the changes in the Earth's magnetic fields on the timescale of years. These changes mostly reflect changes in the Earth's interior, while more rapid changes mostly originate in the ionosphere or magnetosphere. The changes were 1st noted when plotting a graph of the declination in major cities, for example London in 1540. The changes occurring in the direction, declination and magnitude of the field. In order to measure secular change, readings must be taken over a period of many days; the greatest change in the field is that which occurs on a daily basis. An average can then be taken from all these readings so establish how the magnetic field changes over 10 or more years.

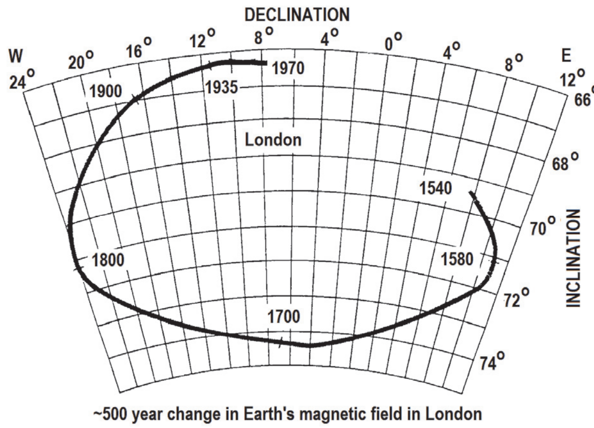

Figure 8a. mapping changes in the Earth's magnetic field in London over 500 years. The westward drift of the earth's magnetic field from observations made in London. Each date represents the direction of the compass needle for that year.

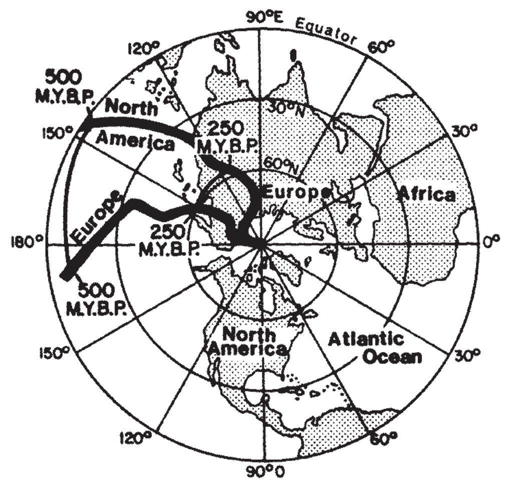

Figure 8b. The Wanderings of the North Pole are traced by the heavy black line. The points are derived from the magnetization of rocks in the British Isles and North America.

The Magnetosphere is a comet shaped region where the charged particles of the solar wind are influenced by the planets magnetic field. It extends to about 65,000 km on the sunward side with a shock front at 100,000 km (Isaacs, 2000). The amplitude of magnetic disturbances is larger at high latitudes because of the presence of the oval bands of enhanced currents around each geo-magnetic pole called auroral electrojets.

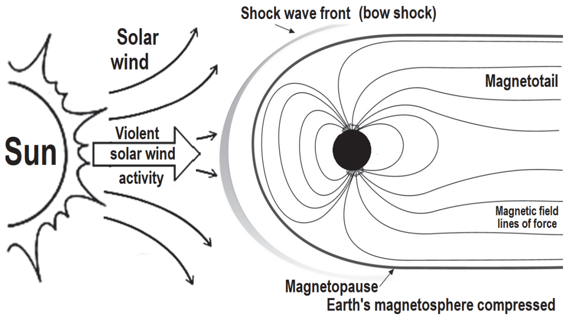

Figure 9. Solar wind and the Earth's magnetosphere. The Earth's geomagnetic field showing the influence of periodic extreme solar activity. The magnetosphere usually extends for about 65,000 km on the sunward side; but severe solar storms might compress the magnetic field to 40,000 km. When conditions in the molten core of the Earth's dynamo correlate, it is believed that cumulate effects (Figs. 8a,b) of such cosmological activity precipitate a reversal of the geomagnetic field (Fig. 8) every ~ 200k yrs. On average.

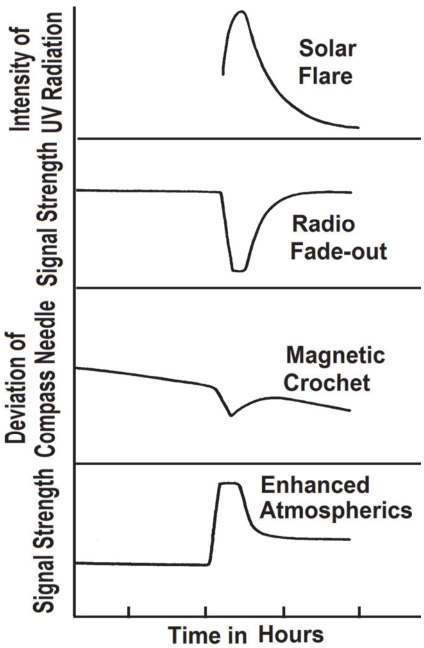

Figure 10. Schematic of instantaneous terrestrial effects of a solar flare.

The most severe magnetic storm in recent times occurred in March 1989 and this had a number of serious impacts on technological systems by generating damaging geomagnetically induced currents [35]. As well as the regular daily variation, the Earth's magnetic field also exhibits irregular disturbances, and when these are large they are called magnetic storms. These disturbances are caused by interaction of the solar wind, and disturbances therein, with the Earth's magnetic field. The solar wind is a stream of charged particles continuously emitted by the Sun and its pressure on the Earth's magnetic field creates a bounded comet-shaped region surrounding the Earth called the magnetosphere. When there is a disturbance in the solar wind the current systems existing within the magnetosphere are enhanced and cause magnetic disturbances and storms. Figure 16 shows a schematic picture of the solar wind and the Earth's magnetosphere.

10. The Origins of Sex

The origin of sex is nearly synonymous with the question of what is life. At a superficial but most fundamental level the thing that separates an ongoing chemical reaction (A continuous supply of material must be available) from a living entity is the domain wall (cell membrane) that separates one reactive system from another. Sexuality is originally a survival mechanism; a form of symbiosis so that missing ingredients can be acquired and that through variation survival optimized. After four billion years of evolution incredibly complex self-organized living systems [36] with sexual reproduction have arisen on Earth [37]. The form of evolution utilized in this manuscript is not random-Darwinian, but a guided evolution to be discussed in detail later.

The self-organization of life is called autopoiesis [38] which means that a living system is able to remain far from equilibrium (death) by the catabolic and anabolic dynamics of metabolism which continuously dissipate the entropy (amount of disorder) that it produces. The three main properties of life are autopoiesis, growth and reproduction. All of these properties may occur in the total absence of sex which was the case through the first three billion years of evolutionary life on Earth.

Individuals of a species may reproduce asexually by replication, a direct copying of genetic material or by sexual reproduction. Sex is the process characteristic of living organisms whereby a genetically new individual is produced from different parents but does not necessarily have to relate to reproduction which is the creation of additional entities. "*Beings can be both new in the sexual sense and additional in the reproductive sense. But this need not be the case. Most organisms in the world in fact reproduce asexually, whether they sexually recombine or not*" [37].

Margulis and Sagan assume that autopoiesis is a prerequisite for reproduction and that reproduction precedes any form of sex. They can also imagine autopoiesis without reproduction which would occur by the uptake of nutrients and the continuous self-maintenance of proteins and nucleic acid. They also wonder "*Why, if asexual beings can have more offspring than sexual ones, are there so many more sexual animals?*" [37]; and conclude we may never know the reason for the origin of reproduction and sexuality because even though many forms of passion are generated, sexuality is not an ultimate biological priority [37]. It seems that there is no scientific reason for sex.

Although speculative, within the teleological framework of a continuous state conscious universe [3, 7, 8] noetic theory is able to offer an obvious theological explanation. The reason for the evolutionary development of complex autopoietic sexually reproducing self-conscious living systems is so that an eternal soul may be packaged into each differentiated entity for passage through an intelligent eternal social progression. As we shall see, a soul adds an additional unified field with an inherent noetic effect involved in 'polarity reversal' of Jungian archetypes [39, 40].

11. Population Genetics

In 1993, it was announced that a gene for homosexuality had been discovered [41]. But it turned out the major researcher's results were eventually shown to be 'fudged' [42] and that "*Hamer was under investigation by the office of research integrity at the Department of Health and Human Services because he may have selectively reported his data*" [43]. But considerable damage was done:

Hammer himself testified as a sworn expert witness to the Colorado court that heard a motion to void the state's 'Proposition 2,' which would have disallowed sexual behavior as a legitimate basis for formal minority status on a par with race. On the basis of his research Hammer testified that he was "*99.5% certain that homosexuality is genetic*." The judge who heard the case ultimately struck down the law [43].

But there is no 'queer' gene. All intensive searches have failed to find a genetic basis for homosexuality so far. Nevertheless, there was a significant element in Hamer's study. He and his colleagues performed a newly widespread type of behavioral genetic experiment called a 'linkage study' where behavioral traits that run in a family are correlated to chromosomal variants found in the genetics of the family [43]. It has also been known for some time that homosexuality has a tendency to run in families. While the incidence of male homosexuality in the general population might be about 5%, having a gay brother increases the chance of homosexuality to about 25% [44]. Hammer was eventually cleared by the U.S. office of Research Integrity; but George Risch of Yale who created the protocol used by Hammer duplicated. Hammer's work with a larger *N* and found no statistical results.

