Forum for Electromagnetic Research Methods and Application Technologies (FERMAT)

Time-Domain Analysis for Transient Electromagnetic Scattering by Conducting-dielectric Objects

Wenjie Chen and Meisong Tong

Department of Electronic Science and Technology Tongji University, Shanghai, China

Abstract: Transient electromagnetic (EM) scattering by conducting-dielectric objects is formulated by time-domain electric field integral equations (TDEFIEs). The TDEFIEs are usually solved by combining the method of moments (MoM) in space domain and march-on-in-time (MoT) scheme in time domain. The space-domain MoM requires two basis functions to represent the electric and magnetic current densities on material interfaces, respectively, leading to an inconvenience in implementation. Also, the MoT scheme has a late-time instability problem which will aggravate in the surface integral equations with penetrable media. In this work, the TDEFIEs are solved by a different approach in which the Nyström method is used to discretize the space domain while the Galerkin method with Laguerre basis and testing functions is employed to discretize the time domain. The proposed approach can overcome the defects of conventional approach as demonstrated by a numerical example.

Key words: Time-domain analysis, transient scattering, integral equation, march-on in degree, conducting-dielectric object.

References:

1. Rao, S. M., Time Domain Electromagnetics, Academic Press, San Diego, 1999.

2. Smith, P. D., “Instability in time marching methods for scattering: Cause and Rectification,” Electromag., Vol. 10, No. 4, 439–451, 1990.

3. Rao, S. M. and T. K. Sarkar, “Numerical solution of time domain integral equations for arbitrarily shaped conductor/dielectric composite bodies," IEEE Trans. Antennas Propagat., Vol. 50, No. 12, 1831–1837, Dec. 2002.

4. Jung, B. H., Y. S. Chung, and T. K. Sarkar, “Time-domain EFIE, MFIE, and CFIE formulations using Laguerre polynomials as temporal basis functions for the analysis of transient scattering from arbitrary shaped conducting structures," Progress In Electromagnetics Research, Vol. 39, 1–45, 2003.

5. Canino, L. S., J. J. Ottusch, M. A. Stalzer, J. L. Visher, and S. Wandzura,

   “Numerical solution of the Helmholtz equation in 2D and 3D using a high-order Nyström discretization," J. Comput. Phys., Vol. 146, 627–663, 1998.

*This use of this work is restricted solely for academic purposes. The author of this work owns the copyright and no reproduction in any form is permitted without written permission by the author. *

Time-Domain Analysis for Transient Electromagnetic Scattering by Conducting-dielectric Objects

Wenjie Chen and Meisong Tong

Department of Electronic Science and Technology
Tongji University, Shanghai, China

August 08, 2016

Outline

- Introduction
- Time-domain combined field integral equations（TDCFIEs）
- Traditional scheme
  - MOM in space domain
  - MOT in time domain
- Hybrid scheme
  - Nystr öm method in space domain
  - Galerkin method（GM）in time domain
- Numerical examples
- Conclusions

Introduction

- Transient electromagnetic（EM）scattering from combined objects widely exists in practical applications and accurate analysis for the phenomenon plays a vital role in the design of many electronic device and systems with an EM infulence.
- The problem can be formulated based on time-domain surface integral equations（TDSIEs）when the involved penetrable materials are homogeneous. And we study the time-domain CFIEs（TDCFIEs）in this work.
- The MoM requires two basis functions to represent two current densities and may be very inconvenient in implementation and the MOT has a well-known late-time instability problem.
- We adopt the Nystr$\ddot{o}$m method in space domain and the Galerkin method（GM）with Laguerre basis and testing functions in time domain.The hybrid scheme possesses the benefits of both methods and can unconditionally yield stable solutions.

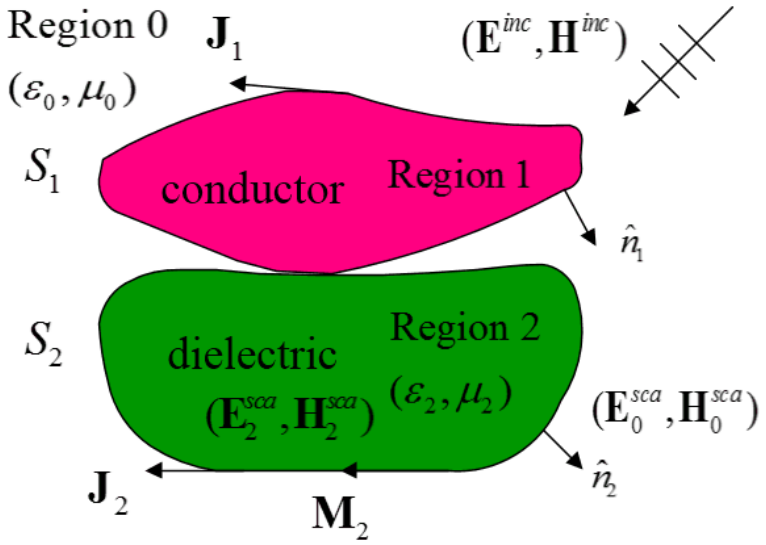
Time-Domain Combined Field Integral Equations (TDCFIEs)

Department of Electronic Science and Technology

$$\left[\mathbf{E}_0^{sca}(\mathbf{J}_1,0)+\mathbf{E}_0^{sca}(\mathbf{J}_2,\mathbf{M}_2)+\mathbf{E}^{inc}\right]_{tan}=0,\quad \mathbf{r}\in S_1$$

$$\left[\mathbf{E}_0^{sca}(\mathbf{J}_1,0)+\mathbf{E}_0^{sca}(\mathbf{J}_2,\mathbf{M}_2)+\mathbf{E}^{inc}\right]_{tan}=0,\quad \mathbf{r}\in S_2^-$$

$$\left[\mathbf{E}_2^{sca}(-\mathbf{J}_2,-\mathbf{M}_2)\right]_{tan}=0,\quad \mathbf{r}\in S_2^+$$



$$\left[\mathbf{H}_0^{sca}(\mathbf{J}_1,0)+\mathbf{H}_0^{sca}(\mathbf{J}_2,\mathbf{M}_2)+\mathbf{H}^{inc}\right]_{tan}=\mathbf{J}_1,\quad \mathbf{r}\in S_1$$

$$\left[\mathbf{H}_0^{sca}(\mathbf{J}_1,0)+\mathbf{H}_0^{sca}(\mathbf{J}_2,\mathbf{M}_2)+\mathbf{H}^{inc}\right]_{tan}=0,\quad \mathbf{r}\in S_2^-$$

$$\left[\mathbf{H}_2^{sca}(-\mathbf{J}_2,-\mathbf{M}_2)\right]_{tan}=0,\quad \mathbf{r}\in S_2^+$$

Time-Domain Combined Field Integral Equations (TDCFIEs)

Department of Electronic Science and Technology

$$\mathbf{E}_j^{sca}[\mathbf{J}_l(\mathbf{r},t)] = -\frac{\partial}{\partial t}\mathbf{A}_j(\mathbf{r},t) - \nabla\Phi_j(\mathbf{r},t)$$

$$\mathbf{E}_j^{sca}[\mathbf{M}_l(\mathbf{r},t)] = -\frac{1}{\varepsilon_j}\nabla\times\mathbf{F}_j(\mathbf{r},t)$$

$$\mathbf{H}_j^{sca}[\mathbf{J}_l(\mathbf{r},t)] = \frac{1}{\mu_j}\nabla\times\mathbf{A}_j(\mathbf{r},t)$$

$$\mathbf{H}_j^{sca}[\mathbf{M}_l(\mathbf{r},t)] = -\frac{\partial}{\partial t}\mathbf{F}_j(\mathbf{r},t) - \nabla\Psi_j(\mathbf{r},t)$$

***j*=0, 2 *l*=1,2**

$$\mathbf{A}_j(\mathbf{r},\mathrm{t}) = \frac{\mu_j}{4\pi}\int_{S_l}\frac{\mathbf{J}_l(\mathbf{r}',\tau_j)}{R}dS'$$

$$\mathbf{F}_j(\mathbf{r},\mathrm{t}) = \frac{\varepsilon_j}{4\pi}\int_{S_l}\frac{\mathbf{M}_l(\mathbf{r}',\tau_j)}{R}dS'$$

$$\Phi_j(\mathbf{r},\mathrm{t}) = \frac{1}{4\pi\varepsilon_j}\int_{S_l}\frac{q_l^e(\mathbf{r}',\tau_j)}{R}dS'$$

$$\Psi_j(\mathbf{r},\mathrm{t}) = \frac{1}{4\pi\mu_j}\int_{S_l}\frac{q_l^m(\mathbf{r}',\tau_j)}{R}dS'$$

$$R = |\mathbf{r}-\mathbf{r}'| \quad \tau_j = t - R/c_j \quad c_j = 1/\sqrt{\varepsilon_j\mu_j}$$

$$\nabla'\cdot\mathbf{J}_l(\mathbf{r}',t) = -\frac{\partial}{\partial t}q_l^e(\mathbf{r}',t)$$

$$\nabla'\cdot\mathbf{M}_l(\mathbf{r}',t) = -\frac{\partial}{\partial t}q_l^m(\mathbf{r}',t)$$

$$\mathbf{J}_l(\mathbf{r}',t) = \frac{\partial}{\partial t}\mathbf{e}_l(\mathbf{r}',t)$$

$$\mathbf{M}_l(\mathbf{r}',t) = \frac{\partial}{\partial t}\mathbf{h}_l(\mathbf{r}',t)$$

Time-Domain Combined Field Integral Equations (TDCFIEs)

Department of Electronic Science and Technology

$$\hat{t}(\mathbf{r})\cdot\left\{\mu_0\int_{S_1}\frac{\partial^2\mathbf{e}_1(\mathbf{r}',\tau_0)}{R\partial t^2}dS'-\frac{1}{\varepsilon_0}\int_{S_1}\overline{\mathbf{G}}\cdot\mathbf{e}_1(\mathbf{r}',\tau_0)dS'+\mu_0\int_{S_2}\frac{\partial^2\mathbf{e}_2(\mathbf{r}',\tau_0)}{R\partial t^2}dS'\right.$$

$$\left.-\frac{1}{\varepsilon_0}\int_{S_2}\overline{\mathbf{G}}\cdot\mathbf{e}_2(\mathbf{r}',\tau_0)dS'+2\pi\hat{n}(\mathbf{r})\times\frac{\partial}{\partial t}\mathbf{h}_2(\mathbf{r}',\tau_0)+\int_{S_2}\nabla\times\left[\frac{\partial}{R\partial t}\mathbf{h}_2(\mathbf{r}',\tau_0)\right]dS'\right\}$$

$$=4\pi\hat{t}(\mathbf{r})\cdot\mathbf{E}^{inc}(\mathbf{r},t),\quad \mathbf{r}\in S_1$$

$$\hat{t}(\mathbf{r})\cdot\left\{\mu_0\int_{S_1}\frac{\partial^2\mathbf{e}_1(\mathbf{r}',\tau_0)}{R\partial t^2}dS'-\frac{1}{\varepsilon_0}\int_{S_1}\overline{\mathbf{G}}\cdot\mathbf{e}_1(\mathbf{r}',\tau_0)dS'+\mu_0\int_{S_2}\frac{\partial^2\mathbf{e}_2(\mathbf{r}',\tau_0)}{R\partial t^2}dS'\right.$$

$$\left.-\frac{1}{\varepsilon_0}\int_{S_2}\overline{\mathbf{G}}\cdot\mathbf{e}_2(\mathbf{r}',\tau_0)dS'+2\pi\hat{n}(\mathbf{r})\times\frac{\partial}{\partial t}\mathbf{h}_2(\mathbf{r}',\tau_0)+\int_{S_2}\nabla\times\left[\frac{\partial}{R\partial t}\mathbf{h}_2(\mathbf{r}',\tau_0)\right]dS'\right\}$$

$$=4\pi\hat{t}(\mathbf{r})\cdot\mathbf{E}^{inc}(\mathbf{r},t),\quad \mathbf{r}\in S_2$$

$$\hat{t}(\mathbf{r})\cdot\left\{\mu_2\int_{S_2}\frac{\partial^2\mathbf{e}_2(\mathbf{r}',\tau_1)}{R\partial t^2}dS'-\frac{1}{\varepsilon_2}\int_{S_2}\overline{\mathbf{G}}\cdot\mathbf{e}_2(\mathbf{r}',\tau_0)dS'-2\pi\hat{n}(\mathbf{r})\times\frac{\partial}{\partial t}\mathbf{h}_2(\mathbf{r}',\tau_0)\right.$$

$$\left.+\int_{S_2}\nabla\times\left[\frac{\partial}{R\partial t}\mathbf{h}_2(\mathbf{r}',\tau_0)\right]dS'\right\}=0,\quad \mathbf{r}\in S_2$$

$$\overline{\mathbf{G}}=\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')=\nabla\nabla(1/R)$$

Time-Domain Combined Field Integral Equations (TDCFIEs)

$$\hat{t}(\mathbf{r})\cdot\left\{2\pi\hat{n}(\mathbf{r})\times\frac{\partial}{\partial t}\mathbf{e}_1(\mathbf{r}',\tau_0)+\int_{S_1}\nabla\times\left[\frac{\partial}{R\partial t}\mathbf{e}_1(\mathbf{r}',\tau_0)\right]dS'-\varepsilon_0\int_{S_2}\frac{\partial^2\mathbf{h}_2(\mathbf{r}',\tau_0)}{R\partial t^2}dS'\right.$$
$$\left.-\frac{1}{\mu_0}\int_{S_2}\overline{\mathbf{G}}\cdot\mathbf{h}_2(\mathbf{r}',\tau_0)dS'+2\pi\hat{n}(\mathbf{r})\times\frac{\partial}{\partial t}\mathbf{e}_2(\mathbf{r}',\tau_0)+\int_{S_2}\nabla\times\left[\frac{\partial}{R\partial t}\mathbf{e}_2(\mathbf{r}',\tau_0)\right]dS'\right\}$$
$$=-4\pi\hat{t}(\mathbf{r})\cdot\mathbf{H}^{inc}(\mathbf{r},t),\quad \mathbf{r}\in S_1$$