11.1. *Complex Multi-Factor Matrix*

The genetic, physiological and environmental situation effecting the homosexual matrix is not simple. If there are genetic variations that are related to homosexuality, why isn't homosexuality genetic? Firstly, relatively few homosexuals have children so a possible gay gene linkage cannot occur directly. The Hamer research group found families in which homosexuality seemed to follow a mother-son linkage. The X chromosome is comprised of about 100 genes; and on region q28 a variation was found that was related to homosexuality [41, 43].

The problem is one of statistical requirements and interpretation. Although a genetic trait can be of high statistical significance in a particular family; in order to be a genetic trait in the general population it must occur in most homosexuals. While Hamer's research techniques and raw data were within acceptable parameters for a genetic linkage study, he made many unscientific assumptions and was severely criticized for the questionable manner in which he used statistics to support a focus on social and political motivations [41, 43].

Complex behavioral traits are the product of multiple genetic and environmental antecedents, with "environment" meaning not only the social environment but also such factors as the "*flux of hormones during development, whether you were lying on your right or left side in the womb and a whole parade of other things* ..." The relationships among genes and environment probably have a somewhat different effect on someone in Salt Lake City than if that person were growing up in New York City. (For example, conservatives in Utah are less likely to become homosexual than liberals in New York) [43].

11.2. *Physiological Changes*

Obviously many but not all gay individuals look quite different from heterosexuals and can be easily recognized by facial characteristics and body language. There are also secondary sociological patterns such as the duration of mutual gaze that extends beyond what is culturally acceptable for heterosexual interaction. It has been said that the eyes are a window to the soul; and it is quite interesting that a mindset, behavior and experience can affect physiological appearance over time. The consequences of one's actions creates physiological changes in both our brain structure and body [19, 45-47]. There are genes associated with these factors and they can be regulated by mental characteristics under certain conditions. This is called gene activation, but is not a 'biologically' inherited trait, rather it is mediated by the external 'psychological environment' and mentally by the disposition and mind set of the personality. This can lead to epigenic gene activation of more superficial and malleable traits. This action occurs at a deeper level than that currently described by the symptomatology of the western medical/psychiatric arena; but is the result of the cosmology of '*soul talk*' [48] as mediated by the unified noetic field [39, 40, 49] and associated noetic effect, which described in more detail later.

11.3. *Prenatal Stress and Increased Incidence of Homosexuality*

There is some evidence that prenatal stress in the mother produces a statistical increase in the incidence of male homosexuality; but experiments are difficult to perform on human subjects and the results remain inconclusive. The stressor is believed to cause a change in fetal hormones that effect brain development [50-52]. A similar effect with much clearer results is shown in rats. If a pregnant rat is stressed late in pregnancy her male progeny show very low male sexual orientation [53].

Williams [54] explored anatomical characteristics to uncover biological origins of human sexual behavior. Men's ring fingers are typically significantly longer than index fingers, believed to be an effect of prenatal testosterone release. For women generally, the two fingers are nearly the same length. Research has suggested that lesbians are exposed to higher prenatal testosterone release than heterosexual women. Williams found a statistically significant number of homosexual women have a male-like index-ring finger ratio [54].

Another curious fact his team found is that men with more than two older brothers have a statistically higher chance of homosexuality. This increases to 50% for men with ten older brothers.

Historically, H. Ellis published Sexual Inversion, the first English medical textbook on homosexuality, in 1897, co-authored with J.A. Symonds, appearing the preceding year in German, banned by Symonds' literary executor; the next edition was banned as an obscene publication in the 'Bedborough Trial'. Finally, a new American edition was released 1901, updated in 1915 to take account of Freudian theories of sexuality. The original 1897 edition contained 33 original case histories of homosexual men and women, and was an important text in the fight against the legal oppression of homosexuality in England. A 2007 edition includes Symonds' appendix on Greek love adapted for the original publication [55].

11.4. *The Modulation of Sexual Brain Dynamics*

The idea of homeostatic balance in living systems goes as far back as Hippocrates. Physiological Homeostasis is an issue central to the study of feedback mechanisms in evolution theory and is defined as the property of a living system to self-regulate itself under conditions of variable inner and outer environments in order to maintain metabolic stability [45].

The form of Genetic Homeostasis coined by Lerner [45], we will call *Strong Genetic Homeostasis*. It refers to standard *mechanistic* [1] Darwinian genetic evolution (statistical probability) mediated by global ecological and sociological conditions in interbreeding Mendelian populations. What we will introduce here for discussion, we call *Noetic Epigenetic Homeostasis* referring to unified field effects on consciousness that do not act directly on the physiology at the biophysical level as associated with current developments in epigenic mechanisms of histone modification in gene activation that modulate specific structures – all said to be caused by various stressors; but instead act on the mind considered to be a physically real field of awareness. It is this experimentally testable concept [4, 39] that allows transmutation of archetypes of Jung's collective unconscious; which in terms of the purpose here, in principle follows the reversal of the Earth's geomagnetic field (Section 9). Universal physical principles are involved in this action that by conformal scale-invariance apply equally to the *Sun-Earth-Dynamo Core* system as to the Mother-Father-Embryo system [56].

Some current thinking classifies stressors related to consciousness in terms of external biological action related to what is currently called neuroplasticity of the brain. This effect suggests that behavior and experience alter brain structure and function so that anatomical and biochemical differences would be expected for homosexuals and heterosexuals. Evidence of this neuroplasticity in terms of sexual preference was researched by Breedlove [57] in terms of the spinal nucleus of bulbocavernosus (SNB). Androgen can permanently masculinize the SNB; but curiously this early influence seems to depend on social factors and there are indications that the plasticity in the SNB system is lifelong.

The internal action of this *noetic effect* that we are introducing can fit under the epigenetic umbrella, but demands a new category since its operation is not in the 4D space of living systems as currently described by biophysics or biochemistry.

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[1] An essential factor of Darwinian evolution is *Biological Mechanism* – The laws of chemistry and physics are sufficient for describing living systems, no additional life principle is required.

The sex organs are indeterminate at conception; certain processes must occur to finalize the genetic traits. All embryos would be feminized without actions occurring at very specific times in embryonic development. One process allows the Y chromosome to form testes, later another action inhibits the formation of fallopian tubes; and finally testosterone stimulates development of the vas deferens. These physiological processes producing the eventual outward manifestation of sexuality in the adult are controlled by genetic mechanisms; but there are environmental, pathological and mutant conditions that can alter these processes to varying degrees. These outward characteristics do not necessarily correspond to sexual preference and are not necessarily pertinent to our discussion about '*Noetic Epigenetic Homeostasis*'.

Section 6 concluded that although the homosexual matrix may have concomitant psychological disorders the primary cause is not mental. Here it is seen also that there is no evidence for a genetic basis for homosexuality; but there is considerable evidence for a *homosexual biology* with prenatal causation by psychologically activated homeostatic mechanisms under genetic control. It has been known for some time that learning, experience or psychotherapy alters the neural pathways of the brain. This means that there is a psychoneurobiology of the brain [58, 59] and that if the sexual tendencies laid down prenatally are acted upon physiological changes will occur in the brain [57]. This sexual dimorphism of the brain has been known for about 10 years in relation to the hypothalamus, the corpus callosum and the amygdala.

The basis of our thesis is that homosexuality is neither psychological or genetic; yet individuals with homosexual tendencies are still *born that way*! If the root of sexual preference is not psychological or genetic; what is it? And how can someone still be *born that way*? The origin of sexual preference is a factor of the unified field of 'consciousness', an aspect of the 'vital force' interacting within the 'psychosphere' [3, 49]. This might not make sense at first glance because the common understanding of consciousness is either:

1. The state of wakefulness or
2. The content of the mind.

This conclusion requires a profound new definition of consciousness and a description of an anthropic cosmology of the universe where it operates.

12. Evolution from Classical Freudian to Integrative Noetic Psychology

The 120-year evolution of Psychoanalysis from its 1895 inception lays one of the major foundations for the basis of the Scientific Origin of Sexual Preference (SOSP). It contrasts the history of psychoanalysis with contemporary psychological concepts that rely on the currently dominant cognitive approach. The cognitive model has sought to understand consciousness as a system of neural correlates in the brain. This is called the mind-brain identity hypothesis. The cognitive approach has failed in providing a complete explanation of the nature of consciousness and is therefore incapable of modelling the SOSP from either a psychiatric or biological perspective. But as we will show in ensuing sections these are not the approaches where the answer lies. Sexual preference relates to the structure and operation of the soul ("*The spirit and the body are the soul of man*") [60] and component Jungian archetypes [13, 14]. By this definition of soul with a component cognitive domain, the psychosphere [3] is a physical field, which as such, is mediated by forces [39].

The currently dominant view of mind and body called Cognitive Theory goes so far as to call understanding the mind a 'hard problem' beyond the current tools of the scientific method [3, 61, 62]. With the recent discovery of the mind by the tenets of noetic cosmology (compatible with LDS doctrine) new principles are introduced applicable to explaining conditions relative to the SOSP. Transpersonal Psychology is the only school of psychology that can currently be merged with anthropic noetic principles as they are excluded from contemporary allopathic (scientific) medicine (which includes psychiatry) and the cognitive approach to biology and psychology. We call the resultant of this long-anticipated evolution in scientific terms - '*Integrative Noetic Psychology*'. Noetic Psychology is a new discipline integrating the soul and spirit of God in addition to principles of biology, transpersonal psychology and a new physical cosmology that includes God. The theological basis of the noetic model can be seen in the 88th section of the Doctrine and Covenants: "*The spirit emanates from the throne of God*…" [60].

13. Advent of the New Science of Consciousness – Psychophysical Bridging

Until now the so-called Neural Correlates of Consciousness (NCC) posited by the cognitive approach to the mind-body problem [12] have remained obscure and the concept of Psychophysical Bridging has remained little more than a philosophical argument supporting the theory. But with the discovery of mind by noetic field theory (NFT) this is about to change [3].

About a hundred years ago when western allopathic (scientific) medicine began to be highly successful that last remnants of a life principle or *spirituality* were removed and banished to primitive foolishness or called *folk medicine*. However as we hope the reader already knows there is significantly more to the human condition than currently covered by the treatment philosophy of the tenets of western allopathic medicine as applied in terms of the pharmacopeia and surgical techniques of psychology and biology.

About a hundred years ago our understanding of the universe switched from Newton's classical mechanics to quantum mechanics creating a new age of discovery. Now we are in the process of incorporating unified field mechanics leading to the next age of discovery which includes 'Discovery of the Mind' [3, 63].

Some of the diseases and abnormal conditions covered, like the approximately 400 autoimmune conditions are currently considered incurable by allopathic techniques; but this is about to change with the empirical introduction of a new nonlocal action principle that effects the evolution of living systems. This new class of diseases will be seen as disorders of consciousness.

Until now it has not been possible to define consciousness (awareness) other than as the abstract content of the mind or the state of wakefulness. The actual nature of 'consciousness' is much deeper than the confines of 'black box' psychology and mechanical and biochemical aspects of biology and will be shown in ensuing chapters to require a new physical principles entailed in a whole new physical cosmology for explanation. Historically just as quantum principles were unavailable to the tools and understanding during the age of Classical Mechanics, likewise the new principles of Unified Field Mechanics are now immanent new tools.

The advent of Noetic Psychology which introduces a better definition of the nature of living systems based on the work of Maturana and Varela [64] called Complex self-organized Living Systems by introducing a cosmological basis for an Anthropic Principle promises to elevate Psychology from an art to a hard, physical science because the life principle is a physically real empirically testable action principle amenable to engineering techniques. The main reason a cosmology of mind allows this to happen is because as philosophers of mind like Chalmers [61, 62] say, 'the problem of qualia is equal to solving the problem of consciousness itself'. Qualia is defined as the feeling of awareness or the sensation of experiencing 'redness' for example. As outlined more clearly in later chapters, the noetic theory is able to physically quantify qualia and go so far as to breakdown the 1st person 3rd person barrier [3, 4, 39, 40].

13.1. *Introduction to Integrative Noetic Psychology*

It could be said that Integrative Noetic Psychology is a branch or unifying force of the school of Transpersonal Psychology. Transpersonal Psychology is the only major school of psychology that has come close to introducing a life principle; however, it is not until now that the life principle can be formalized physically. Transpersonal Psychology is a young discipline that began with founding *The Journal of Transpersonal Psychology* in 1969 and the Association for Transpersonal Psychology in 1971; it draws upon mystical principles drawn from multiple spiritual traditions. Transpersonal psychology attempts to integrate timeless wisdom or the so-called Perennial Philosophy with modern Western psychological methods and translate spiritual principles into scientifically grounded, contemporary language to addresses the full spectrum of human psycho-spiritual development from the most basic fundamental temporal human needs, to the highest existential crisis of the human being, to the most transcendent eternal capacities of the evolution of consciousness. The Perennial Philosophy claims that if there is a God he has provided a path for us to find him. Sadly however, currently the field of Transpersonal Psychology as become saturated with vapid *New Age* pseudoscience, essentially losing the vision of its founding light, A.H. Maslow [65].

Another key element added to Integrative Noetic Psychology is the principle of nonlocal quantum physics related to what physicists call entanglement between quantum states. In terms of Jungian psychology this was the basis of a relationship called Synchronicity [66]. Synchronicity is another key element in our SOSP model as we will show synchronicity in the collective unconscious of husband and wife can have a positive or negative effect on embryonic development.

13.2. *A New Ontology of Awareness*

The perinatal matrix is a four-stage experiential sequence up to and including birth that continues to 'resonate' throughout the lifespan, generating a subliminal bias for conscious action and precipitating re-enactive behavior whenever events in the external world reactivate the unconscious emotional legacy of each critical stage of morphogenesis - source of that unconscious alienation that is our common heritage of birth [67].

It turns out there is significantly more to the human condition than currently covered by the allopathic treatment philosophy of western medicine as applied to human psychology and biology. This monograph introduces another whole class of etiological conditions relating to another whole domain of human ontology with its own set of causal action and resultant effects. We will call these ontological conditions to define a duality between the current outer 'phenomenological' domain and the newly defined inner or deeper 'ontological' domain of UFM. Some of these ontological-autoimmune conditions like colitis or Alzheimer's disease [59, 68, 69] are currently considered incurable; but this will change as noetic theory advances. The new class of diseases will be seen as disorders of consciousness as mediated by the life principle inherent in the unified field [70, 71].

This is partly a definitional problem; consciousness is currently not defined other than as the abstract non-physical content of the mind or the state of wakefulness. The actual nature of 'consciousness' is much deeper than the confines of psychology and biology and will be shown in ensuing sections to be a whole physical cosmology. Let's be very clear here as this is one of the essential points of this paper: Noetic theory introduces a whole new ontological domain of existence not part of current biological and psychological theory which form the basis of medicine. This arena has historically been part of philosophy and theology; but because of the lack of a complete model or empirical evidence it has been marginalized or ignored by scientists, justly so by scientific definition.

The APA for the most part succumbed to political pressure and only by a fluke turned out to be correct; but for the wrong reasons. While sexual orientation turns out not to be a psychiatric condition, as we are about to delineate, certain conditions of familial psychosocial make up still provide key causal factors in creation of the homosexual matrix from the noetic point of view.

Likewise, there are biological traits with associated genetics, but these are more superficial genetic attributes paralleling sexual orientation and related to plasticity - not a genetic cause itself, but heretofore believed to suggest a genetic causality.

The new discovery is that there are factors in the human condition that are deeper than psychology and biology. Factors relating to physically real aspects of consciousness itself. The noumenon of consciousness as opposed to the phenomenological limit considered until now. This is the key foundational issue. While it is currently asked 'what processes in the brain give rise to mental phenomena' [61, 62], a model over 93% of scientists embrace; this is a naive position that makes understanding consciousness impossible [3, 6]. This current model defines mind as an abstract and non-physical emergence of neurodynamics with no extra-corporeal properties [72] a hard problem too difficult to research [61, 62].

Here, according to the noetic theory we are about to introduce, consciousness is more than brain - it is a whole physical cosmology that includes a vital field or *elan vital* with a causal action deeper than that described by the current incarnations of psychology and biology which medical science is based on. It is in this new domain that sexual preference is mediated and can finally be understood!

14. Noetic Field Theory: Foundation for Cosmology of Mind in an Anthropic Multiverse

Recent popular literature [73, 74] lists consciousness among the great unanswered questions; and as science's greatest mystery. "*It will be our proudest achievement if we demystify consciousness*" [75]. To accomplish this the very foundations of science itself need to be ripped apart and reformed [3, 7] to include a broader metaphysics [76, 77] to reach the deeper ontology of the noumenon of consciousness.

Noetic (ancient Greek term *nous* meaning mind), is a discipline that embraces science, philosophy and theology and offers a framework for a potential solution to the problems of consciousness. There is ultimately more to reality than currently allowed by the standard models of science. The deeper Unitarity is not currently accessible because of the measurement problem [78] and calls for a new ontological empirical metaphysics able to access nonlocality experimentally, not just type-I & II as described by the standard model but also a type-III nonlocality of the unified field [84, 85] which is a key element required for understanding the cosmology of consciousness and mind.

14.1. *The Noetic Model of Mind*

The tenets of Noetic Field Theory (NFT) [3-8, 39, 40] suggest that: The mind, $\left|\Psi_M\right\rangle$ is a continuous state cycle with a complementarity of continuous and discrete properties including a dualism of both local temporal and nonlocal eternal aspects [70, 71]. The 5D Planck-scale Kaluza-Klein dimension is cyclical, however, NFT cyclicality is large-scale additional dimensionality (LSXD). This noumenon of consciousness is composed of three (Fig. 11) main integrated dynamic self-organized base states:

1. Elemental intelligence, $\left|\psi_e\right\rangle$, a nonlocal bound of individual existence ("to all things are given a bound or they could not abide" [86].

2. Cosmological ordering principle, $\left|\psi_C\right\rangle$ which is related causally to the vital noetic field that is mediated by a photon/graviton [7, 83, 87] called the noeon. The dynamic flux of the noeon field between the nonlocal eternal bound of elemental intelligence and local quantum brain dynamics provides the UFM spark of life and the light of the mind.