$$\hat{t}(\mathbf{r})\cdot\left\{2\pi\hat{n}(\mathbf{r})\times\frac{\partial}{\partial t}\mathbf{e}_1(\mathbf{r}',\tau_0)+\int_{S_1}\nabla\times\left[\frac{\partial}{R\partial t}\mathbf{e}_1(\mathbf{r}',\tau_0)\right]dS'-\varepsilon_0\int_{S_2}\frac{\partial^2\mathbf{h}_2(\mathbf{r}',\tau_0)}{R\partial t^2}dS'\right.$$
$$\left.-\frac{1}{\mu_0}\int_{S_2}\overline{\mathbf{G}}\cdot\mathbf{h}_2(\mathbf{r}',\tau_0)dS'+2\pi\hat{n}(\mathbf{r})\times\frac{\partial}{\partial t}\mathbf{e}_2(\mathbf{r}',\tau_0)+\int_{S_2}\nabla\times\left[\frac{\partial}{R\partial t}\mathbf{e}_2(\mathbf{r}',\tau_0)\right]dS'\right\}$$
$$=-4\pi\hat{t}(\mathbf{r})\cdot\mathbf{H}^{inc}(\mathbf{r},t),\quad \mathbf{r}\in S_2$$

$$\hat{t}(\mathbf{r})\cdot\left\{\varepsilon_2\int_{S_2}\frac{\partial^2\mathbf{h}_2(\mathbf{r}',\tau_1)}{R\partial t^2}dS'+\frac{1}{\mu_2}\int_{S_2}\overline{\mathbf{G}}\cdot\mathbf{h}_2(\mathbf{r}',\tau_0)dS'+2\pi\hat{n}(\mathbf{r})\times\frac{\partial}{\partial t}\mathbf{e}_2(\mathbf{r}',\tau_0)\right.$$
$$\left.-\int_{S_2}\nabla\times\left[\frac{\partial}{R\partial t}\mathbf{e}_2(\mathbf{r}',\tau_0)\right]dS'\right\}=0,\quad \mathbf{r}\in S_2$$

$$\mathrm{TDCFIE}=\alpha\hat{n}(\mathbf{r})\times\mathrm{TDEFIE}+(1-\alpha)\eta\mathrm{TDMFIE}$$

Hybrid Scheme for Solving the TDCFIEs

In spatial domain（TDEFIEs）—Nyström

Department of Electronic Science and Technology

$$\mathbf{e}_\xi(\mathbf{r}',t)=\sum_{n=1}^{N_1}e_\xi^n(t)\mathbf{J}_\xi^n(\mathbf{r}'),\quad \mathbf{h}_2(\mathbf{r}',t)=\sum_{n=1}^{N_2}h_2^n(t)\mathbf{M}_2^n(\mathbf{r}')$$

$$\hat{t}_{mp}^{(k)}=\hat{t}^{(k)}(\mathbf{r}_{mp})=t_{mp}^{u(k)}\hat{u}+t_{mp}^{v(k)}\hat{v}+t_{mp}^{w(k)}\hat{w}\quad (k=1,2)$$

$$\sum_{n=1}^{N_1}\sum_{q=1}^{Q}\left[\frac{\mu_0 W_{nq}}{R_{mpnq}}\frac{d^2e_1^n(\tau_0)}{dt^2}A_{1mpnq}^{J(l)}-\frac{W_{nq}}{\varepsilon_0}e_1^n(\tau_0)B_{1mpnq}^{J(l)}\right]+\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left[\frac{\mu_0 W_{nq}}{R_{mpnq}}\frac{d^2e_2^n(\tau_0)}{dt^2}A_{2mpnq}^{J(l)}-\frac{W_{nq}}{\varepsilon_0}e_2^n(\tau_0)B_{2mpnq}^{J(l)}\right]$$

$$+\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left[\left(\frac{W_{nq}}{c_0R_{mpnq}}\frac{d}{dt}+\frac{W_{nq}}{R_{mpnq}^2}\right)\frac{dh_2^n(\tau_0)}{dt}C_{2mpnq}^{M(l)}\right]=4\pi\hat{t}_{mp}^{(l)}\cdot\mathbf{E}^{inc}(\mathbf{r}_{mp},t),\quad \mathbf{r}_{mp}\in S_1,$$

$$m=1,2,\cdots,N_1,\quad p=1,2,\cdots,Q$$

$$\sum_{n=1}^{N_1}\sum_{q=1}^{Q}\left[\frac{\mu_0 W_{nq}}{R_{mpnq}}\frac{d^2e_1^n(\tau_0)}{dt^2}A_{1mpnq}^{J(l)}-\frac{W_{nq}}{\varepsilon_0}e_1^n(\tau_0)B_{1mpnq}^{J(l)}\right]+\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left[\frac{\mu_0 W_{nq}}{R_{mpnq}}\frac{d^2e_2^n(\tau_0)}{dt^2}A_{2mpnq}^{J(l)}-\frac{W_{nq}}{\varepsilon_0}e_2^n(\tau_0)B_{2mpnq}^{J(l)}\right]$$

$$+\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left[\left(\frac{W_{nq}}{c_0R_{mpnq}}\frac{d}{dt}+\frac{W_{nq}}{R_{mpnq}^2}\right)\frac{dh_2^n(\tau_0)}{dt}C_{2mpnq}^{M(l)}\right]+2\pi D_{2mp}^{M(l)}\frac{dh_2^m(\tau_0)}{dt}=4\pi\hat{t}_{mp}^{(l)}\cdot\mathbf{E}^{inc}(\mathbf{r}_{mp},t),\quad \mathbf{r}_{mp}\in S_2,$$

$$m=1,2,\cdots,N_2,\quad p=1,2,\cdots,Q$$

In spatial domain（TDEFIEs）—Nyström

$$\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left[\frac{\mu_2 W_{nq}}{R_{mpnq}}\frac{d^2 e_2^n(\tau_0)}{dt^2}A_{2mpnq}^{J(l)}-\frac{W_{nq}}{\varepsilon_2}e_2^n(\tau_0)B_{2mpnq}^{J(l)}\right]+\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left[\left(\frac{W_{nq}}{c_2 R_{mpnq}}\frac{d}{dt}+\frac{W_{nq}}{R_{mpnq}^2}\right)\frac{dh_2^n(\tau_0)}{dt}C_{2mpnq}^{M(l)}\right]$$

$$-2\pi D_{2mp}^{M(l)}\frac{dh_2^n(\tau_0)}{dt}=0,\quad \mathbf{r}_{mp}\in S_2,$$

$$m=1,2,\cdots,N_2,\quad q=1,2,\cdots,Q$$

$$\mathbf{J}_{1nq}^n=\mathbf{J}_1^n(\mathbf{r}'_{nq})=J_{1nq}^{nu}\hat{u}+J_{1nq}^{nv}\hat{v}$$

$$\mathbf{J}_{2nq}^n=\mathbf{J}_2^n(\mathbf{r}'_{nq})=J_{2nq}^{nu}\hat{u}+J_{2nq}^{nv}\hat{v}$$

$$\mathbf{M}_{2nq}^n=\mathbf{M}_2^n(\mathbf{r}'_{nq})=M_{2nq}^{nu}\hat{u}+M_{2nq}^{nv}\hat{v}$$