3. The brain, defined as a classical apparatus or transducer of temporal sensory input, $\left|B\right|\psi_b\rangle$ into the nonlocal seat of the mind.

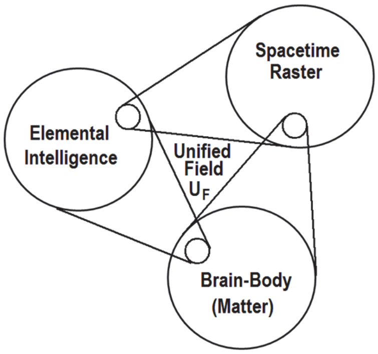

Figure 11. Unlike cognitive theory where mind = brain is a single entity, according to noetic interactive dualism, mind has a triune complementary structure.

The base states of mind interact at the quantum and nonlocal levels as described generally in axiomatic equation (1) or as the sum represented in (2)

$$\Psi_M = \left|B\right|\psi_b\rangle + \left|\overline{\psi}_e\right\rangle + \left|\overline{\psi}_C\right\rangle \quad (1)$$

In equation (2c) N is the superimposed base states over complex spinor space Z.

$$\left|\Psi_M\right\rangle = \sum_i^{Z^\alpha \overline{Z}_\alpha} N_i \left|\Psi_i\right\rangle \quad (2)$$

Most scientists today believe that the brain is the organ of mind with neural processes as the basis of mental life [61, 62, 72]. This is incorrect! [79-81]. The brain is a form of naturally occurring conscious quantum computer [3, 6, 82] *ONLY*:

1. Managing physiology and
2. Acting as a transducer for sensory data to and from the extra-corporeal seat of consciousness like first postulated by Descartes.

Cartesian dualism has been incomplete only able to be rectified now in the comprehensive interactionist 'Noetic Field Theory' (NFT).

The complete arena for consciousness and the process of mentation is called the psychosphere. The structure of the psychosphere is the complex interacting bound or domain walls of the three noetic base states (Fig. 11), This is a 12D hyperstructure of three Minkowski spacetime packages, not just the one 3(4)D spacetime as in the standard model [3, 6, 83]. This domain contains the totality of an individual's mind and consciousness and the extent of its influence. This means that the noetic field $N(f)$ couples and mediates information between both the local brain/body quantum fields, and the nonlocal subspace activity within the boundary conditions of elemental intelligence. The transduction of data or interaction of mind and body occurs through quasi-particle formations that cohere into Bose condensation [78].

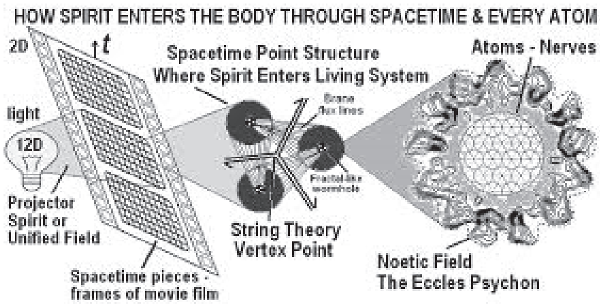

Figure 12. Mind does not reduce to brain, algorithm, or information processing as the current reductionist standard model of existence would have us believe.

Figure 12 is a conceptual view of noetic field theory. In the center we see that singularities do not reduce to Planck scale discrete points; but have a complex HD structure. The 2nd and 3rd 3-spheres are hidden from classical measurement by the uncertainty principle [3, 6, 83]. The domain is in continuous translation like the standing wave produced by plucking a guitar string. But because of the nature of time and the uncertainty principle these components are not readily observed much like the frames of film are not seen while watching a movie.

All matter in the universe is made of either Bosons or Fermions. Fermions are the substance of material objects and Bosons the substance of light-like quanta. Fermions must obey the Pauli exclusion principle (a main reason for uncertainty relations) so only one Fermi particle may occupy a particular point in space at any time. Whereas theoretically an infinite number of Bosons can occupy a particular spacetime position. Interestingly an even number of Fermions can provide a certain type of quasiparticle symmetry so that they can act as if they were Bosons. In simplified terms, this is the mechanism through an intermediary quasiparticle transition whereby external sensory or other physiological information in the Fermi brain states are transduced into the Bose content of the mind [5]. This is the fundamental basis of the substance dualism of the interactionist model. Intentional action of course is the reverse of this process.

This bidirectional pathway is a microcosm of general scale-invariance in the anthropic multiverse. Think of this Bosonic 'grid of light' as loci that pervade the brain/body confines of the individual psychosphere as a 'ball of mental light'. The nonlocal domain of the multiverse [7, 40, 63, 78, 83] is a hyperdimensional ball of light similar to a Wheeler geon [88, 89]. Wheeler proposed a classical geon as an electromagnetic field of sufficient size that it would cohere by its own gravitational self-attraction. In recalling Einstein's famous equation $E = mc^2$ equating energy and matter it might not be as difficult to imagine the supralocal realm as a huge hyper-hologram of pure UFM noeon light energy as a teleological ordering principle of matter, spacetime, and life in our 3D virtual subspace domain of reality.

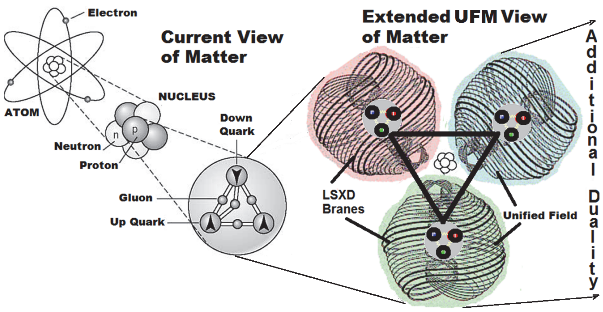

Figure 13. Passing by the 4D limit of the uncertainty principle reveals a radical new UFM M-Theoretic brane picture of matter.

In the standard Darwinian Big Bang model of cosmology, the higher dimensions (XD) were compactified ~15 billion years ago at the beginning of time. In the multiverse compactification is a continuous dynamic process occurring at every singularity in spacetime (Actually creating and recreating spacetime at each moment). This continuous collapse process of the 12D supralocal domain into our perceptual 3(4)D frame allows our temporality to "surf" as it were of the face of the HD supralocal eternity. This is also the reason for the fundamental stochastic barrier of uncertainty that limits our reality to the virtual 4D perceived. It is a veil between us and the higher realm. This also means that there is no need for inflation or expansion of the universe. This is a key element of an anthropic multiverse. Within this continuous cycle of collapse and recollapse, geons of light energy are the little wormholes or singularities that are the entry points of the self-organized vital noetic field providing life, the light of the mind, and order to matter and the large -scale structure of the universe.

For NFT dualism/interactionism is as follows: Eccles postulated the psychon as the fundamental mental unit that coupled mind to the dendron in the brain. Pribram did work suggesting that these dendrons formed a holoscape - a neural grid with holographic properties. Jibu and Yasue said that this grid has bosonic properties. Hameroff and Penrose formed a theory attesting to the quantum properties occurring in association with microtubules related to the neural nets. Walker and Beck said the same quantum processing could occur also at the synapse. All of these researchers except Eccles confine consciousness to the brain. What NFT has done is to integrate all of these pieces into a comprehensive physical theory [3] that formalizes the Eccles psychon. Until now vitalism has been cast out of science because it has not been integrated into a comprehensive dualistic theory. As will soon be more evident dualism with extra-corporeal aspects of mind with a vital noetic field provides the essential foundation for the model of sexual preference presented here. There can be no local domain that contains a Jungian collective unconscious, which as should be obvious by now, is key to understanding the origin of sexual preference.

15. Cosmology of the Noetic Psychosphere

The psychosphere represents the total domain of both mind and conscious awareness as it relates to a living entity [3]. It is a structural-phenomenological domain comprised of:

- A physical noumenon - the cosmological structure; and
- The associated phenomenology - the content of mental activity.

The psychosphere is comprised of a 12D hyperstructure [3] that includes not only the brain holoscape, but also nonlocal domains of elemental intelligence (in this case supralocal because nonlocality is a temporal subspace associated with the Copenhagen regime of quantum theory; and the domain of elemental intelligence is eternal with a root in causal separation from 4D reality). The psychosphere is the structural-phenomenological sum of all boundary conditions housing the mind. It includes the local complex multi-tiered holoscape of brain activity in complementarity with the standard nonlocal and additional unitary elements of mental activity. The noetic field is not just coupled to the brain but all cells, atoms, molecules and spacetime regions associated with a given living system. It is within this complex domain of the psychosphere that qualia can be described in physical terms [3-8].

The central tenets of Noetic Field Theory (NFT) suggest that consciousness is a quantifiable condition of cosmology, with both the mind and thought having complementary features in the sense promoted by Bohr; but as well-known Bohr's rendition of quantum theory was too limited to apply to biological systems. It is for this reason that NFT is required to utilize an extension of all standard model of science.

Equation (2) is a primitive generalization of a mental base state in the 1st term summed over Descartes *res extensa* and *res cogitans*, in the second part a generalization of the three base states comprising the triune nature of the least unit of awareness in noetic cosmology, and the 3rd rendered as a twistor singularity originating in nonlocal projective prespace. The equation shows linear sums for illustration purposes; in reality the expanded equation would have nonlinear characteristics to handle the complex self-organized mental action modalities. An ensuing paper presents a mathematical description of noeon action in terms of the holophote pumping field and includes higher dimensional modes in the light cone of reality [63]. Research avenues for noeon particle isolation are suggested. The mantra of NFT is: If one assumes that qualia is a tensor psychon, the leading light cone singularity is modulated by a phase of the noeon psychon field.

15.1. *Causation*

Causation is a challenging term for the physics of consciousness, particularly in complex systems with feedback loops and multifactor inputs. Furthermore, the agency of change in quantum systems is as fundamental as spacetime itself. The quantum concept of causality thus differs from its classical counterpart in that it necessarily links relationships between spacetime events, and thus has a kind of self-referencing aspect. Worse, it is essentially probabilistic at the outset, which makes cause less tangible. Freeman claims that chaotic dynamics can create information in the Shannon-Weaver sense. This finds its correlate as the Gabor logon. The logon is a kind of quantum of information first defined by Gabor and later used by Pribram [90] in the holoscape. The question remains as to whether consciousness originates from a kind of qualia recall panexperientalism at the level of the quantum domain. There has been general skepticism of quantum effects having any relevance to such a hot entropic matrix as the brain. When a dissipative structure open system such as the brain is pushed to the limit, a new structure can emerge from the fitness landscape. As such, a new template might emerge from lower order inputs, crystallizing into higher order structures which then superimpose limit cycles back on the chaotic regime.

Fröhlich's original idea was that dynamical equilibrium represented by a limit cycle could be tuned by chemical electrical stimulus and cause the collapse of the limit cycle. The triggered release of energy could then be harnessed to invoke large scale molecular events such as changes in the geography of QBD. A precondition for consciousness is the ordering and storing of information in the face of randomization in the quantum heat bath. The challenge is to see if quantum systems self-organize. Bose-Einstein condensates have the unique property of making coherent wholes by summing the behavior of many component parts which feedback on their elements and create a community. This speaks to the binding problem [3, 90] in consciousness where many neurons create a unitary self that doesn't seem reducible to any one part. When cell membranes vibrate sufficiently to be drawn into the Bose-Einstein psychon matrix they are forming a coherent whole which resists degeneration by thermal chaos. That is, something must supply the jiggling and something must supply the ordering - one arises out of the other and then feeds back through the system. If electrical activity of the neuron provides the energy to jiggle molecules which in turn emit photons, then these photons can synchronize jiggling and further photon emissions through superradiance. This is analogous to the pumping of a laser. The shift into the condensed phase depends on this molecular photon interaction. It is here where quantum wholeness radiates out over the entire structure. All this superstructure is built into the self-organized cosmology of living system.

16. Mind - Body: Casimir-Like Role for the Noetic Formalism

Science, physics especially, accepts nothing immeasurable as real. In this section, the Psychon is integrated with Pribram's neural wave equation (which is similar to the Schrödinger equation for a particle moving on a manifold with the addition of a term like the de Broglie/Bohm quantum potential for the neural potential [90] and the fundamental noetic equation [91-93] to provide an interactionist solution to the mind-body problem. Because mind is defined as a physical entity in the noetic formalism the putative interaction is open to empirical tests [94, 95]. Usage of the term Psychon as the unit of mental experience must be expanded because Eccles left the term mostly undefined as an empty philosophical construct. The phenomenological part of the Psychon unit postulated as quantities of Bose noeons gives physicality and work functions to thought processes and the Noetic Effect. The physical unit called the Einstein (one Mole or Avogadro's number - 6.02×10^{23} of noeons) is adapted for use as a measure of awareness signifying Bosons of the unified field which are probably spin 4 noeons [96]. We define the Noetic

Effect as the cosmological complex self-organized mind-body interaction process. This Noetic Effect will one day have profound influence on medicine and psychology [97]. The mathematical method used here to integrate the Eccles Psychon with brain dendrons and the existing holonomic theory of Pribram is a variation of the Lagrange operator of least action utilizing not only the static Casimir effect [99], but a duality between its dynamical counterpart first described by Schwinger [98]. In this context, the noetic effect governs energized boundary conditions as described by the Noetic Field Equation $F_N = \aleph / \rho$ [91-93] which takes the same form as the string tension formalism and can be said to be an alternative derivation of it [92]. F_N is the noetic force, $\aleph$ the nonlocal Casimir energy in UFM noeons and ρ the coherence length of the associated domains.

The thrust of this treatment is to account for the action of spontaneous particle production, in this case Bosons, from the zero-point energy fluctuations of the polarized Dirac vacuum, as noeons, and from the zero 'vacuum' potential as Psychons. These photons, rather non-radiative scattered Bose potentials confined like quarks to the HD topology of the spacetime metric can be constrained in a Gabor-like manner. The Casimir effect was initially used to account for the resonant force between separated parallel uncharged capacitance plates due to an all pervasive electromagnetic field in the interstice. This Casimir-Polder force is one example of a very general phenomenon in which objects impose boundary conditions on the quantized field; but as an extended Bohmian quantum potential (a super quantum potential [100, 101] the noeon of the unified field applies to both quantized and unquantized energy. The ones we are most interested in are the generalization to parallel interfaces between dielectric media as occurring in the holoscape dendrons and topology of spacetime. More recently Schwinger [100-103] has proposed a mechanism for the dynamical counterpart of the static Casimir effect based on the precise measurements of coherent sonoluminescence, where dielectric media are accelerated and emit light. In these experiments, a bubble in water, a hole in a dielectric medium, undergoes contraction and expansion in response to a strong acoustic field. Schwinger's Casimir function formalism utilizes a phase space similar to that of the Gabor function [104] to trap a psychon-noeon bundle and channel it into the previously described quasi states in the quantum holoscape where Bose condensates integrate in the Heisenberg matrix.

Sandwiched dipole polarizations account for pulsatile interactions of neurons. The network is composed of overlapping Gabor elementary functions generating a pixel like lattice of spacetime storing and processing information. The notion of perpendicularly arranged dipoles of polarization generated within dendrons [105] is intriguing. By modifying the Casimir effect [99] we can see parallels between the Eccles' Psychon and Gabor relation. Casimir outlined the influence of retardation on London-van der Waals forces between neutral atoms. Instantaneous dipoles account for interactions between electric double layers separated by large distances. The interaction energy of a neutral atom by analogy, here the Psychon noeon sheaf with a perfectly conducting wall and, the holoscape manifold is given by the atomic dipole with its image. Retardation effects are expected when the distance from the wall becomes large, according to Cavity-Quantum Electro-dynamics (C-QED). The asymptotic expression of ρ contains Planck's constant and the static polarizability of the atom as the only quantities. Casimir confined the neutral atom within a perfectly conducting plane wherein the eigenstates of the electromagnetic field are described by Maxwell's equations and treated as if the atom were a quantum particle in a box. The box in our case of dynamic-static complementarity is a system or domain of fundamental least units (a form of sphere packing tiling the spacetime backcloth) that are the continuously changing boundary conditions of the systems dynamics in FQB Translation. Total energy interaction between the wall and the atom is [99]

$$\Delta_t E = \Delta_d E + \Delta_e E \, . \tag{3}$$

Second order interactions of the atom with a radiation field give vector potentials which can be manipulated with the Heisenberg method where the electromagnetic field is treated as a matrix [99]. Perturbation of the radiation field by a charge assigns vector potentials as elements of the matrix, and uses a simplified wave equation for the oscillating dipoles. This method has been used to account for atomic spectra of helium Rydberg atoms, macroscopic conductors, long range atom surface interactions, dielectrics and liquid thin films. To understand the origin of the Casimir effect requires QED. It is well known that electromagnetic radiation is quantized photons, and that these emitted photons can interact with atoms. Radiation in free space can be thought of as a superposition of many modes of oscillation within a box of arbitrary size. The energy of each mode can be thought of as a harmonic oscillator and restricted with a set of discrete energy values. The level of spacing between energy states corresponds to one photon so that the emission of a photon is simply a process in which the energy field frequency is increased by one unit. We have experimental work in mind looking at the Aharonov-Bohm effect [106] and quantum Hall effect [107].

The quantum mechanical oscillator has energy gaps given by Planck's constant times the oscillator frequency and must have a minimum, called the zero point energy. These fluctuations become apparent in the Lamb shift due to a change in atomic energy levels attributable to proximity to the atomic nucleus. The force arising from vacuum fluctuations has been measured by Sukenik [108] and found to be modified by proximity to a conducting plate with no electric field applied. Near a conducting plate the number of modes of the radiation field are reduced by the boundary condition such that the electric field at the surface must be zero, so the atomic energy decreases close to the conducting surface leading to an attractive force. The van der Waal potential between two atoms, which begins as r^6 becomes a potential that varies as r^7 when the atoms are separated by distances greater than several Bohr radii.