$$R_{nq}=\left|\mathbf{r}-\mathbf{r}'_{nq}\right|$$

$$\hat{R}_{nq}=(\mathbf{r}-\mathbf{r}'_{nq})/R_{nq}$$

$$\overline{\mathbf{G}}_{nq}=\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}'_{nq})$$

$$A_{1mpnq}^{J(l)}=\hat{t}_{mp}^{(l)}\cdot\mathbf{J}_{1nq}^n$$

$$A_{2mpnq}^{J(l)}=\hat{t}_{mp}^{(l)}\cdot\mathbf{J}_{2nq}^n$$

$$B_{1mpnq}^{J(l)}=\hat{t}_{mp}^{(l)}\cdot\overline{\mathbf{G}}_{mpnq}\cdot\mathbf{J}_{1nq}^n$$

$$B_{2mpnq}^{J(l)}=\hat{t}_{mp}^{(l)}\cdot\overline{\mathbf{G}}_{mpnq}\cdot\mathbf{J}_{2nq}^n$$

$$C_{2mpnq}^{M(l)}=\hat{t}_{mp}^{(l)}\cdot\mathbf{M}_{2nq}^n\times\hat{R}_{mpnq}$$

$$D_{2mp}^{M(l)}=\hat{t}_{mp}^{(l)}\cdot\left(\hat{n}_{mp}\times\mathbf{M}_{2mp}^m\right)$$

$$R_{mpnq}=\left|\mathbf{r}_{mp}-\mathbf{r}'_{nq}\right|$$

$$\hat{R}_{mpnq}=(\mathbf{r}_{mp}-\mathbf{r}'_{nq})/R_{mpnq}$$

$$\overline{\mathbf{G}}_{mpnq}=\overline{\mathbf{G}}(\mathbf{r}_{mp},\mathbf{r}'_{nq})$$

Hybrid Scheme for Solving the TDCFIEs

In time domain——Galerkin

$$e_{\xi}^{n}(t)=\sum_{j=0}^{\infty}e_{\xi j}^{n}\phi_{j}(st),\quad h_{2}^{n}(t)=\sum_{j=0}^{\infty}h_{2j}^{n}\phi_{j}(st)$$

$$\xi=1,2 \qquad \phi_{j}(t)=e^{-t/2}L_{j}(t) \qquad \boldsymbol{\phi_{i}(st)}$$

Substituting the above expansions to the TDEFIEs which has been discretized in space domain as shown in previous equations, in time domain, we use a Galerkin method to discretize the TDEFIEs and TDMFIEs. The Laguerre function is used as a basis function to expand the temporal parts of unknown source vectors $e_{\zeta}^{n}(t)$ $h_{2}^{n}(t)$, where $\xi = 1, 2$ and $\phi_{j}(t)=e^{-t/2}L_{j}(t)$ is the Laguerre function in which Lj(t) is the Laguerre polynomial of jth degree. Also, s is the scaling factor used to change the support of expansion.

In time domain ——Galerkin

Department of Electronic Science and Technology

$$\sum_{n=1}^{N_1}\sum_{q=1}^{Q}\left\{\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{4R_{mpnq}}A_{1mpnq}^{J(l)}-\frac{\mathrm{W}_{nq}}{\varepsilon_0}B_{1mpnq}^{J(l)}\right]e_{1i}^{n}\right\}I_{ii}^{(0)}+\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left\{\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{4R_{mpnq}}A_{2mpnq}^{J(l)}-\frac{\mathrm{W}_{nq}}{\varepsilon_0}B_{2mpnq}^{J(l)}\right]e_{2i}^{n}\right\}I_{ii}^{(0)}+$$

$$\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left\{\left(\frac{s^2\mathrm{W}_{nq}}{4c_0R_{mpnq}}+\frac{s\mathrm{W}_{nq}}{2R_{mpnq}^2}\right)C_{2mpnq}^{M(l)}h_{2i}^{n}\right\}I_{ii}^{(0)}=4\pi V_{mpi}^{E(l)}-\sum_{n=1}^{N_1}\sum_{q=1}^{Q}\sum_{j=0}^{i-1}\left\{\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{4R_{mpnq}}A_{1mpnq}^{J(l)}-\frac{\mathrm{W}_{nq}}{\varepsilon_0}B_{1mpnq}^{J(l)}\right]e_{1j}^{n}\right\}I_{ij}^{(0)}-$$

$$\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i-1}\left\{\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{4R_{mpnq}}A_{2mpnq}^{J(l)}-\frac{\mathrm{W}_{nq}}{\varepsilon_0}B_{2mpnq}^{J(l)}\right]e_{2j}^{n}\right\}I_{ij}^{(0)}-\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i-1}\left\{\left(\frac{s^2\mathrm{W}_{nq}}{4c_0R_{mpnq}}+\frac{s\mathrm{W}_{nq}}{2R_{mpnq}^2}\right)C_{2mpnq}^{M(l)}h_{2j}^{n}\right\}I_{ij}^{(0)}-$$

$$\sum_{n=1}^{N_1}\sum_{q=1}^{Q}\sum_{j=0}^{i}\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{R_{mpnq}}A_{1mpnq}^{J(l)}\sum_{k=0}^{j-1}(j-k)e_{1k}^{n}\right]I_{ij}^{(0)}-\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i}\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{R_{mpnq}}A_{2mpnq}^{J(l)}\sum_{k=0}^{j-1}(j-k)e_{2k}^{n}\right]I_{ij}^{(0)}-$$

$$\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i}\left[\frac{s^2\mathrm{W}_{nq}}{c_0R_{mpnq}^2}C_{2mpnq}^{M(l)}\sum_{k=0}^{j-1}(j-k)h_{2k}^{n}\right]I_{ij}^{(0)}-\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i}\left[\frac{s\mathrm{W}_{nq}}{R_{mpnq}^2}C_{2mpnq}^{M(l)}\sum_{k=0}^{j-1}h_{2k}^{n}\right]I_{ij}^{(0)}$$

$$\sum_{n=1}^{N_1}\sum_{q=1}^{Q}\left\{\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{4R_{mpnq}}A_{1mpnq}^{J(l)}-\frac{\mathrm{W}_{nq}}{\varepsilon_0}B_{1mpnq}^{J(l)}\right]e_{1i}^{n}\right\}I_{ii}^{(0)}+\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left\{\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{4R_{mpnq}}A_{2mpnq}^{J(l)}-\frac{\mathrm{W}_{nq}}{\varepsilon_0}B_{2mpnq}^{J(l)}\right]e_{2i}^{n}\right\}I_{ii}^{(0)}+$$

$$\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left\{\left(\frac{s^2\mathrm{W}_{nq}}{4c_0R_{mpnq}}+\frac{s\mathrm{W}_{nq}}{2R_{mpnq}^2}\right)C_{2mpnq}^{M(l)}h_{2i}^{n}\right\}I_{ii}^{(0)}+\pi sD_{2mp}^{M(l)}h_{2i}^{m}I_{ii}^{(0)}=4\pi V_{mpi}^{E(l)}-$$

$$\sum_{n=1}^{N_1}\sum_{q=1}^{Q}\sum_{j=0}^{i-1}\left\{\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{4R_{mpnq}}A_{1mpnq}^{J(l)}-\frac{\mathrm{W}_{nq}}{\varepsilon_0}B_{1mpnq}^{J(l)}\right]e_{1j}^{n}\right\}I_{ij}^{(0)}-\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i-1}\left\{\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{4R_{mpnq}}A_{2mpnq}^{J(l)}-\frac{\mathrm{W}_{nq}}{\varepsilon_0}B_{2mpnq}^{J(l)}\right]e_{2j}^{n}\right\}I_{ij}^{(0)}-$$

$$\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i-1}\left\{\left(\frac{s^2\mathrm{W}_{nq}}{4c_0R_{mpnq}}+\frac{s\mathrm{W}_{nq}}{2R_{mpnq}^2}\right)C_{2mpnq}^{M(l)}h_{2j}^{n}\right\}I_{ij}^{(0)}-\pi sD_{2mp}^{M(l)}\sum_{j=0}^{i-1}h_{2j}^{m}I_{ij}^{(0)}-2\pi sD_{2mp}^{(l)}\sum_{j=0}^{i}\sum_{k=0}^{j-1}h_{2k}^{m}I_{ij}^{(0)}-$$

$$\sum_{n=1}^{N_1}\sum_{q=1}^{Q}\sum_{j=0}^{i}\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{R_{mpnq}}A_{1mpnq}^{J(l)}\sum_{k=0}^{j-1}(j-k)e_{1k}^{n}\right]I_{ij}^{(0)}-\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i}\left[\frac{s^2\mu_0 \mathrm{W}_{nq}}{R_{mpnq}}A_{2mpnq}^{J(l)}\sum_{k=0}^{j-1}(j-k)e_{2k}^{n}\right]I_{ij}^{(0)}-$$

$$\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i}\left[\frac{s^2\mathrm{W}_{nq}}{c_0R_{mpnq}^2}C_{2mpnq}^{M(l)}\sum_{k=0}^{j-1}(j-k)h_{2k}^{n}\right]I_{ij}^{(0)}-\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i}\left[\frac{s\mathrm{W}_{nq}}{R_{mpnq}^2}C_{2mpnq}^{M(l)}\sum_{k=0}^{j-1}h_{2k}^{n}\right]I_{ij}^{(0)}$$

In time domain ——Galerkin

$$\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left\{\left[\frac{s^2\mu_2 \mathrm{W}_{nq}}{4R_{mpnq}}A_{2mpnq}^{J(l)}-\frac{\mathrm{W}_{nq}}{\varepsilon_2}B_{2mpnq}^{J(l)}\right]e_{2i}^{n}\right\}I_{ii}^{(2)}+\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\left\{\left(\frac{s^2\mathrm{W}_{nq}}{4c_2R_{mpnq}}+\frac{s\mathrm{W}_{nq}}{2R_{mpnq}^2}\right)C_{2mpnq}^{M(l)}h_{2i}^{n}\right\}I_{ii}^{(2)}-\pi sD_{2mp}^{M(l)}h_{2i}^{m}I_{ii}^{(2)}$$

$$=4\pi V_{mpi}^{E(l)}-\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i-1}\left\{\left[\frac{s^2\mu_0\mathrm{W}_{nq}}{4R_{mpnq}}A_{2mpnq}^{J(l)}-\frac{\mathrm{W}_{nq}}{\varepsilon_2}B_{2mpnq}^{J(l)}\right]e_{2j}^{n}\right\}I_{ij}^{(2)}-\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i-1}\left\{\left(\frac{s^2\mathrm{W}_{nq}}{4c_2R_{mpnq}}+\frac{s\mathrm{W}_{nq}}{2R_{mpnq}^2}\right)C_{2mpnq}^{M(l)}h_{2j}^{n}\right\}I_{ij}^{(2)}$$

$$+\pi sD_{2mp}^{M(l)}\sum_{j=0}^{i-1}h_{2j}^{m}I_{ij}^{(2)}+2\pi sD_{2mp}^{(l)}\sum_{j=0}^{i}\sum_{k=0}^{j-1}h_{2k}^{m}I_{ij}^{(2)}-\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i}\left[\frac{s^2\mu_2\mathrm{W}_{nq}}{R_{mpnq}}A_{2mpnq}^{J(l)}\sum_{k=0}^{j-1}(j-k)e_{2k}^{n}\right]I_{ij}^{(2)}-$$

$$\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i}\left[\frac{s^2\mathrm{W}_{nq}}{c_2R_{mpnq}^2}C_{2mpnq}^{M(l)}\sum_{k=0}^{j-1}(j-k)h_{2k}^{n}\right]I_{ij}^{(2)}-\sum_{n=1}^{N_2}\sum_{q=1}^{Q}\sum_{j=0}^{i}\left[\frac{s\mathrm{W}_{nq}}{R_{mpnq}^2}C_{2mpnq}^{M(l)}\sum_{k=0}^{j-1}h_{2k}^{n}\right]I_{ij}^{(2)}$$

$$m=1,\cdots,N_2,\quad p=1,\cdots,Q$$

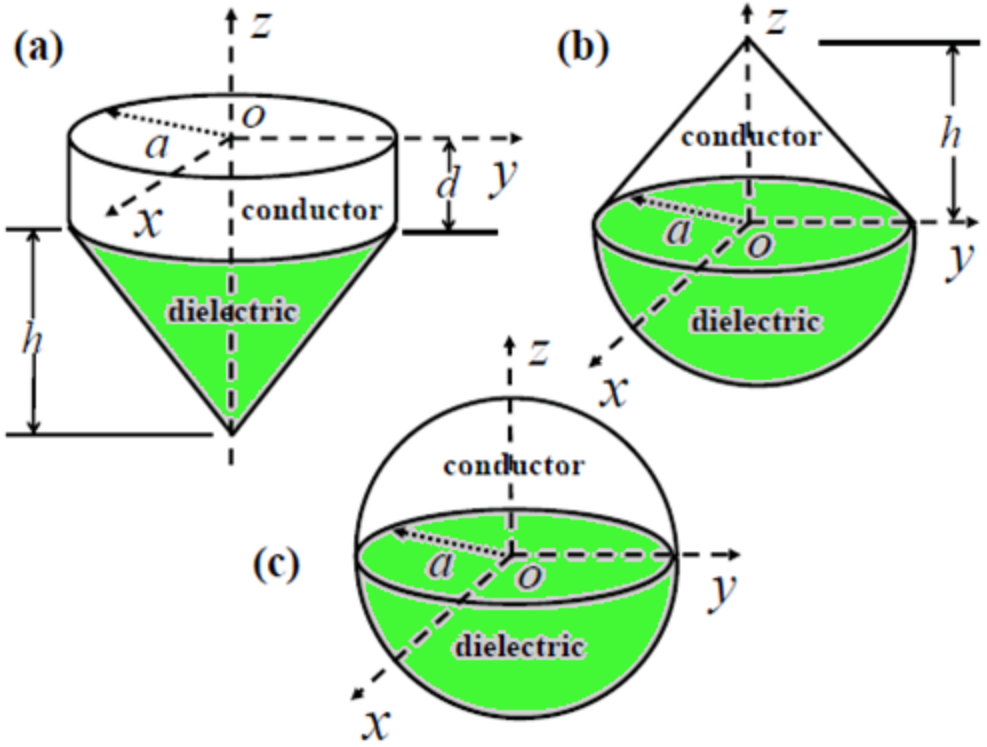
$$I_{ij}^{(0)}=\int_0^{\infty}\phi_i(st)\phi_j\left(st-\frac{sR_{mpnq}}{c_0}\right)d(st)$$

$$I_{ij}^{(2)}=\int_0^{\infty}\phi_i(st)\phi_j\left(st-\frac{sR_{mpnq}}{c_2}\right)d(st)$$

$$V_{mpi}^{E(l)}=\int_0^{\infty}\phi_i(st)\hat{t}_{mp}^{(l)}\cdot\mathbf{E}^{inc}(\mathbf{r}_{mp},t)d(st)$$

Numerical Examples

Department of Electronic Science and Technology



Geometry of composite scatters.