A related prediction is that the interaction between a neutral atom and a conducting wall changes from an initial r^3 to an r^4 potential when an atom is far enough from the wall. This interaction can attract even neutral atoms to each other due to the quantum fluctuations. Classically the electric dipole moment of a neutral spinless atom is exactly zero but in quantum mechanics only the *expectation value* is zero. Probability allows that there can be a nonzero dipole momentarily. If a photon can propagate fast enough between two atoms their instantaneous dipole can be correlated and the result is an attraction or ordering between the two atoms. For distant atoms photon exchange time cannot be ignored. Sufficiently long intervals destroy the dipole correlation.

The limit, as always is set by the uncertainty principle which relates the lifetime of the excited state of the nonzero dipole energy to its energy. Beyond which neutral atoms can still interact via instantaneous polarization of the quantum vacuum. The vacuum fluctuations can be thought of as oscillators with wavelengths long enough to communicate with both atoms. It is no longer wavelengths that mediate the interaction as the distance increases. This separation introduces a 1/*r* multiplier to the potential *r*. The retarded force can also be considered a variance in the zero-point energy, a phenomenon evident in the Lamb effect. More importantly, although QED fluctuations for Maxwell's equations within a box can account for the Casimir force, one can handle these Casimir-Polder interactions with standard methods of quantum mechanical perturbation theory without resorting to zero-point energy. In this case, the long-range Casimir forces depend on the exchange of two photons leading to a format to integrate the Psychon with QBD.

Quantization of the radiation field by means of traveling waves with a period *L* can be written for the vector potential [99]

$$A = \sum_{k,\lambda} cC_k e(K,\lambda) \times \left[AK, \lambda e^{-i(\omega t - kr)} + e^{i(\omega t - kr)} \right] \tag{4}$$

to which the values of the components of the wave vector *k* are restricted. The elementary charge *e* is the perturbation parameter arising from the interaction of *G* of the charged particle with the radiation field. An electron in a stationery state does not radiate. The matrix elements, with *G* as a perturbation operator, can be written for the zero-state consisting of the radiation field and an atom as [99]

$$\Psi'(0;0...) = \Psi(0;0...) + \sum_{n,k,\lambda} \frac{eC_k(e(k,\lambda)p_n;0)}{k_n + k} \Psi(n;0...1_{k\lambda}...). \tag{5}$$

In Anthropic Multiverse cosmology [109, 110] there is an inherent continuous-state acceleration (an alternative interpretation of the Big Bang expansion/inflation scenario) occurring as part of the compactification process of our virtual reality.

The electrostatic interaction between neutral atoms *A* and *B* is shown in eq. (5)

$$Q = \frac{qAqB}{R^3} - \frac{3\left(qA^R\right)\left(qB^R\right)}{R^5} \tag{6}$$

The second order perturbation energy can readily be shown as [99]

$$\Delta_q E = -\frac{1}{R^6} \sum_{1,m} \frac{(q1^x qm^x)^2 + ((q1^y)^2 + 4(q1^z qm^z)^2)}{hc(k1 + km)} \tag{7}$$

As noted, the thrust of this treatment is to account for spontaneous particle production, in this case photons, from the zero-point energy fluctuations of the quantized vacuum. Recently Schwinger [98, 102, 103] has proposed a mechanism for the *dynamical counterpart of the static Casimir effect* based on the precise measurements of coherent sonoluminescence, where dielectric media are accelerated and emit light. The commonality for static and dynamic Casimir effects are probability amplitudes for preserving the photon vacuum state as illustrated in eq. (8) [98].

$$\langle 0t_1 | 0t_2 \rangle = \exp[iW_0] \tag{8}$$

Light emission occurs by the reversible collapse of a cavity in a dielectric medium into a vacuum. Schwinger's starting point is the action W a resultant of scalar electric e and magnetic m fields where X is the spacetime dielectric constant in eq. 8 below [98]

$$W = \int (dX)\left[\frac{1}{2}\varepsilon(X)(\partial_0 A)^2 - \frac{1}{2}\left(\overline{\nabla} A\right)^2 + AJ\right]_e + \int (dX)\left[\frac{1}{2}\left(\partial_0 A\right)^2 - \frac{1}{2}\frac{1}{\varepsilon(X)}\left(\overline{\nabla} A\right)^2 + AJ\right]_m \tag{9}$$

A and J are related by a Green's function which eventually leads to the volume nature of this effect. Conditions under which volume effects dominate surface effects during photon pair production can be formulated by the differential equation [102]

$$\delta W_0 = -T\delta E = \frac{1}{2}Tr\left[\delta_0 \delta\varepsilon \partial_0 G\right], \quad G = \left[\partial_0 \varepsilon \partial_0 - \nabla^2 - i0\right]^{-1}, \tag{10}$$

in which *0* is the toward zero approach from positive values.

The dialectric energy relative to vacuum zero point is derived as [102]

$$E = -V\int \frac{(d\overline{r})(d\overline{k})}{(2\pi)^3}\frac{1}{2}k\left(1 - \frac{1}{(\varepsilon(\overline{r}))^{1/2}}\right) \tag{11}$$

where the Casimir energy is negative for a uniform dialectric medium. The energy relation of the two dielectric regions is proportional to the volumes where $1/e^{1/2}$ demarks the $e > 1$ area from the vacuum.

We propose that The Noetic Effect, through the mediation of the noeon, couples an active psychon to its holoscape dendron, the dielectric medium of the brain. This will release a Casimir energy potential for binding the psychon to a donor acceptor cavity of mixed states akin to that found in spin glasses. The release of the Casimir energy potential parallels the electromagnetic emission of photons or the scattering of photon energy into the oscillating dipole medium during cavity translation. According to Schwinger [102] the average number of photons released for cavity radius R is revealed in the equation

$$N = \frac{4\pi}{3}R^3 \int \frac{\left(d\overline{k}\right)}{(2\pi)^3}\frac{1}{2}\left(e^{1/2} - 1\right) = \frac{1}{9\pi}(RK)^3\left(e^{1/2}\right). \tag{12}$$

A key consideration about this relation is the experimental tact that the force is measurable. It is not merely speculation about quantization of zero point energies. Also, finding a likeness between parallel plates and microtubules is not much of a stretch. Arthur Young's suggestion that the photon as the principle of action is synonymous with purposive behavior is relevant to our discussion. This teleological aspect of light derives from the idea of least action, which in turn comes from 'wholes' and first causes. Action is the whole, of which the three parameters mass, length and time are parts. First promulgated in 1976, we see Young's idea as prescient. Young develops a hierarchy where the uncertainty of the photons, or quantum of action, is its capacity to cause something new, i.e., within light is the essence of causality [111, 112].

17. Epigenetics

Epigenetic mechanisms are affected by several factors and processes including development in utero and in childhood, environmental chemicals, drugs and pharmaceuticals, aging, and diet. DNA methylation is what occurs when methyl groups, an epigenetic factor found in some dietary sources, can tag DNA and activate or repress genes. Histones are proteins around which DNA can wind for compaction and gene regulation.

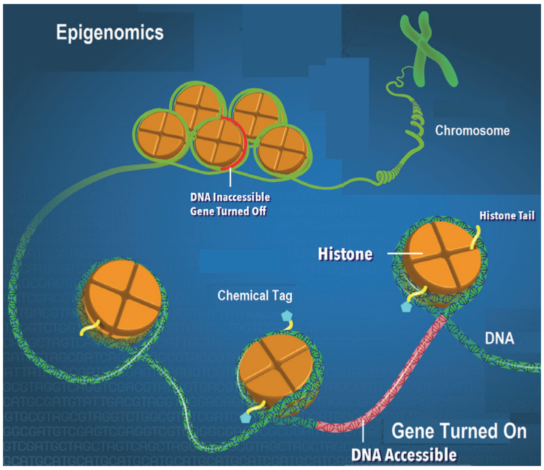

Figure 14. Histone modification occurs when epigenetic factors bind to histone *tails* altering the extent to which DNA is wrapped around histones. NIH PD figure [113].

Histone modification occurs when the binding of epigenetic factors to histone "tails" alters the extent to which DNA is wrapped around histones and the availability of genes in the DNA to be activated. All of these factors and processes can have an effect on health and influence health possibly resulting in cancer, autoimmune disease, mental disorders, or diabetes among other illnesses.

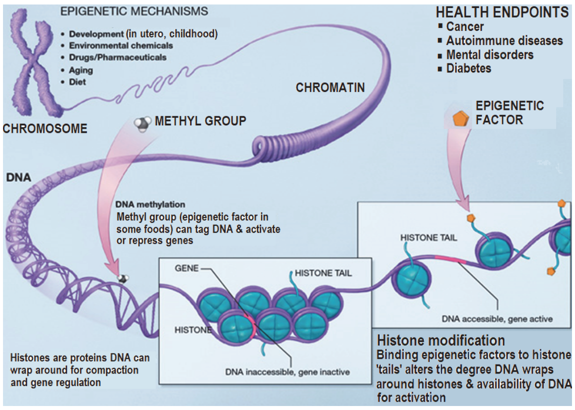

Figure 15. Epigenetic mechanisms occurring when epigenetic factors bind to histone tails altering the extent to which DNA is wrapped around histones. NIH PD figure [113].

Transgenerational epigenetic inheritance is the transmittance of information from one generation of an organism to the next (e.g., parent-child transmittance) that affects the traits of offspring without alteration of the primary structure of DNA (i.e., the sequence of nucleotides) [114, 115] in other words, epigenetically. The less precise term *epigenetic inheritance* may be used to describe both cell-cell and organism-organism information transfer.