(a) A dielectric cone backed with a conducting circular disk.

(b) A conducting cone attached to a dielectric hemisphere.

(c) A conducting hemisphere joint to a dielectric hemisphere.

Parameters

$\varepsilon_r = 3.0 \qquad \mu_r = 1.0$

The first object has a radius $a = 0.25\,\mathrm{m}$**, a thickness** $d = 0.01\,\mathrm{m}$**, and a height** $a = 0.25\,\mathrm{m}$

The second object has a radius $a = 0.2\,\mathrm{m}$ **, a height** $h = 0.5\,\mathrm{m}$

The third object has a radius $a = 0.5\,\mathrm{m}$

A Gaussian plane wave is illuminating the scatterers along the –z direction.

$$\mathbf{E}^{inc}(\mathbf{r},t) = 120\pi\hat{x}\frac{4e^{-\chi^2}}{\sqrt{\pi T}}, \quad \mathbf{H}^{inc}(\mathbf{r},t) = -\frac{\hat{z}}{\eta}\times\mathbf{E}^{inc}(\mathbf{r},t)$$

$$\chi = 4\left(ct - \Delta t + \mathbf{r}\cdot\hat{z}\right)/T$$

T is the duration of Gaussian impulse and Δt is the needed time for the pulse to arrive at the origin. We choose

$T = 4.0$ lm(light meter) and $c\Delta t = 6.0$ lm

A Gaussian plane wave is illuminating the scatterers along the –z direction.

$$\mathbf{E}^{inc}(\mathbf{r},t) = \hat{x}e^{-\left(\frac{\tau-\Delta t}{\sqrt{2}\sigma}\right)^2}\cos\left(2\pi f_0\tau\right)$$

$$\tau = t + \mathbf{r}\cdot\hat{z}/c$$

$$f_0 = 500\ \text{MHz}$$

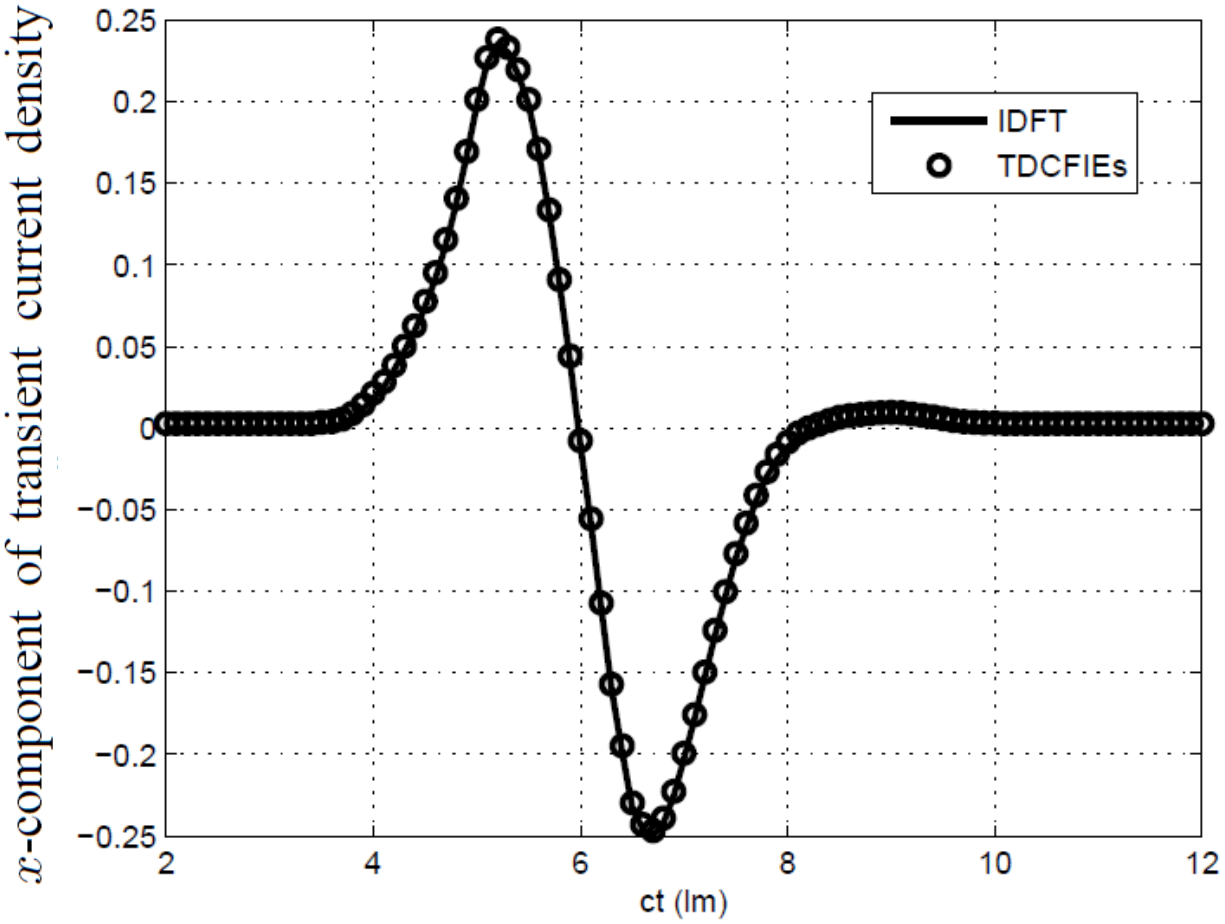
$$\Delta t = 8\sigma$$

$$\sigma = 6/\left(2\pi f_b\right)$$

$$f_b = 1\,\text{GHz}$$

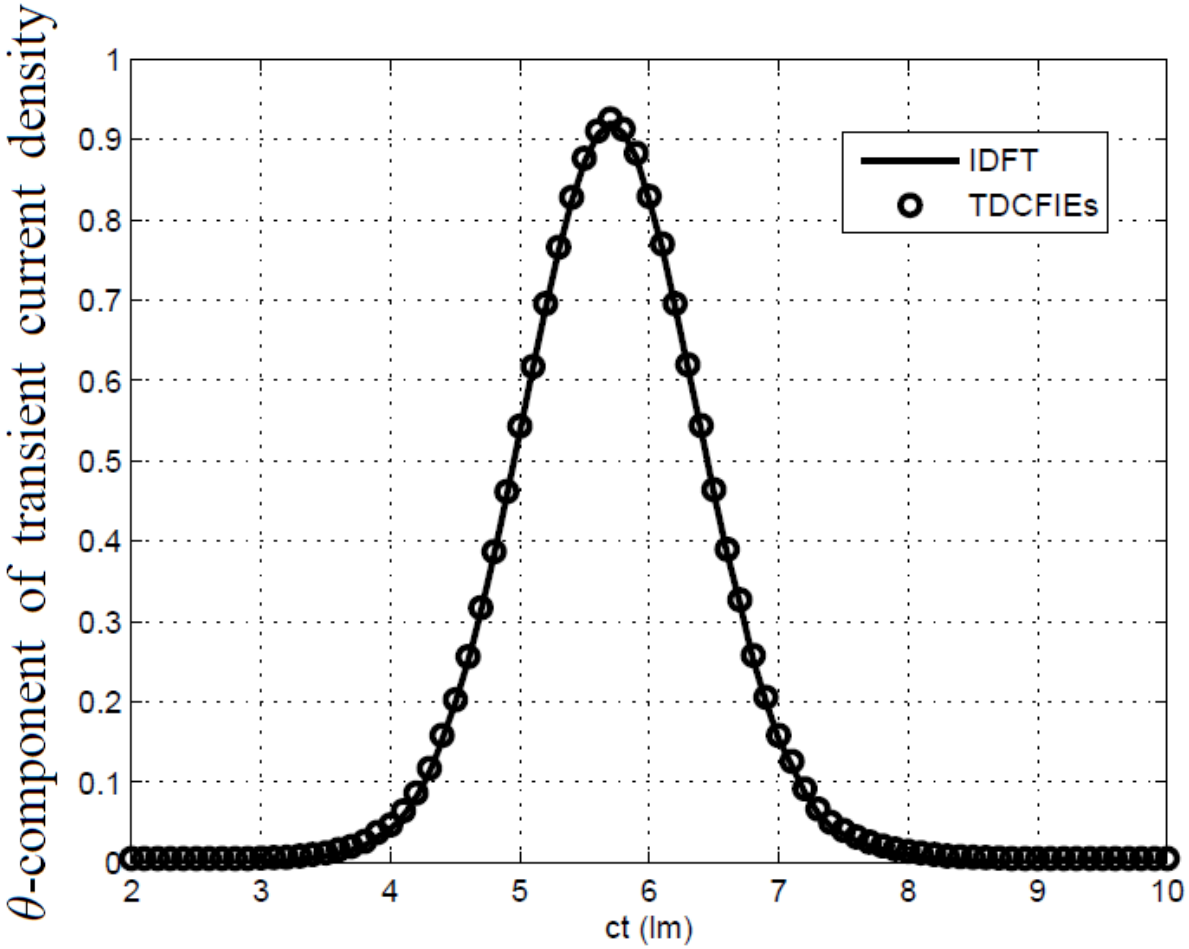
$$s = 10^9 \text{ and } M = 80$$

Result 1



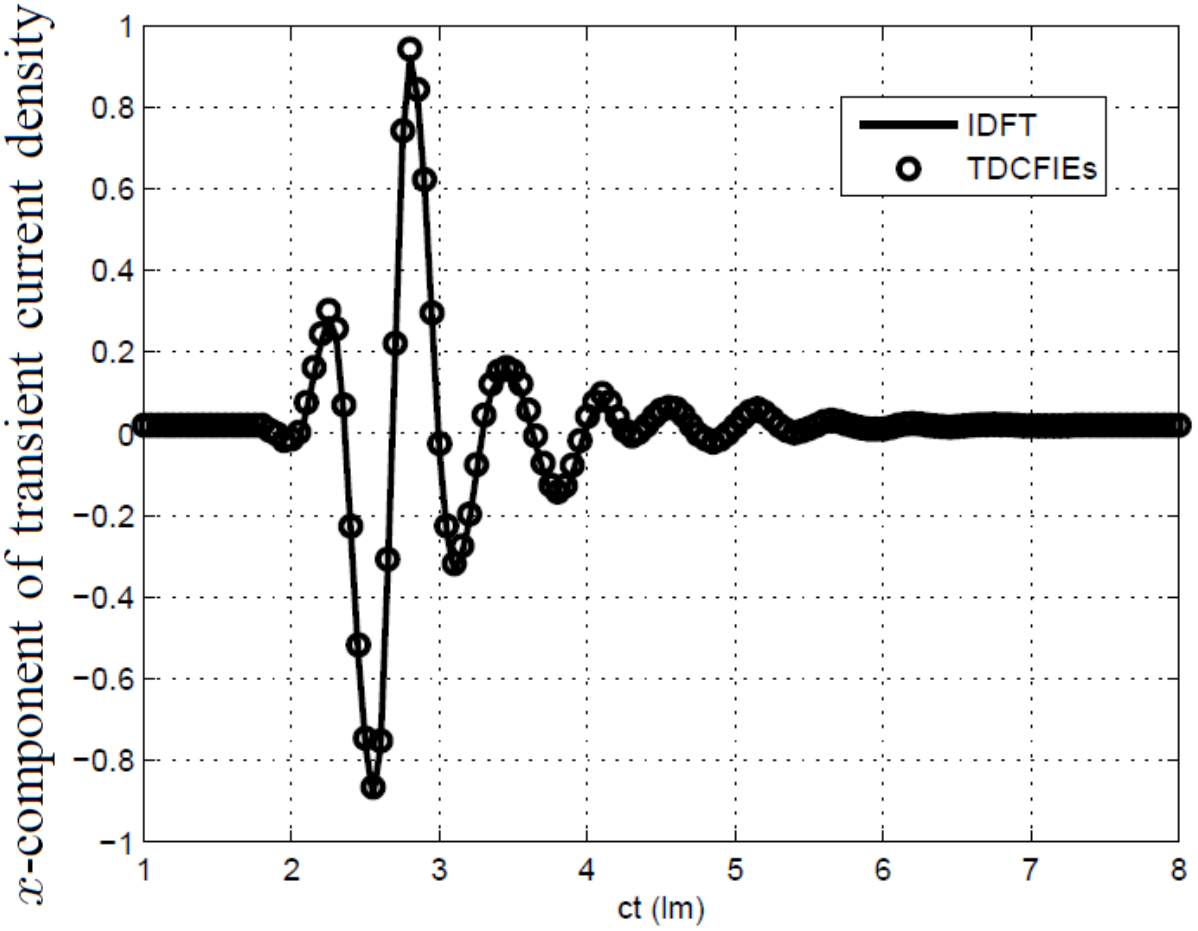
x-component of transient current density sampled at x = z = 0 and y = 0.125 m on the surface of a conducting circular disk attached to a dielectric cone.