Four general categories of epigenetic modification are known: [114]:

1. Self-sustaining metabolic loops, in which an mRNA or protein product of a gene stimulates transcription of the gene;
2. Structural templating in which structures are replicated using a template or scaffold structure on the parent; e.g. the orientation and architecture of cytoskeletal structures, cilia and flagella, proteins that replicate by changing the structure of normal proteins to match their own;
3. Chromatin marks, in which methyl or acetyl groups bind to DNA nucleotides or histones thereby altering gene expression patterns;
4. RNA silencing, in which small RNA strands interfere (RNAi) with the transcription of DNA or translation of mRNA; known only from a few studies.

18. Epigenesis and Beyond: Epigenic Mechanisms of Histone Modification by the Noetic Effect

The epigenome is involved in regulating gene expression, development, tissue differentiation, and suppression of transposable elements. Unlike the underlying genome which is largely static within an individual, the epigenome can be dynamically altered by external and as we propose UFM noetic conditions involved as a form of periconceptive trauma on nonlocal field parameters such as such as physically real aspects of archetypes of Jung's collective unconscious. Periconception, meaning around conception, is usually considered to be 2 to 3 months before and after conception and the beginning of pregnancy. Within the first few days of conception the embryo has been found to be susceptible to the influence of fields.

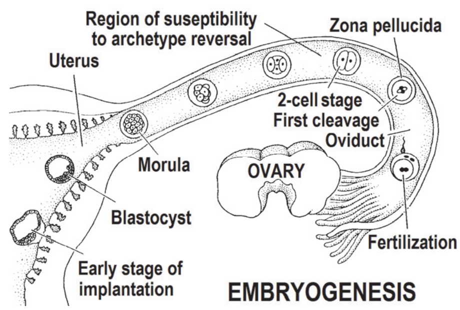

Figure 16. Human embryonic development from fertilization to implantation in the uterus where the egg 'hatches' from the zona pellucida (which probably prevents adhering to the oviduct) when it reaches the uterus. Figure adapted from [116].

Although all cells in the body contain essentially the same genome, the DNA marked by chemical tags on the DNA and histones gets rearranged when cells become specialized. The epigenome can also change throughout a person's lifetime by the mechanisms shown in Fig. 15.

Lifestyle and environmental factors (smoking, diet and infectious disease) cause stressors that producing chemical responses. These responses, can lead to changes in the epigenome, some of which are damaging. But, the ability of the epigenome to adjust to life stressors appears to be required for normal health. Some diseases are caused by malfunctions in the proteins that read and write epigenomic markers.

18.1. *The Force of Dissonance*

Another simple metaphor that helps illustrate the 'attraction - repulsion' affect between people can be elucidated using two magnets. For people, usually opposites attract and like sexes repel; which is also true of magnets. A reversal in the noetic field of the anima - animus produces the tendency to cause persons of the opposite sex to seem repulsive giving rise to the 'apparent' attraction of homosexuals to partners of the same gender. Thus, we get a glimpse of both the complexity and tragic difficulties of the homosexual matrix. (Elements of intelligence cleave unto like elements of intelligence according to certain physical rules.) As in the solar flare metaphor where pulsing magnetic fields acting on the Earth's dynamo core periodically reverse the North and South poles, stones dropped in a pool of water creates smooth ripples; while two stones create areas that enhance each other or destructively interfere with each other.

Human beings have basic innate internal needs as part of both their physiology and psycho-spiritual makeup. These needs are coupled to feelings and sensations; people feel well or have a sense of fullness or happiness when these needs are enhanced or satisfied. When they are not satisfied, people feel down, ill, uncomfortable.

Consciousness contains a real physical field similar to that of the magnets when the spins of the internal atoms are given a specific uniform orientation. Therefore, a gay person does not typically feel comfortable with intimacy with the oppose sex; but rather feels 'attracted' to or 'enhanced' by a same gender individual. This is related to the fact that 'all spirit is matter' which creates a physical presence within us that produces an action similar to that illustrated in the above metaphors. There is a price to pay for this reversal; and many reversed gender individuals are also troubled by emotional issues; which is another complex issue only given brief mention in this paper.

The aversion effect can also be illustrated in terms of dropping two stones in a pool of water. In some places the water waves 'summate' or enhance each other; and in other places there is destructive interference or 'cancellation' of the waves altogether. The summation occurs when the waves are in phase; and the destructive interference occurs when the waves are out of phase. In terms of the noetic field the 'light front' is actually destructively interfered with, which creates the feelings of cognitive dissonance when intimate coupling is attempted with polarity of obverse phase. The current neural basis for mind theories do not contain such telergic effects. More will be said about telergy later.

The author realizes the difficulty in accepting and comprehending the ultimate action of the light of consciousness at this point in the development of noetic theory, especially as experimental confirmation is awaited [4]. There are many levels to the domain wall barriers and gates that mediate the flow of conscious energy. Imagine two individuals approaching a dwelling with the intent on entering and having intimate relations with the occupant. At the point each starts up the walkway there might be no measurable difference between them. One will be welcomed warmly and allowed to share the most intimate aspects of human existence with their partner or spouse; the other a stranger will engender the most violent reaction possible.

We have discussed in general terms the mysteries of handedness and explored the fact that while there seems to be familial predispositions, these dispositions do not appear to be genetic. It is also possible to wear a special set of glasses with lenses that invert the view of the external world. After a relatively short time the mind adapts and the world is again seen right-side up while wearing the inversion glasses. If the basic premise of noetic theory is correct, that a physical unitary vital field not only pervades all matter and spacetime, but also self-organizes it from the hyperdimensional realm; then there is a whole universe of conscious phenomenology to deal with and which clearly has telergic effects to explore and account for.

An instant of telergic commerce also defined as the *Veneration Gap* where subject one, S_1 is noeticly dominant over S_2 which dramatically changes the flux boundaries of the wormholes mediating the loci of the noetic field. This changes the position of the *firmament* between them. People appear separated in the 3-space of our perceptual reality; but in the additional nonlocal dimensions, wormhole-like topological branes make connections that allow for the interpersonal exchange of the noetic field. There is no association or communication without the commerce of the noetic field. The dominant individual changes the energetic boundary conditions that mediate the flow of the noetic field between them. S_2 does not have easy access to the interpersonal higher flux modes; while S_1 has full access to a significant portion of S_2 because of the firmament S_1's dominance creates in the interpersonal nonlocal space. This may seem incomprehensible to many at first because it is outside the realm of experience and not yet incorporated scientifically; but it is an essential aspect of noetic theory. Also when well-known not only will people have to treat each other with more 'veneration', but be expected to do so. This will be left to social mores of the future; at the moment, we are only interested in the aspects of this condition of the 'Law of Hierarchies' that relate to the prenatal reversal of sexual preference.

19. The Ontological Origins of Sexual Preference

A further discussion of the veneration gap is needed before going on. In the ideal case of elemental intelligence where there is no separation or hierarchy, these 'gap' effects do not occur because of the perfect harmony or unity. Meaning that when the veneration gap does not exist, there is perfect unity and balance of the self-organizing teleology of the noetic field. But in the real world with the disparities of the human condition the effect has a range from negligible to dramatic in non-normative interactions of people with personality disorders.

The anima and animus, in Jung's school of analytical psychology are the two primary anthropomorphic archetypes of the unconscious mind. The anima and animus are described by Jung as elements of his theory of the collective unconscious, a domain of the unconscious that transcends the personal psyche. In the unconscious of a man, this archetype finds expression as a feminine inner personality: anima; equivalently, in the unconscious of a woman it is expressed as a masculine inner personality: animus.

The anima and animus can be identified as the totality of the unconscious feminine psychological qualities that a man possesses or the masculine ones possessed by a woman, respectively.

Jung focused more on the man's anima and wrote less about the woman's animus. Jung believed that every woman has an analogous animus within her psyche, this being a set of unconscious masculine attributes and potentials.

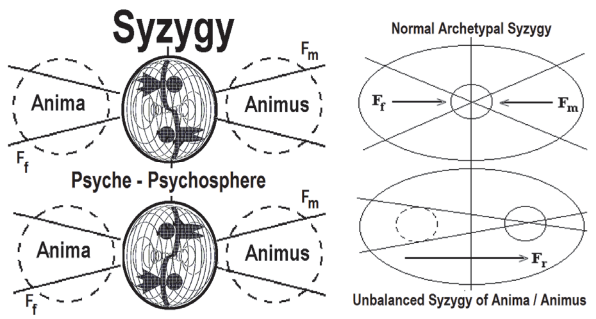

Figure 17. a) Conceptualization of the Psychosphere of a woman top, and man bottom, showing the nonlocal and local hyperstructure where action of the noetic field orders and maintains the dynamics of the psyche and the soul. The Psychosphere is the cosmological bound of an individual's mind - brain, elemental intelligence and spirit or noetic field, which is mediated by the Unified Field through a system of hyperdimensional wormholes. F_f and F_m are noetic force of father and mother respectively. b) Top, normal Syzygy of the Anima & Animus where the psyche, psychosphere and telergic effects are in balance. (Astronomical Syzygy occurs when the Earth and two other celestial bodies are in a straight line of either opposition or conjunction).