Result 2



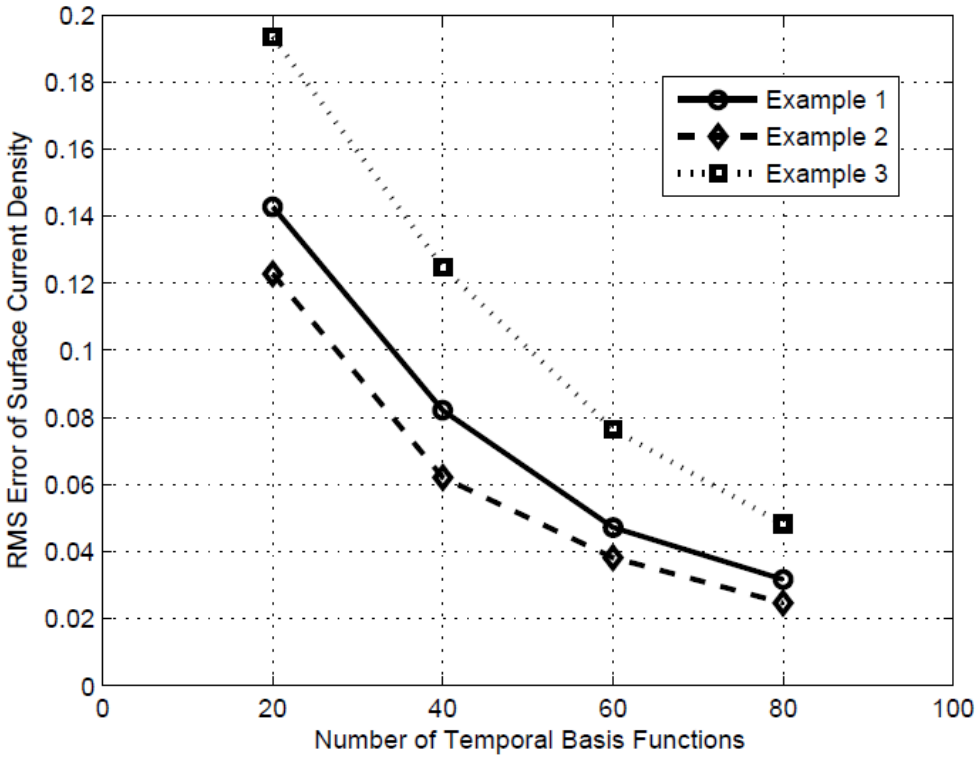
θ-component of transient current density sampled at z = 0.2 m and φ = 180◦ on the surface of a conducting cone attached to a dielectric hemisphere.

Result 3



x-component of transient current density sampled at the north pole on the surface of a conducting hemisphere joint to a dielectric hemisphere.

Comparison



RMS error of surface current density versus the number of temporal basis functions for the three examples.

- The accuracy of numerical solutions is related to the choice of M when geometric discretization is fixed.
- This figure plots the root-mean-square errors, compared to the IDFT counterparts when M= 20, 40, 60 and 80, respectively. These three examples are clearly convergent as M increases.
- Although the computational complexity of the Nyström scheme is equivalent to that of the MoM in space domain, Galerkin method with Laguerre function is better than traditional MoT in time domain.

Conclusions

- We present a hybrid scheme to solve the TDCFIE for transient EM problems.
- Unlike the traditional scheme, we use the Nystr öm method in the spatial domain while employ the Galerkin method（GM）with Laguerre basis and testing functions in time domain.
- The Nystr öm method can simplify the implementation in spatial domain while the Galerkin method can fully solve the instability problem in temporal domain.
- Numerical examples are presented to demonstrate the scheme and good results have been observed.

References

- Rao, S. M., *Time Domain Electromagnetics*, Academic Press, San Diego, 1999.
- Smith, P. D., “Instability in time marching methods for scattering: Cause and Rectification,” *Electromag.*, Vol. 10, No. 4, 439–451, 1990.
- Rao, S. M. and T. K. Sarkar, “Numerical solution of time domain integral equations for arbitrarily shaped conductor/dielectric composite bodies," *IEEE Trans. Antennas Propagat.*, Vol. 50, No. 12, 1831–1837, Dec. 2002.
- Jung, B. H., Y. S. Chung, and T. K. Sarkar, “Time-domain EFIE, MFIE, and CFIE formulations using Laguerre polynomials as temporal basis functions for the analysis of transient scattering from arbitrary shaped conducting structures," *Progress In Electromagnetics Research*, Vol. 39, 1–45, 2003.
- Canino, L. S., J. J. Ottusch, M. A. Stalzer, J. L. Visher, and S. Wandzura, “Numerical solution of the Helmholtz equation in 2D and 3D using a high-order Nyström discretization," *J. Comput. Phys.*, Vol. 146, 627–663, 1998.

Wen Jie Chen received the B.S. degree in electronic science from Xiamen University, Xiamen, China, in 2014. She is currently pursuing the M.S. Degree in electronic science and technology with Tongji University, Shanghai, China.

Her current research interests include computational electromagnetics, antenna design, electromagnetic analysis of interconnect and packaging structures, and inverse scattering for microwave imaging.

Ms. Chen received the National Scholarship of China in 2015.

Mei Song Tong received the Ph.D. degree in electrical engineering from Arizona State University, Tempe, AZ, USA, in 2004.

He is currently a Distinguished Professor and the Chair of the Department of Electronic Science and Technology with the School of Electronics and Information Engineering, Tongji University, Shanghai, China. He was a Research Scientist with the Center for Computational Electromagnetics and the Electromagnetics Laboratory, Department of Electrical and Computer Engineering, University of Illinois at Urbana–Champaign, Urbana, IL, USA. He is also a Visiting Professor with the University of Illinois at Urbana–Champaign, and an Honorary Professor with the University of Hong Kong, Hong Kong. He has authored over 250 papers in refereed journals and conference proceedings, and co-authored three books. His current research interests include electromagnetic field theory, antenna theory and design, simulation and design of RF/microwave circuits and devices, interconnect and packaging analysis, inverse electromagnetic scattering for imaging, and computational electromagnetics.

Prof. Tong is a fellow of the Electromagnetics Academy, a Full Member (Commission B) of the U.S. National Committee for the International Union of Radio Science (URSI), and a member of the Applied Computational Electromagnetics Society and the Sigma Xi Honor Society. He was a recipient of the Visiting Professorship Award from Kyoto University, Kyoto, Japan, in 2012. He advised and co-authored four papers that received the best paper award from the 12th International Workshop on Finite Elements for Microwave Engineering in 2014, the 35th Progress in Electromagnetics Research Symposium in 2014, the 36th Progress in Electromagnetics Research Symposium in 2015, the 37th Progress in Electromagnetics Research Symposium in 2016, respectively. He was also a recipient of the Travel Fellowship Award of USNC/URSI for the 31st General Assembly and Scientific Symposium of URSI in 2014, Science and Technology Advance Award from Shanghai Municipal Government, and many teaching-type awards from Tongji University. He has served as an Associate Editor or a Guest Editor for several well-known international journals, including the *IEEE Antennas and Propagation Magazine*, the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, the IEEE TRANSACTIONS ON COMPONENTS, PACKAGING AND MANUFACTURING TECHNOLOGY, the *International Journal of Numerical Modeling: Electronic Networks, Devices and Fields, Progress in Electromagnetics Research, and the Journal of Electromagnetic Waves and Applications*. He served as a Session Organizer, Chair, and Technical Program Committee Member or Co-Chair for some top conferences in the electromagnetics community, such as the Progress in Electromagnetics Research Symposium, the IEEE International Symposium on Antennas and Propagation, and the USNC/URSI National Radio Science Meeting.