Because of the complexity of the origins of sexual preference; analogy was used to conceptually model aspects of the framework. Firstly, that handedness, a condition traditionally considered genetic, can be changed with concerted effort. It was pointed out that handedness is probably learned under the causal influence of either environmental conditions or the result of prenatal stressors triggering familial dispositions.

Secondly, and most pertinently, we discussed the conditions involved in reversal of Earth's geomagnetic field; and will show how this correlates with the mechanism reversing sexual orientation when described in terms of the Anima & Animus of Jung's collective unconscious. The 3rd metaphor illustrated how interference in the noetic field can create a dissonance that can act as a force to drive the noetic field with a tendency toward polarity reversal. The domination of classical naturalistic science since Galileo has made significant strides in the development of the biological embryology of the day. But noetic theory, the cosmology of consciousness, demands a more comprehensive 'Embryology of the Soul'.

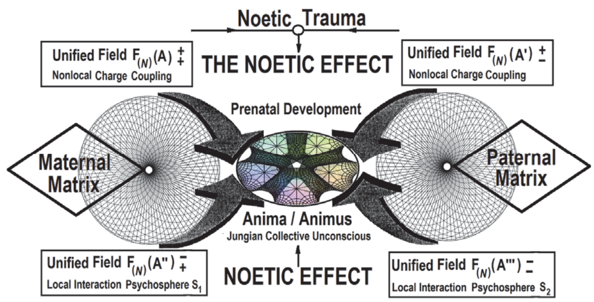

Figure 18. Action of the Noetic Effect on prenatal development is a dynamic resultant of the nonlocal noetic field, and the local action of the psychosphere and telergic factors.

Genetics determines gender upon union of egg and sperm; and after a number of weeks of embryological development the outward physical appearance of the body starts forming. But an understanding of the conscious universe, for the first time, precipitates an understanding of how our eternal spirit enters our body from the supralocal unitary domain of the pre-existence and takes residence within our earthly body to form our soul. Now we are ready to apply the parameters of noetic theory directly to the formation of an individual Psychosphere and discuss how certain critical conditions can cause a reversal of pertinent aspects of the vital noetic field leading to a reversal in the orientation for sexual preference.

The noted psychologist Donald O. Hebb stated:

"*Modern psychology takes completely for granted that behavior and neural function are perfectly correlated ... There is no separate soul or life-force to stick a*

> *finger into the brain now and then and make neural cells do what they would not otherwise...It is quite conceivable that someday the assumption will have to be rejected. (* neural basis) But it is important also to see that we have not reached that day yet...One cannot logically be a determinist in physics and chemistry and biology, and a mystic in psychology*" [117]. (*editorial note added)

The theory to be presented here the 'day' Hebb anticipated. The noetic cosmology of consciousness is a field model, much like electromagnetism; and this similarity is used to illustrate the 'spiritual' aspects of personality traits. The spirit (or *élan vital*) in the body - is the soul of man; and interestingly the Earth system can be used as a metaphor for illustration of its structure: The solar and galactic effects are reminiscent of external influences, the geomagnetosphere represents external manifestations of the psyche and thought processes, the liquid core dynamo is like the Jungian collective unconscious, and the magnetization of the rock in the mantle represents personality traits. This is a reasonable metaphor for illustrating the noetic theory of mind [5, 83], to be further elucidated in future writings.

The crux of the explanation of the origin of sexual preference arises from the action of a stressor called the 'noetic effect' caused by a combination of environmental, spiritual, and psychological factors in a manner surprisingly similar to the factors effecting reversal of the Earth's magnetic field. This 'noetic effect' can cause a torsion driven reversal in the cosmology of the noetic field at a key point in prenatal development. This happens at a stage in the process of hierarchical ordering and polarity formation of the 'spirit in the body' for this particular personality archetype. Thus, causing a man to have a more feminine disposition and needs, and a woman more masculine needs if this ordering of the anima/animus field is reversed.

The new cosmology made it evident that the purpose of the higher or extra 8D is to mediate and interface the dynamics of the eternal-unitary with the subspace of our 4D reality. The significance of this fact is that the complementarity between body and soul at the local and supralocal domains. Because of the nature of time [7] and the complexity of hyperspatial geometry there is an orientation and dynamics to the gate within which energy propagates. This shows the structure of a point or singularity in the cosmology of noetic theory. This is dramatically different than the Planck scale compactification barrier of the current big bang model. As in the new model of the photon, the singularity is a wormhole system open to flux from all dimensions. Most profoundly this 'flux gate' is controlled by the photon-graviton complex and mediated by its teleology!

The noetic personality profile that exists in a family that can produce a noetic stressor; the action of which can prenatally reverse the anima and animus of the collective unconscious during embryonic development. Currently no adequate psychological tests exist for measuring the 'noetic stress potential'. Some existing tests give hints or could possibly be re-analyzed to be applicable for indicating profiles that could produce reversal. Another difficulty or complexity is the telergic component that can act as a power factor for the noetic stressor. This means there is a threshold before the noetic stressor produces an action of the noetic effect.

In terms of our main metaphor of the Earth's geomagnetic field we will now develop the final framework for the development of sexual preference. In the cosmology of consciousness, the psychosphere is the complete domain of an individual's consciousness. The psychosphere is imbedded in the anthropic multiverse [7, 63] and contains the psyche and the collective unconscious. The molten iron core dynamo of the Earth is like the unconscious psyche and the collective unconscious. The magnetized crust, the orientation of which formed when molten magma cooled, represents the metabolism and genetic biochemistry of the body along with current conscious content. The solar wind represents causal action of the noetic field and other interpersonal telergic effects.

The nature of the personality sets the flux loci and boundary conditions of the psychosphere and is governed by a hyperdimensional system of wormholes acting as flux gates for the noetic field. This is a physically real aspect of the 'light front' of consciousness as it constantly propagates within the psychosphere. Every aspect of the psyche and the level of development of its archetypes applies. For example, a child beaten or frightened by an abusive parent over time might develop a timid personality. If a threat is perceived the child will emotionally and perhaps physically flee. Usually this type of personality will yield to superiority. Whereas a an emotionally secure and confident child will pay little attention to such threats. I am referring to posturing here, a glare, a raised fist, a stomping foot for example. This is a primitive example, not complex enough to have significance for a noetic effect; but used to illustrate that any kind of imbalance changes the physical hyperdimensional boundaries for propagation of the noetic field.

Under normal life conditions or fetal development there is a balanced syzygy of all aspects of the psyche, anima and animus. This means that there is no potential for a noetic effect because $f(A) = 0$. But with sufficient imbalance a stress potential arises which at a certain threshold will produce a force potential to drive the action of the noetic effect.

If the shielding is removed from a television set and a fairly powerful magnet is held near the picture tube a dramatic distortion of the images occur much like those reflected in a fun-house mirror. Under the proper boundary conditions of the propagation of the noetic field, the force of dissonance will act in a similar way to distort the field. If this force of dissonance is maintained at a sufficient level for a sufficient duration at the critical point in embryonic development a reversal in the normal dominance of the anima / animus occurs.

The noetic energy of a normal psychosphere is in balance, with no force potential from telergic effects from other individuals or from deleterious family history as indicated by cancellation of F(f) the female component and F(m) the male component. Certain personality types will create an imbalance in the psychosphere that creates a force that can reverse the polarity of the field. This effect can become fixed in the psyche of the individual if it occurs for sufficient duration and with a sufficient threshold force during a key time in prenatal development. Two infinitesimally separated particle paths remain separated until acted upon by an external force. In the standard physical model these pathways cannot merge without collapse of the quantum wave function. This is not true in noetic theory where extra degrees of freedom arise because of photon mass. This allows an energyless interaction that creates a superposition of the two paths without collapse of the wave function [3]. In this case, it is the action of the noetic field that produces the deviation in the loci of the geodesic.

Only a generalization of the personality types that produce the geodesic deviation [3] in the loci of noetic field propagation can be given here because only observational evidence has been explored at this point in time. This is research that will be done with more sophistication in the future. There are certain personality types like that of the borderline personality disorder, which is fairly common, that demand extreme dominance. Other dominant types which also include manipulation, as does the borderline, are the narcissistic, sociopathic or obsessive compulsive. When these individuals pick a spouse, it is often someone they can manipulate and who will submit to their control. This submission is often more at the noetic level. On the interpersonal level, there will be typically common episodes of anger and argument that will occur in cycles of 'war and peace'. The type of submissive personalities are outwardly quiet and meek. They might be shy and introverted. Many of the paranoid, schizophrenic and schizotypal personalities would match this profile.

20. Parting Remarks

The theory presented based proposes a new epigenic class of stressor based on the interaction of topological charge in M-Theoretic brane related to Jungian archetypes, building upon recent work on extended models in a number of related disciplines. Science usually advances with hundreds or even thousands of constant tiny steps; and then occasionally every decade, fifty or hundred years as the case may be dramatic advances occur leading to new paradigm shifts in the understanding of fundamental theory. The noetic theory is a vanguard of such a moment in the history of human epistemology to a 3rd regime of Natural Science accessing to Descartes *res cogitans*. It will become one of the greatest shifts because it will revolutionize so many aspects of life all at once. Noetic theory is not raw speculation as some would surmise, but is empirically testable; which will happen soon enough.

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